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PHILOSOPHICAL TRANSACTIONS.

I. *Note on Professor* Sylvester's *representation of the Motion of a free rigid Body by that of a material Ellipsoid whose centre is fixed, and which rolls on a rough Plane.*

By the Rev. N. M. Ferrers, *Fellow and Tutor of Gonville and Caius College, Cambridge. Communicated by Professor* J. J. Sylvester, *F.R.S.*

Received May 29,—Read June 17, 1869.

In a paper published in the Transactions of the Royal Society for 1866, Professor Sylvester has given an important extension of Poinsot's representation of the motion of a freely rotating rigid body, by means of the momental ellipsoid. He has proved that if a material ellipsoid, similar in form to the momental ellipsoid, and so constituted that its principal moments of inertia, A, B, C, are connected with its semiaxes a, b, c by the relation $Aa^4(b^2-c^2)+Bb^4(c^2-a^2)+Cc^4(a^2-b^2)=0$, be made to roll in contact with a perfectly rough plane, the motion of this material ellipsoid will be precisely the same as that of the momental ellipsoid of the rigid body; the rough plane taking the place of the geometrical fixed plane, in contact with which the momental ellipsoid is supposed to roll. He has also investigated expressions for the pressure and friction between the ellipsoid and the rough plane, in terms of the angular velocity of the ellipsoid, and of the length of its axes, and the distance of the centre from the rough plane. In investigating independently the values of these forces, I have been led to a somewhat different treatment of the same problem, in the course of which some theorems have presented themselves which may be not without interest.

The notation which I adopt is as follows: ω_1, ω_2, ω_3 represent the component angular velocities of the ellipsoid about its principal axes, which, as proved by Professor Sylvester, are connected by the following equations:—

$$\frac{d\omega_1}{dt}=a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)\omega_2\omega_3,\quad \frac{d\omega_2}{dt}=b^2\left(\frac{1}{c^2}-\frac{1}{c^2}\right)\omega_3\omega_1,\quad \frac{d\omega_3}{dt}=c^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\omega_1\omega_2. \quad . \quad . \quad . \quad (1)$$

The distance from the centre of the ellipsoid to the rough plane is denoted by p, and the component angular velocity of the ellipsoid about the normal to the rough plane, which is known to be constant, by λ. The component angular velocity about the projection of the instantaneous axis on the rough plane is denoted by μ, and the whole angular velocity by ω, so that we have

$$\omega_1^2+\omega_2^2+\omega_3^2=\lambda^2+\mu^2=\omega^2. \qquad (2)$$

We have also the following relations:—

$$\frac{\omega_1^2}{a^2}+\frac{\omega_2^2}{b^2}+\frac{\omega_3^2}{c^2}=\frac{\lambda^2}{p^2}, \qquad (3)$$

$$\frac{\omega_1^2}{a^4}+\frac{\omega_2^2}{b^4}+\frac{\omega_3^2}{c^4}=\frac{\lambda^2}{p^4}. \qquad (4^*)$$

And the principal moments of inertia are represented by

$$\left(\frac{\mathrm{G}}{a^2}-\frac{\mathrm{H}}{p^2}\right)b^2c^2,\ \left(\frac{\mathrm{G}}{b^2}-\frac{\mathrm{H}}{p^2}\right)c^2a^2,\ \left(\frac{\mathrm{G}}{c^2}-\frac{\mathrm{H}}{p^2}\right)a^2b^2,$$

which will be found, when substituted for A, B, C, to satisfy the relation already stated. In the case of a uniform ellipsoid, we have

$$\mathrm{G}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=\frac{\mathrm{H}}{p^2}.$$

---

* [The equations (1), (2), (3), (4), and the invariability of λ, may also be proved as follows. If x, y, z be the coordinates of the point of contact, referred to the principal axes, we have

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ \ \frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}=\frac{1}{p^2}.$$

Also

$$\frac{x}{\omega_1}=\frac{y}{\omega_2}=\frac{z}{\omega_3}=\frac{p}{\lambda},$$

whence

$$\frac{\omega_1^2}{a^2}+\frac{\omega_2^2}{b^2}+\frac{\omega_3^2}{c^2}=\frac{\lambda^2}{p^2},\ \ \frac{\omega_1^2}{a^4}+\frac{\omega_2^2}{b^4}+\frac{\omega_3^2}{c^4}=\frac{\lambda^2}{p^4}.$$

Multiply these equations by $-\frac{\mathrm{H}}{p^2}$, G, and add, then

$$a^2b^2c^2\left\{\left(\frac{\mathrm{G}}{a^2}-\frac{\mathrm{H}}{p^2}\right)b^2c^2\omega_1^2+\left(\frac{\mathrm{G}}{b^2}-\frac{\mathrm{H}}{p^2}\right)c^2a^2\omega_2^2+\left(\frac{\mathrm{G}}{c^2}-\frac{\mathrm{H}}{p^2}\right)a^2b^2\omega_3^2\right\}=(\mathrm{G}-\mathrm{H})\frac{\lambda^2}{p^4},$$

or

$$\lambda^2=\frac{p^4}{a^2b^2c^2}\cdot\frac{\textit{vis viva} \text{ of the ellipsoid}}{\mathrm{G}-\mathrm{H}},$$

and is therefore constant.

Hence the direction-cosines of the perpendicular to the fixed plane are

$$\frac{\omega_1}{a^2}\frac{p^2}{\lambda},\ \frac{\omega_2}{b^2}\frac{p^2}{\lambda},\ \frac{\omega_3}{c^2}\frac{p^2}{\lambda};$$

and since this line is fixed,

$$\frac{1}{a^2}\frac{p^2}{\lambda}\frac{d\omega_1}{dt}-\frac{\omega_2}{b^2}\frac{p^2}{\lambda}\omega_3+\frac{\omega_3}{c^2}\frac{p^2}{\lambda}\omega_2=0,$$

or

$$\frac{1}{a^2}\frac{d\omega_1}{dt}-\left(\frac{1}{b^2}-\frac{1}{c^2}\right)\omega_2\omega_3=0.$$

Similarly

$$\frac{1}{b^2}\frac{d\omega_2}{dt}-\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\omega_3\omega_1=0,$$

$$\frac{1}{c^2}\frac{d\omega_3}{dt}-\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\omega_1\omega_2=0.$$ —February 1870.]

[We shall first investigate the value of h_λ, the component angular momentum of the ellipsoid about the normal to the rough plane. The cosines of the inclinations of this line to the principal axes of the ellipsoid are $\frac{p^2}{\lambda}\frac{\omega_1}{a^2}, \frac{p^2}{\lambda}\frac{\omega_2}{b^2}, \frac{p^2}{\lambda}\frac{\omega_3}{c^2}$ respectively; and the component angular momenta about the principal axes are

$$\left(\frac{\mathrm{G}}{a^2}-\frac{\mathrm{H}}{p^2}\right)b^2c^2\omega_1,\ \left(\frac{\mathrm{G}}{b^2}-\frac{\mathrm{H}}{p^2}\right)c^2a^2\omega_2,\ \left(\frac{\mathrm{G}}{c^2}-\frac{\mathrm{H}}{p^2}\right)a^2b^2\omega_3$$

respectively. Hence

$$h_\lambda=\frac{p^2}{\lambda}\left\{\left(\frac{\mathrm{G}}{a^2}-\frac{\mathrm{H}}{p^2}\right)\frac{b^2c^2}{a^2}\omega_1^2+\left(\frac{\mathrm{G}}{b^2}-\frac{\mathrm{H}}{p^2}\right)\frac{c^2a^2}{b^2}\omega_2^2+\left(\frac{\mathrm{G}}{c^2}-\frac{\mathrm{H}}{p^2}\right)\frac{a^2b^2}{c^2}\omega_3^2\right\}$$

$$=\frac{p^2}{\lambda}a^2b^2c^2\left\{\mathrm{G}\left(\frac{\omega_1^2}{a^6}+\frac{\omega_2^2}{b^6}+\frac{\omega_3^2}{c^6}\right)-\frac{\mathrm{H}}{p^2}\left(\frac{\omega_1^2}{a^4}+\frac{\omega_2^2}{b^4}+\frac{\omega_3^2}{c^4}\right)\right\}.$$

Now, by multiplying (2), (3), (4) by $\frac{1}{a^2b^2c^2}$, $-\left(\frac{1}{b^2c^2}+\frac{1}{c^2a^2}+\frac{1}{a^2b^2}\right)$, $\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$ respectively, and adding, we see that

$$\frac{\omega_1^2}{a^6}+\frac{\omega_2^2}{b^6}+\frac{\omega_3^2}{c^6}=\frac{\lambda^2}{p^6}-\lambda^2\left(\frac{1}{p^2}-\frac{1}{a^2}\right)\left(\frac{1}{p^2}-\frac{1}{b^2}\right)\left(\frac{1}{p^2}-\frac{1}{c^2}\right)+\frac{\mu^2}{a^2b^2c^2}.$$

Hence

$$h_\lambda=\frac{p^2}{\lambda}\left[\mathrm{G}\lambda^2\left\{\frac{a^2b^2c^2}{p^6}-\left(\frac{a^2}{p^2}-1\right)\left(\frac{b^2}{p^2}-1\right)\left(\frac{c^2}{p^2}-1\right)\right\}+\mathrm{G}\mu^2-\mathrm{H}\frac{\lambda^2}{p^6}a^2b^2c^2\right]$$

$$=(\mathrm{G}-\mathrm{H})\lambda\frac{a^2b^2c^2}{p^4}+\mathrm{G}\lambda p^2\left(1-\frac{a^2}{p^2}\right)\left(1-\frac{b^2}{p^2}\right)\left(1-\frac{c^2}{p^2}\right)+\mathrm{G}\frac{p^2}{\lambda}\mu^2. \quad \ldots \quad (5)$$

We shall next investigate a relation between the component angular momentum of the ellipsoid about any axis through its centre, and that, about the same axis, of a particle of mass G, situated at the point of contact of the ellipsoid and rough plane, and moving as that point moves. If l, m, n be the direction-cosines of the axis referred to the principal axes, the component angular momentum of the body about it is

$$\left(\frac{\mathrm{G}}{a^2}-\frac{\mathrm{H}}{p^2}\right)b^2c^2l\omega_1+\left(\frac{\mathrm{G}}{b^2}-\frac{\mathrm{H}}{p^2}\right)c^2a^2m\omega_2+\left(\frac{\mathrm{G}}{c^2}-\frac{\mathrm{H}}{p^2}\right)a^2b^2n\omega_3,$$

or

$$a^2b^2c^2\left\{\mathrm{G}\left(\frac{l\omega_1}{a^4}+\frac{m\omega_2}{b^4}+\frac{n\omega_3}{c^4}\right)-\frac{\mathrm{H}}{p^2}\left(\frac{l\omega_1}{a^2}+\frac{m\omega_2}{b^2}+\frac{n\omega_3}{c^2}\right)\right\}. \quad \ldots \quad (6)$$

Again, the coordinates of the point of contact are $\frac{p}{\lambda}\omega_1, \frac{p}{\lambda}\omega_2, \frac{p}{\lambda}\omega_3$ respectively. Hence its component velocities, parallel to the axes, are

$$\frac{p}{\lambda}\frac{d\omega_1}{dt},\ \frac{p}{\lambda}\frac{d\omega_2}{dt},\ \frac{p}{\lambda}\frac{d\omega_3}{dt};$$

and its component angular momenta are therefore

$$\mathrm{G}\frac{p^2}{\lambda^2}\left(\omega_2\frac{d\omega_3}{dt}-\omega_3\frac{d\omega_2}{dt}\right),\ \mathrm{G}\frac{p^2}{\lambda^2}\left(\omega_3\frac{d\omega_1}{dt}-\omega_1\frac{d\omega_3}{dt}\right),\ \mathrm{G}\frac{p^2}{\lambda^2}\left(\omega_1\frac{d\omega_2}{dt}-\omega_2\frac{d\omega_1}{dt}\right).$$

Now, by equations (1), we see that

$$\omega_2\frac{d\omega_3}{dt}-\omega_3\frac{d\omega_2}{dt}=c^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\omega_1\omega_2^2-b^2\left(\frac{1}{c^2}-\frac{1}{a^2}\right)\omega_1\omega_3^2$$

$$=b^2c^2\omega_1\left\{\frac{1}{a^2}\left(\frac{\omega_2^2}{b^2}+\frac{\omega_3^2}{c^2}\right)-\left(\frac{\omega_2^2}{b^4}+\frac{\omega_3^2}{c^4}\right)\right\}$$

$$=b^2c^2\omega_1\left(\frac{1}{a^2}-\frac{1}{p^2}\right)\frac{\lambda^2}{p^2}\text{ by (3) and (4).}$$

The other component angular momenta being similarly transformed, we obtain, for the component angular momentum about the assigned axis,

$$\mathrm{G}a^2b^2c^2\left\{\frac{l\omega_1}{a^2}\left(\frac{1}{a^2}-\frac{1}{p^2}\right)+\frac{m\omega_2}{b^2}\left(\frac{1}{b^2}-\frac{1}{p^2}\right)+\frac{n\omega_3}{c^2}\left(\frac{1}{c^2}-\frac{1}{p^2}\right)\right\},$$

or

$$\mathrm{G}a^2b^2c^2\left\{\frac{l\omega_1}{a^4}+\frac{m\omega_2}{b^4}+\frac{n\omega_3}{c^4}-\frac{1}{p^2}\left(\frac{l\omega_1}{a^2}+\frac{m\omega_2}{b^2}+\frac{n\omega_3}{c^2}\right)\right\}. \quad \ldots\ldots\ldots \quad (7)$$

Comparing this with the expression already obtained for the angular momentum of the body, we see that the two expressions are equal if

$$\frac{l\omega_1}{a^2}+\frac{m\omega_2}{b^2}+\frac{n\omega_3}{c^2}=0,$$

i. e. if the axis be parallel to the rough plane; and generally that the angular momentum of the ellipsoid, that of the particle,

$$=(\mathrm{G}-\mathrm{H})\frac{a^2b^2c^2}{p^2}\left(\frac{l\omega_1}{a^2}+\frac{m\omega_2}{b^2}+\frac{n\omega_3}{c^2}\right),$$

which, if the axis be perpendicular to the rough plane, becomes $(\mathrm{G}-\mathrm{H})\frac{a^2b^2c^2}{p^4}\lambda$, a constant.—February 1870].

Now, let h_t denote the angular momentum of the ellipsoid about an axis through its centre, parallel to the rough plane, and at right angles to the instantaneous axis, and h_ρ about an axis through its centre parallel to the rough plane and at right angles to this last. The radius vector of the point of contact measured from the foot of the perpendicular on the rough plane is $\frac{p}{\lambda}\mu$; and hence if n denote the angular velocity of this radius vector in space, the radial and transversal component velocities of the point of contact will be $\frac{p}{\lambda}\frac{d\mu}{dt}$, $\frac{p}{\lambda}n\mu$ respectively. To obtain the component angular momenta of the particle we have only to multiply the expressions by G. Hence, by the theorem just proved,

$$h_t=\mathrm{G}\frac{p^2}{\lambda}\frac{d\mu}{dt},\ h_\rho=\mathrm{G}\frac{p^2}{\lambda}n\mu. \quad \ldots\ldots\ldots \quad (8)$$

We can now calculate the values of P and F, the pressure and friction of the ellipsoid against the plane. Taking moments about axes through the centre of the ellipsoid, lying in the rough plane and perpendicular to the direction of P and F respectively, each of these forces acts at an arm $\frac{p}{\lambda}\mu$. Hence

$$\begin{aligned}\mathrm{F}\frac{p}{\lambda}\mu&=\frac{dh_\lambda}{dt}\\&=\mathrm{G}\frac{2p^2}{\lambda}\mu\frac{d\mu}{dt}\text{ by (7);}\end{aligned}$$

$$\therefore\quad \mathrm{F}=2\mathrm{G}p\frac{d\mu}{dt}. \quad\quad (9)$$

It is worthy of notice that $\mathrm{F}=\frac{2\lambda}{p}h_t$.

Again,

$$\begin{aligned}\mathrm{P}\frac{p}{\lambda}\mu&=\frac{dh_t}{dt}-nh_e\\&=\mathrm{G}\frac{p^2}{\lambda}\left(\frac{d^2\mu}{dt^2}-n^2\mu\right);\end{aligned}$$

$$\therefore\quad \mathrm{P}=\mathrm{G}p\left(\frac{1}{\mu}\frac{d^2\mu}{dt^2}-n^2\right). \quad\quad (10)$$

It may be desirable to replace these expressions by others in which μ shall be the only variable quantity, and which shall be free from differential coefficients. This may be done as follows. Writing, for shortness, α, β, γ in place of

$$1-\frac{a^2}{p^2},\ 1-\frac{b^2}{p^2},\ 1-\frac{c^2}{p^2}$$

respectively, it may be proved, from equations (1), (2), (3), (4), that

$$\left(\mu\frac{d\mu}{dt}\right)^2=-(\mu^2+\beta\gamma\lambda^2)(\mu^2+\gamma\alpha\lambda^2)(\mu^2+\alpha\beta\lambda^2). \quad\quad (11)$$

Again, it is proved by POINSOT, 'Sur la Rotation des Corps,' p. 130 (see also Quarterly Journal of Pure and Applied Mathematics, vol. vii. p. 74), that

$$p=\lambda+\alpha\beta\gamma\frac{\lambda^3}{\mu^2}, \quad\quad (12)$$

a result which also follows from (5), (6), (7), remembering that the angular momentum of the particle about the normal to the rough plane is $\mathrm{G}\frac{p^2}{\lambda^2}\mu^2 n$.

Now, differentiating (11),

$$\begin{aligned}\frac{d}{dt}\left(\mu\frac{d\mu}{dt}\right)=&-(\mu^2+\gamma\alpha\lambda^2)(\mu^2+\alpha\beta\lambda^2)-(\mu^2+\alpha\beta\lambda^2)(\mu^2+\beta\gamma\lambda^2)\\&-(\mu^2+\beta\gamma\lambda^2)(\mu^2+\gamma\alpha\lambda^2);\end{aligned}$$

$$\begin{aligned}\therefore\ \mu\frac{d^2\mu}{dt^2}=&-(\mu^2+\gamma\alpha\lambda^2)(\mu^2+\alpha\beta\lambda^2)-(\mu^2+\alpha\beta\lambda^2)(\mu^2+\beta\gamma\lambda^2)-(\mu^2+\beta\gamma\lambda^2)(\mu^2+\gamma\alpha\lambda^2)\\&+\frac{(\mu^2+\beta\gamma\lambda^2)(\mu^2+\gamma\alpha\lambda^2)(\mu^2+\alpha\beta\lambda^2)}{\mu^2},\end{aligned}$$

and

$$n^2\mu^2 = \left(\lambda\mu + \alpha\beta\gamma\frac{\lambda^2}{\mu}\right)^2;$$

$$\therefore \mu^2\left(\frac{1}{\mu}\frac{d^2\mu}{dt^2} - n^2\right) = -2\mu^4 - (1+\beta\gamma+\gamma\alpha+\alpha\beta)\lambda^2\mu^2 - 2\alpha\beta\gamma\lambda^4 - \alpha^2\beta^2\gamma^2\frac{\lambda^6}{\mu^2},$$

$$\therefore \mathrm{P} = \mathrm{G}p\left\{-2\mu^2 - (1+\beta\gamma+\gamma\alpha+\alpha\beta)\lambda^2 - 2\alpha\beta\gamma\frac{\lambda^4}{\mu^2} - \alpha^2\beta^2\gamma^2\frac{\lambda^6}{\mu^4}\right\}. \quad . \quad . \quad . \quad . \quad (13)$$

Again,

$$\mathrm{F} = \mathrm{G}p\,\frac{d\mu}{dt},$$

$$= \frac{\mathrm{G}p}{\mu}\left\{-(\mu^2+\beta\gamma\lambda^2)(\mu^2+\gamma\alpha\lambda^2)(\mu^2+\alpha\beta\lambda^2)\right\}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

[The theorem contained in equations (5) and (6) may perhaps receive additional illustration by a comparison of the moments, about the principal axes, of the forces acting on the ellipsoid, and of those acting on the particle coinciding with the point of contact. Since the component angular momenta of the ellipsoid about the principal axes are $\left(\frac{\mathrm{G}}{a^2} - \frac{\mathrm{H}}{p^2}\right)b^2c^2\omega_1$, $\left(\frac{\mathrm{G}}{b^2} - \frac{\mathrm{H}}{p^2}\right)c^2a^2\omega_2$, $\left(\frac{\mathrm{G}}{c^2} - \frac{\mathrm{H}}{p^2}\right)a^2b^2\omega_3$, it follows that the moment of the forces about one of the principal axes is

$$\left(\frac{\mathrm{G}}{a^2} - \frac{\mathrm{H}}{p^2}\right)b^2c^2\frac{d\omega_1}{dt} - \left\{\left(\frac{\mathrm{G}}{b^2} - \frac{\mathrm{H}}{p^2}\right)c^2a^2 - \left(\frac{\mathrm{G}}{c^2} - \frac{\mathrm{H}}{p^2}\right)a^2b^2\right\}\omega_2\omega_3,$$

or

$$\mathrm{G}\left\{\frac{b^2c^2}{a^2}\frac{d\omega_1}{dt} - \left(\frac{c^2a^2}{b^2} - \frac{a^2b^2}{c^2}\right)\omega_2\omega_3\right\} - \frac{\mathrm{H}}{p^2}\left\{b^2c^2\frac{d\omega_1}{dt} - (c^2a^2 - a^2b^2)\omega_2\omega_3\right\}$$

$$= \mathrm{G}\left\{\frac{b^2c^2}{a^2}\frac{d\omega_1}{dt} - \left(\frac{c^2a^2}{b^2} - \frac{a^2b^2}{c^2}\right)\omega_2\omega_3\right\} \text{ by (1),}$$

or

$$\mathrm{G}\,\frac{(b^2-c^2)(c^2a^2+a^2b^2-b^2c^2)}{b^2c^2}\,\omega_2\omega_3,$$

a result independent of H. Now, if we refer to equation (7) we shall see that the angular momenta of the particle only differ from those of the ellipsoid by having G written in place of H; consequently the moments of the forces, since they do not involve H, must be the same for the particle and the ellipsoid. It follows of course that the moments of the forces about any other axis must be the same in both cases.

In the above investigation of the value of P, I have followed Professor SYLVESTER, in assuming that the friction acts wholly in a direction perpendicular to the instantaneous axis. The other component of the friction is necessarily indeterminate, since any force in the direction of the instantaneous axis may be combined with it, without altering its effect. I have assumed this component of the friction to be zero; if it be taken to be equal to an arbitrary force F′, the value of P above investigated must be increased by $\frac{\mathrm{F}'\lambda}{\mu}$. The values of the moments of the forces are not, of course, affected by this supposition; and if F′ be so chosen that the pressure between the ellipsoid and the rough plane may be zero, the forces acting on the body will become absolutely identical with those acting on the particle G,—that is, we shall have $F' = G\frac{p}{\lambda}\left(\frac{d^2\mu}{dt^2} - n^2\omega\right)$, and F, as before, $= 2Gp\frac{d\mu}{dt}$.—February 1870.]

It may be worth while to point out that the correlated and contrarelated bodies treated of in the latter part of Professor SYLVESTER's paper include, as a particular case, POINSOT's "rolling and sliding cone;" for the equation of that cone is

$$\frac{x^2}{a^2-p^2}+\frac{y^2}{b^2-p^2}+\frac{z^2}{c^2-p^2}=0,$$

which is asymptotic to the two following surfaces:—

$$\frac{x^2}{a^2-p^2}+\frac{y^2}{b^2-p^2}+\frac{z^2}{c^2-p^2}=1,$$

$$\frac{x^2}{p^2-a^2}+\frac{y^2}{p^2-b^2}+\frac{z^2}{p^2-c^2}=1,$$

the former of which is confocal, the latter contrafocal, to the momental ellipsoid of the free body. Hence, since the difference between the squares on corresponding semiaxes is in this case p^2, each of these hyperboloids will roll on the invariable plane through the fixed point, which will be asymptotic to it, while the plane itself rotates with uniform angular velocity λ. Hence the asymptotic cone will move in exactly the same manner.

II. *On the Refraction-Equivalents of the Elements.*

By J. H. Gladstone, *Ph.D.*, *F.R.S.*

Received June 17,—Read June 17, 1869.

In our paper "On the Refraction, Dispersion, and Sensitiveness of Liquids," Mr. Dale and I pointed out a property of bodies which we termed their "specific refractive energy." It is the refractive index minus unity, divided by the density, or in symbolical language $\frac{\mu-1}{d}$. We found that this is a constant unaffected by temperature, and that the specific refractive energy of a mixture is the mean of the specific refractive energies of its constituents. At the same time, however, we admitted that in both cases our numbers were not in perfect accordance with theory, there being some unknown cause which affected them to a slight extent. These conclusions, both in regard to the general law and its qualification, have been since confirmed by continental physicists, and especially by the late rigorous experiments of Wüllner*.

In the same paper we ventured also on the generalization that "every liquid has a specific refractive energy composed of the specific refractive energies of its component elements, modified by the manner of combination." Later research has confirmed this also, extending it to conditions of matter other than liquid, and showing more clearly when such modifications occur, and what is their nature. Professor Landolt, of Bonn, has greatly advanced our knowledge of the subject, and has simplified the calculations by adopting what he terms the refraction-equivalent, that is, the specific refractive energy multiplied by the atomic weight, or $P\frac{\mu-1}{d}$. Recent investigations in fact tend to the general conclusion that the refraction-equivalent, not only of mixtures, but of every compound body, is the sum of the refraction-equivalents of the elements that compose it.

Were this perfectly true, like the statement "the atomic weight of a compound is the sum of the atomic weights of its constituents," it would be a simple matter to determine the refraction-equivalents of all the elements; and then we should be in a position to calculate the effect of every transparent body of known composition on the rays of light transmitted by it. But it is not absolutely true: even in Landolt's first paper it is evident that there are exceptions; the unknown cause which modifies the refraction of mixtures probably acts in cases of more perfect chemical combination; and the conviction has grown that some elements have two or more refraction-equivalents.

I have continued from time to time to make observations on this subject, and the period seems now to have come at which it is wise to put on permanent record the results at which I have hitherto arrived. I shall give therefore first the data, and then the deductions in reference to each element examined.

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* Pogg. Annalen, vol. cxxxiii. p. 1.

The Data.

These consist of the observations of DULONG[*] on the refraction of gases, of MALUS, BREWSTER, and others[†] on solids, and of DELFFS[‡], JAMIN, SAUBER[§], LANDOLT[||], HAAGEN[¶], and KETTELER[**] on liquids, in addition to such determinations as I have myself made, whether previously published[††] or not.

Most of my fresh experiments have been made with a new instrument constructed by Mr. BROWNING, with a horizontal instead of a vertical circle, and several other improvements.

I have continued to measure the solar lines A, D, and H, whilst LANDOLT has preferred the three bright lines of the hydrogen spectrum. I have calculated the refraction for the line A, as being the most free from whatever influence there may be connected with dispersion, and the German professor has reckoned for hydrogen α, which is identical with the solar C. These rays are so near together that the difference can scarcely affect the first place of decimals in a refraction-equivalent. When the determinations are made with greater precision, it will be for physicists to decide which shall be finally adopted.

In the subjoined Tables the actual refractive indices are given, and the refraction-equivalents as calculated from them. For the complete data, I must refer to the papers of the several observers, and to Appendices I., II., and III., where my own new experiments are tabulated.

TABLE I.—Simple Elements.

Elementary substance.	Condition.	Part of spectrum.	Refractive index.	Authority for index.	Refraction-equivalent.
Carbon	Diamond	Bright	2·470	Brewster, &c.	5·18
„	„	Red	2·4606	Schrauf.	4·85
Sulphur	Solid	Bright	2·03	Various.	16·0
„	Liquid	A	1·9024	Gladstone and Dale.	15·98
„	„	D	1·9295	„	16·47
Phosphorus	Solid	D	2·1168	„	18·98
„	Liquid	A	2·0389	„	18·27
„	„	D	2·0746	„	18·89
Bromine	„	A	1·6260	Gladstone.	16·23
Chlorine	Gaseous	Bright	1·000779	Dulong.	8·87
Hydrogen	„	„	1·000138	„	1·53
Oxygen	„	„	1·000272	„	3·04
Nitrogen	„	„	1·900300	„	3·30

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[*] Annales de Chimie, xxxi. p. 154.

[†] Ency. Brit., Article "Optics."

[‡] Pogg. Annalen, lxxxi. p. 470.

[§] Ibid. cxvii. 577.

[||] Ibid. cxvii. 353, cxxii. 545, cxxiii. 595.

[¶] Ibid. cxxiii. 125.

[**] Ibid. cxxiv. 390.

[††] The papers of Mr. DALE and myself in the Philosophical Transactions, 1858, p. 887, and 1863, p. 317; Phil. Mag. July 1859; also Brit. Assoc. Report, 1863, Trans. Sec. p. 12; my paper in Journal Chem. Soc. 1865, p. 108.

TABLE II.—Binary Compounds.

Substance.	Formula.	Part of Spectrum.	Refractive index.	Authority for index.	Refraction-equivalent.
Bisulphide of Carbon	CS_2	A	1·6121	Gladstone and Dale	36·67
,, ,,	,,	C	1·61846	Wüllner	37·20
,, ,,	,,	D	1·6299	Gladstone and Dale	37·73
,, ,, Gaseous	,,	Bright	1·001500	Dulong	33·21
Carbonic Oxide	CO	,,	1·000340	,,	7·53
,, Acid Gas	CO_2	,,	1·000449	,,	10·03
Cyanogen	CN	,,	1·000834	,,	9·18
Tetrachloride of Carbon	CCl_4	A	1·4560	Gladstone	44·2
,, ,,	,,	C	1·45789	Haagen	44·21
Olefiant Gas	C_2H_4	Bright	1·000678	Dulong	15·09
Amylene	C_5H_{10}	A	1·3844	Gladstone and Dale	37·63
Oil of Turpentine	$C_{10}H_{16}$	A	1·4614	,, ,,	72·9
Hydride of Œnanthyle	C_7H_{16}	A	1·3899	,, ,,	55·0
,, Capryl	C_8H_{18}	A	1·3973	,, ,,	62·95
Sulphurous Acid Gas	SO_2	Bright	1·000665	Dulong	14·91
,, ,, Liquid	,,	D	1·33835	Ketteler	14·59
Hydrosulphuric Acid	H_2S	Bright	1·000644	Dulong	14·28
Chloride of Sulphur	S_2Cl_2	C	1·64368	Haagen	51·64
Water, Solid	H_2O	D	1·3089	Gladstone and Dale	6·05
,, Liquid	,,	A	1·32924	Gladstone	5·926*
,, ,,	,,	D	1·33328	,,	5·999*
,, ,,	,,	H	1·34393	,,	6·191*
,, ,,	,,	C	1·33111	Landolt	5·96
,, ,,	,,	D	1·33250	Kühlmann	6·006
,, Gaseous	,,	Bright	1·000260	Dulong	5·778
Hydrochloric Acid	HCl	,,	1·000449	,,	10·71
Ammonia	NH_3	,,	1·000385	,,	8·60
Nitrous Oxide	N_2O	,,	1·000503	,,	11·22
Nitric ,,	NO	,,	1·000303	,,	6·74
Terchloride of Phosphorus	PCl_3	A	1·5062	Gladstone and Dale	48·3
Terbromide ,,	PBr_3	A	1·6730	,, ,,	63·36
Arsenious Anhydride	As_2O_3	...	1·748	Descloizeau	40·03
Terchloride of Arsenic	$AsCl_3$	C	1·5920	Haagen	49·59
Pentachloride of Antimony	$SbCl_5$	C	1·5845	,,	74·61
,, ,,	,,	A	1·5739	Gladstone	74·03
Tetrachloride of Silicon	$SiCl_4$	C	1·4119	Haagen	47·06
,, Tin	$SnCl_4$	C	1·5070	,,	59·05
,, ,,	,,	A	1·5035	Gladstone	58·76
,, Titanium	$TiCl_4$	A	1·5856	,,	65·08
Chloride of Sodium	$NaCl$	A	1·5369	,,	15·02
Calomel	Hg_2Cl_2	Bright	1·970	Brewster	65·46
Fluor-spar	CaF_2	,,	1·436	,,	10·76
Quartz, ordinary ray	SiO_2	,,	1·5484	Malus	12·41
,, extraordinary ray	,,	,,	1·5582	,,	12·63

* These are the numbers adopted throughout the series of aqueous solutions, with the new instrument, recorded in Appendix II. The water was purposely not deprived of air.

TABLE III.—Ternary and other Compounds.

Substance.	Formula.	Part of spectrum.	Refractive index.	Authority for index.	Refraction-equivalent.
Alcohol	C_2H_6O	A	1·3615	Gladstone	20·857*
,,	,,	D	1·3655	,,	21·084*
,,	,,	H	1·3769	,,	21·745*
Methylated Acetone	C_4H_8O	A	1·3817	,,	33·89
Butyrone	$C_7H_{14}O$	A	1·4073	,,	56·29
Laurostearate of Ethyl	$C_{12}H_{23}(C_2H_5)O_2$	Red	1·4240	Delffs	111·5
Cyanide ,,	C_2H_5CN	B	1·362552	Sauber	25·57
Nitrate ,,	$C_2H_5NO_3$	B	1·3768	,,	30·94
,, ,,	,,	Red	1·381	Jamin	31·17
,, Amyl	$C_5H_{11}NO_3$	A	1·4065	Gladstone and Dale	54·01
Carbonic Ether	$(C_2H_5)_2CO_3$	A	1·3779	,,	45·86
Silicic ,,	$(C_2H_5)_4SiO_4$	A	1·3781	,,	84·38
Boracic ,,	$(C_2H_5)_3B_2O_3$	A	1·3676	,,	65·95
Triethylarsine	$As(C_2H_5)_3$	A	1·4597	,,	64·69
Mercuric Methyl	$Hg(CH_3)_2$	A	1·5229	Gladstone	40·54
,, Ethyl	$Hg(C_2H_5)_2$	A	1·5162	Gladstone and Dale	54·48
Cane-sugar	$C_{12}H_{22}O_{11}$	Bright	1·541	Brewster	119·3
Phosgene	$COCl_2$	,,	1·001159	Dulong	25·64
Chloroform	$CHCl_3$	A	1·4400	Gladstone and Dale	35·25
Chlorobenzole	C_6H_5Cl	A	1·5135	,,	52·13
Trichlorobenzole	$C_6H_3Cl_3$	A	1·5563	,,	69·62
Bichloride of Ethylene	$C_2H_4Cl_2$	C	1·44201	Haagen	34·84
,, Chlorethylene	$C_2H_3Cl_3$	A	1·4619	Gladstone and Dale	43·51
Bibromide ,,	$C_2H_3ClBr_2$	A	1·5430	,,	53·73
,, Bromethylene	$C_2H_3Br_3$	A	1·5809	,,	59·28
Bromoform	$CHBr_3$	A	1·5554	,,	53·31
Bromide of Ethyl	C_2H_5Br	C	1·42132	Haagen	31·46
,, ,,	,,	B	1·4138	Sauber	32·15
,, Amyl	$C_5H_{11}Br$	C	1·43856	Haagen	54·98
Bibromide of Ethylene	$C_2H_4Br_2$	C	1·53389	,,	45·98
,, ,,	,,	Red	1·532	Jamin	45·82
Iodide of Methyl	CH_3I	A	1·5171	Gladstone and Dale	33·50
,, ,,	,,	C	1·52434	Haagen	32·89
,, Ethyl	C_2H_5I	C	1·50812	,,	40·96
,, ,,	,,	Red	1·503	Jamin	40·81
,, ,,	,,	A	1·5026	Gladstone and Dale	40·78
,, Propyl	C_3H_7I	A	1·4934	,,	48·99
,, Amyl	$C_5H_{11}I$	A	1·4804	,,	63·62
,, ,,	,,	C	1·48714	Haagen	65·46
Chloral	C_2HCl_3O	Red	1·461	Jamin	45·3
Phosphite of Ethyl	$3(C_2H_5)PO_3$	A	1·3996	Gladstone	61·76
Sulphuric Acid	H_2SO_4	A	1·4205	,,	21·90
Nitric Acid	HNO_3	A	1·4017	,,	16·46
Chloride of Ammonium	NH_4Cl	Bright	1·625	Brewster	22·08
Oxychloride of Phosphorus	$POCl_3$	A	1·4810	Gladstone and Dale	43·79
,, Vanadium	$VOCl_3$	A	1·6143	Gladstone	57·96
,, Chromium	CrO_2Cl_2	A	1·5177	,,	42·08
Nitrate of Lead	$Pb\,2NO_3)$	Bright	1·758	Brewster	57·0
Sulphate of Barium (ord. ray)	$BaSO_4$	,,	1·6352	Malus	33·29
,, ,, (ext. ray)	,,	,,	1·6468	,,	33·90
Kryolite	Na_3AlF_6	,,	1·346	Brewster	24·63
Alum	$KAl, 2SO_4, 12H_2O$	,,	1·458	Wollaston	126·78
Zircon, least	$ZrSiO_4$	,,	1·961	Brewster	39·22
,, greatest	,,	,,	2·015	,,	41·42
Borax	$Na_2B_4O_7, 10H_2O$	,,	1·475	,,	45·8+60
Ferrocyanide of Potassium	$K_4Fe(CN)_6, 3H_2O$	,,	1·586	,,	117·28+18

Solutions.

If the refraction-equivalent of a mixture or of a chemical compound be the sum of the refraction-equivalents of its constituents, the same may be expected to hold good in the case of a solution. This consideration led me to examine a large number of aqueous solutions of salts, bodies which in their solid state are generally doubly refracting, and necessarily present difficulties that are not met with in the examination of liquids.

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* These are the numbers employed in calculating the alcoholic solutions given in Appendix III.

The method usually adopted was as follows:—An amount of salt representing the atomic weight was dissolved in *n* atoms of water, and the refractive index and density of the solution were taken. From these was reckoned the refraction-equivalent, and subtracting from this *n* times the refraction-equivalent of water for the solar line A, there remained the refraction-equivalent of the dissolved salt for that part of the spectrum. Thus to take an actual instance: 1 atom or 58·5 parts of chloride of sodium were dissolved in 12 atoms or 216 parts of water. The refractive index of the solution for A was 1·3683, and the specific gravity at the same temperature was 1·168; $\frac{\mu A-1}{d}$ therefore was 0·3154, and $P\frac{\mu A-1}{d}$ was $0·3154\times(58·5+216)$, that is, 86·57. From this, the refraction-equivalent of the whole compound system, 12 times 5·926 (*i. e.* 71·12) the refraction-equivalent of water was subtracted, leaving 86·57−71·12, or 15·45, as the refraction-equivalent of chloride of sodium. That a number so arrived at fairly represents the action exerted by the chemical compound on light, is evident from the following considerations.

1st. The refraction-equivalent 15·45 closely approximates to that previously determined for chloride of sodium from the examination of solid rock-salt, namely 15·02. Similarly, cane-sugar dissolved in water gave 119·0, while from BREWSTER's observation of the crystallized solid it should be 119·3 (see Table III.). Again, crystallized borax, after making allowance for the refraction due to the water of crystallization, gave 45·8, while from its aqueous solution its equivalent was determined at 45·9. Chloride of ammonium, solid and in solution, gave respectively 22·08 and 22·33.

2nd. The refraction-equivalents of several solid organic bodies, as determined from their aqueous solutions, agree closely with what might be calculated from LANDOLT's values for C, H, and O. Thus,

	Experiment.	Calculation.
Citric Acid	60·89	61·4
Racemic Acid	45·54	45·8
Tartaric Acid	45·29	45·8

3rd. The refraction-equivalent as reckoned from a solution is not affected by varying the amount of water. This has been proved in the case of the chlorides of sodium, potassium, strontium, and copper, iodide of sodium, sulphate of ammonium, and other salts, and even in the case of the combinations of water with strong acids, such as sulphuric and nitric acids. The following experiment on chloride of sodium will serve as an illustration.

Composition of solution.	Refraction-equivalent of Na Cl.	Variation from mean.
1 atom Chloride of Sodium + 10·74 atoms Water	15·33	−0·07
„ + 12 „	15·45	+0·05
„ + 12 „	15·51	+0·11
„ + 12 „	15·51	+0·11
„ + 14 „	15·26	−0·14
„ + 16 „	15·32	−0·08
„ + 18 „	15·47	+0·07
„ + 20 „	15·43	+0·03
„ + 22 „	15·55	+0·15
„ + 24 „	15·51	+0·11
„ + 26 „	15·37	−0·03
„ + 34 „	15·30	−0·10

This shows also that under favourable circumstances a refraction-equivalent may be depended on to the first place of decimals, but not to the second.

4th. The calculated refraction-equivalent is the same whether water or alcohol be the solvent employed. This was tested in the following cases, the actual observations for which are given in Appendix III.

TABLE IV.

Substance.	Aqueous solution.	Alcoholic solution.
Cobalt Chloride	32·02	32·38
Copper Chloride (line D)...............	34·00	34·72
Mercuric Chloride	41·23	35·59
Potassium Iodide	35·72	35·1
Potassium Sulphocyanide	33·47	33·7
Ammonia	9·49	8·97

The mercuric salt appears to be exceptional, but this metal will be seen later on to be anomalous.

But whatever may be the worth of these considerations, an examination of some corresponding series of salts in solution, viz. the chlorides, bromides, and iodides, convinced me at once that we thus obtain numbers made up of two component parts, the one due to the base, the other to the radical with which it is combined; and the multiplication of these experiments on a large variety of salts has only served to deepen this conviction.

The actual observations will be found in the Appendix, but the refraction-equivalents thus arrived at are given in the following Table.

As the determination of the refraction-equivalent of a salt in solution depends on the difference between it and the refraction-equivalent of water, it is evident that experimental errors will be multiplied undesirably if the water be large in quantity as compared with the salt. Hence the most soluble salts give the most trustworthy results. In some instances the solubility of the salts depended on the addition of some other salt or acid to the solution; in such cases the refraction due to the salt or acid, as well as that due to the water, has been deducted, and in the following Tables the number so arrived at has been marked with an asterisk (*).

TABLE V.—Refraction-equivalents of Compounds in Solution.

Substance.	Atomic weight.	Monobasic.							Bibasic.		
		Chloride.	Bromide.	Iodide.	Nitrate.	Formiate.	Acetate.	Cyanide.	Sulphate.	Hyposulphite.	Lactate.
UNIVALENT.											
Potassium	39·1	18·83	25·09	35·72	22·11	20·24	27·78	17·23	33·11	47·88	76·25
Sodium	23	15·40	21·89	32·52	18·89	......	24·34	......	26·92	41·80	69·45
Lithium............	7	14·86	20·56	31·49	......	......	23·25	......	24·26	......	......
Cæsium	133	24·4	......	......	......	......	......	......	......	......	......
Rubidium	85·4	24·28	......	......	......	......	......	......	45·95*	......	......
Silver	108	......	......	......	27·44	......	......	25·52*	......	66·3*	......
Thallium	204	......	......	......	37·2	32·88	40·45	......	......	......	......
Ammonium	18	22·33	28·53	38·90	25·44	......	31·57	......	39·22	......	......
Hydrogen	1	14·22	20·65	31·17	16·50	13·81	21·29	......	22·45	......	......
BIVALENT.											
Barium	137	37·32	50·72	69·72	43·26	39·82	56·44	......	......	......	78·98
Strontium	87·5	35·04	47·52	......	41·68	39·04	......	......	......	......	78·2
Calcium............	40	32·28	44·32	64·66	38·66	33·94	49·80	......	......	42·52	......
Magnesium	24	28·88	......	......	35·92	30·80	46·02	......	24·18	38·52	......
Cerium	92	35·08	......	......	......	......	......	......	......	......	......
Didymium.........	96	34·18	......	......	......	......	......	......	......	......	......
Zinc	65·2	30·76	43·96	65·67	38·52	34·95	48·80	26·2*	27·61	......	......
Cadmium	112	35·32	47·88	......	41·38	......	......	......	30·34	......	......
Copper	63·4	33·40	......	......	40·04	......	......	......	27·80	......	......
Iron	56	32·86	46·26	......	......	......	......	......	29·09	......	......
Nickel	58·8	31·39*	......	......	38·30	......	......	......	28·16	......	......
Cobalt	58·8	32·02	......	......	38·48	......	50·06	......	29·01	......	......
Manganese.........	55	33·58	47·24	65·14	40·22	......	51·34	......	27·83	......	73·8
Lead	207	......	......	......	52·56	......	64·52	......	......	......	......
Mercury	200	41·23	......	......	48·80*	......	......	34·16	......	......	......
Palladium	106·5	43·90*	......	......	......	......	......	......	......	......	......
TRIVALENT.											
Aluminium	27·4	40·5*	......	......	......	......	......	......	67·7	......	......
Iron	56	51·27	......	......	62·20*	......	......	......	90·9*	......	......
Chromium.........	52·2	48·2*	......	......	......	......	......	......	82·46*	......	......
Gold	196·7	56·11*	......	......	......	......	......	......	......	......	......
Rhodium	104·4	65·08*	......	......	......	......	......	......	......	......	......
QUADRIVALENT.											
Platinum	197·4	71·06*	......	......	......	......	......	......	......	......	......

TABLE V.—Supplementary.

Substance.	Atomic weight.	Monobasic.					Bibasic.				
		Alcoholate.	Hydrate.	Silicate.	Hypophosphite.	Nitrite.	Tartrate.	Chromate.	Bichromate.	Oxalate.	Carbonate.
UNIVALENT.											
Potassium	39·1	28·10	12·61	31·03*	27·28	19·31	57·87	51·50	82·55	37·71	28·77
Sodium	23	24·61	9·20	27·28	......	15·50	50·85	46·08	74·87	......	22·35
Lithium	7	......	......	......	......	......	......	43·62	72·60	......	......
Ammonium ...	18	......	15·42	......	......	......	......	58·12	87·06	42·22	......
Hydrogen	1	20·857	5·92632	......	......	......	45·29	......	......	23·44	......
BIVALENT.											
Calcium	40	......	......	......	46·88	......	......	......	......	......	......
Manganese.. ...	24	......	......	......	48·10	......	......	......	......	......	......

Substance.	Atomic weight.	Monobasic.			Bibasic.			Tribasic.	Quadribasic.
		Fluoride.	Arsenite.	Sulphocyanide.	Sulphite.	Biborate.	Permanganate.	Ferricyanide.	Ferrocyanide.
Potassium	39·1	9·55	......	33·47	35·10*	......	91·8	102·05	114·72
Sodium	23	......	26·14*	......	......	45·9	......	......	......

A glance at this Table will be enough to show that the numbers are not independent of one another, but that there is a remarkable relation between them. Thus the bromides are between six and seven higher than the chlorides corresponding to them in the case of the univalent metals, and double that number in the case of the bivalent; again, the line of sodium salts consists of numbers from three to four lower than the corresponding potassium salts in the monobasic series, and double that number in the bibasic. This kind of relation is precisely what was to be expected if the refraction-equivalent of a salt is really made up of the refraction-equivalents of its constituents. These differences are drawn out in the following Tables. Table VI. exhibits the differences between the refraction-equivalent of potassium indicated by the letter A, and those of the other metals, together with ammonium and hydrogen, the radicals with which they are combined being indicated by Greek letters. Table VII. shows the differences between the refraction-equivalent of chlorine, represented by α, and those of the other radicals, the refraction-equivalent of each metal being represented by a different Roman letter.

TABLE VI.

Substance.	Chloride.	Bromide.	Iodide.	Nitrate.	Formiate.
UNIVALENT.					
Potassium	$A+\alpha$	$A+\beta$	$A+\gamma$	$A+\delta$	$A+\epsilon$
Sodium	$A-3{\cdot}43+\alpha$	$A-3{\cdot}20+\beta$	$A-3{\cdot}20+\gamma$	$A-3{\cdot}22+\delta$	....
Lithium...............	$A-3{\cdot}97+\alpha$	$A-4{\cdot}53+\beta$	$A-4{\cdot}23+\gamma$	....	....
Cæsium	$A+5{\cdot}6\ +\alpha$	....	....	....	....
Rubidium	$A+5{\cdot}45+\alpha$	....	....	....	....
Silver	....	....	....	$A+5{\cdot}33+\delta$	....
Thallium	....	....	....	$A+15{\cdot}1+\delta$	$A+12{\cdot}64+\epsilon$
Ammonium	$A+3{\cdot}50+\alpha$	$A+3{\cdot}44+\beta$	$A+3{\cdot}18+\gamma$	$A+3{\cdot}33+\delta$	....
Hydrogen	$A-4{\cdot}61+\alpha$	$A-4{\cdot}44+\beta$	$A-4{\cdot}55+\gamma$	$A-5{\cdot}61+\delta$	$A-6{\cdot}43+\epsilon$
BIVALENT.					
Barium	$2(A-0{\cdot}17+\alpha)$	$2(A+0{\cdot}27+\beta)$	$2(A-0{\cdot}86+\gamma)$	$2(A-0{\cdot}48+\delta)$	$2(A-0{\cdot}33+\epsilon)$
Strontium	$2(A-1{\cdot}31+\alpha)$	$2(A-1{\cdot}33+\beta)$	....	$2(A-1{\cdot}27+\delta)$	$2(A-0{\cdot}72+\epsilon)$
Calcium...............	$2(A-2{\cdot}69+\alpha)$	$2(A-2{\cdot}93+\beta)$	$2(A-3{\cdot}39+\gamma)$	$2(A-2{\cdot}78+\delta)$	$2(A-3{\cdot}27+\epsilon)$
Magnesium	$2(A-4{\cdot}39+\alpha)$	....	....	$2(A-4{\cdot}15+\delta)$	$2(A-4{\cdot}84+\epsilon)$
Cerium	$2(A-1{\cdot}29+\alpha)$	....	....	....	....
Didymium............	$2(A-1{\cdot}74+\alpha)$	....	....	....	....
Zinc	$2(A-3{\cdot}45+\alpha)$	$2(A-3{\cdot}11+\beta)$	$2(A-2{\cdot}89+\gamma)$	$2(A-2{\cdot}85+\delta)$	$2(A-2{\cdot}77+\epsilon)$
Cadmium	$2(A-1{\cdot}17+\alpha)$	$2(A-1{\cdot}15+\beta)$	....	$2(A-1{\cdot}42+\delta)$	....
Copper	$2(A-2{\cdot}13+\alpha)$	.. ..	....	$2(A-2{\cdot}09+\delta)$	....
Iron	$2(A-2{\cdot}40+\alpha)$	$2(A-1{\cdot}96+\beta)$	....	....	....
Nickel	$2(A-3{\cdot}13+\alpha)$	....	....	$2(A-2{\cdot}96+\delta)$	....
Cobalt	$2(A-2{\cdot}82+\alpha)$	....	....	$2(A-2{\cdot}87+\delta)$	....
Manganese	$2(A-2{\cdot}04+\alpha)$	$2(A-1{\cdot}47+\beta)$	$2(A-3{\cdot}15+\gamma)$	$2(A-2{\cdot}00+\delta)$	....
Lead	....	....	....	$2(A+4{\cdot}17+\delta)$	....
Mercury...............	$2(A+1{\cdot}78+\alpha)$	....	....	$2(A+2{\cdot}29+\delta)$	....
Palladium	$2(A+3{\cdot}12+\alpha)$	....	....	....	....
TRIVALENT.					
Aluminium	$3(A-5{\cdot}3\ +\alpha)$	....	....	$3(A-1{\cdot}38+\delta)$	....
Iron	$3(A-1{\cdot}59+\alpha)$	....	....	....	....
Chromium	$3(A-2{\cdot}8_{,}+\alpha)$	....	....	....	....
Gold	$3(A-0{\cdot}13+\alpha)$	....	.. ..	....	....
Rhodium	$3(A+2{\cdot}86+\alpha)$	....	....	....	....
QUADRIVALENT.					
Platinum	$4(A-1{\cdot}65+\alpha)$	....	....	....	....

TABLE VI.—(continued.)

Substance.	Acetate.	Cyanide.	Sulphate.	Hyposulphite.	Lactate.
UNIVALENT.					
Potassium	$A+\zeta$	$A+\eta$	$2A+\theta$	$2A+\iota$	$2A+\kappa$
Sodium	$A-3{\cdot}44+\zeta$	....	$2(A-3{\cdot}10)+\theta$	$2(A-3{\cdot}04)+\iota$	$2(A-3{\cdot}40)+\kappa$
Lithium	$A-4{\cdot}53+\zeta$	....	$2(A-4{\cdot}42)+\theta$	....	....
Cæsium	....	....	....	....	....
Rubidium	....	....	$2(A+6{\cdot}42)+\theta$	....	....
Silver	....	$A+8{\cdot}29+\eta$	....	$2(A+9{\cdot}2)+\iota$	....
Thallium	$A+12{\cdot}67+\zeta$	....	....	....	....
Ammonium	$A+3{\cdot}79+\zeta$	....	$2(A+3{\cdot}05)+\theta$	....	....
Hydrogen	$A-6{\cdot}49+\zeta$	....	$2(A-5{\cdot}33)+\theta$	....	....
BIVALENT.					
Barium	$2(A+0{\cdot}44+\zeta)$	....	....	....	$2(A+1{\cdot}30)+\kappa$
Strontium	....	....	....	....	$2(A+0{\cdot}9\ )+\kappa$
Calcium	$2(A-2{\cdot}88+\zeta)$	....	....	$2(A-2{\cdot}68)+\iota$	....
Magnesium	$2(A-4{\cdot}77+\zeta)$	....	$2(A-4{\cdot}46)+\theta$	$2(A-4{\cdot}68)+\iota$	....
Cerium	....	....	....	....	....
Didymium	....	....	....	....	....
Zinc	$2(A-3{\cdot}35+\zeta)$	$2(A-4{\cdot}13+\eta)$	$2(A-2{\cdot}75)+\theta$	....	....
Cadmium	....	....	$2(A-1{\cdot}38)+\theta$	....	....
Copper	....	....	$2(A-2{\cdot}65)+\theta$	....	....
Iron	....	....	$2(A-2{\cdot}01)+\theta$	....	....
Nickel	....	....	$2(A-2{\cdot}47)+\theta$	....	....
Cobalt	$2(A-2{\cdot}75+\zeta)$	....	$2(A-2{\cdot}05)+\theta$	....	....
Manganese	$2(A-2{\cdot}11+\zeta)$	....	$2(A-2{\cdot}64)+\theta$	....	$2(A-1{\cdot}3\ )+\kappa$
Lead	$2(A+4{\cdot}48+\zeta)$	....	....	....	....
Mercury	....	$2(A-0{\cdot}15+\eta)$	....	....	....
Palladium	....	....	....	....	....
TRIVALENT.					
Aluminium	....	....	$3(2(A-5{\cdot}3\ )+\theta)$	....	....
Iron	....	....	$3(2(A-1{\cdot}4\ )+\theta)$	....	....
Chromium	....	....	$3(2(A-2{\cdot}81)+\theta)$	....	....
Gold	....	....	....	....	....
Rhodium	....	....	....	....	....
QUADRIVALENT.					
Platinum	....	....	....	....	....

TABLE VI.—Supplementary.

Substance.	Alcoholate.	Hydrate.	Silicate.	Hypophosphite.	Nitrite.
UNIVALENT.					
Potassium	$A+\lambda$	$A+\mu$	$A+\nu$	$A+\xi$	$A+o$
Sodium	$A-3{\cdot}52+\lambda$	$A-3{\cdot}41+\mu$	$A-3\ 75+\nu$	....	$A-3{\cdot}81+o$
Lithium	....	....	....	....	....
Ammonium	....	$A+2{\cdot}81+\mu$	....	....	....
Hydrogen	$A-7{\cdot}24+\lambda$	$A-6{\cdot}68+\mu$	....	....	....
BIVALENT.					
Calcium	....	....	....	$2(A-3{\cdot}84+\xi)$	....
Manganese	....	....	....	$2(A-3{\cdot}23+\xi)$	....

Substance.	Tartrate.	Chromate.	Bichromate.	Oxalate.	Carbonate.
UNIVALENT.					
Potassium	$2A+\pi$	$2A+\rho$	$2A+\sigma$	$2A+\tau$	$2A+\upsilon$
Sodium	$2(A-3{\cdot}51)+\pi$	$2(A-2{\cdot}71)+\rho$	$2(A-3{\cdot}84)+\sigma$	....	$2(A-3{\cdot}21)+\upsilon$
Lithium	....	$2(A-3\ 94)+\rho$	$2(A-4{\cdot}97)+\sigma$	....	....
Ammonium	....	$2(A+3{\cdot}31)+\rho$	$2(A+2{\cdot}25)+\sigma$	$2(A+2{\cdot}25)+\tau$	....
Hydrogen	$2(A-6{\cdot}29)+\pi$	....	....	$2(A-7{\cdot}13)+\tau$	....
BIVALENT.					
Calcium	....	....	....	....	....
Manganese	....	....	....	....	....

TABLE VII.

Substance.	Chloride.	Bromide.	Iodide.	Nitrate.	Formiate.
UNIVALENT.					
Potassium	$A+\alpha$	$A+\alpha+6{\cdot}26$	$A+\alpha+16{\cdot}89$	$A+\alpha+3{\cdot}28$	$A+\alpha+1{\cdot}41$
Sodium	$B+\alpha$	$B+\alpha+6{\cdot}49$	$B+\alpha+17{\cdot}12$	$B+\alpha+3{\cdot}49$	....
Lithium	$C+\alpha$	$C+\alpha+5{\cdot}70$	$C+\alpha+16{\cdot}63$	....	....
Rubidium	$D+\alpha$	....	....	....	....
Silver	....	....	....	$E+\beta$	....
Thallium	....	....	....	$F+\beta$	$F+\beta-4{\cdot}3$
Ammonium	$G+\alpha$	$G+\alpha+6{\cdot}20$	$G+\alpha+16{\cdot}57$	$G+\alpha+3{\cdot}11$	....
Hydrogen	$H+\alpha$	$H+\alpha+6{\cdot}43$	$H+\alpha+16{\cdot}95$	$H+\alpha+2{\cdot}28$	$H+\alpha-0{\cdot}41$
BIVALENT.					
Barium	$I+2\alpha$	$I+2(\alpha+6{\cdot}70)$	$I+2(\alpha+16{\cdot}20)$	$I+2(\alpha+2{\cdot}97)$	$I+2(\alpha+1{\cdot}25)$
Strontium	$K+2\alpha$	$K+2(\alpha+6{\cdot}24)$	....	$K+2(\alpha+3{\cdot}32)$	$K+2(\alpha+2{\cdot}00)$
Calcium	$L+2\alpha$	$L+2(\alpha+6{\cdot}02)$	$L+2(\alpha+16{\cdot}19)$	$L+2(\alpha+3{\cdot}19)$	$L+2(\alpha+0{\cdot}83)$
Magnesium	$M+2\alpha$	....	....	$M+2(\alpha+3{\cdot}52)$	$M+2(\alpha+0{\cdot}96)$
Zinc	$N+2\alpha$	$N+2(\alpha+6{\cdot}60)$	$N+2(\alpha+17{\cdot}45)$	$N+2(\alpha+3{\cdot}88)$	$N+2(\alpha+2{\cdot}09)$
Cadmium	$O+2\alpha$	$O+2(\alpha+6{\cdot}28)$	....	$O+2(\alpha+3{\cdot}03)$	....
Copper	$P+2\alpha$	....	....	$P+2(\alpha+3{\cdot}32)$	....
Iron	$Q+2\alpha$	$Q+2(\alpha+6{\cdot}70)$	....	....	....
Nickel	$R+2\alpha$	....	....	$R+2(\alpha+3{\cdot}45)$	....
Cobalt	$S+2\alpha$	....	....	$S+2(\alpha+3{\cdot}23)$	....
Manganese	$T+2\alpha$	$T+2(\alpha+6{\cdot}83)$	$T+2(\alpha+15{\cdot}78)$	$T+2(\alpha+3{\cdot}32)$	....
Lead	....	....	....	$V+2\beta$	....
Mercury	$W+2\alpha$	....	....	$W+2(\alpha+3{\cdot}78)$	....
TRIVALENT.					
Aluminium	$X+3\alpha$	....	....	....	....
Iron	$Y+3\alpha$	....	....	$Y+3(\alpha+3{\cdot}64)$	....
Chromium	$Z+3\alpha$	....	....	....	....

Substance.	Acetate.	Cyanide.	Sulphate.	Hyposulphite.	Lactate.
UNIVALENT.					
Potassium	$A+\alpha+8{\cdot}95$	$A+\alpha-1{\cdot}60$	$2(A+\alpha)-4{\cdot}55$	$2(A+\alpha)+10{\cdot}22$	$2(A+\alpha)+38{\cdot}59$
Sodium	$B+\alpha+8{\cdot}94$	....	$2(B+\alpha)-3{\cdot}88$	$2(B+\alpha)+11{\cdot}00$	$2(B+\alpha)+38{\cdot}65$
Lithium	$C+\alpha+8{\cdot}39$	....	$2(C+\alpha)-5{\cdot}46$	....	....
Rubidium	....	....	$2(D+\alpha)-2{\cdot}61$	....	....
Silver	....	$E+\beta-1{\cdot}92$	....	$2(E+\beta)+11{\cdot}4$	....
Thallium	$F+\beta+3{\cdot}25$	....	....	....	....
Ammonium	$G+\alpha+9{\cdot}24$	....	$2(G+\alpha)-5{\cdot}44$	....	....
Hydrogen	$H+\alpha+7{\cdot}07$	....	$2(H+\alpha)-5{\cdot}99$	....	....
BIVALENT.					
Barium	$I+2(\alpha+9{\cdot}56)$	....	....	....	$I+2\alpha+41{\cdot}66$
Strontium	....	....	....	....	$K+2\alpha+43{\cdot}2$
Calcium	$L+2(\alpha+8{\cdot}76)$	....	....	$L+2\alpha+10{\cdot}24$	....
Magnesium	$M+2(\alpha+8{\cdot}57)$	....	$M+2\alpha-4{\cdot}70$	$M+2\alpha+9{\cdot}64$	....
Zinc	$N+2(\alpha+9{\cdot}02)$	$N+2(\alpha-2{\cdot}28)$	$N+2\alpha-3{\cdot}15$	....	....
Cadmium	....	....	$O+2\alpha-4{\cdot}98$	....	....
Copper	....	....	$P+2\alpha-5{\cdot}60$	....	....
Iron	....	....	$Q+2\alpha-3{\cdot}77$	....	....
Nickel	....	....	$R+2\alpha-3{\cdot}23$	....	....
Cobalt	$S+2(\alpha+9{\cdot}02)$	....	$S+2\alpha-3{\cdot}01$	....	....
Manganese	$T+2(\alpha+8{\cdot}88)$	....	$T+2\alpha-5{\cdot}75$	....	$T+2\alpha+40{\cdot}2$
Lead	$V+2(\beta+5{\cdot}98)$	....	....	....	....
Mercury	....	$W+2(\alpha-3{\cdot}53)$	....	....	....
TRIVALENT.					
Aluminium	....	....	$2(X+3\alpha)-3(4{\cdot}5)$	....	....
Iron	....	....	$2(Y+3\alpha)-3(3{\cdot}9)$	....	....
Chromium	....	....	$2(Z+3\alpha)-3(5{\cdot}41)$	....	....

TABLE VII.—Supplementary.

Substance.	Alcoholate.	Hydrate.	Silicate.	Hypophosphite.	Nitrite.
UNIVALENT.					
Potassium	$A+\alpha+9{\cdot}27$	$A+\alpha-6{\cdot}22$	$A+\alpha+12{\cdot}20$	$A+\alpha+8{\cdot}45$	$A+\alpha+0{\cdot}48$
Sodium	$B+\alpha+9{\cdot}21$	$B+\alpha-6{\cdot}20$	$B+\alpha+11{\cdot}88$	....	$B+\alpha+0{\cdot}10$
Lithium	....	....	....	....	....
Ammonium	....	$G+\alpha-6{\cdot}91$	....	....	....
Hydrogen	$H+\alpha+6{\cdot}637$	$H+\alpha-8{\cdot}294$	....	....	....
BIVALENT.					
Calcium	....	....	....	$L+2(\alpha+7{\cdot}30)$	....
Manganese............	....	....	....	$T+2(\alpha+7{\cdot}26)$	....

Substance.	Tartrate.	Chromate.	Bichromate.	Oxalate.	Carbonate.
UNIVALENT.					
Potassium	$2(A+\alpha)+20{\cdot}21$	$2(A+\alpha)+13{\cdot}84$	$2(A+\alpha)+44{\cdot}89$	$2(A+\alpha)+0{\cdot}05$	$2(A+\alpha)-8{\cdot}89$
Sodium	$2(B+\alpha)+20{\cdot}05$	$2(B+\alpha)+15{\cdot}28$	$2(B+\alpha)+44{\cdot}07$	....	$2(B+\alpha)-8{\cdot}45$
Lithium	....	$2(C+\alpha)+13{\cdot}90$	$2(C+\alpha)+42{\cdot}88$	....	....
Ammonium	....	$2(G+\alpha)+13{\cdot}46$	$2(G+\alpha)+42{\cdot}40$	$2(G+\alpha)-2{\cdot}44$	....
Hydrogen	$2(H+\alpha)+16{\cdot}85$	....	....	$2(H+\alpha)-5{\cdot}00$	....
BIVALENT.					
Calcium	....	....	....	....	....
Manganese............	....	....	....	....	....

TABLE VII.—Supplementary.

Substance.	Fluoride.	Arsenite.	Sulphocyanide.	Sulphite.
Potassium	$A+\alpha-9{\cdot}28$	....	$A+\alpha+14{\cdot}64$	$2(A+\alpha)-2{\cdot}56$
Sodium	....	$B+\alpha+10{\cdot}70$	....	....

Substance.	Biborate.	Permanganate.	Ferricyanide.	Ferrocyanide.
Potassium	....	$2(A+\alpha)+54{\cdot}1$	$3(A+\alpha)+46{\cdot}56$	$4(A+\alpha)+39{\cdot}40$
Sodium	$2(B+\alpha)+15{\cdot}1$	....	....	....

The differential numbers along a line in Table VI., or down a column in Table VII., are sufficiently near to show that we are dealing with a reality; but they are sufficiently wide apart to show that we must rely upon the average of the numbers and not on any single experiment, if we wish to get a refraction-equivalent true to the first place of decimals. Unfortunately all experimental errors fall upon this residuary number.

The only exception to this regularity which is worth notice, is in the case of hydrogen, which is $A-4{\cdot}5$ or thereabouts in the hydracids, but drops to somewhere about $A-6{\cdot}7$ in the organic acids. This seems to indicate that in the first group hydrogen has a refraction-equivalent somewhere about 2·2 higher than in the other.

Though these Tables alone do not afford us the means of determining a single refraction-equivalent of a metal or of any other element, it is evident that the refraction-equivalents of the whole would be a simple matter of calculation if we could determine with certainty the value of any letter, Roman or Greek, that is, the refraction-equivalent of any one of the constituents. The means of arriving at this will be explained in the second part of this paper under the head of Potassium.

Deductions.

Carbon.—Crystallized carbon (that is diamond) has a refraction-equivalent of about 5·0; the same number was arrived at by LANDOLT from a consideration of a multitude of organic substances. If we compare together the two gaseous oxides, CO, 7·53, and CO_2, 10·03, it is clear that the second atom of oxygen is represented by 2·5, and taking the first atom at the same it leaves 5·03 for carbon.

If, indeed, anything is certain in this whole subject, it is that carbon, whether pure or in combination with other elements, and thus forming solid liquid or gaseous bodies, exerts the same influence on the rays of light transmitted by it, and that this influence may be expressed by the number 5·0; but the cumulative evidence on which this conviction rests is derived from the whole range of organic bodies, and from many other compounds of carbon that will be considered under other headings. The apparent exceptions, such as the aromatic series of organic compounds, may be accounted for by a part of the hydrogen having a higher refraction-equivalent than it usually exhibits*.

Hydrogen.—According to DULONG's observations hydrogen gas has a refraction-equivalent of 1·53, and it seems to have the same in water; LANDOLT, however, has shown that in the large majority of the organic compounds examined by him, it does not exceed 1·3. This is confirmed by such observations as those on the new ketones, or on laurostearate of ethyl, given in Table III. LANDOLT examined no hydrocarbons, but assuming $C=5·0$, the series in Table II. give the following values for H:—

Olefiant Gas	gives	1·27
Amylene	„	1·26
Oil of Turpentine . . .	„	1·43
Hydride of Œnanthyl .	„	1·25
Hydride of Capryl . .	„	1·27

This is the value of H in acetic, formic, tartaric, and oxalic acids; but from Table VI. it would appear that the hydrogen in hydrochloric, hydrobromic, and hydriodic acids has a value about 2·2 higher than in these organic acids; it must therefore be about 3·5. The same element in nitric and sulphuric acids seems to have a value intermediate between these.

Oxygen.—Gaseous oxygen, according to DULONG, is 3·04; and LANDOLT found that 3·0 suited well for calculating the refraction-equivalent of the great group of organic compounds. There is, however, more uncertainty about this number; most of the substances examined by the German professor contained comparatively little of the element, and his best comparisons give a somewhat lower figure.

Assuming $C=5·0$, and $H=1·3$,

Sugar	gives	$O=2·8$
Carbonic Acid . . .	„	$O=2·5$
Carbonic Oxide . .	„	$O=2·5$
Oxalic Acid	„	$O=2·7$

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* See Postscript.

Formic Acid . . .	gives	O=3·1
Tartaric Acid . . .	„	O=2·9
Citric Acid	„	O=2·9

On comparing nitrate of potassium, KNO_3, 22·11, with nitrite of potassium, KNO_2, 19·31, we deduce for O the value 2·8.

From this diversified evidence, 2·9 may be fairly taken as the probable value of oxygen.

Sulphur.—The pure element, whether solid or liquid, has a refraction-equivalent of 16·0 or 16·3; as deduced from CS_2, 36·7, it will be 15·85. Again, the difference between KCNS, 33·47, and KCN, 17·23, gives S=16·24; it will be seen that it has a similar value in chloride of sulphur. It is evident, however, that in the two gases, H_2S, 14·28, and SO_2, 14·91, or in liquified SO_2, 14·59, it cannot be 16; nor yet in its other oxygen compound, H_2SO_4, 21·9.

Phosphorus.—The refraction-equivalent for this very dispersive elementary body is 18·3 for the line A. In its compounds with the halogens it seems to exert the same influence on light, but in phosphoric acid its refractive energy must be greatly diminished.

Chlorine.—The gas itself has the refraction-equivalent of 8·87, as reckoned from DULONG's experiments, and the same figure represents it in gaseous phosgene; but a somwehat higher number is arrived at when liquid compounds are examined. Thus, taking the numbers previously given for carbon, hydrogen, sulphur, and phosphorus, we find—

From Tetrachloride of Carbon . . .	Chlorine	= 9·8
„ Chloroform	„	= 9·6
„ „ (HAAGEN) . . .	„	= 9·7
„ Bichloride of Chlorethylene .	„	= 9·9
„ Chloride of Sulphur	„	= 9·8
„ Terchloride of Phosphorus . .	„	=10·0
„ Bichloride of Ethylene . . .	„	= 9·8
„ Chloral	„	=10·4

Moreover the substitution of chlorine for hydrogen in benzole gives for each Cl—H 8·7, that is Cl=10·0. The mean of these numbers is 9·9.

Bromine.—The liquid element has a refraction-equivalent of 16·23. The determinations of compounds of carbon, hydrogen, and phosphorus give:—

From Bromoform	Bromine	=15·7
„ Bibromide of Bromethylene .	„	=15·1
„ Bibromide of Chlorethylene .	„	=15·0
„ Terbromide of Phosphorus .	„	=15·0
„ Bromide of Ethyl	„	=15·0
„ Bromide of Amyl	„	=15·7
„ Bibromide of Ethylene . .	„	=15·4

The average of these numbers is 15·3.

Iodine.—Solutions of iodine have hitherto given results which are not comparable among themselves.

From Iodide of Methyl	Iodine	=24·6
,, ,, ,, Ethyl	,,	=24·3
,, ,, ,, Propyl	,,	=24·9
,, ,, ,, Amyl	,,	=24·3

The average of these is 24·5; HAAGEN gives 24·87 for the line C, as deduced from the same series of compounds.

It would appear that the differences between the three halogens are as follows: Br=Cl+5·4, and I=Cl+14·6. This does not exactly agree with the differences between the three series of dissolved haloid salts, where Br=Cl+6·36, and I=Cl+16·46.

Potassium.—The number of potassium salts in solution whose refraction-equivalents have been determined is 26. There are two ways of arriving at the equivalent of the metal itself from these data. 1st. If we know the value of any of the radicals conjoined with potassium (expressed in Table VI. by Greek letters), it is a simple question of subtraction. 2nd. If we know the value of any other capital letter in Table VII., we have merely to add to it, or subtract from it, the mean number representing the difference between it and A, and we arrive at A itself, that is the refraction-equivalent of potassium.

For calculation by the first method, the numbers already arrived at may be employed, namely, C=5, H=1·3, O=2·9, S=16; and from the ethyl compounds in Table III. the following values may also be accepted, NO_3=14·4, SiO_4=18·4, CO_3=12·9, and CN=9·1, the two latter numbers corresponding with those of carbonic anhydride and cyanogen gas in Table II. We obtain

From the Formiate	Potassium	=8·14
,, Acetate	,,	=8·08
,, Oxalate	,,	=8·05
,, Alcoholate	,,	=8·72
,, Lactate	,,	=7·92
,, Tartrate	,,	=7·63
,, Nitrate	,,	=7·71
,, Silicate	,,	=8·30
,, Carbonate	,,	=7·93
,, Cyanide	,,	=8·13
,, Sulphocyanide . . .	,,	=8·37
	Mean . . .	8·09

Deductions from the chloride, bromide, and iodide are omitted from this list, because, as has been shown already, the differences between the refraction-equivalents of these halogens in dissolved salts must be somewhat greater than we find them to be in organic compounds. It is true this *à priori* objection does not lie against the chloride itself, but the close analogy between its properties and those of the two other halogens renders it open to suspicion. If, indeed, we assume Cl=9·9, we obtain K=8·9, a higher number than any of the above. Considering the whole scope of the evidence, I would rather determine chlorine from potassium, than potassium from chlorine.

By the second method, assuming H in water=1·5, and in the hydracids 3·5, we obtain from a comparison of hydrate of potassium with water K=8·18, and from a comparison of the potassium salts with the hydracids K=8·03.

The numbers thus arrived at range from 7·6 to 8·7; but the determinations most to be relied on are a little above 8·0, and the whole concurrent testimony points to 8·1 as the most probable number.

Having determined 8·1 as the refraction-equivalent of potassium (the A of Table VI.), it is perfectly simple to calculate the refraction-equivalent of every other metal in that Table. It is only necessary to add to, or subtract from, 8·1 the mean of the figures in each line; but, inasmuch as some observations deserve more confidence than others, the exact mean was not always followed, but rather what was thought to be the most trustworthy number.

From this it results that

Sodium	=A− 3·3,	that is	4·8
Lithium	=A− 4·3	,,	3·8
Cæsium	=A+ 5·6	,,	13·7
Rubidium	=A+ 5·9	,,	14·0
Silver	=A+ 7·6	,,	15·7
Thallium	=A+13·5	,,	21·6
Barium	=2(A−0·2)	,,	15·8
Strontium	=2(A−1·3)	,,	13·6
Calcium	=2(A−2·9)	,,	10·4
Magnesium	=2(A−4·6)	,,	7·0
Cerium	=2(A−1·3)	,,	13 6
Didymium	=2(A−1·7)	,,	12·8
Zinc	=2(A−3·0)	,,	10·2
Cadmium	=2(A−1·3)	,,	13·6
Copper	=2(A−2·3)	,,	11·6
Iron	=2(A−2·1)	,,	12·0
Nickel	=2(A−2·9)	,,	10·4
Cobalt	=2(A−2·7)	,,	10·8
Manganese	=2(A−2·0)	,,	12·2
Lead	=2(A+4·3)	,,	24·8
Mercury	=2(A+2·0)	,,	20·2
Palladium	=2(A+3·1)	,,	22·4
Aluminium	=3(A−5·3)	,,	8·4
Iron	=3(A−1·4)	,,	20·1
Chromium	=3(A−2·8)	,,	15·9

Gold	$=3(A-0{\cdot}1)$	that is	24·0
Rhodium	$=3(A+5{\cdot}2)$	,,	39·9
Platinum	$=4(A-1{\cdot}6)$	,,	26·0

Assuming these to be the correct numbers, we are in a position to assign values to all the inorganic radicals of the salts comprised in Table VII. We have:—

TABLE VIII.

Radical.	From the potassium salt.	From the sodium salt.	From the mean of all salts.
Cl	10·7	10·6	10·7
Br	17·0	17·1	17·0
I	27·6	27·7	27·2
NO_3	14·0	14·1	14·0
CN	9·1	......	......
SO_4	16·85	17·3	17·0
S_2O_3	31·6	32·2	31·7
H_2SiO_4	22·9	22·5	......
PH_2O_2	19·2	......	18·4
NO_2	11·2	10·7	......
CrO_4	35·3	36·5	35·5
Cr_2O_7	66·35	65·3	65·0
CO_3	12·6	12·75	......
F	1·45	......	......
AsO_2	......	21·3	......
SCN	25·4	......	......
SO_3	18·9	......	......
B_4O_7	......	36·3	......
Mn_2O_8	75·6	......	......
FeC_6N_6 (Ferricy.)	77·75	......	......
,, (Ferrocy.)	82·3	......	......

This Table shows that the three halogens, chlorine, bromine, and iodine, have higher refraction-equivalents in these mineral salts than they have in their organic compounds, and that the divergence increases as we advance in the series*.

	In organic compounds.	In mineral salts.
Chlorine	9·9	10·7
Bromine	15·3	17·0
Iodine	24·5	27·2

It also gives us additional information respecting the refraction-equivalents of some of the metals.

Iron.—This metal in combination with cyanogen in the ferrocyanide and ferricyanide of potassium has apparently a higher equivalent than in the compounds where it plays the part of a base.

Manganese.—This element exists in a highly oxidized condition in permanganate of potassium. If O be taken at 2·9, the refraction-equivalent of manganese will be 26·2.

Chromium.—This also exists in combination with oxygen in the chromates and bichromates. There it has a refraction-equivalent of about 23.

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[*] In estimating metals from their chlorides, the equivalent 9·9 has been taken where the chloride is decomposed by water, 10·7 where it is soluble without decomposition. This arbitrary distinction seems to have a foundation in fact.

In the oxychloride (Table III.) it seems to have about the same power as in the chromium salts, viz. 16·5.

Mercury.—This metal presents greater difficulties in the estimation of its refraction-equivalent than any other in the list, and a comparison of its value, as deduced from all its compounds, only increases the difficulty.

From the Chloride in Water	Mercury	=19·8
,, ,, Alcohol	,,	=14·2
,, Nitrate	,,	=20·8
,, Cyanide	,,	=16·0
,, Crystallized Calomel	,,	=22·0
,, Mercuric Methide	,,	=22·7

These differences are beyond what may be due to errors of experiment.

There are some other elementary bodies the refraction-equivalents of which may be deduced from the observations recorded.

Tin.—From the Tetrachloride	Sn=19·2
Titanium.—From the Tetrachloride	Ti=25·5
Arsenic.—From the Tetrachloride	As=19·9 (line C)
,, Arsenious Anhydride	,, =15·7
,, Sodium Arsenite	,, =15·5
,, Cacodylic Acid	,, =15·2
,, Triethylarsine	,, =15·2

Arsenic acid and some arseniates have been examined, but the results are discordant, showing, however, always a lower equivalent than 15.

Antimony.—From the Terchloride	Sb=31·8
,, Pentachloride	,, =24·5
Vanadium.—From the Oxychloride	V=25·3

Nitrogen.—From Dulong's numbers for gaseous nitrogen the refraction-equivalent is 3·30, but no other means of calculation give so low a figure. The gaseous compounds afford the following results:—

From Cyanogen	N=4·18
,, Nitrous Oxide	N=4·16
,, Nitric Oxide	N=3·84
,, Ammonia	N=4·10

These point clearly to 4·0 or 4·1. The hydrogen in ammonia has been taken as gaseous hydrogen, viz. 1·5. It is to be remarked that ammonium in the series of salts is 11·5; but it is impossible to calculate N from this, as the refraction-equivalent of hydrogen is uncertain. Cyanogen in its compounds is 9·1; hence the nitrogen is also 4·1 in this combination. In the nitrates and nitrites, however, it seems to have a greater influence on light.

From NO_3 . . . . . . $N=5·3$
,, NO_2 . . . . . . $N=5·3$

Silicon.—From the tetrachloride, $Si=7·5$. Silicic acid, $Si\,O_2$, in the form of quartz has the refraction-equivalent 12·4 and 12·6 for the ordinary and extraordinary rays respectively; as deduced from silicic ether it is 12·6, and from the soluble silicates 12·6. Therefore $Si=6·8$.

Boron.—$B_2\,O_3$, as deduced from boracic ether, is 16·45, from crystallized borax 16·85, and from borax in solution 16·7. Taking the mean of these values, $B=4·0$.

Zirconium.—From zircon, $Zr=20·8$ or 22·6, according as we reckon from the ordinary or extraordinary ray.

Fluorine.—From potassium fluoride $F=1·45$. The numbers given for fluor-spar and kryolite confirm this very small value, or rather indicate that this body has scarcely any influence on the rays of light.

SUMMARY.

The general results of the foregoing deductions give the following numbers as the refraction-equivalents of the elements, already determined more or less accurately.

Element.	Atomic weight.	Refraction-equivalent.	Specific refractive energy.
Aluminium	27·4	8·4	0·307
Antimony	122	24·5 ?	0·201 ?
Arsenic	75	15·4 (other values ?)	0·205
Barium	137	15·8	0·115
Boron	11	4·0	0·364
Bromine	80	15·3, in dissolved salts 16·9	0·191 or 0·211
Cadmium	112	13·6	0·121
Cæsium	133	13·7 ?	0·103 ?
Calcium	40	10·4	0·260
Carbon	12	5·0	0·417
Cerium	92	13·6 ?	0·148 ?
Chlorine	35·5	9·9, in dissolved salts 10·7	0·279 or 0·301
Chromium	52·2	15·9 (in chromates 23 ?)	0·305 or 0·441 ?
Cobalt	58·8	10·8	0·184
Copper	63·4	11·6	0·183
Didymium	96	12·8 ?	0·133 ?
Fluorine	19	1·4 ?	0·073 ?
Gold	197	24·0 ?	0·122 ?
Hydrogen	1	1·3, in hydracids 3·5	1·3 or 3·5
Iodine	127	24·5, in dissolved salts 27·2	0·193 or 0·214
Iron	56	12·0 in ferrous, 20·1 in ferric salts	0·214 or 0·359
Lead	207	24·8	0·120
Lithium	7	3·8	0·543
Magnesium	24	7·0	0·292
Manganese	55	12·2 (26·2 ? in permanganate)	0·222 or 0·476 ?
Mercury	200	20·2 ?	0·101 ?
Nickel	58·8	10·4	0·177
Nitrogen	14	4·1, or 5·3 in higher oxides	0·293 or 0·379
Oxygen	16	2·9	0·181
Palladium	106·5	22·4 ?	0·210 ?
Phosphorus	31	18·3 (other values ?)	0·590
Platinum	197·4	26·0	0·132
Potassium	39·1	8·1	0·207
Rhodium	104·4	24·2 ?	0·232 ?
Rubidium	85·4	14·0	0·164
Silicon	28	7·5 ? (6·8 in oxygen compounds)	0·268 ? or 0·243
Silver	108	15·7 ?	0·145 ?
Sodium	23	4·8	0·209
Strontium	87·5	13·6	0·155
Sulphur	32	16·0 (other values ?)	0·500
Thallium	204	21·6 ?	0·106 ?
Tin	118	19·2 ?	0·163 ?
Titanium	50	25·5 ?	0·510 ?
Vanadium	51·2	25·3 ?	0·494 ?
Zinc	65·2	10·2	0·156
Zirconium	89·6	21·0 ?	0·234 ?

In the above Table those equivalents are marked ? where they have been deduced from only one compound, or where the different determinations are not fairly accordant.

At some future time I hope to reexamine each of the doubtful points, and to extend the observations to the whole range of the chemical elements. The question of dispersion-equivalents is also of interest: the data for an investigation of the matter are given in the Appendix, since the refractive index has been calculated for the lines D and H, as well as the line A; but I have avoided encumbering the present paper with any remarks on this subject.

The specific refractive energy of a body is in some respects worthy of more consideration than the refraction-equivalent, for it is a physical property independent of chemical theories. If these energies in the preceding Table are compared with one another several suggestive facts may be observed.

1st. Hydrogen has more than double the energy of any other element, even in the lowest number that can be assigned to it.

2nd. Phosphorus, vanadium, titanium, and sulphur have singularly high energies, and they are substances that present certain chemical analogies.

3rd. There are several pairs of analogous elements having the same, or nearly the same, energy; thus, bromine and iodine, arsenic and antimony, potassium and sodium, manganese and iron, nickel and cobalt.

4th. An element in altering its quantivalence alters its energy.

5th. If those metals that form the soluble salts of Table V. be arranged in the order of their energies, it will be seen that, with a few exceptions, they are in the inverse order of their combining proportions. This is shown in the annexed Table, where the third column gives the actual weight of the metal that combines with 35·5 of chlorine.

Element.	Specific refractive energy.	Combining proportion.	Element.	Specific refractive energy.	Combining proportion.
Hydrogen	1·300	1	Nickel	·177	29·4
Lithium	·540	7	Rubidium	·164	85·4
Aluminium	·307	9·1	Zinc	·156	32·6
Chromium	·305	17·4	Strontium	·155	43·8
Magnesium	·292	12	Cerium	·148 ?	46
Calcium	·260	20	Silver	·145 ?	108
Zirconium	·234 ?	22·4	Didymium	·133 ?	47·5
Rhodium	·232 ?	34·8	Platinum	·132 ?	49·3
Manganese	·222	27·5	Gold	·122 ?	65·7
Iron	·214	28	Cadmium	·121	56
Palladium	·210 ?	53·2	Lead	·120	103·5
Sodium	·209	23	Barium	·115	68·5
Potassium	·207	39·1	Thallium	·106 ?	204
Cobalt	·184	29·4	Cæsium	·103 ?	133
Copper	·183	31·7	Mercury	·101 ?	100

This has not the regularity of a physical law, but it clearly points to some connexion between the power of a metallic body to saturate the affinities of other elements, and its power to retard the rays of light.

APPENDIX.

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I. Refractive Indices.

Substance.	Formula.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Bromine	Br	3·085	12	1·6260	....	....	16·23
Water	H_2O	1·0	15	1·32924	1·33328	1·34393	5·92632
Rock-salt	$NaCl$	2·086	......	1·5369	1·5443	1·5685	15·02
Carbon Tetrachloride	CCl_4	1·5888	......	1·4560	....	....	44·2
Tin ,,	$SnCl_4$	2·231	20	1·5035	1·5124	1·5429	58·76
Titanium ,,	$TiCl_4$	1·727	......	1·5856	1·6039	....	65·08
Antimony Pentachloride	$SbCl_5$	2·322	17·5	1·5739	1·5871	....	74·03
Vanadium Oxychloride	$VOCl_3$	1·841	12	1·6143	1·6416	....	57·96
Chromium ,,	CrO_2Cl_2	1·908	23	1·5177	1·5242	....	42·08
Sulphuric Acid	H_2SO_4	1·882	11·5	1·4205	1·4251	1·4366	21·90
Nitric Acid	HNO_3	1·549	13	1·4047	1·4115	....	16·46
Alcohol	C_2H_6O	0·797	12	1·3615	1·3655	1·3769	20·857
Methylated Acetone	C_4H_8O	0·811	13	1·3817	1·3860	1·3991	33·89
Butyrone	$C_7H_{14}O$	0·825	13	1·4073	1·4116	1·4262	56·29
Ethylic Phosphite	$(C_2H_5)_3PO_3$	1·074	20	1·3996	1·4032	1·4160	61·76
Mercuric Methyl	$Hg(CH_3)_2$	2·966	30	1·5229	1·5336	1·5683	40·54

II. Aqueous Solutions.

Substance.	Formula.	Equivalents of water.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Potassium Chloride	KCl	20	1·119	15·5	1·3535	1·3581	1·3704	18·80
,, ,,	,,	22	1·108	15·5	1·3511	1·3560	1·3682	18·74
,, ,,	,,	24	1·100	15·5	1·3493	1·3542	1·3662	18·63
,, ,,	,,	26	1·093	15·5	1·3482	1·3527	1·3646	18·80
,, ,,	,,	28	1·087	16·5	1·3478	1·3521	1·3638	19·12
,, ,,	,,	13	1·167	13	1·3625	1·3670	1·3800	18·84
,, ,,	,,	30	1·082	15	1·3462	1·3506	1·3625	18·89
,, ,,	,,	15	1·148	13	1·3591	1·3637	1·3768	18·84
,, Bromide	KBr	13	1·292	12·5	1·3747	1·3801	1·3953	25·09
,, Iodide	KI	13	1·421	13·5	1·4006	1·4073	1·4287	35·72
,, Nitrate	KNO_3	25	1·125	13·5	1·3476	1·3519	1·3647	22·11
,, ,,	,,	24·77	1·125	14·5	1·3476	1·3522	1·3647	22·10
,, Nitrite	KNO_2	17	1·136	24·5	1·3487	1·3528	.....	19·31
,, Formiate	$KCHO_2$	9·37	1·220	17	1·3647	1·3692	1·3818	20·03
,, ,,	,,	11·71	1·179	9	1·3591	1·3636	1·3758	20·46
,, Acetate	$KC_2H_3O_2$	10	1·189	14	1·3721	1·3765	1·3891	27·78
,, Lactate	$K_2C_6H_{10}O_6$	11·04	1·316	8·5	1·4103	1·4145	1·4297	76·37
,, ,,	,,	36·55	1·162	10·5	1·3725	1·3771	1·3897	76·14
,, Cyanide	KCN	6·13	1·195	13·5	1·3648	1·3694	1·3822	17·23
,, Sulphocyanide	$KCNS$	9	1·208	14·5	1·4017	1·4105	1·4297	33·47
,, Chromate	K_2CrO_4	16	1·231	14·5	1·3853	1·3924	......	51·50
,, Hydrate	KHO	5·47	1·368	11·5	1·3987	1·4040	1·4187	12·61
,, Fluoride	KF	20	1·111	18	1·3403	1·3448	1·3555	9·55
,, Sulphate	K_2SO_4	107·5	1·069	11·5	1·3397	1·3441	1·3551	33·11
,, Bisulphate	$KHSO_4$	38·5	1·128	13	1·3472	1·3514	1·3623	27·00
,, Tartrate	$K_2C_4H_4O_6$	19	1·300	17·5	1·3900	1·3947	1·4074	57·87
,, Oxalate	$K_2C_2O_4$	54·28	1·112	10·5	1·3497	1·3541	1·3658	37·71
,, Ferrocyanide	$K_4C_6N_6Fe$	45·86	1·125	13	1·3667	1·3719	1·3866?	114·24
,, ,,	,,	53·68	1·113	14·5	1·3635	1·3682	1·3819	115·20
,, Ferricyanide	$K_3C_6N_6Fe$	77	1·109	25·5	1·3606	1·3657	......	101·44
,, ,,	,,	68·47	1·110	13	1·3616	1·3669	......	102·73
,, ,,	,,	53·99	1·149	......	1·3728	1·3783	......	101·99
,, Permanganate	$KMnO_4$	162	1·027	11	1·3360	......	......	45·9
,, Hyposulphite	$K_2S_2O_3$	13·07	1·397	......	1·4118	1·4174	1·4335	47·88
,, Silicate	$KH_3SiO_4-0{\cdot}07K_2O$	8·18	1·334	17	1·3795	1·3842	1·3962	29·68
,, Carbonate	K_2CO_3	26	1·222	14·5	1·3681	1·3731	1·3855	28·54
,, ,,	,,	15·66	1·327	18	1·3847	1·3894	1·4018	28·97
,, ,,	,,	28·17	1·203	20	1·3649	1·3695	1·3811	28·79

II. Aqueous Solutions (continued).

Substance.	Formula.	Equivalents of water.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Potassium Hypophosphite	KPH_2O_2	11	1·222	12·5	1·3733	1·3778	1·3908	27·09
" "	"	8·65	1·243	22	1·3768	1·3821	1·3946	27·48
" Sulphite	$K_2SO_3+0·488\,SO_2$	21·62	1·280	22	1·3771	1·3824	1·3957	42·28
" "	" $+0·29\,SO_2$	24·20	1·296	22	1·3831	1·3878	1·4013	40·18
" Bichromate	$K_2Cr_2O_7$	120·93	1·085	21·5	1·3511	1·3564	......	82·5
" "	"	215	1·051	15·5	1·3424	1·3472	......	82·6
Sodium Chloride	$NaCl$	10·74	1·183	13·5	1·3710	1·3763	1·3901	15·33
" "	"	12	1·168	9·5	1·3683	1·3736	1·3864	15·45
" "	"	12	1·166	14	1·3680	1·3734	1·3868	15·51
" "	"	12	1·162	12·5	1·3667	1·3717	1·3849	15·51
" "	"	14	1·148	9·5	1·3632	1·3681	1·3814	15·26
" "	"	16	1·132	9	1·3598	1·3649	1·3771	15·32
" "	"	18	1·118	15·5	1·3570	1·3621	1·3748	15·47
" "	"	20	1·109	9	1·3549	1·3596	1·3712	15·43
" "	"	22	1·099	15	1·3529	1·3576	1·3701	15·55
" "	"	24	1·091	14·5	1·3509	1·3555	1·3678	15·51
" "	"	26	1·085	14·5	1·3492	1·3540	1·3661	15·37
" "	"	34	1·066	15	1·3446	1·3491	1·3610	15·30
" Bromide	$NaBr$	13	1·294	11·5	1·3801	1·3855	1·4013	21·89
" Iodide	NaI	15·1	1·373	11·5	1·3968	1·4037	1·4241	32·41
" "	"	27	1·221	11·5	1·3699	1·3752	1·3917	32·63
" Nitrate	$NaNO_3$	13	1·202	12	1·3615	1·3664	1·3803	18·89
" Nitrite	$NaNO_2$	6·49	1·284	14·5	1·3731	1·3782	1·3931	15·50
" Acetate	$NaC_2H_3O_2$	13	1·141	14	1·3668	1·3712	1·3837	24·55
" "	"	12	1·151	11·5	1·3678	1·3724	1·3852	24·13
" Hydrate	$NaHO$	10·21	1·206	11·5	1·3759	1·3812	1·3946	9·20
" Chromate	Na_2CrO_4	16	1·218	12	1·3889	1·3956	......	46·08
" Sulphate	Na_2SO_4	45	1·132	23	1·3491	1·3535	1·3645	26·92
" "	"	63	1·100	18·5	1·3451	1·3494	1·3600	26·91
" Arsenite	$NaAsO_2-0·2\,Na_2O$	5·9	1·727	15	1·4564	1·4638	1·4858	23·65
" Arseniate	$Na_2HAsO_4-0·64\,Na_2O$	32·68	1·195	16	1·3619	1·3662	1·3781	28·77
" Tartrate	$Na_2C_4H_4O_6$	30	1·204	17	1·3750	1·3797	1·3922	50·85
" Carbonate	Na_2CO_3	23·34	1·216	24	1·3709	1·3753	1·3876	22·07
" "	"	26·77	1·192	14·5	1·3676	1·3722	1·3847	22·63
" Biborate	$Na_2B_4O_7$	467	1·020	18	1·3334	1·3377	1·3487	45·9
" Hyposulphite	$Na_2S_2O_3$	25	1·235	14	1·3858	1·3906	1·4051	41·80
" Bichromate	$Na_2Cr_2O_7$	100	1·103	11	1·3570	1·3623	......	74·87
" Silicate	NaH_3SiO_4	7·96	1·384	16·5	1·3943	1·3990	1·4119	27·28
" Lactate	$Na_2C_6H_{10}O_6$	40·11	1·137	20	1·3693	1·3740	1·3860	69·45
Lithium Chloride	$LiCl$	13	1·090	9	1·3623	1·3670	1·3801	14·86
" Bromide	$LiBr$	13	1·242	12·5	1·3776	1·3826	1·3893	20·56
" Iodide	LiI	13	1·365	......	1·4014	1·4085	1·4306	31·49
" Nitrate	$LiNO_3$	6	1·189	14	1·3683	1·3731	1·3874	19·28
" Acetate	$LiC_2H_3O_2$	10	1·110	14·5	1·3723	1·3774	1·3887	23·25
" Chromate	Li_2CrO_4	22	1·225	14·5	1·4051	1·4142	......	43·62
" Bichromate	$Li_2Cr_2O_7$	72	1·097	14·5	1·3589	1·3651	......	72·60
" Sulphate	Li_2SO_4	26	1·173	13	1·3619	1·3662	1·3771	24·26
Cæsium Chloride	$CsCl$	169	1·042	18	1·3330	1·3374	1·3486	24·4
Rubidium "	$RbCl$	23·77	1·168	17	1·3515	1·3560	1·3682	24·28
" Sulphate	$Rb_2SO_4+1·04\,H_2SO_4$	68·58	1·177	17·5	1·3495	1·3536	1·3646	69·30
Silver Nitrate	$AgNO_3$	13	1·512	13·5	1·3920	1·3974	1·4138	27·68
" "	"	8·63	1·721	18	1·4150	1·4212	1·4398	27·28
" "	"	8·71	1·718	17	1·4152	1·4212	1·4399	27·35
" Cyanide	$AgCN+KCN$	60·22	1·110	20	1·3455	1·3502	1·3622	42·75
" Hyposulphite	$\left\{Ag_2S_2O_3+2\,NaCl,\ 2·6\,Na_2S_2O_3\right\}$	149·36	1·212	19·5	1·3732	1·3784	1·3933	205·8
Thallium Nitrate	$ThNO_3$	252·6	1·049	13	1·3347	1·3390	1·3502	37·7
" "	"	243·14	1·050	18	1·3342	1·3385	1·3493	36·0
" "	"	231·18	1·052	20	1·3348	1·3392	1·3503	37·8
" Formiate	$ThCHO_2$	39·85	1·283	22	1·3572	1·3624	1·3752	32·88
" Acetate	$ThC_2H_3O_2$	73·24	1·154	10	1·3463	1·3508	1·3634	40·45
Ammonium Chloride	NH_4Cl	13	1·054	14·5	1·3647	1·3695	1·3828	22·44
" "	"	13	1·055	12·5	1·3645	1·3694	1·3825	22·23
" Bromide	NH_4Br	13	1·194	9	1·3797	1·3855	1·4010	28·53
" Iodide	NH_4I	13	1·313	21·5	1·4012	1·4081	1·4292	38·90
" Nitrate	NH_4NO_3	13	1·110	12	1·3623	1·3672	1·3807	25·44
" Hydrate	NH_4HO	12·5	0·971	18	1·3324	1·3366	1·3475	15·17
" "	"	6	0·952	14	1·3409	1·3452	1·3570	15·65
" "	"	1·23	0·894	15	1·3475	1·3519	1·3647	15·44

II. Aqueous Solutions (continued).

Substance.	Formula.	Equivalents of water.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Ammonium Acetate	$NH_4C_2H_3O_2$	7	1·079	14·5	1·3883	1·3929	1·4670	31·57
" Chromate	$(NH_4)_2CrO_4$	60	1·076	14·5	1·3613	1·3672	......	58·12
" Bichromate	$(NH_4)_2Cr_2O_7$	65·38	1·107	15	1·3676	1·3740	......	87·06
" Sulphate	$(NH_4)_2SO_4$	26	1·129	11·5	1·3645	1·3687	1·3801	39·63
" "	"	26·03	1·129	12·5	1·3637	1·3682	1·3795	39·19
" "	"	30	1·114	13	1·3600	1·3646	1·3761	39·26
" "	"	46	1·082	12·5	1·3510	1·3555	1·3668	38·81
" Oxalate	$(NH_4)_2C_2O_4$	75·22	1·019	26·5	1·3364	1·3408	1·3517	42·22
Hydrochloric Acid	HCl	5	1·143	15	1·3991	1·4053	1·4215	14·53
" "	"	5	1·144	11·5	1·3954	1·4010	1·4168	14·08
" "	"	6	1·127	14	1·3924	1·3983	1·4136	14·75
" "	"	4·16	1·163	22	1·4031	1·4096	1·4268	13·96
" "	"	4·19	1·165	17·5	1·4035	1·4094	1·4270	13·93
" "	"	4·5	1·156	17	1·4007	1·4072	1·4239	14·07
Hydrobromic "	HBr	13	1·217	11·5	1·3774	1·3830	1·3991	20·65
Hydriodic "	HI	13	1·340	23	1·3996	1·4067	......	31·17
Nitric "	HNO_3	0·28	1·521	13	1·4076	1·4147	......	16·60
" "	"	0·9	1·460	12	1·4003	1·4067	1·4252	16·33
" "	"	2·69	1·376	13	1·3975	1·4040	1·4220	16·59
Formic "	$HCHO_2$	3·47	1·103	21·5	1·3499	1·3542	1·3662	13·85
" "	"	26·22	1·022	22	1·3342	1·3385	1·3495	14·00
" "	"	26·68	1·023	19	1·3338	1·3381	1·3491	13·57
Acetic "	$HC_2H_3O_2$	7·44	1·042	19·5	1·3513	1·3557	1·3671	21·29
Sulphuric "	H_2SO_4	0·42	1·843	12	1·4291	1·4340	1·4466	22·09
" "	"	1	1·767	13	1·4332	1·4384	1·4513	22·50
" "	"	1	1·781	14·5	1·4344	1·4396	1·4521	22·37
" "	"	2	1·659	9	1·4230	1·4285	1·4410	22·32
" "	"	3	1·557	14	1·4117	1·4150	1·4301	22·41
" "	"	5	1·430	9	1·3949	1·4003	1·4127	22·28
" "	"	11	1·256	9	1·3714	1·3765	1·3881	22·33
" "	"	25	1·118	9	1·3499	1·3544	1·3657	23·33
" "	"	25	1·127	13	1·3514	1·3559	1·3672	22·70
Tartaric "	$H_2C_4H_4O_6$	12·86	1·210	23	1·3851	1·3899	1·4022	45·17
" "	"	12·86	1·213	19·5	1·3869	1·3913	1·4039	45·42
Racemic "	"	40	1·077	24·5	1·3498	1·3544	1·3656	45·51
Oxalic "	$H_2C_2O_4$	86·94	1·034	18	1·3368	1·3413	1·3520	23·44
Citric "	$H_3C_6H_5O_7$	22	1·161	23	1·3777	1·3820	1·3944	60·89
Cacodylic "	$AsC_2H_7O_2$	55·39	1·059	14·5	1·3438	1·3482	1·3596	40·09
Barium Chloride	$BaCl_2$	60	1·166	13	1·3557	1·3601	1·3722	37·32
" Bromide	$BaBr_2$	26	1·503	8·5	1·4024	1·4080	1·4242	50·72
" Iodide	BaI_2	56	1·321	22	1·3784	1·3842	1·4008	69·72
" Nitrate	$Ba(NO_3)_2$	200	1·055	22	1·3356	1·3398	1·3508	43·26
" Formiate	$Ba(CHO_2)_2$	41·34	1·225	24·5	1·3593	1·3637	1·3759	39·82
" Acetate	$Ba(C_2H_3O_2)_2$	26	1·314	14·5	1·3826	1·3872	1·4006	56·44
" Lactate	$BaC_6H_{10}O_6$	84·5	1·137	8	1·3592	1·3635	1·3751	78·98
Strontium Chloride	$SrCl_2$	22·90	1·299	16	1·3883	1·3931	1·4074	34·88
" "	"	28	1·255	16	1·3801	1·3847	1·3987	34·78
" "	"	32	1·224	10·5	1·3731	1·3782	1·3912	34·28
" "	"	36	1·199	16	1·3698	1·3742	1·3869	35·42
" "	"	40	1·180	16	1·3660	1·3703	1·3829	35·46
" "	"	44	1·167	11·5	1·3628	1·3670	1·3797	34·80
" "	"	48	1·152	16	1·3606	1·3647	1·3772	35·70
" Bromide	$SrBr_2$	26	1·399	9	1·3941	1·3997	1·4158	47·52
" Nitrate	$Sr(NO_3)_2$	60	1·147	12	1·3527	1·3574	1·3698	41·68
" Formiate	$Sr(CHO_2)_2$	78·96	1·089	21·5	1·3454	1·3497	1·3606	39·04
" Lactate	$SrC_6H_{10}O_6$	320·66	1·028	8	1·3370	1·3410	1·3524	78·2
Calcium Chloride	$CaCl_2$	26	1·172	13·5	1·3774	1·3824	1·3960	32·28
" Bromide	$CaBr_2$	26	1·306	25	1·3874	1·3926	1·4087	44·32
" Iodide	CaI_2	52	1·241	23	1·3747	1·3805	1·3970	64·66
" Nitrate	$Ca(NO_3)_2$	26	1·226	12	1·3740	1·3788	1·3931	38·66
" Formiate	$Ca(CHO_2)_2$	52·44	1·090	21 5	1·3499	1·3543	1·3658	33·94
" Acetate	$Ca(C_2H_3O_2)_2$	32	1·131	12	1·3689	1·3736	1·3860	49·80
" Hypophosphite	$Ca(PH_2O_2)_2$	64	1·089	12 5	1·3510	1·3549	1·3666	46·88
" Hyposulphite	CaS_2O_3	31	1·210	12	1·3855	1·3908	1·4047	42·52
Magnesium Chloride	$MgCl_2$	26	1·156	14·5	1·3757	1·3805	1·3943	28·88
" Nitrate	$Mg(NO_3)_2$	26	1·194	8 5	1·3683	1·3736	1·3872	35·92
" Formiate	$Mg(CHO_2)_2$	63·10	1·062	24·5	1·3439	1·3482	1·3595	30·80
" Acetate	$Mg(C_2H_3O_2)_2$	20	1·170	13·5	1·3835	1·3881	1·4010	46·02
" Sulphate	$MgSO_4$	31·18	1·187	15·5	1·3643	1·3681	1·3805	24·18

II. Aqueous Solutions (continued).

Substance.	Formula.	Equivalents of water.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Magnesium Hyposulphite	$Mg S_2 O_3$	26	1·225	13	1·3906	1·3958	1·4099	38·52
Cerium Chloride	$Ce Cl_2$	75·68	1·110	12·5	1·3519	1·3566	1·3686	35·08
Didymium ,,	$Di Cl_2$	126·23	1·061	15	1·3407	1·3450	1·3566	34·18
Zinc Chloride	$Zn Cl_2$	8·84	1·519	12	1·4271	1·4331	1·4502	30·58
,, ,,	,,	10·68	1·447	9	1·4147	1·4204	1·4365	30·81
,, ,,	,,	13·88	1·360	14·5	1·3983	1·4038	1·4191	30·74
,, ,,	,, +0·054 $Zn O$	7·96	1·563	12	1·4338	1·4402	1·4578	31·51
,, Bromide	$Zn Br_2$	26	1·363	9	1·3895	1·3954	1·4113	43·96
,, Iodide	$Zn I_2$	21·44	1·564	20	1·4276	1·4352	......	65·67
,, Nitrate	$Zn (NO_3)_2$	26	1·297	12·5	1·3803	1·3855	1·4003	38·52
,, Formiate	$Zn (C H O_2)_2$	152·28	1·037	22	1·3357	1·3401	1·3510	34·8
,, ,,	,,	193·32	1·032	22	1·3352	1·3395	1·3506	35·1
,, Acetate	$Zn (C_2 H_3 O_2)_2$	34	1·163	13	1·3662	1·3708	1·3835	48·80
,, Cyanide	$Zn (C N)_2$ + 1·58 $K C N$	141·72	1·044	23·5	1 3366	1·3408	1·3512	53·4
,, Sulphate	$Zn S O_4$	104	1·082	12·5	1·3426	1·3474	1·3581	27·61
Cadmium Chloride	$Cd Cl_2$	26	1·304	12	1·3796	1·3849	1·3991	35·32
,, Bromide	$Cd Br_2$	26	1·438	24	1·3910	1·3968	1·4133	47·88
,, Nitrate	$Cd (N O_3)_2$	26	1·369	12·5	1·3801	1·3853	1·3999	41·38
,, Sulphate	$Cd S O_4$	28·56	1·346	25·5	1·3721	1·3767	1·3891	30·34
Copper Chloride	$Cu Cl_2$	12·466	1·432	21	......	1·4354	......	D 34·30
,, ,,	,,	20	1·290	23	......	1·4012	......	D 33·78
,, ,,	,,	40	1·155	23	......	1·3698	......	D 33·82
,, ,,	,,	60	1·105	23	......	1·3587	1·3704	D 34·32
,, Nitrate	$Cu (N O_3)_2$	17·96	1·392	17	......	1·4044	1·4211	D 40·64
,, Sulphate	$Cu S O_4$	59·78	1·149	13	1·3554	1 3600	1·3713	27·80
Ferrous Chloride	$Fe Cl_2$	28·58	1·195	24	1·3767	1·3815	1·3949	32·86
,, Bromide	$Fe Br_2$	36·58	1·249	25·5	1·3758	1·3812	1·3959	46·26
,, Sulphate	$Fe SO_4$	37	1·212	21·5	1·3672	1·3712	1·3837	28·79
,, ,,	,,	40·8	1·186	20·5	1·3630	1·3671	1·3792	29·39
Nickel Chloride	$Ni Cl_2$ + 0·09 $H Cl$	23·22	1·273	24	......	1·3987	......	D 33·26
,, Nitrate	$Ni (N O_3)_2$	26	1·298	15·5	......	1·3890	......	D 39·14
,, Sulphate	$Ni SO_4$	30	1·264	13·5	1·3747	1·3796	......	28·16
Cobalt Chloride	$Co Cl_2$	24	1·254	15	1·3889	1·3945	1·4086	32·02
,, Nitrate	$Co (N O_3)_2$	30	1·259	15·5	1·3767	1·3818	1·3957	38·48
,, Acetate	$Co (C_2 H_3 O_2)_2$	44	1·124	15	1·3607	1·3659	1·3772	50·18
,, ,,	,,	38·42	1·133	20	1·3623	1·3666	1·3790	49·94
,, Sulphate	$Co SO_4$	32	1·238	15·5	1·3704	1·3748	1·3867	29·01
Manganese Chloride	$Mn Cl_2$	26	1·194	13	1·3774	1·3824	1·3964	33·58
,, Bromide	$Mn Br_2$	26	1·323	12·5	1·3899	1·3954	1·4117	47·24
,, Iodide	$Mn I_2$	159·14	1·090	20	1·3463	1·3510	1 3642	65·14
,, Nitrate	$Mn (N O_3)_2$	26	1·251	12·5	1·3757	1·3807	1·3952	40·22
,, Acetate	$Mn (C_2 H_3 O_2)_2$	32	1·153	13	1·3710	1·3763	1·3893	51·34
,, Hypophosphite	$Mn (P H_2 O_2)_2$	54	1·104	13	1·3512	1·3568	1·3674	48·10
,, Sulphate	$Mn S O_4$	23·68	1·297	21·5	1·3784	1·3828	1·3949	28·07
,, ,,	,,	26	1·277	15·5	1·3748	1·3794	1·3915	27·60
,, Lactate	$Mn C_6 H_{10} O_6$	124·2	1·054	8	1·3459	1 3502	1·3618	73·8
Lead Nitrate	$Pb (N O_3)_2$	50	1·297	11	1·3676	1·3731	1·3876	53·56
,, Acetate	$Pb (C_2 H_3 O_2)_2$	64	1·192	15	1·3576	1·3625	1·3755	64·52
Mercuric Chloride	$Hg Cl_2$	241·4	1·049	23	1·3342	1·3385	1·3492	39·8
,, ,,	,,	274·52	1·044	14·5	1·3347	1·3389	1·3499	42·6
,, ,,	,,	279·04	1·043	18	1·3341	1·3383	1·3492	40·54
,, ,,	,, +0·96 $Na Cl$	25·88	1·491	11·5	1·3924	1 3987	1·4170	55·34
,, ,,	,, +2·15 $Na Cl$. 0·16 $Na_2 S O_4$	57·64	1·285	13	1·3704	1·3765	1·3912	80·14
,, Nitrate	$Hg (N O_3)_2$ + 1·14 $H N O_3$	25·06	1·556	12	1·3972	1 4033	1·4207	67·60
,, Cyanide	$Hg (C N)_2$	146	1·071	25·5	1·3342	1·3385	1·3495	33·60
,, ,,	,,	158·88	1 067	18	1·3349	1·3390	1·3703	34·74
Palladium Chloride	$Pd Cl_2$ + 1·18 $H Cl$	28·10	1·267	11·5	1·3962	......	......	60·66
Aluminium Chloride	$Al Cl_3$ + 0·182 $Al_4 O_3$	41·82	1·165	9	1·3771	1 3820	1·3958	45·23
,, Sulphate	$Al_2 (S O_4)_3$	104	1·165	25	1·3584	1·3627	1·3739	67·7
Ferric Chloride	$Fe Cl_3$	13	1·177	22	1·3827	1·3888	......	52·16
,, ,,	,,	22·44	1·273	24·5	1·4141	1·4221	......	51·27
,, Nitrate	$Fe (N O_3)_3$ + 2·5 $H N O_3$	59·5	1·212	24·5	1·3760	1 3814	......	103·45
,, Sulphate	$Fe_2 (S O_4)_3$ + 1·9 $H_2 S O_4$	141·6	1·168	24	1·3624	1·3673	1·3814?	133·5
Chromium Chloride	$Cr Cl_3$ + 0·56 $Cr_2 O_3$	67·64	1·165	13	1·3769	1·3826?	......	71·90
,, Sulphate	$Cr_2 (S O_4)_3$ − 1·41 $H_2 S O_4$	70·58	1·231	18	1·3790	......	......	50 81
Gold Chloride	$Au Cl_3$ + 0·868 $H Cl$	24·33	1·483	11·5	1·4079	1·4160	......	68·45
Rhodium ,,	$Rh Cl_3$ + 3 $Na Cl$	91·97	1·172	11·5	1·3717	1·3771	......	102·48
Platinum ,,	$Pt Cl_4$ + 0·94 $H Cl$	20·30	1·665	11	1·4610	1·4709	......	84·34

III. Alcoholic Solutions.

Substance.	Formula.	Equivalents of alcohol.	Specific gravity.	Temperature Centigrade.	μA.	μD.	μH.	Refraction-equivalent.
Potassium Alcoholate	$K C_2 H_5 O$	5·28	0·915	12·5	1·3871	1·3916	1·4046	28·22
,, ,,	,,	5·355	0·911	20·5	1·3852	1·3896	1·4026	28·03
,, Iodide	$K I$	217·8	0·799	19·5	1·3591	1·3632	1·3743	35·1
,, Sulphocyanide	$K S C N$	46·9	0·813	19	1·3649	1·3693	1·3812	33·7
Sodium Alcoholate	$Na C_2 H_5 O$	5·88	0·875	24	1·3807	1·3856	1·3985	24·61
Copper Chloride	$Cu Cl_2$	11·90	0·961	12	……	1·4026	……	D 34·72
Cobalt ,,	$Co Cl_2$	24·34	0·873	17	1·3769	1·3814	1·3937	31·72
,, ,,	,,	35·04	0·851	12	1·3733	……	1·3883	33·06
Mercury ,,	$Hg Cl_2$	16·2	1·043	19·5	1·3826	1·3872	1·4006	35·10
,, ,,	,,	20·24	0·995	12	1·3795	1·3839	1·3970	36·08
Ammonia	$N H_3$	5·0	0·780	20	1·3576	1·3618	1·3736	8·97

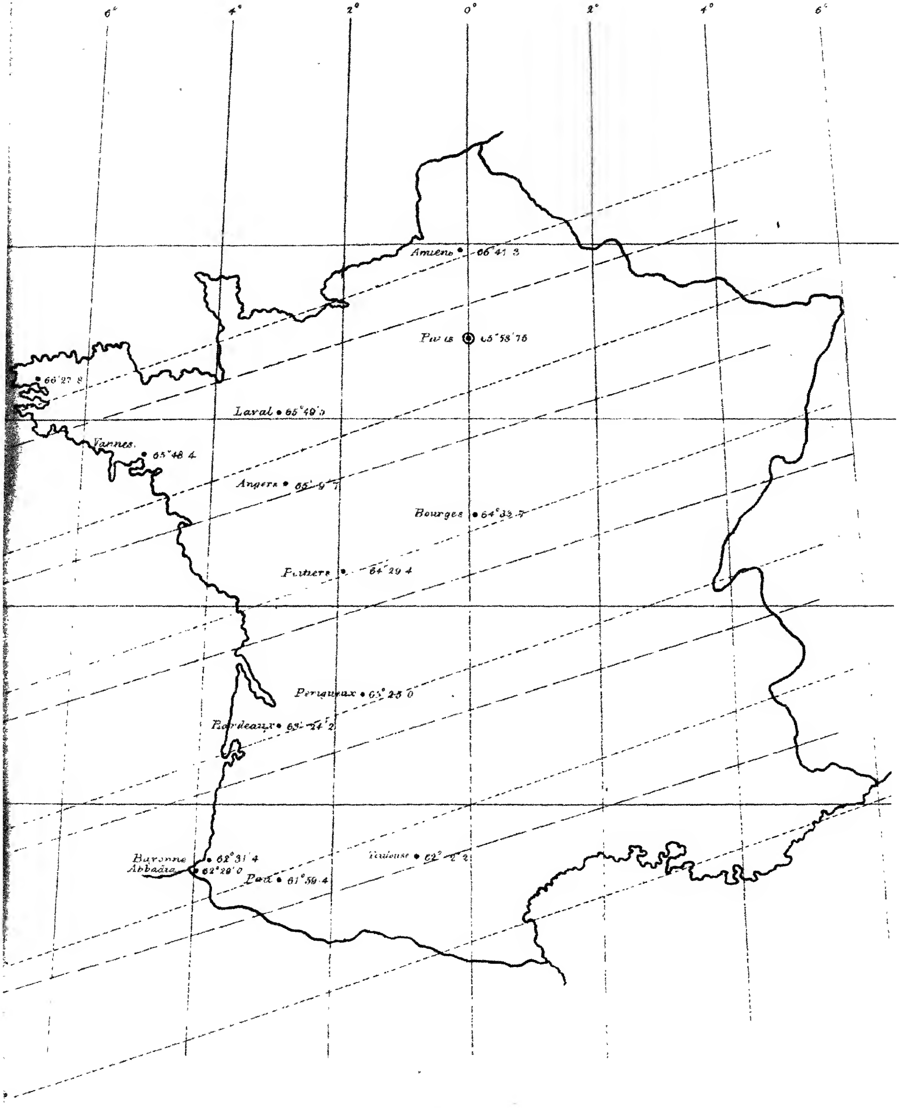
I am indebted to friends for the use of some of the above compounds. Professor FRANKLAND kindly lent me the mercuric methide and the ketones; Professor WILLIAMSON the titanium tetrachloride, chromium oxychloride, and ethylic phosphite; Professor ROSCOE the vanadium oxychloride; and Mr. PERKIN the cacodylic acid.

POSTSCRIPT, 21st March, 1870.

Since writing the statement about the aromatic compounds under the head of carbon, I have satisfied myself that their exceptionally high refractive power cannot be explained by the higher equivalent of hydrogen.

More recent experiments have confirmed the numbers provisionally assigned to antimony and gold, and led to some augmentation in those for didymium and zirconium.

Magnetic Dip


6°
4°
2°
0°
2°
4°
6°
Amiens • 66°41 3
Paris ◉ 65°58 76
• 66°27 8
Laval • 65°49 5
Vannes. • 65°48 4
Angers • 65° 9 7
Bourges • 64°33 7
Poitiers • 64°29 4
Periqueux • 63°25 0
Bordeaux • 63°24 2
Bayonne • 62°31 4
Abbadia • 62°28 0
Pau • 61°59 4
Toulouse • 62° 2 2

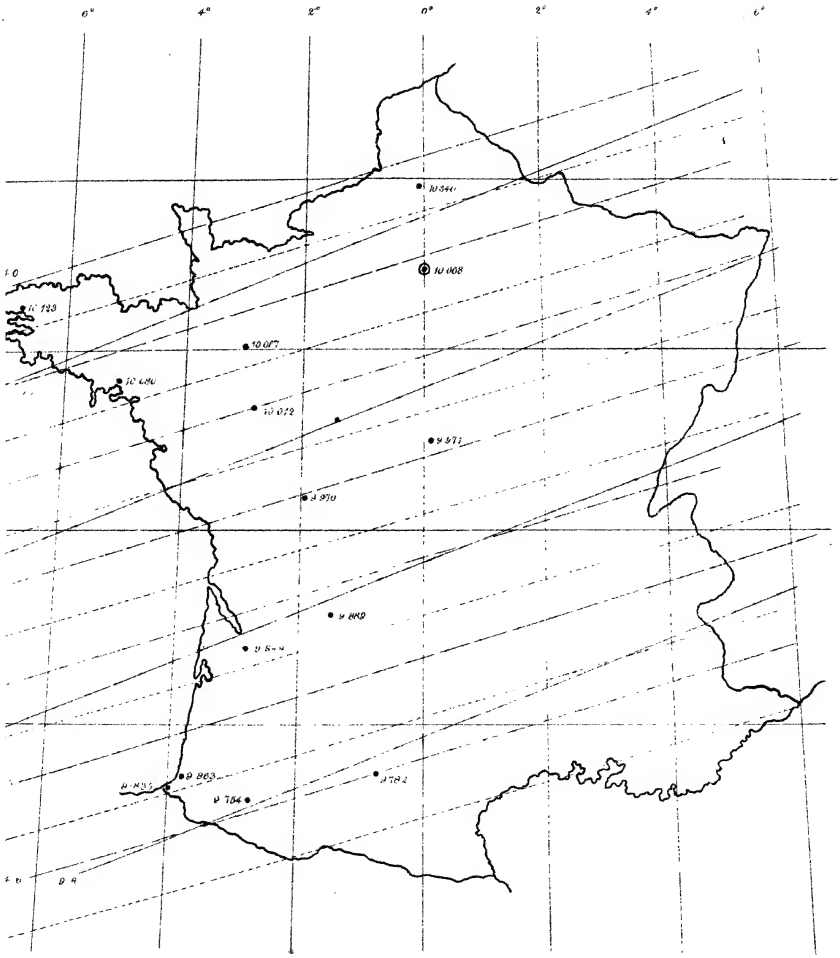
Isoclinals, Epoch 1858 . . . .

1868 — — — — —

eeley lith.

W We

Magnetic Intensity


6°
4°
2°
0°
2°
4°
6°
10 340
10 068
10 123
10 077
10 080
10 072
9 971
9 970
9 889
9 848
9 863
9 835
9 754
9 782
10
9 9

Isodynamics, Epoch 1868 ————————

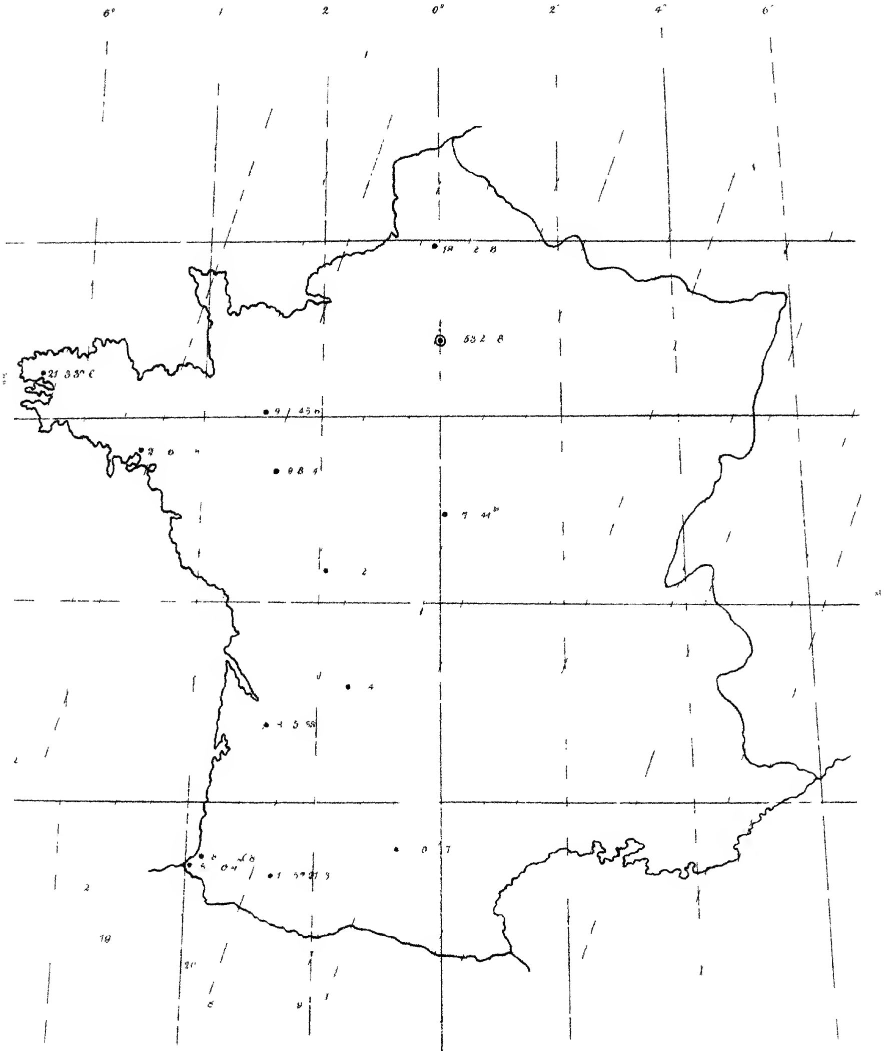
Lines of equal Horizontal Force, " *1858* - - - - - - - -

" " " " *1868* - · - · - · - ·

Wesley, lith.

W. W. [illegible]

Magnetic Declination



Isogonics Epoch 1858 ....

1868 — — —

III. *Magnetic Survey of the West of France, 1868. By the Rev.* STEPHEN J. PERRY, *S.J., F.R.A.S., F.M.S., &c. Communicated by the President.*

Received June 3,—Read June 17, 1869.

THE Magnetic Survey of the West of France, of which the following is a report, was undertaken by the Rev. W. SIDGREAVES and myself at the request of the authorities of Stonyhurst College, who generously undertook to defray all the expenses of the expedition. A similar survey of the East of France will be made during the coming autumn, so as to complete the series of observations of the magnetic elements for the whole of France.

The instruments employed were those which have been in constant use at Stonyhurst Observatory for the determination of the monthly absolute values of the Dip, Declination, and Intensity. They consisted of a dip-circle by BARROW, No. 32, a unifilar by JONES, and FRODSHAM'S marine chronometer, No. 3148. A beautiful transit-theodolite and an aneroid barometer were kindly placed at our disposal by the late Mr. COOKE.

The dip-circle was provided with three needles of the ordinary construction, the length of each being 3·54 inches, and the maximum breadth of Nos. 1 & 2 0·21 inch, whilst that of No. 3 was 0·32 inch. For the unifilar there were five magnets—two for declination observations, a third, No. 7, for vibrating and deflecting, and the remaining two, Nos. 9 & 10, for suspension in the deflection experiments. The same declination magnet was used throughout the whole series of observations, and the only deflected magnet employed was No. 9.

The reduced observations are given at considerable length in the following Tables, in order that the accuracy of the conclusions may be more reliable, and the results be more easily compared with those of past and future observers.

The moment of inertia of the deflecting magnet, No. 7, with its stirrup, for different degrees of temperature, and the coefficients in the corrections required for the effects of temperature and of terrestrial magnetic induction on the magnetic moment of the magnet, were determined at the Kew Observatory by the late Mr. WELSH.

The moment of inertia of the magnet, with its stirrup, using the grain and foot as the units of mass and of linear measure, is 5·27303. Its rate of increase for increase of temperature is 0·00073 for every 10° of FAHR.

The weight of the magnet, with its stirrup, is approximately 825 grains, and the length of the magnet is nearly 3·94 inches. The moment of inertia was determined, independently of the weight and dimensions, by the method of vibration, with and without a known increase of the moment of inertia.

The observations will, for clearness' sake, be arranged under the successive heads of Dip, Intensity, and Declination.

The Magnetic Dip.

The method of observing differed in nothing from that usually employed. The magnetic prime vertical was first found by four readings of the needle when vertical, and the circle then set so that the needle might oscillate in the magnetic meridian. Each end of the needle was read with the face of the circle first East and then West; next, the axis of the needle was changed on the agate planes, and the same observations again taken; and lastly, the whole was repeated after reversing the poles of the needle. Each pair of readings was repeated after lifting the axis of the needle on its Ys, and if a difference of 2′ was found, fresh sets were observed.

The results are contained in the following Table (I.).

Station.	Date.	G.M.T.	Number of readings.			Dip.			Mean.	Place of observation.
			1.	2.	3.	No. 1.	No. 2.	No. 3.		
Paris	1868. Aug. 12	h m 11 26 9 43	36	...	34	65° 50′·2	° ′	65° 55′·2	65° 52′·7	Garden of Coll. Vaugirard.
Laval	" 14	5 45 P.M. 10 28 A.M.	...	32	32	....	65 50·3	65 49·9	65 49·5	Maison St. Michel.
"	" 15	5 44 P.M.	32	...	...	65 48·3	....	....		
Brest	" 18	11 9 0 4 3 41 P.M.	36	34	36	66 29·5	66 23·9	66 30·1	66 27·8	3 Rue d'Aiguillon.
Vannes	" 22	2 59 P.M. 3 24 P.M. 10 49 A.M.	31	44	44	65 51·4	65 45·3	65 39·5	65 48·4	Garden of College.
Angers	" 24	6 9 P.M. 4 9 P.M.	34	44	...	65 12·0	65 8·7	....	65 9·7	8 Rue du Faubourg St. Michel.
"	" 25	7 51 A.M.	...	...	32	....	....	65 8·5		
Poitiers	" 26	3 8 4 9	34	...	34	64 33·9	....	64 26·5	64 29·4	Meadow of the College.
"	" 27	10 0 4 13	42	34	...	64 32·9	64 26·3	....		
Bordeaux ...	" 29	9 3 10 3	34	32	34	63 25·6	63 22·0	63 22·6	63 24·2	Grounds of College.
" ...	" 30	9 3	32	...	...	63 26·6	....	....		
Abbadia	Sept. 1	3 18	44	...	..	62 30·9	....	....	62 29·0	Magnetic Observatory.
"	" 2	....	34	36	...	62 32·7	62 23·3	....		
Bayonne......	" 5	5 30 P.M. 4 20	32	...	30	62 30·2	....	62 32·5	62 31·4	Lamont's Station.
Pau...........	" 7	5 28 P.M. 3 25	44	..	31	62 3·5	....	61 53·4	61 59·4	
"	" 8	....	...	40	...	....	62 1·4	....		
Toulouse ...	" 9	6 10 P.M.	34	...	...	62 4·1	....	....	62 2·2	Grounds of College.
" ...	" 10	9 13 A.M. 10 40	...	40	38	....	62 1·2	62 1·2		
Périgueux ...	" 12	11 23 5 28	36	...	40	63 27·8	....	63 22·1	63 25·0	Garden of the Grand Séminaire.
Bourges	" 14	11 3 2 58	32	...	42	64 34·6	....	64 32·7	64 33·7	Rue neuve St. Laurent.
Paris	" 16	9 28 10 38 11 43	40	34	36	65 55·3	65 56·9	65 57·0	65 54·8	Garden of Coll. Vaugirard.
"	" 17	9 3	44	...	...	65 50·1	....	....		
Amiens	" 20	10 35 4 38	38	...	16	66 41·3	....	66 36·5	66 41·3	Garden of St. Acheul.

In the above Table the G.M.T. is arranged in the same order as the observations. The exact site of each separate observation has been carefully noted, but the details are herein omitted for brevity's sake. The fullest inquiries have invariably been made as to the exact position of any underground iron pipes that might interfere with the observations.

In the following calculation of the most probable dip at each station, I have omitted the observations taken at Vannes with needle No. 3, since these were made during heavy rain and under a lofty but covered ball court. The readings of No. 3 at Amiens have also been excluded, since they were only taken as a verification of the results obtained with No. 1, and were only single readings.

The latitudes and longitudes in Table II. are taken from the 'Connaissance des Temps,' and the differences between the latitude and longitude of Paris, and those of the other stations, have been calculated in geographical miles by aid of the Table in LOOMIS's 'Practical Astronomy,' and each verified by measurement on the geological map of MM. DUFRÉNOY et ELIE DE BEAUMONT.

TABLE II.

Station.	Latitude.	Longitude.	Difference of Latitude.	Difference of Longitude.
	° ′ ″	h m s	geogr. miles.	geogr. miles.
Paris	48 50 11	0 0 0		
Laval	48 4 7	12 27 w.	− 53	+143
Brest	48 23 32	27 19	− 31	+318
Vannes	47 39 30	20 23	− 81	+235
Angers	47 28 17	11 34	− 94	+134
Poitiers	46 34 55	7 59	−156	+ 93
Bordeaux	44 50 19	11 40	−276	+138
Abbadia	43 23 7	16 21	−376	+196
Bayonne	43 29 29	15 16	−369	+183
Pau	43 17 44	10 51	−383	+130
Toulouse	43 36 33	3 35	−361	+ 43
Périgueux	45 11 4	6 28	−252	+ 76
Bourges	47 4 59	15 E.	−121	− 3
Amiens	49 53 43	8 w.	+ 73	+ 2

For the latitude and longitude of the Magnetic Observatory at Abbadia I am indebted to the kindness of M. D'ABBADIE, Membre de l'Institut, who rendered us every assistance during our stay in the South of France. The pillar on which most of the observations at this station were taken stands at about 800 yards N. of the Astronomical Observatory.

In forming the equations of condition, by which the most probable value of the dip at each station can be determined by the method of least squares, I have chosen Paris as the origin of coordinates for several reasons. The chief of these arises from the fact that Paris is practically the centre of France, and thus observations could easily be made there at the beginning and end of the Survey of 1868, and also of 1869. Add to this that frequent observations have in past times been made there, and that the able staff under the direction of M. LE VERRIER keep up a continued series of determinations of the magnetic elements. Lastly, the nature of the soil guarantees a perfect freedom from the disturbing influence of igneous rocks, &c.

Table III. contains the equations of condition formed from the data in Tables I., II.

$$\begin{aligned}
4{\cdot}825&=\delta+143x+53y\\
5{\cdot}497&=\delta+318x+31y\\
4{\cdot}807&=\delta+235x+81y\\
4{\cdot}162&=\delta+134x+94y\\
3{\cdot}490&=\delta+93x+156y\\
2{\cdot}403&=\delta+138x+276y\\
1{\cdot}483&=\delta+196x+376y\\
1{\cdot}523&=\delta+183x+369y\\
0{\cdot}990&=\delta+130x+383y\\
1{\cdot}037&=\delta+43x+361y\\
2{\cdot}417&=\delta+76x+252y\\
3{\cdot}562&=\delta-3x+121y\\
5{\cdot}688&=\delta+2x-73y
\end{aligned}$$

In these equations δ=the dip at the central station diminished by 61°; and $x=r\cos u$, $y=r\sin u$, where u is the angle which the isoclinal lines make with the meridian, and r is the increase in the angle of the dip for every change of a geographic mile in the direction normal to the isoclinal lines.

Solving these equations by the method of least squares, we obtain the following equations:—

$$\left\{\begin{aligned}
41{\cdot}884&=13\delta+1690x+2480y\\
5718{\cdot}258&=1690\delta+319466x+326949y\\
4912{\cdot}379&=2480\delta+326949x+757700y.
\end{aligned}\right.$$

These give

$$\left\{\begin{aligned}
487347{\cdot}924&=-59137\delta+239731870x\\
19552806{\cdot}880&=3699700\delta+469679480x;
\end{aligned}\right.$$

$$\therefore\ 44585338=9147115\delta,\quad \therefore\ \delta=4{\cdot}874.$$

Hence the most probable dip at Paris derived from the observations taken at the other stations is 65°·874=65° 52′·44.

By substitution in the above equations we find the values of x, y, u, and r,

$$x=0{\cdot}0032352,\quad y=-0{\cdot}0108651,\quad u=-73^\circ\,25'{\cdot}10,\quad r=0^\circ{\cdot}0113.$$

The isoclinal lines are therefore in the direction

N. 73° 25′ 10″ E. to S. 73° 25′ 10″ W.,

and the distance between the lines representing the difference of 30′ in the dip will be 44·25 geographical miles.

The substitution of these values of x, y, and δ in our original equations will give us the values of the computed dip at each station.

TABLE IV.

Station.	Computed Dip.	Observed Dip.	Obs.—Comp.	(Error)².
Paris	65° 52′·44	65° 53′·75	+1′·31	
Laval	65 46·02	65 49·5	+3·48	12·1104
Brest	66 33·96	66 27·8	−6·16	37·9456
Vannes	65 45·24	65 48·4	+3·16	9·9856
Angers	65 17·16	65 9·7	−7·46	55·6516
Poitiers	64 28·80	64 29·4	+0·60	0·3600
Bordeaux	63 19·32	63 24·2	+4·88	23·8141
Abbadia	62 25·38	62 29·0	+3·62	13·1044
Bayonne	62 27·42	62 31·4	+3·98	15·8404
Pau	62 7·98	61 59·4	−8·58	73·6164
Toulouse	62 5·46	62 2·2	−3·26	10·6276
Périgueux	63 22·92	63 25·0	+2·08	4·3264
Bourges	64 33·00	64 33·7	+0·70	0·4900
Amiens	66 40·44	66 41·3	+0·86	0·7396

$[\nu^2]=258{\cdot}6121$

The most probable error for any single observation is then given by the formula $r=g.\sqrt{\frac{[\nu^2]}{m-1}}$, where $g=0{\cdot}6745$, and $m=13$; $\therefore$ Probable error $=3'{\cdot}13$.

The disturbing influence from geological causes may be judged of by Table V., which is drawn up from the map of MM. DUFRÉNOY and ELIE DE BEAUMONT, Ingénieurs des Mines.

TABLE V.

Station.	Error.	Geological nature of the position.
Pau	8′·58	Terrain tertiaire supérieur.
Angers	7·46	Terrain de transition.
Brest	6·16	Terrain crystallisé, primitif; gneiss.
Bordeaux	4·88	Dépôts postérieurs aux dernières dislocations du sol. Terr. tert. sup.
Bayonne	3·98	,, ,, ,,
Abbadia	3·62	Terrain crétacé inférieur.
Laval	3·48	Terrain de transition.
Toulouse	3·26	Dépôts postérieurs.
Vannes	3·16	Terrain crystallisé, primitif.
Périgueux	2·08	Terrain crétacé inférieur.
Paris	1·31	Terrains tertiaires inférieurs.
Amiens	0·86	Dépôts postérieurs.
Bourges	0·70	Terrain Jurassique, système oolitique.
Poitiers	0·60	,, ,,

The error at Pau was most probably due to the fine dust that filled the air, the observations having been taken not far from a building in the course of erection. The amount of error at Angers and at Brest might also partly be attributed to the less favourable situation in which the instruments were used. Most of the stations were, however, well adapted to magnetic observations, being on sedimentary rocks or later deposits, and equally free from all disturbing influences.

In constructing a map of the isoclinal lines from the above values of u and r, it must be borne in mind that the assumptions of these lines being straight, of their parallelism, or of a uniform rate of increase of the dip, are only first approximations to the truth in so extensive a tract of country as that covered by the Survey. The curvature of the lines towards the north as we approach the west should be very considerable.

The secular variation of the dip has not been taken into account in the above calculations, since the time occupied in the survey was not long enough to cause any considerable change to take place during the interval. The epoch for all the observations may therefore be taken as September 1st, 1868. Table VI. contains a comparison between the above results, and those published by Dr. LAMONT in his 'Erdmagnetismus.'

TABLE VI.

Station.	Dip, Jan. 1, 1858.	Sept. 1, 1868.	Diff. of Epoch.	Diff. of Dip.	Yearly rate of decrease.	Dip on Jan. 1, 1869.
Paris	66° 26′·5	65° 53′·75	10·8	−32′·75	3′·03	65° 52′·5
Angers	65 55·9	65 9·7	„	−46·2	4·28	65 8·4
Poitiers	65 8·3	64 29·4	„	−38·9	3·60	64 28·1
Bordeaux ...	64 5·8	63 24·2	„	−41·6	3·85	63 23·0
Bayonne......	63 6·8	62 31·4	„	−35·4	3·28	62 30·2
Toulouse ...	62 46·1	62 2·2	„	−43·9	4·06	62 1·1
				Mean	3·68	

Supposing the same rate of decrease to hold for the remaining stations, where LAMONT did not observe this magnetic element, we have for the 1st of Jan. 1869 the following results:—

Laval	65° 48′·1	Pau	61° 58′·2
Brest	66 26·4	Périgueux	63 23·9
Vannes	65 47·1	Bourges	64 32·6
Abbadia	62 27·8	Amiens	66 40·3

The dip was observed on March 3rd, 1866, by M. D'ABBADIE at his magnetic observatory, and found to be 62° 39′·15, which gives 4′·22 for the annual decrease.

Referring, now, to former determinations of the secular decrease of the magnetic dip at Paris, we find, from a memoir by M. G. AIMÉ, "Sur le magnétisme terrestre"*, that the mean annual decrease between the years

1671 and 1754 was 1′·7
1754 and 1780 „ 1·0
1780 and 1806 „ 6·0.

From 1780 to 1830 the yearly diminution in the decrease amounted to 0′·051, as we learn from a Table in General SABINE's article in the Report of the British Association for 1838.

Comparing, now, the results of the Dip Observations taken in 1858 and 1868 with the mean of those obtained by ARAGO, HUMBOLDT, and MATHIEU from 1825 to 1830, we find the annual decrease of the dip at Paris to be 2′·82 for 1843, whilst that for 1863–4 is 3′·68, which shows that there is at present a gradual acceleration in the decrease of this element amounting to about 0′·043 per annum. Dr. LAMONT gives 2′·7 as the annual diminution for 1858, which is somewhat smaller than the amount found above.

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* Published in the Annales de Chimie et de Physique, 3^{e} série, t. xvii.

The Magnetic Intensity.

The method invariably adopted for determining the horizontal component of the earth's magnetic force was that of vibrations and deflections.

The horizontal, vertical, and total forces are calculated to English measure, one foot, one second of mean solar time, and one grain being assumed as the units of space, of time, and of mass.

The vertical and total forces are obtained from the absolute measure of the horizontal force and the dip.

The observed times of vibration are entered in the Table without correction.

The time of one vibration has been obtained from the mean of twelve determinations of the time of 100 or of 200 vibrations.

In deducing from the observed vibrations and deflections the product and ratio of the magnetic moment of the magnet and the earth's horizontal magnetic intensity, the induction and temperature corrections have always been applied, and the observed time of vibration has been corrected for the effect of torsion of the suspending thread.

The induction coefficient μ is 0·000244.

The temperature corrections have always been obtained from the formula

$$q(t_0-35^\circ)+q'(t_0-35^\circ)^2,$$

where t_0 is the observed temperature, and 35° FAHR. the adopted standard temperature. The values of the coefficients q and q' are respectively 0·0001128 and 0·000000436.

The correction for error of graduation of the deflection bar at 1 foot is +0·00004 ft., and at 1·3 foot +0·000064 ft.

It has been found necessary to apply the correction $\left(1-\frac{S}{86400}\right)$ for the rate of the chronometer at two stations only, *i. e.* at Laval and at Bordeaux, where the rate was respectively $+2^s{\cdot}68$ and $+2^s{\cdot}18$; at the other stations it was always less than 2^s.

In the calculations of the ratio $\frac{m}{X}$, the third and subsequent terms of the series $1+\frac{P}{r^2}+\frac{Q}{r^4}+\ldots$ have always been omitted.

The value of the constant P was found to be −0·002797.

The angular measure of one division of the scale in the vibration-apparatus was found to be $=2'{\cdot}26$.

The value of π^2K at 90° is 1·71636; this was deduced by Mr. WELSH of Kew, from observations made with three inertia cylinders.

No correction has been applied for semiarcs of vibration, which were always small.

TABLE VII.

Station.	Date 1868.	G.M.T.	Distance of centres of magnets.	Temp.	Observed deflection.	Log $\frac{m}{X}$.	Date.	G.M.T.	Temp.	Time of one vibration.	Log mX.
		h m		°	° ′ ″			h m	°		
Paris	Aug. 11	10 48	1·0	77·8	13 30 22	9·07147	Aug. 10	6 9 P.M.	85·8	5·11042	0·30100
		11 19	1·0	80·5	13 29 43	9·07134	11	9 23	78·3	5·11138	0·30029
		11 57	1·3	79·2	6 6 27	9·07135		9 31	......	5·11294	0·30003
										5·11173	0·30023
Laval	„ 15	11 47	1·0	72·5	13 28 40	9·07016	„ 15	8 5	69·0	5·10733	0·30037
		3 21	1·0	77·9	13 27 26	9·06992	......	9 13	......	5·10745	0·30035
		3 44	1·3	78·2	6 5 35	9·07025	......	......	......	5·10745	0·30035
								9 46	70·2	5·10717	0·30048
								9 55	70·8	5·10676	0·30060
									70·5	5·10694	0·30055
Brest	„ 20	7 53	1·0	61·5	13 47 33	9·07919	„ 20	6 28 A.M.	58·8	5·15046	0·29259
		8 18	1·3	61·9	6 14 28	9·07941	......	6 36	58·8	5·15042	0·29260
										5·15044	0·29259
Vannes	„ 21	10 52	1·0	66·0	13 29 47	9·07026	„ 21	3 46	68·0	5·09825	0·30202
		11 27	1·3	65·5	6 6 4	9·06985	......	3 54	66·9	5·09760	0·30206
		11 31	1·0	65·7	13 29 10	9·06994	......	......	67·9	5·09783	0·30209
							......	6 37	62·0	5·09384	0·30237
Angers	„ 24	9 18	1·0	65·2	13 13 44	9·06168	„ 24	2 24	70·2	5·05212	0·30995
		9 49	1·0	63·8	13 13 44	9·06157					
Poitiers	„ 26	5 33	1·0	66·0	12 52 31	9·05017	„ 26	6 13	64·2	5·01525	0·31585
								6 21	63·7	5·01475	0·31586
									63·9	5·01500	0·31583
Bordeaux ...	„ 29	2 29	1·0	70·2	12 28 5	9·03671	„ 29	4 8	71·0	4·94079	0·32922
		3 12	1·3	70·5	5 38 44	9·04311	„ 30	5 22	71·2	4·94679	0·32817
								6 46	70·5	4·94417	0·32856
Abbadia	Sept. 1	5 10	1·0	73·9	12 8 1	9·02540	Sept. 1	3 54	76·8	4·87851	0·34064
		5 55	1·3	73·5	5 30 9	9·02575	......	4 20	75·5	4·87675	0·34085
								4 28	75·5	4·88079	0·34013
										4·88044	0·34019
Bayonne......	„ 5	2 43	1·0	89·5	12 5 37	9·02522	„ 5	1 35	95·0	4·88179	0·34140
								1 54	95·2	4·88275	0·34124
Pau............	„ 7	11 26	1·0	81·5	12 1 14	9·02199	„ 7	10 2	79·0	4·86188	0·34379
								10 10	79·0	4·86113	0·34392
										4·86150	0·34386
Toulouse ...	„ 9	2 10	1·0	84·8	12 2 7	9·02277	„ 9	11 5	80·9	4·85392	0·34534
		2 40	1·3	85·9	5 27 1	9·02260	......	11 28	82·2	4·85368	0·34547
							„ 10	2 52	87·6	4·85375	0·34587
								3 16	87·2	4·85504	0·34560
								3 24	87·5	4·85396	0·34583
								3 28	87·5	4·85283	0·34603
Périgueux ...	„ 12	3 48	1·0	80·7	12 29 1	9·03809	„ 12	2 35	80·8	4·94071	0·32995
Bourges	......	......	......	......	......	......	„ 14	4 30	70·0	5·01896	0·31554
Paris	„ 16	5 58	1·0	66·7	13 29 38	9·07024	„ 16	2 34	72·4	5·12921	0·29683
							„ 18	8 17 A.M.	62·2	5·11971	0·29775
Amiens	„ 20	2 58	1·0	70·5	13 47 11	9·07967	„ 20	8 36 A.M.	60·2	5·18455	0·28667
		3 16	1·3	69·1	6 14 28	9·07994	......	8 40	60·4	5·18425	0·28673
								8 44	60·9	5·18392	0·28683

The values of the total force and of its components, as also the changes in the magnetic moment of the deflecting magnet, are at once deduced from the above Table.

TABLE VIII.

Station.	Horizontal Force.	Vertical Force.	Total Force.	Moment of Magnet.
Paris	4·1162	9·1926	10·0721	0·4852
Laval	4·1226	9·1840	10·0668	0·4845
Brest	4·0424	9·2807	10·1228	0·4852
Vannes	4·1310	9·1948	10·0802	0·4854
Angers	4·2088	9·0926	10·0196	0·4851
Poitiers	4·2938	8·9979	9·9701	0·4820
Bordeaux ...	4·4093	8·8064	9·8487	0·4834
Abbadia	4·5439	8·7225	9 8351	0·4820
Bayonne......	4·5504	8·7498	9·8626	0·4823
Pau............	4·5807	8·6113	9·7539	0·4819
Toulouse ...	4·5867	8·6397	9·7818	0·4833
Périgueux ...	4·4252	8·8432	9·8887	0·4831
Bourges	4·2830	9·0045	9·9713	0·4828
Paris...	4·1070	9·1869	10·0631	0·4828
Amiens	4·0129	9·3128	10·1403	0 4823

Deflections not having been observed at Bourges, it was necessary to compute the value of m, in order to eliminate it from the product $m\mathrm{X}$ obtained by the vibrations. The mean value of m during the survey was 0·48335 at epoch Sept. 1st, 1868; the mean for the remainder of 1868 was 0·48037, therefore the variation per month $=\frac{298}{2\cdot5}=119\cdot2$, ∴ the value on Sept. 14 should have been 0·48283. This agrees well with the mean value of m at Périgueux on the 12th and Paris on the 16th, *i. e.* 0·48294. Forming, now, our equations of condition from the above data, and solving them by the method of least squares, precisely in the same manner as in the case of the dip, we obtain the equations

$$12\cdot3419=13f+1690x+2480y, \text{ \&c.},$$

where $f=$ the total force diminished by 9.

From these we obtain

$$6279\cdot2889=-59137f+239731870x,$$
$$4089465\cdot7980=3699700f+469679480x;$$
$$\therefore\ 977426255=914711558f \text{ and } f=1\cdot06857.$$

This gives for the most probable computed value of the total force at Paris 10·0686, whereas the mean of the direct observations was 10·0676; difference =0·0010.

By substitution we at once get the quantities

$$x=0\cdot00028979,\ y=-0\cdot0008223;\ \therefore\ r=0\cdot000872, \text{ and } u=-70^\circ\ 34'\ 13''\cdot1.$$

Hence the direction of the isodynamic lines is

N. 70° 34′ 13″·1 E. to S. 70° 34′ 13″·1 W.,

and the total force changes 0·1 for every 115 geographical miles along the normal to these lines.

We can now draw up a Table of the computed errors in the observations.

TABLE IX.

Station.	Computed T. F.	Observed T. F.	Obs. – Comp.	(Error)².
Paris	10·0686	10·0676	–0·0010	
Laval, ...	10·0670	10·0668	–0·0002	0·00000004
Brest	10·1352	10·1228	–0·0124	0·00015376
Vannes	10·0706	10·0802	+0·0096	0·00009216
Angers	10·0301	10·0196	–0·0105	0·00011025
Poitiers	9·9672	9·9701	+0·0029	0·00000841
Bordeaux	9·8816	9·8487	–0·0329	0·00108241
Abbadia	9·8162	9·8351	+0·0189	0·00035721
Bayonne...........	9·8172	9·8626	+0·0454	0·00206116
Pau.................	9·7913	9·7539	–0·0374	0·00139876
Toulouse	9·7842	9·7818	–0·0024	0·00000576
Périgueux	9·8834	9·8887	+0·0053	0·00002809
Bourges	9·9682	9·9713	+0·0031	0·00000961
Amiens	10·1292	10·1403	+0·0111	0·00012321

$[\nu^2]=0\cdot00543083$

The most probable error of any one observation will therefore be

$$=r=0\cdot6745\sqrt{\frac{[\nu^2]}{12}}=0\cdot0144.$$

For the sake of comparison with the results of former observations, and in particular of those of Dr. LAMONT, we will determine the direction of the lines of equal intensity for the horizontal component of the earth's magnetism.

By a process identical with that employed for the Total Force, we now find that at our central station the Horizontal Force$=4\cdot1156$,

$$x=-0\cdot00037605,\ y=0\cdot0013401,\ \text{and}\ \therefore\ r=0\cdot0013919,\ \text{and}\ u=-74^\circ\,19'\,30''\cdot4\,;$$

or the direction sought is

N. 74° 19′ 30″·4 E. to S. 74° 19′ 30″·4 W.;

the probable error in any single observation being now 0·0067.

We will next place side by side the values of the Horizontal Force for 1858 and 1868.

TABLE X.

Station.	Jan. 1st, 1858.	Sept. 1st, 1868.	Difference.
Amiens	3·9639	4·0129	+0·0490
Angers	4·1450	4·2088	+0·0638
Bayonne	4·4875	4·5504	+0·0628
Bordeaux	4·3730	4·4093	+0·0363
Paris	4·0685	4·1116	+0·0431
Périgueux	4·3699	4·4252	+0·0553
Poitiers	4·2341	4·2938	+0·0597
Toulouse	4·5243	4·5867	+0·0624

Hence the yearly increase of the Horizontal Force in the West of France $=0\cdot0050$.

Dr. LAMONT gives 0·0048 as the value for 1858, therefore we may conclude that there is a slight acceleration of about 0·00002 per annum in the increase of this element.

The Magnetic Declination.

The method adopted in the determination of this element was the following.

The azimuth of a fixed horizontal mark, situated at a considerable distance, was first read with Cooke's transit theodolite, and then a transit of both limbs of the sun taken with the same instrument, the time being noted by Frodsham's chronometer. The azimuth circle having been read, the theodolite was removed, and Jones's unifilar placed on the same tripod-stand. The reading of the fixed mark and of the collimator magnet completed the observation. The torsion of the silk thread was removed entirely, as far as that was possible, before each observation.

Brest was the only station at which the above method was departed from, and there, on account of the confined space in which the observations were taken, a distant mark could not be sighted. It was therefore considered that more accurate results would be obtained by observing the sun's transit by reflection from the mirror of the unifilar, according to Dr. Lloyd's method. The chief reason for not using this method at other stations was that the line of collimation of the telescope was not perpendicular to the axis of the mirror, and the motion in azimuth of the axis was not sufficient to correct this error.

The correction to be applied to the azimuth reading of the sun on account of this position of the mirror is given by the formula $x = \frac{2m \sin^2 \frac{\alpha}{2}}{\cos \alpha}$, where the constant $2m = \frac{\cos \alpha' \cos \alpha''}{\sin \frac{\alpha' + \alpha''}{2} \sin \frac{\alpha' - \alpha''}{2}}$, α, α', α'' being the altitudes of the sun at the times of observation.

The formula $\cos a = \frac{\sin h \cos \delta}{\sin A}$ was used for determining the altitudes.

The determination of the meridian line, or of the azimuth of the sun, at each observation depends mainly for its correctness upon the knowledge of the error and rate of the chronometer employed. No pains were therefore spared to prevent all jolting of the instrument during the voyage, and no opportunity was missed of comparing it with other chronometers. The following Table is the result of these comparisons, made always with the greatest courtesy by those in charge at the several observatories and dépôts de chronomètres.

The chronometer is a large-size marine instrument by Frodsham, No. 3148; it has been in constant use at Stonyhurst Observatory since the beginning of 1863, and its daily rate is found by the observation of clock stars on every favourable night. Its mean daily rate previous to the journey was $0^{\mathrm{s}}{\cdot}57$, and afterwards it increased to $0^{\mathrm{s}}{\cdot}61$. The rate is found to vary somewhat during the year. Mr. Frodsham kindly examined the instrument before it was taken to France and immediately after its return, and declared it to be in perfect order.

TABLE XI.

Station.	Date.	G. M. T.	Error.	Daily rate.
		h m	m s	s
Stonyhurst Observatory	July 26	......	+0 59·14	+0·57
London, Frodsham	Aug. 7	......	+1 13·0	+1·8
Paris Observatory	„ 11	3 30	+1 15·63	+0·66
Brest Observatory	„ 17	2 15	+1 31·71	
	„ 18	2 15	+1 32·88	+1·17
	„ 19	2 15	+1 34·05	+1·17
Bordeaux, Dépôts des Chronomètres	„ 29	1 13	+1 29·0	
Abbadia Observatory	Sept. 2	......	+1 37·7	+2·18
Toulouse Observatory	„ 9	......	+	small
Paris Observatory	„ 17	......	+1 51·5	+0·92
London, Frodsham	„ 24	......	+0 28·0	+1·0
Stonyhurst Observatory	„ 30	......	+0 34·35	
	March 14	......	+2 15·58	+0·61

At Toulouse the chronometer was only compared as to rate by the Director of the Observatory, the absolute times of comparison not being registered.

Altitudes of the sun were taken at nearly all the stations with COOKE's small altazimuth, but these were only used as a check on the above results and for intermediate stations. These direct determinations of the rate of our instrument were exceedingly useful, from showing us that the two principal disturbances probably do not interfere in the least with the results obtained, since the first took place between Poitiers and Bordeaux, and the second between Amiens and London.

In the following Table of observations the first line of azimuths at each station was read on the theodolite circle, and the second line on the unifilar. The centre division of the magnet was always made to coincide with the centre line of the telescope, except at Vannes, where it was 0·1 div. to the apparent left. The zero of the scale of the collimator magnet was +7·2 div., each scale-division being =2′ 9″·805.

At Abbadia the azimuth of the mark A, *i.e.* the Biarritz Lighthouse, was accurately determined by M. ANTOINE D'ABBADIE, Président de la Société Météorologique de France.

TABLE XII.

Station.	Date.	Chronometer.	Error at Noon.	Daily Rate.	Azimuth of Sun.	Azimuth of Mark.	Azimuth of Magnet.
		h m s	m s	s			
Paris	Aug. 12	5 25 59·375	+1 16·19	+0·66	261° 5′ 10″	187° 45′ 20″	
		6 15 0	....	....	....	240 37 25	201° 40′ 25″
					,,	108 45 20	
						161 37 55	,,
Laval	,, 14	2 26 55·75	+1 23·67	+2·68	110 12 12·5	144 12 10	
		3 10	....	....	....	257 23 15	152 4 45
Brest	,, 20	10 3 17·75	+1 35·22	+1·17	252 6 12·5		
		10 26	....	....	....	....	100 57 40
Vannes	,, 21	4 59 49·5	+1 36·39	+1·17	107 46 55	142 21 25	
		5 45	....	....	....	106 3 50	147 12 0
	,, 22	5 34 24·08	+1 37·56	+1·17	215 55 13·3	244 26 30	
		6 38	....	....	....	101 21 10	142 20 0
					,,	244 31 20	
						101 26 50	,,
Angers	,, 24	5 8 21·5	+1 39·90	+1·17	254 37 40	252 55 45	
		5 52	....	....	....	127 49 15	203 16 25
					,,	198 30 25	
						253 24 5	,,
Poitiers	,, 26	10 10 5·08	+1 42·24	+1·17	155 27 18·3	115 23 10	
		11 11	....	....	....	202 38 25	266 16 45
Bordeaux ...	,, 29	5 35 47·56	+1 29·0	+2·18	228 17 43·8	139 51 30	
						92 12 40	250 38 30
Abbadia	Sept. 1	3 5	+1 35·52	+0·92	....	217 21 15	146 5 40
Bayonne......	,, 5	11 18 16·13	+1 40·46	+0·92	223 25 15	135 55 55	
		12 50	....	....	....	127 15 15	215 23 45
Pau......... ...	,, 7	8 24 41·88	+1 42·30	+0·92	246 29 21·7	193 19 20	
		8 48	....	....	....	93 47 35	196 50 50
Toulouse ...	,, 9	9 37 11·18	+1 44·14	+0·92	264 15 45	176 7 25	
		10 12	....	....	....	194 9 0	132 39 35
					,,	220 37 25	
						238 38 20	,,
Périgueux ...	,, 12	9 9 42·18	+1 46·90	+0 92	127 41 58·3	250 53 57·5	
		10 4	....	....	....	206 46 45	119 21 25
Bourges	,, 14	8 49 22·38	+1 48·74	+0·92	261 31 15	118 15 45	
		9 45	....	....	....	162 43 15	164 7 35
Paris	,, 16	3 30 32·76	+1 50·58	+0·92	139 23 3·3	165 54 15	
		4 35 30	....	....	....	151 51 20	222 36 40
Amiens . ..	,, 20	1 32 38·75	+1 54·26	+0·92	118 42 5	234 19 0	
		2 12	....	....	....	100 14 55	112 19 35

The azimuth of the sun at the time of observation, calculated by the formula $\cot A = \cot h \dfrac{\cos(mP+\lambda)}{\sin mP}$, where $mP = \tan^{-1}(\cos h \cot \delta)$, gives the results in Table XIII.

TABLE XIII.

Station.	Azimuth of the Sun.	Declination corrected for scale-reading.
Paris	94° 13′ 13″·9	17° 48′ 1″·5
,, ...	....	17 48 31·5
Laval	51 55 12·3	19 7 45·6
Brest	50 39 10·9	21 3 32·6
Vannes	84 6 11·4	19 55 47·0
,, ...	89 36 59·1	20 37 19·6
,, ...	....	20 38 9·6
Angers	86 50 31·4	19 8 39·0
,, ...	....	19 8 49·0
Poitiers	42 11 13·4	18 21 27·1
Bordeaux	91 29 15·4	18 15 33·8
Abbadia	....	18 17 6·4
Bayonne........................	19 21 5·4	18 26 20·8
Pau................	68 1 9·2	17 52 21·3
Toulouse	47 48 10·3	17 10 0·7
Périgueux	48 36 28·1	17 43 34·3
Bourges	55 27 8·7	17 2 44 1
Paris	64 29 14·6	17 58 39·1
Amiens	33 41 22·6	18 21 27·8

An application of the method of least squares, similar in all details to that made use of above, furnishes the equations

$$20·125=13D+1690x+2480y, \text{ \&c.}$$

Hence

$$D=0·92921, \quad x=0·01056199, \quad y=-0·0039534;$$

$\therefore r=0°·011278$ per geographical mile, *i.e.* the rate of change in the declination along the normal to the isogonic lines is 1° for every 88·7 geographical miles, and $u=-20° 31' 16''$, or the direction of the isogonic lines is

N. 20° 31′ 16″ E. to S. 20° 31′ 16″ W.

We might naturally expect that observations of this magnetic element would lead to much less satisfactory results than those of the Dip or Total Force; for the almost continuous, and often energetic, action of disturbing forces render any limited number of absolute readings of the declination at different stations a very uncertain guide in calculating the exact direction taken by the isogonic lines; whereas in the case of the vertical and horizontal components of the total force the daily disturbances are of a much less exaggerated character, and the maximum perturbations of not frequent occurrence. The subjoined Table will show the probable amount of error at each station.

TABLE XIV.

Station.	Computed Declination.	Observed Declination.	Diff.
Paris	17°·929	17°·891	− 2′·28
Laval	19·251	19·129	− 7·32
Brest	21·165	21·059	− 6·36
Vannes	20·091	20·279	+11·28
Angers	18·973	19·146	+10·38
Poitiers	18·295	18·358	+ 3·78
Bordeaux	18·296	18·259	− 2·22
Abbadia	18·513	18·285	−13·68
Bayonne	18·403	18·439	+ 2·16
Pau	17·788	17·873	+ 5·10
Toulouse	17·044	17·167	+12·66
Périgueux	17·736	17·726	− 0·60
Bourges	17·419	17·047	−22·32
Amiens	18·239	18·358	+ 7·14

ıe probable error of any single observation will therefore be 6′·9549. We are now able to determine the secular variation of the declination.

TABLE XV.

Station.	Epoch January 1st. 1858.	September 1st. 1868.	Variation.
Amiens	19° 56′ 18″	18° 21′ 27″·8	−1° 34′ 50″·2
Angers	20 16 18	19 8 44·0	−1 7 34·0
Bayonne	19 57 48	18 26 20·8	−1 31 27·2
Bordeaux	20 0 12	18 15 33·8	−1 44 38·2
Paris	19 36 18	17 53 27·8	−1 42 50·2
Périgueux	19 26 30	17 43 34·3	−1 42 55·7
Poitiers	19 56 24	18 21 27·1	−1 34 56·9
Toulouse	18 45 0	17 10 0·7	−1 34 59·3

At Angers an iron pipe was lying N. and S. at a distance of about 11 yards from the place of observation. Neglecting, therefore, this station, we find the yearly decrease of the Declination in the West of France to be 9′ 4″·9.

M. G. Aimé, in the "Mémoire" already quoted, gives the mean annual variation of the Declination at Paris for past years.

Epoch.	Declination.	Yearly variation.
1663 to 1767	0° 0′ to 19° 16′	+11′·0
1767 to 1785	19 16 to 22 0	+ 9·0
1785 to 1805	22 0 to 22 5	+ 0·2
1805 to 1817	22 5 to 22 19	+ 1·1
1817 to 1825	22 19 to 22 22	+ 0·3

Continuing this summary a step further we have

1825 to 1858, 22° 22′ to 19° 36′, yearly diminution 5′·0,
1858·0 to 1868·7, 19° 36′ to 17° 53′, yearly diminution 9′·6,

whence it appears that the declination is rapidly on the decrease, with a mean yearly acceleration in the decrease of 0′·22.

The value of the yearly diminution, as given by Dr. Lamont, is 7′·6 for 1858, which shows a steadiness in the variation of this magnetic element.

A glance at the maps which accompany this report will show at once the changes that ten years have produced in the position of the lines of equal declination, dip, and intensity. The distance between the lines remains in all cases almost constant for the same element, and the amount is moreover identical for the isoclinal and isogonic lines, the values of r being respectively 0·678 and 0·677. The angle moved through by the lines is more considerable in the cases of the isoclinal lines than for the others, and the direction of this motion is away from the astronomical meridian in the case of the isogonics and isoclinals, and scarcely perceptibly towards the astronomical meridian for the lines of equal horizontal force.

In the maps of Lamont's 'Erdmagnetismus' the lines are curved, and the epochs for the dip, horizontal force, and declination are severally August 1848, June 1848, and March 1854; whereas I have taken January 1st, 1858, as the common epoch, and drawn the lines straight, for the sake of comparison with the lines for 1868, which are laid down without any modification from the calculated values of u and r. The dotted lines, which belong to the survey of 1858, can therefore only be looked on as first approximations to the results derived from Lamont's observations. The broken lines are those obtained by the above calculations.

The numbers marked at each station are the means of the observed values, and they serve to show the degree of approximative correctness attained by the adopted method of reduction. A comparison of these maps with those of the latest surveys of the British Islands manifests a striking resemblance between the isoclinal lines in France and England. For in the Report by General SABINE for the British Association 1861, we find that the direction of the isoclinals in England was −65° 5′ in 1837, and −71° 22′ in 1860; therefore, supposing the rate of change to remain constant, we have −73° 33′·2 as the value for 1868, −73° 25′·2 being the amount found for France in August and September of the same year. The values of r, 0·678 and 0·644, show, however, that the lines are closer packed in France than in England, and this is confirmed by the observations taken with needle No. 1 at Kew and at Stonyhurst immediately after our return from France; for if the angles be computed on the assumption of the same data holding in England as in France, we shall have:

	Observed.	Computed.	Difference.
	°	°	′
Kew	68·087	68·241	− 9·24
Stonyhurst	69·707	70·306	−35·94

The isogonics of France are nearly in the same direction as those of Scotland, as seen in Dr. STEWART's Report of Mr. WELSH's Survey; but they are nearly twice as close in Scotland as in France, the values of r being 0′·677 in 1868 for the latter country, and 1′·465 for the former in 1858.

The greatest difference is in the isodynamics, their angle for England being −58° 32′·7, and r=0·00106, whilst for France at the same epoch, 1868, the values are −70° 34′ 13″·1 and 0·00087.

The following is a complete Table of all the elements at the Epoch Jan. 1st, 1869.

TABLE XVI.

Station.	Dip.	Declination.	Horizontal Force.
Abbadia	62° 27′·8	18° 14′·10	4·5456
Amiens	66 40·3	18 18·96	4·0143
Angers	65 8·4	19 5·58	4·2106
Bayonne	62 30·2	18 23·46	4·5520
Bordeaux	63 23·0	18 12·54	4·4110
Bourges	64 32·6	17 0·18	4·2845
Brest	66 26·4	21 0·30	4·0442
Laval	65 48·1	19 4·38	4·1245
Paris	65 52·5	17 50·46	4·1133
Pau	61 58·2	17 49·50	4·5823
Périgueux	63 23·9	17 40·92	4·4268
Poitiers	64 28·1	18 18·36	4·2955
Toulouse	62 1·1	17 7·32	4·5883
Vannes	65 47·1	20 13·50	4·1328
Annual Variation	−3·68	−9·1	+0·0050
Acceleration	0·043	0·19	0·00002

In none of the previous reductions have I ventured to introduce a correction for any magnetic disturbance that might have existed at the time of observation, or for diurnal range, since there were no self-recording magnetographs in France by which these corrections could be accurately determined, and any correction founded on the supposition of the simultaneous similar action of the disturbing forces in England and in France might appear somewhat arbitrary. That a correction, however, might be applied with advantage is rendered more than probable by the results of the comparison of the Kew and Lisbon magnetograms, and by the great similarity between the daily range in England and in Italy which I remarked in some of the Florence declination-curves sent by Signor DONATI. It would seem, from the comparisons made by Dr. STEWART and Senhor CAPELLO, that the declination and horizontal-force disturbances at Kew and Lisbon are simultaneous, in the same direction, and in the proportion of 1·6 to 1 for the declination, and 1·8 to 1 for the horizontal force; whilst, on the other hand, Dr. STEWART remarks that there is "very little likeness between the vertical-force curves."

We will therefore assume that the perturbations of the declination and horizontal force are simultaneous in England and in France, and in the proportion of 1·3 to 1 and 1·4 to 1 respectively, and we will take the corrections from the magnetograms obtained at Stonyhurst Observatory during the survey, since the Kew and Stonyhurst curves may be considered as almost identical.

From hourly measurements of the undisturbed portions of our declination-curves from Dec. 16th, 1868 to Jan. 16th, 1869, we obtain 2·147 as the mean reading of the ordinate for Jan. 1st, 1869; and taking the difference between this and the ordinate at the time of each observation, we obtain a number which, when divided by 1·3 and multiplied into 28′ 38″·875, the coefficient of the declination magnetograms, gives us the correction in arc for reducing the French observations to their mean value on Jan. 1st, 1869. No correction for torsion of thread has been found necessary.

The results obtained from the corrected observations compared with the values found above, will best show what is gained by this correction.

	From uncorrected observations.	From corrected observations.
$x=$	0·010562	0·010385
$y=$	−0·003953	−0·003833
$u=$	−20° 31′ 16″	−20° 15′ 35″
$r=$	0·011278	0·01107
Error of Paris	−2′·28	1′·26
Probable error of any observation	6′·95	7·15.

These figures of themselves would scarcely justify us in giving much weight to this correction.

The horizontal-force magnetogram seems to offer a surer means of improving our results. The standard ordinate has been obtained from hourly measurements of the curves on undisturbed days in Aug. and Sept. 1868, the gradual loss of magnetism of the suspended magnet being taken into account. The coefficient of the curve is 0·02689. In this case the amount of difference between the mean observed and the computed values at each station is generally diminished by the correction; but, owing to the smallness of these differences and to the trace of the magnetograph having been lost at the time of the observations at Angers and Bordeaux, where the differences were greatest, the total change of the values is inconsiderable.

The following comparison will give an idea of the results:—

	From uncorrected observations.	From corrected observations.
$x=$	$-0{\cdot}00037605$	$-0{\cdot}00037562$
$y=$	$0{\cdot}0013401$	$0{\cdot}0013398$
$u=$	74° 19′ 30″·4	74° 20′ 19″·5
$r=$	$0{\cdot}0013919$	$0{\cdot}0013914$
Error at Paris	0·0040	0·0034
Mean probable error . . .	0·0067	0·0065

A similar correction might be applied to the Total Force and the Dip; but since the simultaneous variations of the vertical force are less understood than those of the horizontal force or the declination, the attempt could scarcely be expected to lead to any satisfactory result in the present state of our knowledge.

APPENDIX.

The observations made with the same instruments at Loyola on Sept. 3 and 4 are not taken into account in the above reductions, since we wished to confine ourselves entirely to the West of France. It may, however, be of some use to subjoin a synopsis of the results obtained.

Sept. 3rd, $3^h\ 11^m\ 9^s{\cdot}26$ G.M.T. Azimuth of sun 165° 35′ 46″·7. Azimuth of mark A 141° 53′ 40″, read with the altazimuth.

$3^h\ 43^m$ Azimuth of magnet 229° 49′ 5″. Azimuth of mark A 206° 42′ 40″, read with the unifilar.

Correction for zero-point of scale of collimator magnet −15′ 34″·6.

The declination is not deduced from want of sufficiently reliable data as to latitude and longitude.

Sept. 3rd, $6^h\ 26^m$, temp. 71°·6 Fahr. Observed deflection 12° 7′ 2″. Distance between centres of magnets 1 ft.

„ $4^h\ 54^m$, „ 77°·0 Fahr. Observed time of one vibration $4^s{\cdot}87359$.

Hence

Moment of magnet . . . . . . =0·48205
Horizontal force . . . . =4·5546
Vertical force . . . . =8·7245
Total force . . . . =9·8419

Sept. 4. No. 1 Needle. Observed dip 62° 26′.

IV. *A Memoir on Abstract Geometry.* *By Professor* CAYLEY, *F.R.S.*

Received October 14,—Read December 16, 1869.

I SUBMIT to the Society the present exposition of some of the elementary principles of an Abstract m-dimensional Geometry. The science presents itself in two ways,—as a legitimate extension of the ordinary two- and three-dimensional geometries; and as a need in these geometries and in analysis generally. In fact whenever we are concerned with quantities connected together in any manner, and which are, or are considered as variable or determinable, then the nature of the relation between the quantities is frequently rendered more intelligible by regarding them (if only two or three in number) as the coordinates of a point in a plane or in space: for more than three quantities there is, from the greater complexity of the case, the greater need of such a representation; but this can only be obtained by means of the notion of a space of the proper dimensionality; and to use such representation, we require the geometry of such space. An important instance in plane geometry has actually presented itself in the question of the determination of the number of the curves which satisfy given conditions: the conditions imply relations between the coefficients in the equation of the curve; and for the better understanding of these relations it was expedient to consider the coefficients as the coordinates of a point in a space of the proper dimensionality.

A fundamental notion in the general theory presents itself, slightly in plane geometry, but already very prominently in solid geometry; viz. we have here the difficulty as to the form of the equations of a curve in space, or (to speak more accurately) as to the expression by means of equations of the twofold relation between the coordinates of a point of such curve. The notion in question is that of a k-fold relation,—as distinguished from any system of equations (or onefold relations) serving for the expression of it, and as giving rise to the problem how to express such relation by means of a system of equations (or onefold relations). Applying to the case of solid geometry my conclusion in the general theory, it may be mentioned that I regard the twofold relation of a curve in space as being completely and precisely expressed by means of a system of equations ($P=0$, $Q=0$, … $T=0$), when no one of the functions P, Q, … T is a linear function, with constant or variable *integral* coefficients, of the others of them, and when *every surface whatever* which passes through the curve has its equation expressible in the form $U=AP+BQ\ldots+KT$, with constant or variable integral coefficients, A, B, … K. It is hardly necessary to remark that all the functions and coefficients are taken to be rational functions of the coordinates, and that the word integral has reference to the coordinates.

General Explanations; Relation, Locus, &c.—Article Nos. 1 to 36.

1. Any m quantities may be represented by means of $\overline{m+1}$ quantities as the ratios of m of these to the remaining $\overline{m+1}$th quantity, and thus in place of the absolute values of the m quantities we may consider the ratios of $\overline{m+1}$ quantities.

2. It is to be noticed that we are *throughout* concerned with the ratios of the $\overline{m+1}$ quantities, not with the absolute values; this being understood, any mention of the ratios is in general unnecessary; thus I shall speak of a relation between the $\overline{m+1}$ quantities, meaning thereby a relation as regards the ratios of the quantities; and so in other cases. It may also be noticed that in many instances a limiting or extreme case is sometimes included, sometimes not included, under a general expression; the general expression is intended to include whatever, having regard to the subject-matter and context, can be included under it.

3. Postulate. We may conceive between the $\overline{m+1}$ quantities a *relation**.

4. A relation is either *regular*, that is, it has a definite manifoldness, or, say, it is a k-fold relation; or else it is *irregular*, that is, composed of relations not all of the same manifoldness. As to the word "composed," see *post*. No. 14.

5. The ratios are determined (not in general uniquely) by means of a m-fold relation; and a relation cannot really be more than m-fold. But the notion of a more than m-fold relation has nevertheless to be considered. A relation may be, either in mere appearance or else according to a provisional conception thereof, more than m-fold, and be really m-fold or less than m-fold. Thus a relation expressed by $\overline{m+1}$ or more equations is in general and *primâ facie* more than m-fold; but if the equations are not independent, and equivalent to m or fewer equations, then the relation will be m-fold or less than m-fold. Or the given relation may depend on parameters, and so long as these are arbitrary be really more than m-fold; but the parameters may have to be, and be accordingly, so determined that the relation shall be m-fold or less than m-fold. A more than m-fold relation is said to be superdeterminate.

6. A system of any number of onefold relations, whether independent or dependent, and if more than m of them, whether compatible or incompatible, is termed a 'Plexus,' viz. if the number of onefold relations be $=\theta$, then the plexus is θ-fold. A θ-fold plexus constitutes a relation which is at most θ-fold, but which may be less than θ-fold.

7. Every relation whatever is expressible, and that *precisely*, by means of a plexus; but for the expression of a k-fold relation we may require a more than k-fold plexus.

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* The whole difficulty of the subject is (so to speak) in the analytical representation of a relation; without solving it, the theories of the text cannot be exhibited analytically with equivalent generality; and I have for this reason presented them in an abstract form without analytical expression or commentary. But it is perfectly easy to obtain analytical illustrations; a onefold relation is expressed by an equation $P=0$; and (although a k-fold relation is not in general expressible by k equations) any k independent equations $P=0$, $Q=0$, &c. constitute a k-fold relation. Thus (No. 4), an instance of an irregular relation is $MP=0$, $MQ=0$, viz. this is satisfied by the satisfaction either of the onefold relation $M=0$, or of the twofold relation $P=0$, $Q=0$. And *post*, Nos. 14 and 21, the relation composed of the two onefold relations $P=0$ and $Q=0$ is the onefold relation $PQ=0$; the relation aggregated of the same two relations is the twofold relation $P=0$, $Q=0$.

8. **Postulate.** We may conceive a m-dimensional space, the indetermination of the ratios of $\overline{m+1}$ coordinates, and *locus in quo* of the point, the unique determination of these ratios. More generally we may conceive any number of spaces, each of its own dimensionality, and existing apart by itself.

9. Conversely, any $\overline{m+1}$ quantities may be taken as the coordinates of a point in a m-dimensional space.

10. The $\overline{m+1}$ coordinates may have a k-fold relation; it appears (*ante*, No. 5) that the case $k>m$, or where the relation is more than m-fold, is not altogether excluded; but this is not now under consideration. The two limiting cases $k=0$ and $k=m$ will be presently mentioned; the remaining case is $k>0<m$; the system of points the coordinates of which satisfy such a relation constitutes a k-fold or $(m-k)$dimensional locus. And k is the manifoldness, $\overline{m-k}$ the dimensionality, of the locus.

11. If $k=m$, that is, if the ratios are determined, we have the point-system, which, if the determination be unique, is a single point. The expression "a locus" may extend to include the point-system, and therefore also the point. If $k=0$, that is, if the coordinates are not connected by any relation, we have the original m-dimensional space.

12. We may say that the m-dimensional space is the *locus in quo* not only of the points in such space, but of the locus determined by any relation whatever between the coordinates; and in like manner that any $(m-k)$dimensional locus in such space is a $(m-k)$-dimensional space, a *locus in quo* of the points thereof, and of every locus determined by a relation between the coordinates, implying the k-fold relation which corresponds to the $(m-k)$dimensional locus.

13. There is not any locus corresponding to a relation which is really more than m-fold; hence in speaking of the locus corresponding to a given relation, we either assume that the relation is not more than m-fold, or we mean the locus, *if any*, corresponding to such relation.

14. Any two or more relations may be *composed* together, and they are then factors of a single *composite* relation; viz. the composite relation is a relation satisfied if, and not satisfied unless, some one of the component relations be satisfied.

15. The foregoing notion of composition is, it will be noticed, altogether different from that which would at first suggest itself. The definition is defective as not explaining the composition of a relation any number of times with itself, or elevation thereof to power; which however must be admitted as part of the notion of composition.

16. A k-fold relation which is not satisfied by any other k-fold relation, and which is not a power, is a *prime* relation. A relation which is not prime is composite, viz. it is a relation composed of prime factors each taken once or any other number of times; in particular, it may be the power of a single prime factor. Any prime factor is single or multiple according as it occurs once or a greater number of times.

17. A relation which is either prime or else composed of prime factors each of the same manifoldness is a regular relation; a k-fold relation is *ex vi termini* regular. An irregular

relation is a composite relation, the prime factors whereof are not all of the same manifoldness.

18. A prime k-fold relation cannot be implied in any prime k-fold relation different from itself. But a prime k-fold relation may be implied in a prime more-than-k-fold relation,—or in a composite relation, regular or irregular, each factor whereof is more than k-fold; and so also a composite relation, regular or irregular, each factor whereof is at most k-fold, may be implied in a composite relation, regular or irregular, each factor whereof is more than k-fold. In a somewhat different sense, each factor of a composite relation implies the composite relation.

19. A composite relation is satisfied if any particular one of the component relations is satisfied; but in order to exclude this case we may speak of a composite relation as being satisfied *distributively*; viz. this will be the case if, in order to the satisfaction of the composite relation, it is *necessary* to consider *all* the factors thereof, or, what is the same thing, when the reduced relation obtained by the omission of any one factor whatever is *not always satisfied*. And when the composite relation is satisfied distributively, the several factors thereof are satisfied *alternatively*; viz. there is no one which is throughout unsatisfied.

20. A composite onefold relation is never distributively implied in a prime k-fold relation—that is, a prime k-fold relation implies only a prime onefold relation, or at least only implies a composite onefold relation improperly, in the sense that it implies a certain prime factor of such composite onefold relation. Conversely, every k-fold relation which implies distributively a composite onefold relation is composite.

21. Any two or more relations may be *aggregated* together into, and they are then constituents of, a single *aggregate* relation; viz. the aggregate relation is only satisfied when all the constituent relations are satisfied. The aggregate relation implies each of the constituent relations.

22. There is no meaning in aggregating a relation with itself; such aggregation only occurs accidentally when two relations aggregated together become one and the same relation; and the aggregate of a relation with itself is nothing else than the original relation.

23. A onefold relation is not an aggregate, but is its own sole constituent; a more than onefold relation may always be considered as an aggregate of two or more constituent relations. The constituent relations determine, they in fact constitute, the aggregate relation; but the aggregate relation does not in any wise determine the constituent relations. Any relation implied in a given relation may be considered as a constituent of such given relation.

24. The aggregate of a k-fold and a l-fold relation is in general and at most a $(k+l)$-fold relation; when it is a $(k+l)$fold relation, the constituent relations are independent, but otherwise, viz. if the aggregate relation is, or has for factor, a less than $(k+l)$fold equation, the constituent relations are dependent or interconnected.

25. Passing from relations to loci, we may say that the composition of relations corresponds to the *congregation* of loci, and the aggregation of relations to the *intersection* of loci.

26. For, first, the locus (if any) corresponding to a given composite relation is the congregate of the loci corresponding to the several prime factors of the given relation, the locus corresponding to a single factor being taken once, and the locus corresponding to a multiple factor being taken a number of times equal to the multiplicity of the factor.

27. And, secondly, the locus (if any) corresponding to a given aggregate relation is the locus common to and contained in each of the loci corresponding to the several constituent relations respectively; or, what is the same thing, it is the intersection of these several loci.

28. It may be remarked that a k-fold locus and a l-fold locus where $k+l>m$ (or where the aggregate relation is more than m-fold) have not in general any common locus.

29. Any onefold relation implied in a given k-fold relation is said to be in *involution* with the k-fold relation, and so in a system of onefold relations, if any relation be implied in the other relations, or, what is the same thing, in the relation aggregated of the other relations, then the system is said to be in *involution*; a system not in involution is said to be *asyzygetic*.

30. Consider a given k-fold relation, and, in conjunction therewith, a system of any number of onefold relations each implied in the given k-fold relation. We may omit from the system any relation implied in the remaining relations, and so successively until we arrive at an asyzygetic system. Consider now any other onefold relation implied in the given k-fold relation; this is either implied in the system of onefold relations, and it is then to be rejected, or if it is not implied in the system, it is to be added on to and made part of the system. It may happen that, in the system thus obtained, some one relation of the original system is implied in the remaining relations of the new system; but if this is so the implied relation is to be rejected; the new system will in this case contain only as many relations as the original system, and in any case the new system will be asyzygetic. Treating in the same manner every other onefold relation implied in the given k-fold relation, we ultimately arrive at an asyzygetic system of onefold relations, such that every onefold relation implied in the given k-fold relation is implied in the asyzygetic system. The number of onefold relations will be at least equal to k (for if this were not so we should have the given k-fold relation as an aggregate of less than k onefold relations); but it may be greater than k, and it does not appear that there is any superior limit to the number of onefold relations of the asyzygetic system.

31. The system of onefold relations is a precise equivalent of the given k-fold relation. Every set of values of the coordinates which satisfies the given k-fold relation satisfies the system of onefold relations; and reciprocally every set of values which satisfies the system of onefold relations satisfies the given k-fold relation. But if we omit any one or more of the onefold relations, then the reduced system so obtained is not a precise equi-

valent of the given k-fold relation; viz. there exist sets of values satisfying the reduced system, but not satisfying the given k-fold relation.

32. In fact consider a k-fold relation the aggregate of less than all of the onefold relations of the asyzygetic system, and in connexion therewith an omitted onefold relation; this omitted relation is not implied in the aggregate, and it constitutes with the aggregate not a $(k+1)$fold, but only a k-fold relation. This happens as follows, viz. the omitted relation is a factor of a composite onefold relation distributively implied in the aggregate; hence the aggregate is composite, and it implies distributively a composite onefold relation composed of the omitted relation and of an associated onefold relation; that is, the aggregate will be satisfied by values which satisfy the omitted relation, and also by values which (not satisfying the omitted relation) satisfy the associated relation just referred to.

33. Selecting at pleasure any k of the onefold relations of the asyzygetic system, being such that the aggregate of the k relations is a k-fold relation, we have a composite k-fold relation wherein each of the remaining onefold relations is alternatively implied; viz. each remaining onefold relation is a factor of a composite onefold relation implied distributively in the composite k-fold relation. Hence considering the $\overline{k+1}$ onefold relations, viz. any $\overline{k+1}$ relations of the asyzygetic system, each one of these is implied alternatively in the aggregate of the remaining k relations; and we may say that the $\overline{k+1}$ onefold relations are in *convolution*.

34. More generally any $\overline{k+1}$ or more, or all the relations of the asyzygetic system are in *convolution*, that is, any relation of the system is alternatively implied in the aggregate of the remaining relations, or indeed in the aggregate of any k relations (not being themselves in convolution) of the remaining relations of the asyzygetic system. It may be added that, besides the relations of the system, there is not any onefold relation alternatively implied in the asyzygetic system.

35. The foregoing theory has been stated without any limitation as to the value of k, and it has I think a meaning even when k is $>m$; but the ordinary case is $k \not> m$. Considering the theory as applying to this case, I remark that the last proposition, viz. that no reduced system is a precise equivalent of the given k-fold relation, is generally true only on the assumption of the existence or quasi-existence of sets of values satisfying a more than m-fold relation. For let k be $\not> m$, and, on the contrary, assume, as we usually do, that it is not in general possible to satisfy a more than m-fold relation between the coordinates; the number of relations in the system may be $>m+1$; and if this is so, then selecting any $\overline{m+1}$ relations of the system, it may very well happen that the given k-fold relation is not satisfied by any sets of values other than those which satisfy the $\overline{m+1}$ relations,—that is, that the $\overline{m+1}$ relations are a precise equivalent of the given k-fold relation. But even in this case the consideration of the entire system of the onefold relations is not the less advantageous; and I say in general that the given k-fold relation has for its precise and complete equivalent the asyzygetic system of onefold relations.

36. [In illustration of the foregoing Nos. 29 to 35, I remark that, for the functions or equations $P=0$, $Q=0$, $R=0$, &c., if we have identically $AP+BQ+CR+\ldots=0$, where the factors A, B, C, ... are integral functions of the coordinates, and where some one of these factors, say, A, is a constant (or if we please $=1$), then the system of functions or equations is in involution; or, to speak more accurately, the function or equation $P=0$ is in involution with the remaining functions or equations $Q=0$, $R=0$, ... But when the factors A, B, C, ... are no one of them constant, then we have a convolution. If $P=0$ is in involution with the remaining equations $Q=0$, $R=0$, ..., then $P=0$ is implied in these equations, and the relations $(Q=0, R=0, \ldots)$ and $(P=0, Q=0, R=0, \ldots)$ are equivalent to each other. But in the case of a convolution where

$$AP+BQ+CR+\ldots=0,$$

then the relation the equations $Q=0$, $R=0$, ... imply $AP=0$, that is. $A=0$ or else $P=0$; or, what is the same thing, the relation $(Q=0, R=0, \ldots)$ is a relation composed of the two relations $(A=0, Q=0, R=0, \ldots)$ and $(P=0, Q=0, R=0, \ldots)$. In the k-fold relation expressed by the more than k equations $(P=0, Q=0, R=0, \ldots)$, selecting any k of these equations which are not in convolution, and uniting thereto any one of the remaining equations, we have a convolution of $\overline{k+1}$ equations; and when a k-fold relation is precisely expressed by means of a system of k or more equations $(P=0, Q=0, \ldots)$, then every equation $\Omega=0$ implied in the given relation, or, what is the same thing, the equation of any onefold locus passing through the locus given by the k-fold relation is in involution with the equations $P=0$, $Q=0$, ..., that is, we have identically $\Omega=AP+BQ+CR+\ldots$, A, B, C, ... being integral functions of the coordinates.]

Omal Relation; Order. Article Nos. 37 to 42.

37. A k-fold relation may be linear or omal. If $k=m$, the corresponding locus is a point; if $k<m$ the locus is a k-fold, or $(m-k)$dimensional omaloid; the expression omaloid used absolutely denotes the onefold or $(m-1)$dimensional omaloid; the point may be considered as a m-fold omaloid.

38. A m-fold relation which is not linear or omal is of necessity composite, composed of a certain number M of m-fold linear or omal relations; viz. the m-fold locus corresponding to the m-fold relation is a point-system of M points, each of which may be considered as given by a separate m-fold linear or omal relation; each which relation is a factor of the original m-fold relation. The given m-fold relation, and the point-system corresponding thereto, are respectively said to be of the order M.

39. The order of a point-system of M points is thus $=$M, but it is of course to be borne in mind that the points may be single or multiple points; and that if the system consists of a point taken α times, another point taken β times, &c., then the number of points and therefore the order M of the system is considered to be $=\alpha+\beta+\ldots$

40. If to a given k-fold relation $(k<m)$ we unite an absolutely arbitrary $(m-k)$fold linear relation, so as to obtain for the aggregate a m-fold relation, then the order M of this m-fold relation (or, what is the same thing, the number M of points in the corresponding

point-system) is said to be the order of the given k-fold relation. The notion of order does not apply to a more than m-fold relation.

41. The foregoing definition of order may be more compendiously expressed as follows: viz.

Given between the $\overline{m+1}$ coordinates a relation which is at most m-fold; then if it is not m-fold, join to it an arbitrary linear relation so as to render it m-fold; we have a m-fold relation giving a point-system; and the order of the given relation is equal to the number of points of the point-system.

42. The relation aggregated of two or more given relations, when the notion of order applies to the aggregate relation, that is, when it is not more than m-fold, is of an order equal to the product of the orders of the constituent relations; or, say, the orders of the given relations being μ, μ', ..., the order of the aggregate relation is $=\mu\mu'\dots$

Parametric Relations. Article Nos. 43 and 44.

43. We have considered so far relations which involve only the coordinates $(x, y, \dots)$*; the coefficients are purely numerical, or, if literal, they are absolute constants, which either do or do not satisfy certain conditions; if they do not, the relation assumed in the first instance to be k-fold is really k-fold, or, as we may express it, the relation is really as well as formally k-fold; if they do satisfy certain relations in virtue whereof the formally k-fold relation is really less than k-fold, say, it is $(k-l)$fold, then the relation is in fact to be considered *ab initio* as a $(k-l)$fold relation: there is no question of a relation being in general k-fold and becoming less than k-fold, or suffering any other modification in its form; and the notion of a more than m-fold relation is in the preceding theory meaningless.

44. But a relation between the coordinates $(x, y, \dots)$ may involve parameters, and so long as these remain arbitrary it may be really as well as formally k-fold; but when the parameters satisfy certain conditions, it may become $(k-l)$fold, or may suffer some other modification in its form. And we have to consider the theory of a relation between the coordinates $(x, y, \dots)$, involving besides parameters which may satisfy certain conditions, or, say simply, a relation involving variable parameters. If the number of the parameters be m', then these parameters may be regarded as the ratios of m' quantities to a remaining $\overline{m'+1}$th quantity, and the relation may be considered as involving *homogeneously* the $\overline{m'+1}$ parameters $(x', y', \dots)$. And these may, if we please, be regarded as coordinates of a point in their own m'-dimensional space, or we have to consider relations between the $\overline{m+1}$ cordinates $(x, y, \dots)$ and the $\overline{m'+1}$ (parameters or) coordinates $(x', y', \dots)$. It is to be added that a relation may involve distinct sets of parameters, say, we have besides the original set of parameters, a set of $\overline{m''+1}$ parameters $(x'', y'', \dots)$ involved homogeneously. But this is a generalization the necessity for which has hardly arisen.

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* The only exception is *ante*, No. 5, where, in illustration of the notion of a more than m-fold relation, mention is made of "parameters."

Quantics, Notation, &c. Article Nos. 45 to 55.

45. A homogeneous function of the coordinates $(x, y, \ldots)$ is represented by a notation such as

$$(*\between x, y, \ldots)^{(\cdot)}$$

(where $(*)$ indicates the coefficients and $(\,\cdot\,)$ the degree), and it is said to be a quantic; and in reference to the quantic the quantities or coordinates $(x, y, \ldots)$ are also termed *facients*. More generally a quantic involving two or more sets of coordinates, or facients, is represented by the similar notation

$$(*\between x, y, \ldots)^{(\cdot)}(x', y', \ldots)^{(:)}\ldots$$

46. The quantic is unipartite, bipartite, tripartite, &c., according as the number of sets is one, two, three, &c.; and with respect to any set of coordinates, it is binary, ternary, quaternary, ... $(m+1)$ary, according as the number of the coordinates is two, three, four, or $m+1$; and it is linear, quadric, cubic, quartic, ..., according as the degree in regard to the coordinates in question is 1, 2, 3, 4 ...

47. A quantic involving two or more sets of coordinates, and linear in regard to each of them, is said to be tantipartite; or, in particular, when there are only two sets, it is said to be lineo-linear; we may even extend the epithet lineo-linear to the case of any number of sets.

48. Instead of the general notation

$$(*)(x, y, \ldots)^{(\cdot)}(x', y', \ldots)^{(:)}\ldots$$

we may write

$$(a, \ldots)(x, y, \ldots)^{\mu}(x', y', \ldots)^{\mu'}, \ldots,$$

where the coefficients are now indicated by $(a, \ldots)$, and the degrees are $\mu, \mu', \ldots$

49. In the cases where the particular values of the coefficients have to be attended to, we write down the entire series of coefficients, or at least refer thereto by the notation $(a, \ldots)$; and it is to be understood that the coefficients expressed or referred to are each to be multiplied by the appropriate numerical coefficient, viz. for the term $x^{\alpha}y^{\beta}\ldots x'^{\alpha'}y'^{\beta'}\ldots$ this numerical coefficient is

$$=\frac{[\mu]^{\mu}[\mu']^{\mu'}\ldots}{[\alpha]^{\alpha}[\beta]^{\beta}\ldots[\alpha']^{\alpha'}[\beta']^{\beta'}\ldots}.$$

50. It is sometimes convenient not to introduce these numerical multipliers, and we then use the notation

$$(a, \ldots \between x, y, \ldots)^{\mu}(x', y', \ldots)^{\mu'}\ldots,$$

or

$$(a, \ldots \between x, y, \ldots \rceil^{\mu}(x', y', \ldots \rceil^{\mu'}\ldots$$

In particular $(a, b, c \between x, y)^2$, $(a, b, c, d \between x, y)^3$ &c. denote respectively

$$ax^2+2bxy+cy^2,$$

$$ax^3+3bx^2y+3cxy^2+dy^3,$$

&c.;

but $(a, b, c \between x, y)^2$, $(a, b, c, d \between x, y)^3$, &c. denote

$$ax^2+bxy+cy^2,$$
$$ax^3+bx^2y+cxy^2+dy^3,$$
&c.,

and so $(a, b, c, f, g, h \between x, y, z)^2$ and $(a, b, c, f, g, h \between x, y, z)^2$ denote respectively

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy$$

and

$$ax^2+by^2+cz^2+\ fyz+\ gzx+\ hxy.$$

51. To show which are the coefficients that belong to the several terms respectively, it is obviously proper that the quantic should be once written out at full length; thus, in speaking of a ternary cubic function, we say let $\mathrm{U}=(a, \ldots \between x, y, z)^3$

$$\begin{aligned}&=(a, b, c, f, g, h, i, j, k, l \between x, y, z)^3\\&=ax^3+by^3+cz^3\\&\quad+3(fy^2z+gz^2x+hx^2y+iyz^2+jzx^2+kxy^2)\\&\quad+6lxyz,\end{aligned}$$

and the like in other cases.

52. A onefold relation between the coordinates is expressible by means of an equation of the form

$$(\ast \between x, y, \ldots)^{(\mu)} =0.$$

53. The expression "an equation" used without explanation may be taken to mean an equation of the form in question, viz. the equation obtained by putting a quantic equal to zero; the quantic is said to be the *nilfactum* of the equation. We may consequently say simply that a onefold relation between the coordinates is always expressible by an equation.

54. It is frequently convenient to denote the quantic or nilfactum by a single letter, and to use a locution such as "the equation $\mathrm{U}=(\ast \between x, y \,..,)^{(\mu)}=0$," which really means that the single letter U stands for the quantic $(\ast \between x, y, \ldots)^{(\mu)}$, so that we are afterwards at liberty to write $\mathrm{U}=0$ as an abbreviated expression for $(\ast \between x, y, \ldots)^{\mu}=0$. We may also speak of the equation or function $\mathrm{U}=0$, meaning thereby the equation $\mathrm{U}=0$, or the function U.

55. A k-fold relation between the coordinates is (as has been shown) equivalent to a system of k or more onefold relations; each of these is expressible by an equation $\mathrm{U}=0$, and the k-fold relation is thus expressible by a system of k or more such equations. Representing by $\big((\mathrm{U})\big)$ the system of functions which are the nilfacta of these equations respectively, the k-fold relations may be represented thus, $\big((\mathrm{U})\big)=0$; or more completely, the relation being k-fold, and the number of equations being $=s$, by the notation

$$\big((\mathrm{U})s\big)(k\text{-fold})=0.$$

We may also speak of the system or relation $\big((\mathrm{U})\big)=0$, meaning thereby the system of functions $\big((\mathrm{U})\big)$, or the relation $\big((\mathrm{U})\big)=0$.

Resultant, Discriminant, &c. Article Nos. 56 to 62.

56. In the case $k>m$, a given k-fold relation between the $\overline{m+1}$ coordinates $(x, y, \ldots)$ and the parameters $(x', y', \ldots)$ leads to a $(k-m)$fold relation between the parameters. This is termed the *resultant relation* of the given k-fold relation, or when the additional specification is necessary, the *resultant relation* obtained by elimination of the coordinates $(x, y, \ldots)$.

57. Consider a k-fold relation between the $\overline{m+1}$ coordinates $(x, y, \ldots)$ and the $\overline{m'+1}$ coordinates $(x', y', \ldots)$. If $k \not> m$, then, considering the $(x, y, \ldots)$ as coordinates and the $(x', y', \ldots)$ as parameters, we have corresponding to the given relation a k-fold locus in the m-space; and so if $k \not> m'$, then, considering the $(x', y', \ldots)$ as coordinates, but the $(x, y, \ldots)$ as parameters, we have corresponding to the given relation a k-fold locus in the m'-space.

58. If $k>m$, but if the $(k-m)$fold resultant relation is satisfied, then the given k-fold relation becomes a m-fold linear relation between the coordinates $(x, y, \ldots)$, and is consequently satisfied by a single set of values of the coordinates. Hence, considering the given k-fold relation as implying the $(k-m)$fold resultant relation, the k-fold relation will represent a single point in the m-space, say, the *common point*.

59. A m-fold relation, or the locus, or point-system thereby represented, may have a *double* or *nodal* point, viz. two of the points of the point-system may be coincident. More generally a k-fold relation $(k \not> m)$, or the locus thereby represented, may have a *double* or *nodal* point; for let the relation if less than m-fold be made m-fold by adjoining to it a linear $(m-k)$fold relation satisfied by the coordinates of the point in question but otherwise arbitrary, then, if the point in question be a double or nodal point of the m-fold relation, or of the point-system thereby represented, the point is said to be a double or nodal point of the original k-fold relation, or of the locus thereby represented.

60. A given k-fold relation $(k \not> m)$ between the $\overline{m+1}$ coordinates, or the locus thereby represented, has not in general a nodal point. But if the relation involve the $\overline{m'+1}$ parameters $(x', y', \ldots)$, then, if a certain onefold relation be satisfied between the parameters, there will be a nodal point. The onefold relation between the parameters is the *discriminant relation* of the given k-fold relation.

61. In the case in question, $k \not> m$, the discriminant relation is the resultant relation of a $(m+1)$fold relation which is the aggregate of the given k-fold relation with a certain relation called the *Jacobian relation*, or when the distinction is required, the *Jacobian relation* in regard to the $(x, y, \ldots)$.

62. Consider a k-fold relation $(k \not> m, \not> m')$ between the $\overline{m+1}$ coordinates $(x, y, \ldots)$ and the $\overline{m'+1}$ coordinates $(x', y', \ldots)$. It has been seen that to a given set of values of

the $(x', y', \ldots)$ or, say, to a given point in the m'-space, there corresponds a k-fold locus in the m-space, and that to a given set of values of the $(x, y, \ldots)$, or to a given point in the m-space, there corresponds a k-fold locus in the m'-space. The k-fold locus in the m'-space may have a nodal point; this will be the case if there is satisfied between the $(x, y, \ldots)$ a certain onefold relation, the discriminant relation of the given k-fold relation in regard to the $(x', y', \ldots)$. This onefold relation represents in the m-space a onefold locus, the *envelope* of the k-fold loci in the m-space corresponding to the several points of the m'-space. The property of the envelope is that to each point thereof there corresponds in the m'-space a k-fold locus having a nodal point.

Consecutive Points; Tangent Omals. Article Nos. 63–69.

63. As the notions of proximity and remoteness have been thus far altogether ignored, it seems necessary to make the following

Postulate. We may conceive a point consecutive (or indefinitely near) to a given point.

64. If the coordinates of the given point are $(x, y, \ldots)$, those of the consecutive point may be assumed to be $(x+\delta x, y+\delta y, \ldots)$, where $\delta x, \delta y, \ldots$ are indefinitely small in regard to $(x, y, \ldots)$.

65. It may be remarked that, taking the coordinates to be $(x+\mathrm{X}, y+\mathrm{Y}, \ldots)$, there is no obligation to have $(\mathrm{X}, \mathrm{Y}, \ldots)$ indefinitely small; in fact whatever the magnitudes of these quantities are, if only $\mathrm{X} : \mathrm{Y} : \ldots = x : y : \ldots$, then the point $(x+\mathrm{X}, y+\mathrm{Y}, \ldots)$ will be the very same with the original point, and it is therefore clear that a consecutive point may be represented in the same manner with magnitudes, however large, of $\mathrm{X}, \mathrm{Y}, \ldots$ But we *may* assume them indefinitely small, that is, the ratios $x+\delta x : y+\delta y, \ldots$, where $\delta x, \delta y, \ldots$ are indefinitely small in regard to $(x, y, \ldots)$, will represent any set of ratios indefinitely near to the ratios $(x : y, \ldots)$.

The foregoing quantities $(\delta x, \delta y, \ldots)$ are termed the increments.

66. Consider a k-fold relation between the $\overline{m+1}$ coordinates $(x, y, \ldots)$, $k \ngtr m$; the increments $(\delta x, \delta y, \ldots)$ are connected by a linear k-fold relation.

The linear k-fold relation is satisfied if we assume the increments proportional to the coordinates—this is, in fact, assuming that the point remains unaltered. We may write $(\delta x, \delta y, \ldots) = (x, y, \ldots)$, since in such an equation only the ratios are attended to. But it may be preferable to write $(\delta x, \delta y, \ldots) = \lambda(x, y, \ldots)$. In particular if $k=m$, then the increments are connected by a linear m-fold relation; that is, the ratio of the increments is uniquely determined; and as the relation is satisfied by taking the increments proportional to the coordinates, it is clear that the values which the linear m-fold relation gives for the increments are in fact proportional to the coordinates: viz. there is not in this case any consecutive point.

67. Considering the k-fold relation as belonging to a k-fold locus in the m-space, so that $(x, y, \ldots)$ are the coordinates of a point on this locus, then if in the linear k-fold relation between the increments these increments are replaced by the coordinates $(\mathrm{x}, \mathrm{y}, \ldots)$ of a point in the m-space, then considering the original coordinates $(x, y, \ldots)$ as para-

meters, the locus of the point (x, y, . . .) is a k-fold omal locus: it is to be observed that, by what precedes, the linear k-fold relation is satisfied by writing therein the values x : y, . . . $=x : y$, . . . , that is, the k-fold omal locus passes through the original point ($x, y, \ldots$); the k-fold omal locus is said to be the *tangent-omal* of the original k-fold locus at the (point $x, y, \ldots$), which point is said to be the *point of contact.*

68. If in the original k-fold locus we replace ($x, y, \ldots$) by (x, y, . . .), and combine therewith the k-fold linear relation, we have between the coordinates (x, y, . . .) a $2k$-fold relation (containing as parameters the coordinates ($x, y, \ldots$)); these parameters satisfy the original k-fold relation, and in virtue hereof the $2k$-fold relation (whether $2k$ is or is not greater than m) is satisfied by the values x, y, . . . $=x : y : \ldots$; and not only so, but the point in question is a nodal or double point on the $2k$-fold locus. It also follows that the tangent-omal locus, considering in the k-fold linear relation ($x, y, \ldots$) as parameters satisfying the original k-fold relation, has for its envelope the k-fold locus.

69. We thus arrive at the notion of the double generation of a k-fold locus, viz. such locus is the locus of the points, or, say, of the *ineunt-points* thereof; and it is also the envelope of the tangent-omals thereof. We have thus a theory of duality; I do not at present attempt to develope the theory, but it is necessary to refer to it, in order to remark that this theory is essential to the systematic development of a m-dimensional geometry; the original classification of loci as onefold, twofold, . . .($m-1$)fold is incomplete, and must be supplemented with the loci reciprocally connected with these loci respectively. And moreover the theory of the singularities of a locus can only be systematically established by means of the same theory of duality; the singularities in regard to the ineunt-point must be treated of in connexion with the singularities in regard to the tangent-omal. These theories (that is, the classification of loci, and the establishment and discussion of the singularities of each kind of locus), vast as their extent is, should in the logical order precede that which for other reasons it may be expedient next to consider, the theory of Transformation, as depending on relations involving simultaneously the $\overline{m+1}$ coordinates ($x, y, \ldots$) and the $\overline{m'+1}$ coordinates ($x', y', \ldots$).

V. *On Remains of a large extinct Lama* (Palauchenia magna, Ow.) *from Quaternary Deposits in the Valley of Mexico. By Professor* Owen, *F.R.S. &c.*

Received March 22,—Read April 22, 1869.

In the second of Dr. Lund's communications to the Danish Academy of Sciences, entitled "Survey of the extinct species of Mammalia which inhabited the Highlands of tropical Brazil previously to the last Geological Revolution," he includes amongst the *Ruminantia* two kinds of *Camelidæ*, observing:—"Of the genus *Camelus* I possess the remains of two species, one exceeding a horse in size, the other a little less. To which of the two subgroups of this genus the fossils belong, that is, whether to the modern inhabitant of the warm regions of the old world, *Camelus*, Ill., or to that now found in the chain of the Andes, *Auchenia*, Ill., my insufficient means of comparison will not allow me to decide"*.

Professor Pictet, in his comprehensive 'Traité de Paléontologie,' refers the fossils so indicated by Lund to the genus *Auchenia*, but without, apparently, any additional facts or evidence to guide him to this decision†.

Professor De Blainville, in the Fasciculus of his 'Ostéographie' relating to the genus *Camelus*, remarks:—

"Parmi les pièces fossiles déjà assez nombreuses provenant du Brésil, que nous possédons dans les collections du Muséum, je n'en ai encore rencontré aucune qui puisse être rapportée aux Lamas, et je ne vois pas que dans ses Mémoires, publiés dans les 'Actes de l'Académie Royale des Sciences de Copenhague,' M. Lund ait fait connaître, soit par des descriptions, soit par des figures, les pièces qui ont servi de base à ces assertions; elles n'en ont pas moins été reprises cependant dans toutes les compilations paléontologiques" (p. 123).

I may remark that M. Lund does not commit himself to any assertion of the particular genus of *Camelidæ* to which his fossils belonged, whatever parts he might have obtained from the caves at the date of the Memoir above alluded to.

In his later communication to the Danish Academy, of November 1844, published in the twelfth volume of the Transactions above quoted, there appears in the list, p. 86, "No. 12, *Auchenia*;" but all that is subsequently added upon this subject is as follows:—

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* Det Kongelige Danske Videnskabernes Selskabs Naturvidens. og Mathem. Afhandlinger, 1838. (Translated and published in the 'Magazine of Natural History,' by the Rev. W. Bilton, M.A., 1840, New Series, p. 1.)

† "Les Lamas (*Auchenia*, Ill.) paraissent avoir habité l'Amérique méridionale pendant l'époque diluvienne, comme de nos jours. M. Lund en a trouvé deux espèces dans les cavernes du Brésil; l'un d'elles surpassait le cheval par sa taille."—8vo Ed. 1853, tome i. p. 345.

"The remains of this genus were most numerous in this Cavern ('Lapa d'Anna Felicia'), and showed evidences of six individuals, mostly young animals"*.

Professor Gervais, in the Anatomical Part or Appendix to Count Castelnau's Expedition to the central parts of South America†, refers to the genus *Auchenia* some fossil bones of the feet found by Mr. Weddell in deposits at Tarija, in Bolivia, associated with *Mastodon* and other extinct species. They exceeded in size those of the largest Lama (*Auchenia lama*), being intermediate in size between that and the Camel. M. Gervais observes that it may be, perhaps, the species said by Lund "to exceed a horse in size"‡; he figures the fragmentary fossils as belonging to an *Auchenia Weddellii*.

More decisive evidences of *Auchenia* from the deposits at Tarija were afforded by fossil teeth. The first of these is a part of the upper jaw with the four chief molars ("portant encore les quatre paires de molaires principales pour chaque côte," Gervais, *ut suprà*, p. 41), having a longitudinal extent nearly the same as that in the Lama. (This fossil is not figured, nor are the dimensions of the teeth given.) Next are mentioned portions of mandibles "with the four molars in place." These are figured of half the natural size§.

The longitudinal extent of the series is 85 millims. (=3″ 4‴), while that in a large Lama in the Museum of Comparative Anatomy in the Jardin des Plantes is 75 millims. (=2″ 11‴).

In a specimen of *Auchenia lama* before me, the longitudinal extent of the four lower grinders is 3 inches, or 76 millims. The anterior molar (=*p* 4) of the fossil is stronger ('plus forte') than its correspondent in the living Lamas, and its anterior fold is much more marked. These fossils, with an astragalus and calcaneum, are referred to *Auchenia Castelnaudii*, Gerv.

A fragment of the right ramus of the mandible with the molar *m* 2 in place and the alveoli of *m* 1 and *p* 4, inferior in size to that of *Auchenia Castelnaudii*, is indicative of an animal less than the domestic Lama but greater than the Vicugna, and it is referred, with a tibia which reproduces that of an *Auchenia* by its forms‖, to the *Auchenia intermedia*, Gerv.

To the evidences of extinct *Camelidæ* in the tertiaries and post-tertiaries of North America, for which science is indebted to Professor Leidy, I shall refer in the sequel.

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* "Levningerne af denne Slægt hörte til de talrigere i denne Hule og antydede vel sex Individer, hvoraf de fleste unge Dyr."—*Op. cit.* 12 Deel (1846) p. 89.

† Expédition dans les parties centrales de l'Amérique du Sud, &c., sous la direction du Comte Francis de Castelnau: 'Anatomie,' par M. Paul Gervais, &c., 4to, 1855, p. 41.

‡ "L'animal qui a laissé ces différents ossements était trop supérieur en dimensions aux Lamas actuels pour que l'on suppose qu'il a pu être de la même espèce qu'eux, et il ne me paraît pas douteux que l'examen de nouveaux débris montrera entre eux et lui de nouvelles différences; c'est peut-être cette espèce ou une peu différente par les dimensions que M. Lund a comparé au Cheval. Elle approchait du Chameau sous le même rapport, mais sans être cependant aussi grande, et elle tenait le milieu entre lui et le Paco ou Guanaco."

§ *Op. cit.* plate 10. figs. 1 & 2.

‖ *Op. cit.* p. 43.

Professor BURMEISTER states that M. BRAVARD has deposited in the Public Museum of Buenos Ayres part of a mandible with the three posterior molars, which seemed identical with those of *Auchenia intermedia*, Gerv.* No other fossil evidence of an *Auchenia* had come to Professor BURMEISTER's knowledge in 1867.

In that year (1867) I was favoured by receiving from Don ANTONIO DEL CASTILLO, Mining Engineer of Mexico, through the kind intermedium of R. T. C. MIDDLETON, Esq[illegible] Sec. to Her Majesty's Legation, Mexico, photographs and casts of six of the cervic[illegible] vertebræ, and photographs of the lower molar series and canines of an *Auchenia*, much exceeding in size any remains suggesting an animal intermediate between a Lama and a Camel. Without knowing the degree in which the fossil Cameline remains from the Brazilian cavern "exceeded a horse in size," one cannot judge of the difference or resemblance in that character between LUND's fossil and those about to be described; but the probability is in favour of Professor GERVAIS's estimate, as exemplified in his *Auchenia Weddellii*. According thereto the extinct species of Lama from Mexico must greatly exceed in size any of which we have had previous indications. The evidences above specified were found by Don ANTONIO DEL CASTILLO, in or beneath volcanic tufa, in the valley of Mexico, associated with remains of *Elephas* and *Mastodon*.

Don ANTONIO DEL CASTILLO informs me that "the teeth lay, when exposed, in their natural position; but much of the jaw had crumbled or dissolved away after entombment."

In that position his photographs of the inside view, and of an oblique upper and outside view were taken, and with these I received admeasurements of the several teeth in millimeters.

The teeth consist of the series of grinders, in number five, not four as in *Camelus* and *Auchenia*, also of a minute caniniform premolar rising about halfway in the long diastema between the molar series and the canine; this tooth was likewise present, small, compressed, subrecurved. Of the incisors I have received no information.

Concluding that they existed in the number common to the *Camelidæ*, the dental formula of the mandibular ramus, in the present fossil, would be:—*i* 3, *c* 1, *p* 3, *m* 3 =10 (Plate IV. fig. 3). The series of five molars (ib. figs. 1 & 2) includes *p* 3, *p* 4, *m* 1, *m* 2, *m* 3; the advanced rudimental premolar may be *p* 1, fig. 3†; then, with a shorter interval, comes the canine, *c*.

It is rare to find in any *Auchenia*, still more rare in *Camelus*, the lower penultimate premolar (p 3) retained after m 3 has risen into place. The only example of *Auchenia* (an *A. vicugna*) in which I have observed this condition I figured, on that account, in my 'Odontography,' plate 133. fig. 2. The proportions of the retained p 3 in the great fossil Lama resemble those in that *Auchenia vicugna*, but the functional and commonly retained last premolar (p 4) is relatively larger in *Palauchenia* (compare figs. 3, *Palauchenia*, and 4, *Auchenia vicugna*, both reduced to $\frac{2}{3}$, nat. size, in Plate IV.).

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* G. BURMEISTER, M. & Phil. D. Anales del Museo Público de Buenos Aires, 4to (Entrega Quarta), 1867, p. 234.

† The decomposed state of the jaw photographed makes the precise position of this rudimental tooth somewhat uncertain; but of its existence in the alveolar part of the long diastema there is no doubt.

[I regret that the political troubles in Mexico, followed by the withdrawal of our Legation, and an anarchical condition of the Capital, suspended my relations with the accomplished discoverer of the fossils described in the present paper.—February 1870.]

A caniniform premolar is commonly present in the lower jaw of the Camel and Dromedary in the long diastema between the retained last functional premolar (p 4) and the canine: I have not observed such premolar in any *Auchenia*, nor is any mention of such made by CUVIER or DE BLAINVILLE (*op. cit.* p. 95). In the lower jaw of an *Auchenia vicugna* in the British Museum (675 *a*), there is a small hard tuberosity on the alveolar border, a little way behind the canine, which may indicate the former existence of a rudiment of a premolar (Plate IV. fig. 4, p 1,?) answering to that developed in *Camelus*.

In *Palauchenia* the rudimental caniniform premolar (ib. fig. 3, p 1) seems to be situated as in *Camelus*, but is relatively much smaller than is the caniniform premolar (p 1 or p 2) in that genus.

The penultimate premolar (Plate IV. figs. 1, 2, 3, p 3) is a longish, slender, straight, obtusely pointed cone, rising in contact with p 4, but not attaining the level of the grinding-surface; though small (see Table of Dimensions, p. 69) it is relatively larger than its homologue, the exceptionally developed rudiment, in the *Vicugna* (ib. fig. 4, p 3), and it may be therefore inferred to have been more constantly developed and present in *Palauchenia*.

What is more certain is the larger proportional size of the last or functional premolar (ib. ib. p 4) in *Palauchenia* than in *Auchenia*, the fossil in that respect more resembling *Camelus*, but with a larger size and difference of form of the tooth in question. In *Camelus* the fore end of p 4 is narrow, the outer surface curving inward to meet the inner one at a ridge which forms the fore part of that surface. In *Palauchenia* the fore part of p 4 is as thick or broad as the back part, and is flattened,—a modification which adds to the probability of the constancy of p 3 in the grinding-series of *Palauchenia*. In *Camelus* p 4 has a posterior portion or lobule marked off by an external and an internal longitudinal or vertical groove, and the corresponding part of the grinding-surface shows, after moderate attrition, a distinct small island of enamel. One cannot help recognizing this hind part as the rudimental homologue of the second lobe in the true molars. There is a slight indication of the outer posterior groove in *Palauchenia* (ib. figs. 1 & 3, p 4), but no corresponding inner one, and no distinct posterior islet of enamel. One long slightly curved fold (Plate IV. fig. 1, p 4) penetrates the grinding-surface, and the tooth represents, as in *Auchenia* and the true Ruminants, the half, or a single lobe, of the true molars.

These (Plate IV. figs. 1, 2, 3, m 1–3) adhere to the type in *Camelidæ*, with minor modifications resembling those in *Auchenia*. The outer sides of the lobes (ib. figs. 1 & 3) are not subangular prominences as in *Camelus*, but are convex as in *Auchenia*, yet in a markedly minor degree; and, concurrently, the worn edge of parietal enamel is more even, rises less angularly, than in existing *Camelidæ*. The small anterior lobule ("filet verticale," CUVIER) is present in the penultimate and last molars, as in *Auchenia*, but is relatively less, and is indicated only on the outer side of the tooth (ib. fig. 1, *m* 2, *x*, and *m* 3, *x*). The third or posterior lobule of *m* 3 (ib. *z*) is relatively less developed than in either *Auchenia* or *Camelus*.

The canine, 1 inch 1 line in length of crown, 6 lines in fore-and-aft breadth, is compressed, recurved, and retains more of the shape and proportions of that tooth in *Auchenia* than in *Camelus*.

The following are dimensions of the series of grinders and of the individual teeth of *Palauchenia magna*:—

Teeth, Lower jaw.		*Palauchenia.*		*Camel.*		*Lama.*	
		m. in.	lines.	in.	lines.	in.	lines.
	Length of series of *p* 3–*m* 3	0·168=6	7	6	9	3	0
p 3.	Antero-posterior breadth of base	0·007=0	3				
p 4.	Antero-posterior breadth *	0·029=1	1½	1	0	0	5½
	Transverse breadth *	0·023=0	11	0	7	0	2
	Length of crown †	0·034=1	4	1	2	0	6
m 1.	Antero-posterior breadth ‡	0·039=1	6	1	6	0	7
	Transverse breadth	0·024=0	11½	0	9‡	0	5‡
	Length of crown ‡	0·034=1	4	1	3	0	5
m 2.	Antero-posterior breadth	0·045=1	9	1	11	0	9
	Transverse breadth	0·024=0	11½	0	9½	0	6
	Length of crown	0·035=1	4½	1	4	0	6
m 3.	Antero-posterior breadth	0·048=1	10½	2	4	1	0½
	Transverse breadth	0·024=0	11½	0	9	0	5
	Length of crown	0·050=2	0	1	5	0	7

Cervical Vertebræ. Plates V., VI., VII., and Plate IV. figs. 5 & 6 (reduced view).

With the decayed portions of mandible and the teeth above described were found parts of the skeleton, of which I have been favoured by Don ANTONIO with plaster-casts and photographs of six vertebræ. They are more or less mutilated, but sufficiently entire to show that they are the six consecutive vertebræ of the neck succeeding the atlas, and of one and the same quadruped (Plate IV. fig. 6, 2 to 7).

These cervical vertebræ present the character of the intraneural vertebrarterial canal characteristic of the *Camelidæ* among existing Ungulates; and, as this character is only known among extinct species in the perissodactyle genus *Macrauchenia*, the field of comparison is restricted, and the results confirm the inference, from juxtaposition of the fossils, that the vertebræ in question belong to the same animal as the teeth above described.

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* Across the middle of the grinding-surface.

† To the origin of the roots.

‡ Across the middle of the anterior lobe.

An oblique upper view, one-fourth the natural size, of the second to the seventh cervical vertebræ inclusive, coarticulated, is given in Plate IV. fig. 6; an oblique under view of the third and fourth cervicals, coarticulated, is given, similarly reduced, at fig. 5. Figures of three of the more characteristic vertebræ are given, of the natural size, with the answerable ones in *Auchenia* and *Camelus*, in Plates V. & VI., and alone in Plate VII.

The axis vertebra (Plate IV. fig. 6, 2, Plate V. fig. 1) wants the major part of the odontoid process, *o*, the right diapophysis, the end of the left one (Plate V. fig. 1, *d*), and the left postzygapophysis (restored in ib. fig. 1, from the right side at *z'*). This vertebra yields an entire length of 8 inches, the breadth anteriorly is 4 inches, the breadth of the posterior surface of the centrum is 2 inches 5 lines; with the odontoid complete the length of this vertebra would be nearly 9 inches.

The vertebrarterial canal commences within the anterior half of the neural canal, and emerges at the lower and lateral part of the fore end of the centrum within the exit canal (*e*, *e'*) of the second cervical nerve, near the confluence of the odontoid process; a forward continuation of the almost subsided ridge from the diapophysis (*d*) divides the nerve-outlet into an upper issue (*e*) for the dorsal, and a lower issue (*e'*) for the sternal divisions of that nerve, near the latter of which outlets the arterial canal opens, being concealed from outer view by the bony bridge.

Now this is precisely the condition of the nervous and arterial foramina in the vertebra dentata of *Auchenia* (Plate V. fig. 2, *e*, *e'*). In *Camelus* (ib. fig. 3) the dividing bridge is wanting; the antero-lateral part of the centrum of the second cervical, at its confluence with the odontoid, presents a large longitudinally elliptic depression, *e*, into the hind part of which opens the vertebrarterial canal, and into the fore part the wider nerve-foramen. The part of the neurapophysis anterior to this is narrower in *Camelus*, relatively much narrower than in *Auchenia*, with which in this respect *Palauchenia* agrees. In the second cervical of *Palauchenia* the hypapophysial ridge commences at the middle of the centrum, gradually deepening backward, and rather abruptly expanding into an obtuse subelongate tuberosity (ib. fig. 1, *hy*) reaching the posterior articular surface of the centrum. In this character *Palauchenia* agrees with *Auchenia* (ib. fig. 2, *hy*); in *Camelus* the hypapophysis (ib. fig. 3, *hy*) expands posteriorly and divides into a pair of tuberosities. The bridge dividing the upper and lower nerve-outlets is continued backwards, in *Palauchenia*, into the angle dividing the lower lateral from the upper lateral surfaces of the vertebra, and this obtuse ridge or angle is gradually produced outward to form the long but low diapophysis, *d*. In *Camelus* the lower border of the elliptic depression (Plate V. fig. 3, *e*) is produced, sharpened, and continued backward into the diapophysis (ib. *d*).

The neural spine (*n.s.*) in *Palauchenia* is a long low ridge, expanding posteriorly into a largish irregularly rough surface, which seems to have been simple or subbifid as in *Auchenia*; it is certainly not so abruptly expanded, nor is it so distinctly bituberculate as in *Camelus* (the contour of the neural spine of which is indicated by the dotted line in fig. 3, to *n.s.*).

The postzygapophyses are small in *Palauchenia* (Plate V. fig. 1, *z'*), showing similar proportions to those in *Auchenia* (fig. 2, *z'*); in *Camelus* (fig. 3, *z'*) they are more abruptly expanded and, as it were, pedunculate. From the outer side of the postzygapophysis a low ridge (fig. 1, *r*) extends obliquely downward and forward to the fore part of the diapophysis; in *Auchenia* this ridge is directed toward the upper nerve-outlet (fig. 2, *e*), and subsides before it attains thereto; in *Camelus* (fig. 3) the ridge is wanting. The loftier and stronger neural arch and spine of the second cervical in *Camelus* (fig. 3, *n.s.*) give it proportions more like those of ordinary Ruminants; in the longer and more slender form of the vertebra *Palauchenia* resembles *Auchenia.*

In an old Lama I have seen a pair of sharp longitudinal ridges (fig. 2, *s*) at the under part of the second cervical centrum, midway between the beginnings of the hypapophysis (*hy*) and the diapophysis (*d*); a low ridge on each side the beginning of the hypapophysis indicates the same relation to muscular attachments in *Palauchenia*: there is no trace of this character in *Camelus.*

The third cervical vertebra of *Palauchenia* (Plate IV. figs. 5 & 6, $_3$, and Plate VI. fig. 1) lacks, accidentally, the hind half of the neural arch with the postzygapophysis and the right pleurapophysis; sufficient, however, remains to well test the degree of its correspondence, respectively, with the same vertebra in *Auchenia* (Plate VI. fig. 2) and in *Camelus* (ib. fig. 3). In these existing genera the contrast and conformity respectively with the fossil, in the proportions of the third cervical, are greater than that in the second, the Camel's vertebra assuming more breadth and height in relation to its length. *Palauchenia* strikingly resembles the Auchenian type in the general shape and proportions of the third cervical. The length of the centrum is 7 inches 3 lines; the vertical dimension at the ends of the pleurapophyses (*pl*) is 3 inches 8 lines: in *Camelus* the length of the centrum is 6 inches 9 lines; the vertical dimension at the ends of the pleurapophyses (*pl*) is 5 inches 6 lines; as this is the place of greatest vertical diameter of the third cervical in both Camel and Lama, the mutilation of the fossil vertebra does not affect the application of this comparative admeasurement. The third cervical of a large Lama (ib. fig. 2) yields 4 inches 6 lines, and 2 inches 3 lines, in the above dimensions.

The fore part of the centrum is convex in *Palauchenia* in the degree it presents in *Auchenia*, and is less convex than in *Camelus*; in each of these the ball does not fit into a corresponding cup at the back of the second cervical centrum, but works in one contributed chiefly by the intervertebral concentric ligamentous substance. In the flatness of the hind surface of the centrum *Palauchenia* agrees with *Auchenia*; in *Camelus* it is convex at the periphery, slightly depressed at the centre.

The hypapophysial ridge (Plate IV. fig. 5, *hy*, & Plate VI. fig. 1, *hy*) commences in *Palauchenia* more in advance than in *Auchenia* (fig. 2, *hy*), which otherwise it seems to resemble; but the terminal tuberosity is broken off in the fossil.

The pleurapophysis (Plate VI. fig. 1, *pl*) resembles in position, direction, and length that of *Auchenia*; in *Camelus* (ib. fig. 3, *pl*) it is much produced, and is more inclined downward, the pair of these processes bounding a much deeper concavity at the lower half of the vertebra.

Both in *Auchenia* and *Camelus* the sharp hind border of the process (*pl*) is concave, and directly continued backward into the diapophysis, *d*. In *Palauchenia* the hind border appears to be less sharp, less concave; and if this be in any way due to abrasion, a more marked difference is plainly inherent; the border, passing backward, subsides on the under part of the centrum; the diapophysial ridge (Plate VI. fig. 1, *d*) is continued forward 7 lines above the pleurapophysial one, and subsides midway between the pleurapophysis (*pl*) and the base of the prozygapophysis, *z*. This process is less expanded at its articular part in *Palauchenia* (Plate VI. fig. 1, *z*) than in *Camelus* (ib. fig. 3, *z*), and resembles in shape and in the extent of the infero-external strengthening ridge that in *Auchenia*. The notch between the prozygapophyses is wider in proportion to its depth in *Palauchenia* and *Auchenia* than in *Camelus*; the depression above the base of the process in both existing genera is greater than in *Palauchenia*.

The diapophysis (*d*) has a long thickened outer border parallel with its base in *Palauchenia* as in *Auchenia*; in *Camelus* it is more triangular in form, terminating in a thick obtuse apex. The anterior outlet of the neural canal of *Palauchenia* resembles that in *Camelus*, being less elevated than in *Auchenia*.

The succeeding cervical vertebræ of *Palauchenia* (Plate IV. fig. 6, $_{4}$, $_{5}$, $_{6}$, $_{7}$) show the same general modifications as in existing *Camelidæ*, due, viz. to progressive expansion and shortening of the centrum to the seventh inclusive, to progressive elongation or enlargement of the pleurapophyses to the sixth inclusive, to the development of a pair of parapophyses on this cervical, and to the suppression of par- and pleur-apophyses on the seventh, with the superadded partial articulations (ib. $_{7}$, *ff*) on the hind surface of the centrum for the heads of the first pair of dorsal ribs.

In the minor characteristics differentiating *Auchenia* from *Camelus*, *Palauchenia* agrees with the former; of the instances of which may be noted the following:—the shape and greater relative size of the diapophyses (*d*) of the fifth cervical, the non-confluence in the sixth cervical (Plate VII.) of the parapophyses (*p*) with the pleurapophyses (ib. *pl*), which confluence in *Camelus* forms a single large deflected plate coextensive with the centrum.

Of modifications peculiar to *Palauchenia*, the thicker, more tuberous and more anteriorly directed pleurapophyses (*pl*) of the fourth, fifth, and sixth cervicals (Plate IV. fig. 6) are remarkable. In the fourth is repeated the character of the distinction and wide interval between the hind terminal ridge of the pleurapophyses (Plate IV. fig. 5, *pl*) and the front terminal ridge of the diapophysis (ib. *d*): in the fifth cervical a wide and deep depression marks this interval, of which there is no trace in the corresponding part of that vertebra in *Auchenia* or *Camelus*. In the sixth cervical the bases of the pleur- and par-apophyses (Plate VII. *pl*, *p*) coalesce beneath the diapophysis (ib. *d*) in a degree which makes a step toward the Cameline modification. The diapophyses are imperforate in the seventh cervical of *Palauchenia*: in that vertebra of the Dromedary's skeleton in the British Museum (673 *a*) the vertebral artery traverses lengthwise the base of each diapophysis; but in a specimen in the College of Surgeons' Museum (No. 3455) the vertebral arteries do not perforate any part of the vertebra. In both, as in the skeletons of the Camels, Dromedaries, and Lamas at the Jardin des Plantes, where I first (in 1831) observed the fact, the vertebral arteries enter the neural canal of the sixth cervical and perforate the neurapophyses, emerging forward at the inner side of the base of each prezygapophysis. In the seventh cervical vertebra of a Lama's skeleton in the British Museum, as in the corresponding one of that in the Surgeons' Museum (Catalogue of the Osteology, 4to, 1853, p. 578, No. 3487), the right diapophysis is perforated by the vertebral artery, the left one not.

In size and general shape the cervical vertebræ of *Palauchenia* recall those of *Macrauchenia*, but detailed comparison brings out greater differences than any of those above noted in *Auchenia* and *Camelus*.

The vertebra dentata of *Palauchenia* differs from that of *Macrauchenia** in being more slender, in having a lower neural spine, in the shorter diapophyses, and in the non-bifurcation posteriorly of the hypapophysis.

The third and fourth cervicals of *Palauchenia* differ from those of *Macrauchenia* in the longer, narrower, but thicker and more tuberous pleurapophyses, in the convexity of the anterior surface of the centrum; and this latter character distinguishes the succeeding cervicals from the corresponding vertebræ of *Macrauchenia*, in which that surface is less convex, being nearly flat at the middle part.

Thus, the general result of the comparison of characters of the vertebræ of the neck concurs with that of the dental characters in demonstrating the former existence in America of a Cameline Ruminant as large as the largest variety of living Camel or Dromedary, with closer affinities to the Lamas and Vicugnas, yet with such departures from the dental and osteological characters of *Auchenia* as seem to justify their indication by the generic or subgeneric term *Palauchenia*, here proposed for such extinct form of American Cameline quadruped.

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* Plate 6 & 7, Fossil Mammalia of the Voyage of the 'Beagle,' 4to, 1860.

Table of Dimensions of the Cervical Vertebræ in *Palauchenia*, *Auchenia*, and *Camelus*.

	Palauchenia.		*Auchenia**.		*Camelus*†.	
	in.	lines.	in.	lines.	in.	lines.
Second Cervical Vertebra.						
Length	8	10‡	5	3	9	6
Length of centrum	7	6	4	3	7	6
Vertical diameter at highest part of neural spine	3	7	2	0	4	8
Transverse diameter of hind end of centrum	2	5	1	2	2	5
Third Cervical Vertebra.						
Length of centrum	7	3	4	6	6	9
Breadth of hind end of centrum	2	6	1	3	2	7
Breadth across diapophyses§	4	8§	3	0	5	3
Fourth Cervical Vertebra.						
Length	8	9‖	5	4¶	7	9
Length of centrum	6	9	4	3	6	7
Breadth of hind end of centrum	2	9	1	4½	2	9¶
Fifth Cervical Vertebra.						
Length	9	6‖	5	0¶	7	10¶
Length of centrum	6	10	4	0	6	6
Breadth of hind end of centrum	3	0	1	6	3	3
Breadth across diapophyses	6	6	3	1½	5	3
Sixth Cervical Vertebra.						
Length	9	3‖	4	5¶	6	10¶
Length of centrum	5	9	3	6	5	6
Length of hind end of centrum	3	4	1	7	3	0
Breadth across diapophyses	7	3	3	4½	5	0
Seventh Cervical Vertebra.						
Length of centrum	4	4	2	10	5	0
Breadth of hind end of centrum	4	0	2	1	3	9
Breadth across diapophyses§	7	0	3	7	6	3

Professor LEIDY, as was remarked in the preliminary historical sketch of the discovery of fossil Camelines in America, has recorded interesting indications of the extension of the former range of this family of Ruminants northward of Central America.

The first of these indications to which I shall refer relates to the extinct form called "Procamelus."

The species on which the genus is founded is described as follows:—

Procamelus occidentalis, LEIDY.—"This genus and species are founded on several fragments of jaws, with teeth, of several individuals of an animal allied to the Camel, and about two-thirds its size. The posterior fragment of a lower jaw presents the same general form as in the corresponding part of the Camel, but is broader at the ramus in relation with its height than in the latter. The posterior coronoid process is well developed, and the upper part of the ramus is more strongly depressed externally than in the Lama (*Auchenia*). The body of the lower jaw is relatively deeper than in the Camel, though not so robust; and the two sides are co-ossified by a comparatively short symphysis.

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* Full-sized old *Auchenia lama*. † Full-grown large var. of *Camelus dromedarius*.

‡ Including the odontoid process, which is restored according to the proportions of that in *Auchenia*.

§ These are the only admeasurements which the state of the fossil vertebræ enables me to take for the purpose of comparison.

‖ From the fore end of the pleurapophysis to the hind end of the postzygapophysis.

¶ From the end of the prozygapophysis to that of the postzygapophysis, the pleurapophyses not extending, as in *Palauchenia*, in advance of the prozygapophyses.

"Six molar teeth form a closed row in the lower jaw, being two additional to the number in the Camel and Lama. The true molars and the last premolar have nearly the same form as the corresponding teeth of the Camel. The second premolar is a reduced one from that behind it; and the first premolar [*p* 2] has a laterally compressed ovate crown implanted by two fangs.

"In a small fragment of a lower jaw, in the middle of the hiatus, in advance of the closed row of molars, there is the fang of a tooth which appears to have been a caniniform premolar [*p* 1]. The mental foramen is just in advance and below the position of this tooth. A foramen likewise exists before the third premolar [*p* 4] of the closed row of teeth, corresponding to that more posteriorly situated in the Camel and Lama.

"As in the lower jaw, six molar teeth form a closed row in the upper jaw. The true molars, though much mutilated in the specimens under examination, appear to possess the same form as those of the Camel. The last premolar [*p* 4] is also like the corresponding tooth of the latter. The second premolar [*p* 3] is like the first one of the Camel, with the exception that it has the antero-internal fold of its crown as well developed as the posterior fold, which it joins at the base. The first premolar [*p* 2] is like the first one of the series in the Lama, having a trilobate, flattened oval crown.

	in.	lines.
"Length of upper molar series	4	8
Length of lower molar series	4	10
Length of upper true molar series	3	0
Length of lower true molar series	3	5."

(Leidy, 'Proceedings of the Academy of Natural Sciences of Philadelphia,' March 1858, p. 23.)

The Cameloid fossil from the "probably pliocene freshwater deposits of the Niobrara Valley, Nebraska," is thus shown to be not only of markedly smaller size than the Mexican *Palauchenia*, but to differ therefrom, as from *Auchenia* and *Camelus*, in a more important character, viz. the number of contiguous teeth forming the molar series in both upper and lower jaws. In this respect the *Procamelus* offers an interesting link in the transition from the cameline to the true *Ruminantia*, and an additional illustration of an approach to a more generalized type of dentition in a species existing at a tertiary period anterior in time to that in which the remains of *Palauchenia* were found. The dentition of *Palauchenia*, in the number of the molar series shown in the fossil from the Mexican post-tertiaries, exemplifies an intermediate step between the dentition of *Procamelus*, Leidy, and that of the existing *Auchenia*, Illig.

The second cameline subgenus indicated by the accomplished and assiduous American palæontologist is the "*Camelops kansanus*, Leidy" (Proc. Acad. Nat. Sc. Philadelphia, vol. vii. p. 172); it is described as follows:—"This genus and species are established upon a fragment of the anterior extremity of an upper jaw of an animal of the Camel family, discovered by Mr. HENRY PRATTEN, of New Harmony, Indiana, in the gravel drift of Kansas Territory.

"The specimen consists of portions of the left maxillary and intermaxillary bones, the latter of which contains the fang of a transformed incisor, or functional canine tooth as in the Lama. The intermaxillary bone is of very much more robust proportions than in the Lama or Camel. The inclination of its nasal border approaches more the horizon than in the Lama or Camel, apparently indicating the animal to have possessed a lower and perhaps a longer face than in either of the latter genera. The gingeval border is rugged as in its congeners, and it presents two irregular pits, apparently the alveoli of incisive germs. The fang of the functional canine contained in the intermaxillary bone is laterally compressed, conical, and is an inch and a half in length. From the orifice of its alveolus it is strongly curved upward and backward, nearly on a line parallel with the curved palatal margin of the bone.

"The crown of the tooth was directed downward and outward, and at the base it is ovate in section, with the narrow end posteriorly; it measures six lines and three-fourths wide, and three lines and three-fourths transversely. A small portion of remaining enamel indicates this to have been thin and smooth.

"The small remaining fragment of the maxillary bone attached in the fossil exhibits at its broken margin the portion of an alveolus, situated an inch and three-fourths behind the tooth contained in the intermaxillary bone. It has been about four lines in transverse diameter, apparently had a direction curving downward, forward, and outward from its bottom, and probably accommodated a true canine tooth, although the position is usually far back, a necessary condition, however, in the *Camelops* from the distance to which the fang of the functional canine tooth extended backward"*.

From the foregoing account of his materials much remains to be discovered in order to yield the generic and specific characters of *Camelops Kansanus*, Leidy. It is obvious that the description affords no grounds for identifying them with the Mexican fossils, the subjects of the present memoir. But it is of great interest to have such indications as Professor LEIDY has recorded of the geographical extension of Cameloid forms into the Nebraska and Kansas territories of the great northern division of the American Continent, and more especially after Don A. DE CASTILLO's discovery of Cameline remains in a locality of Central America. At the present day, as is well known, this restricted family of Ungulates is represented, in the New World, exclusively by the small Lamas, Pacos, and Vicugnas, wild or domesticated, in South America.

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* Journal of the Academy of Natural Sciences of Philadelphia, 2nd Series, vol. iii. 1856, p. 166.

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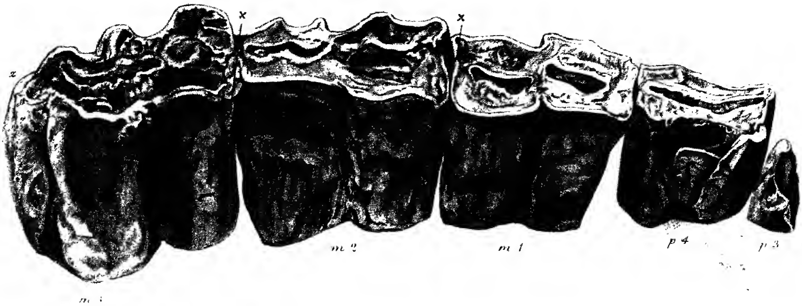
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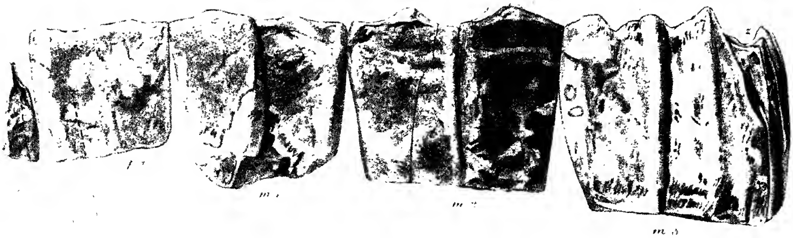
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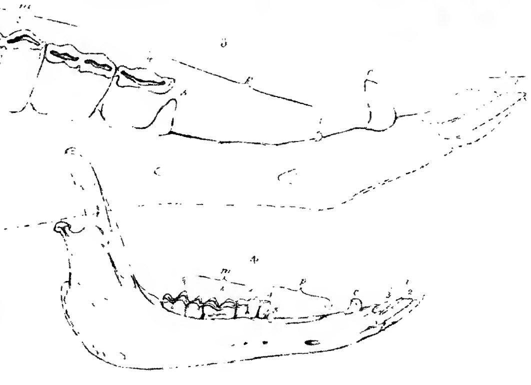
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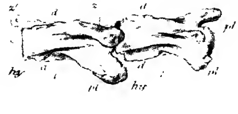
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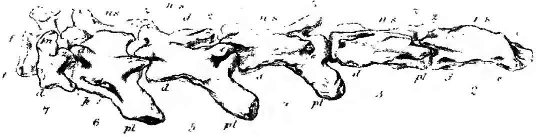
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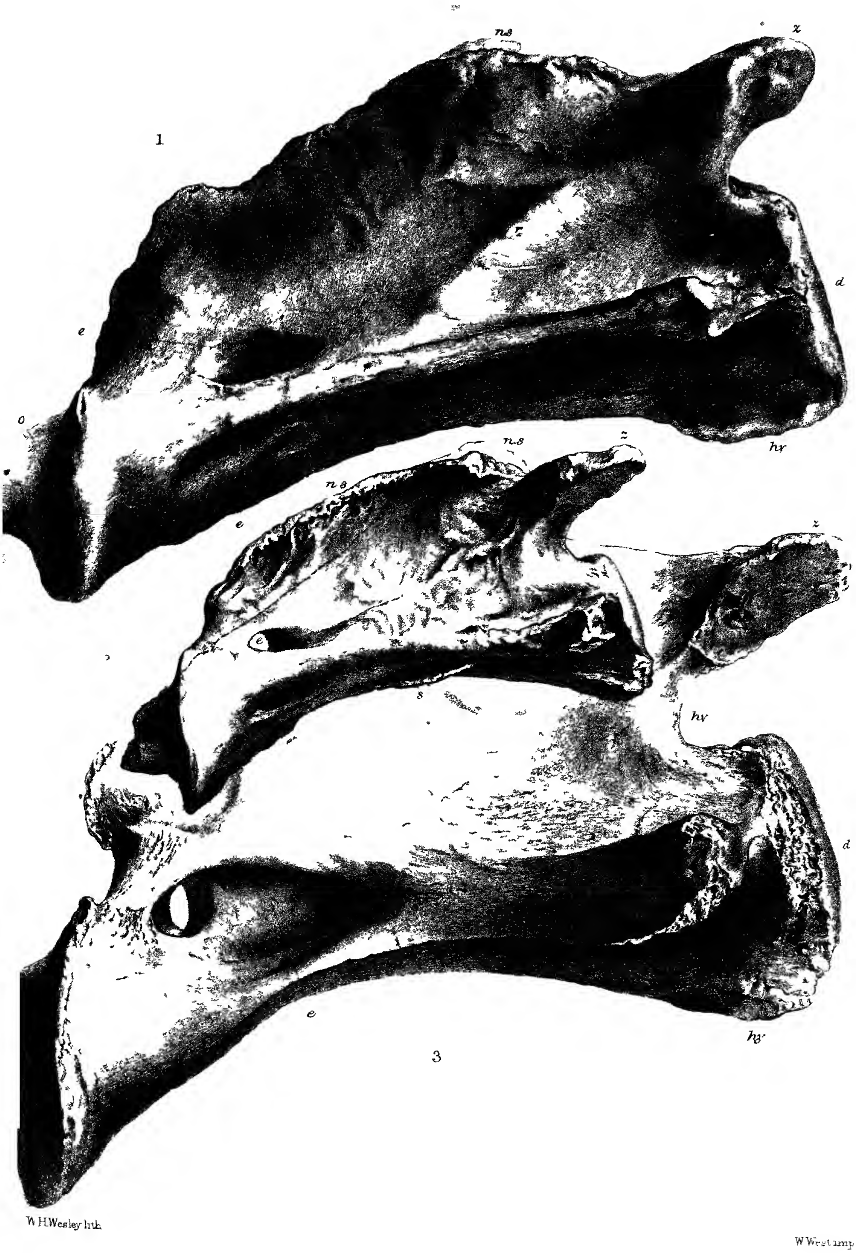
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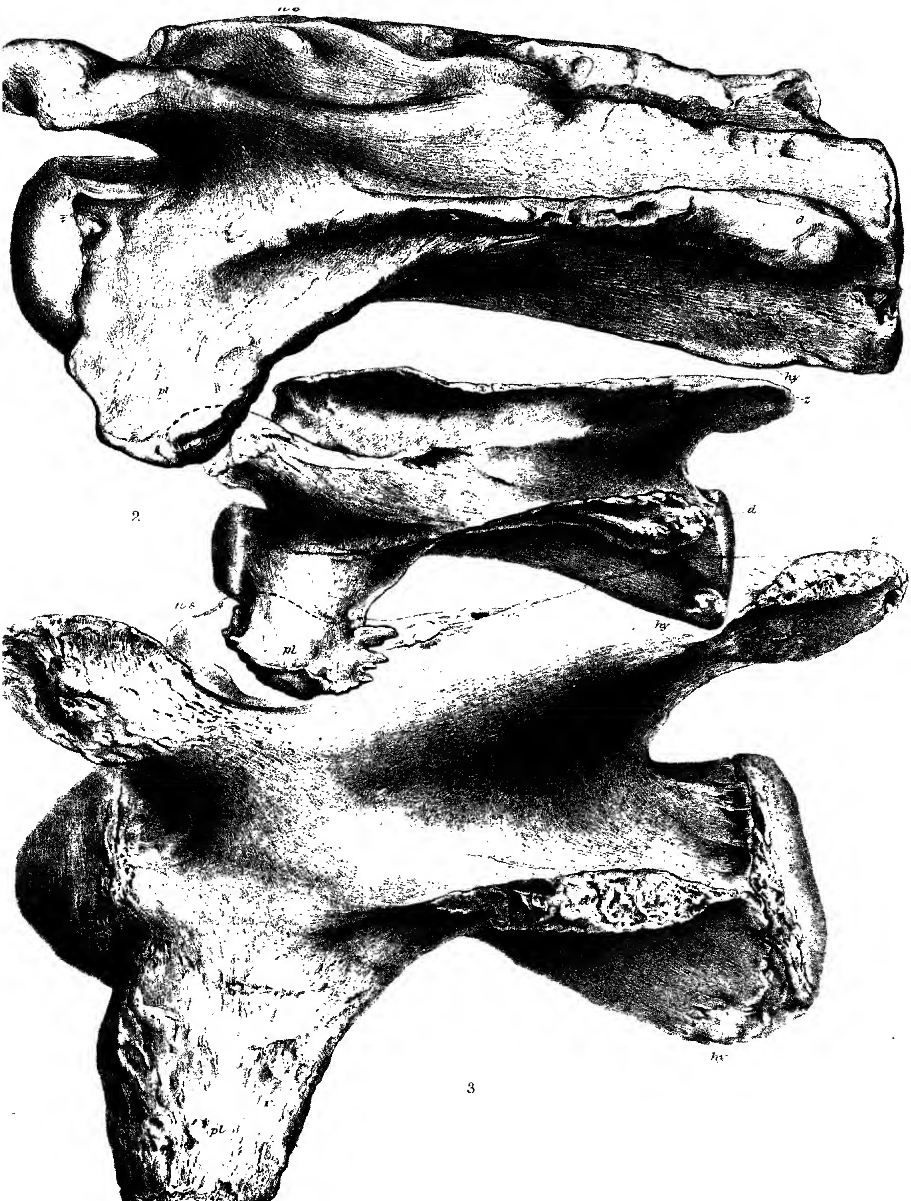
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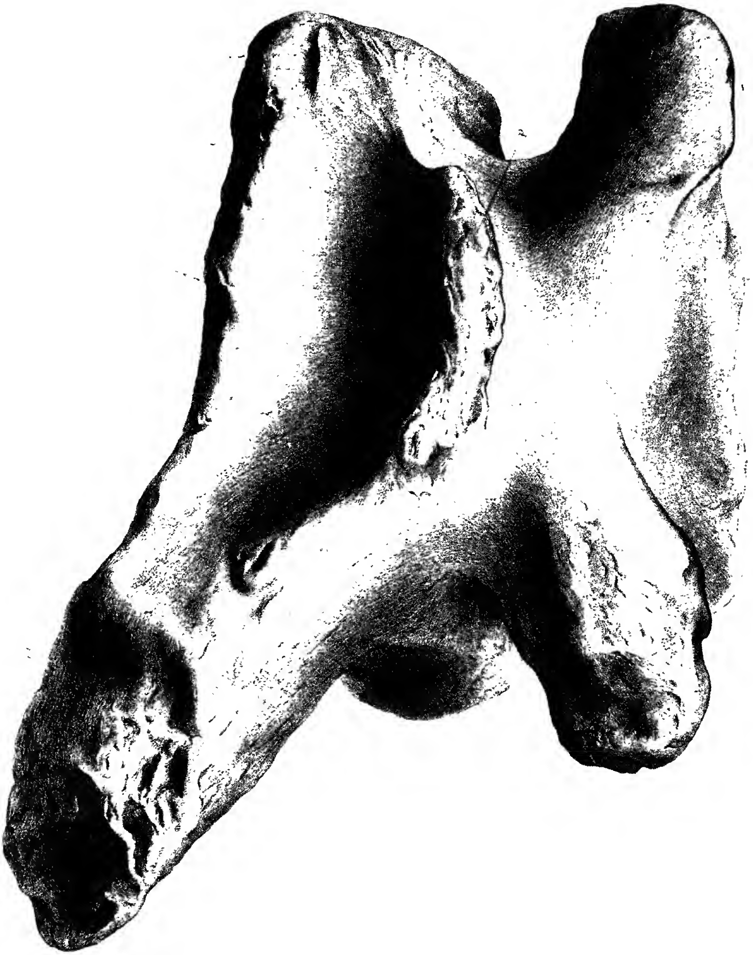

6th *cerv vert, Palauchenia*

Description of the Plates.

PLATE IV.

Fig. 1. Oblique view of the outer and grinding-surfaces of the molar series of teeth, *Palauchenia magna*, nat. size.

Fig. 2. Inner view of the same teeth, nat. size.

Fig. 3. Outer view of mandibular teeth (with mandible and incisors restored) of *Palauchenia magna*, two-fifths of nat. size.

Fig. 4. Outer view of mandible and mandibular teeth (*Auchenia vicugna*), two-fifths of nat. size.

Fig. 5. Oblique under view of the third and fourth cervical vertebræ of *Palauchenia magna* (one-eighth nat. size).

Fig. 6. Oblique upper view of the second to the seventh cervical vertebræ of *Palauchenia magna* (one-eighth nat. size).

PLATE V.

Fig. 1. Side view of vertebra dentata, *Palauchenia magna.*

Fig. 2. Side view of vertebra dentata, *Auchenia lama.*

Fig. 3. Side view of vertebra dentata, *Camelus dromedarius.*

PLATE VI.

Fig. 1. Side of third cervical vertebra, *Palauchenia magna.*

Fig. 2. Side of third cervical vertebra, *Auchenia lama.*

Fig. 3. Side of third cervical vertebra, *Camelus dromedarius.*

PLATE VII.

Side view of sixth cervical vertebra, *Palauchenia magna.*

The figures in Plates V.–VII. are of the nat. size. The letters &c. are explained in the text.

VI. *On the Molar Teeth, Lower Jaw, of* Macrauchenia patachonica, *Ow.*

By Professor OWEN, *F.R.S.*

Received April 21,—Read June 10, 1869.

THE dentition of a Mammal so rare and interesting as the *Macrauchenia* deserves better illustrations than the single reduced view of the lower molars given in 1845*, and the still more reduced figures of both upper and lower teeth lithographed by BRAVARD†.

The intention to communicate to the Royal Society a description with figures of the natural size of the specimen of mandible and teeth, still unique, in the British Museum, has been deferred in the hope of acquiring from South America other fossil remains, especially the upper jaw and teeth of *Macrauchenia patachonica*; but such fossils have not yet come under my observation. The recently obtained knowledge, however, of the former existence of another large quadruped in America, with cameline characteristics of the cervical vertebræ like those in *Macrauchenia*, coupled with true cameline affinities, as exemplified by the dentition of the lower jaw in *Palauchenia*, induces me no longer to delay the adequate record of the characters which so strikingly distinguish the perissodactyle from the artiodactyle forms of hoofed quadrupeds with the intraneural course of the vertebral arteries in the region of the neck.

The specimen here described formed part of a series of fossils from Buenos Ayres, purchased for the British Museum in 1845. I was requested by Mr. KÖNIG, the then Keeper of the Department of Mineralogy, to examine and report on that Collection, which chiefly consisted of Megatherian remains‡, and I was led by the conclusions which I had formed of the pachydermal affinities of the genus *Macrauchenia*, based on bones of the trunk and limbs described in the 'Fossil Mammalia of the Voyage of the Beagle'§, to recognize the mandibular specimen with teeth as belonging to that genus, and I accordingly figured it as such in the concluding part of my 'Odontography.'

The specimen (Plate VIII. figs. 1–3) consists of the part of the left ramus of the lower jaw of a full-grown individual, with six consecutive grinders, anterior to which the jaw is broken away, as is also the hind end of the ramus about 3 or 4 inches behind the last grinder. The first tooth in place answers to the second premolar (Plate. VIII. figs. 1–3, *p* 2) of the typical series. It is implanted by two fangs, supporting a lamelliform crown, the compression being from side to side, or from within outwards, and the extension of the crown from before backwards. In this direction the crown expands as it rises to an antero-posterior breadth of 30 millims. (=1 inch 2 lines), whence it contracts, rising to a submedian obtuse apical summit. The outer side of the crown is convex from its fore margin to two-thirds of the way back, then becomes concave to a vertical ridge, *b*, marking off a short posterior tract of the crown which inclines inward and is almost flat. From this tract the continuation of enamel bends abruptly inward and forward (fig. 2, *c*), rapidly sinking to a mere basal ridge, continued along the inner side of the crown into the similarly bent anterior border of the crown, *e*. The concavity bounded by those inwardly inflected borders of the crown is divided into two by the prominence of the thickened mid parts (*d*) of the crown forming its apex, *a*. The abraded surface of this tooth forms a sinuous tract of dentine, thickest at the middle, thinnest behind (fig. 3, *p* 2).

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* OWEN's 'Odontography,' pl. 135. fig. 7, p. 602.

† Published by BURMEISTER, in the 'Anales del Museo Público de Buenos Aires,' Entrega Primera, 4to, 1864, pl. 1.

‡ See OWEN's 'Memoir on the Megatherium,' 4to, 1860, p. 11.

§ 4to, 1840, pp. 35–56, pls. vi.–xv.

The next tooth (*p* 3) resembles *p* 2, with increase of thickness, but none of fore-and-aft extent. The facet of the crown behind the outer ridge (fig. 3, *b*) passes more directly inward, so as to form the posterior part of the crown. The inner wall (*c*) is more abruptly continued from it, subsiding to the ridge crossing the base of the mid inner convexity (*d*), to which the anterior inflected fold of enamel (*e*) is continued. The convexity (*d*) is broader and rather flattened between the better defined hollows of that surface of the crown. This tooth is implanted by two fangs.

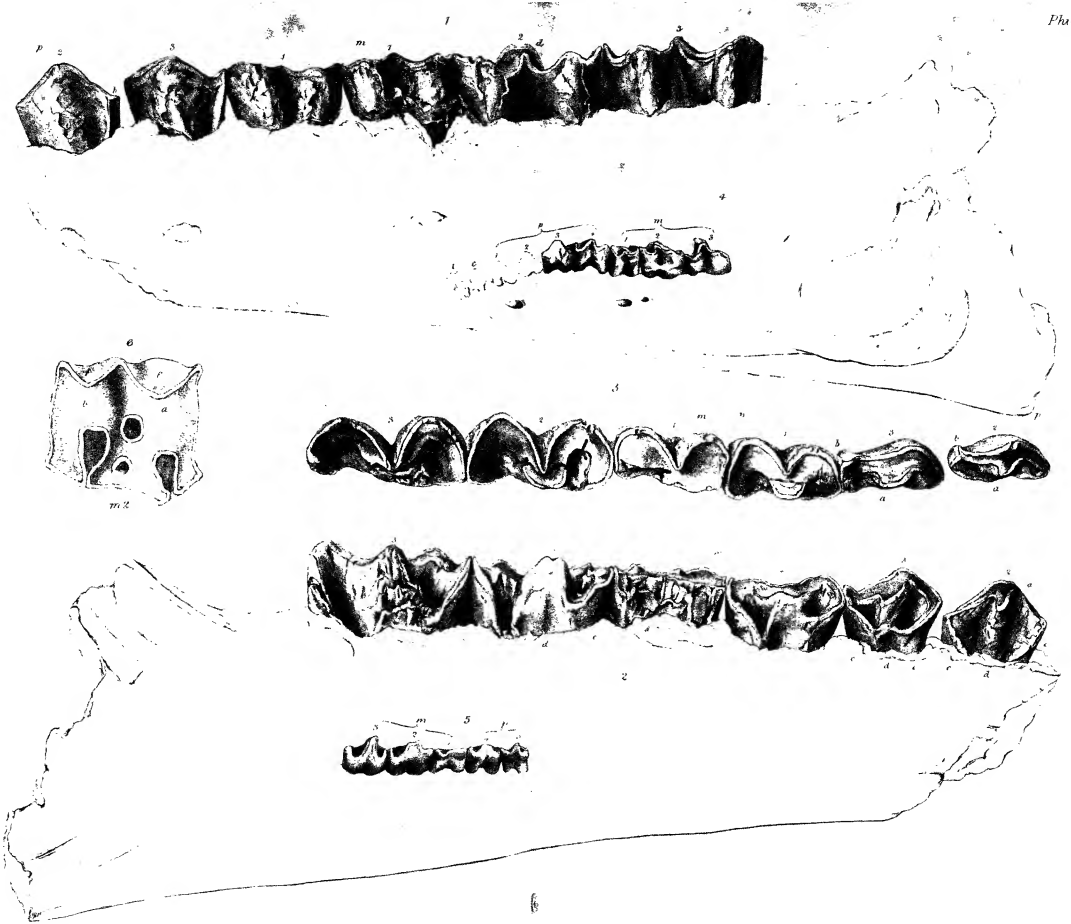
The third tooth (*p* 4) adds increase of fore-and-aft extent to that of thickness of the crown, and also resembles the succeeding true molars in being divided into two lobes by a vertical indent of the outer surface (fig. 1, *p* 4) entering the inner mid convexity, fig. 3, *d*. This gives to the grinding-surface (*p* 4, fig. 3) the form of two consecutive crescents, of which the hinder one is the largest; such being the type of lower grinder common to *Rhinoceros* and *Palæotherium* with this singular South-American Perissodactyle.

The first of the true molars (*m* 1) exemplifies its earlier development and longer usage by having the crown worn down below the convexity and concavities on the inner surface; a broad bilobed tract of dentine shows the outer mid indent, with a remnant of the postinternal cavity (fig. 3, *m* 1): the enamel is now reduced to a very thin line along the anterior and inner sides of the crown.

In the penultimate molar (*m* 2) the sinuous configuration of the inner surface of the crown is preserved, the enamel of the mid convexity rising to form the most prominent part of the grinding-surface (figs. 1 & 2, *d*). The tooth is increased in all dimensions.

The last molar (*m* 3), with a fore-and-aft extent of crown of 48 millims. (=1″ 10‴), preserves the same bilobed type as in *Rhinoceros*, without any trace of a third posterior lobe or 'talon,' as in *Palæotherium*. The lobes have been worn by mastication to a breadth of two or three lines, and as the inner enamel-wall is continued by the inflected and subsiding fore and hind borders to the inner basal ridge, the crescents of the masticating surface have an enamel border only on their outer or convex sides.

Thus *Macrauchenia* differs from *Rhinoceros* and *Palæotherium* in the limitation of


1
p
2
3
1
m
1
2
d
3
4
p
m
1
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assumption by the premolars of the molar type of grinding-surface to the last of the premolar series, the antecedent premolars retaining the single-lobed crown; from *Palæotherium* it further differs, as before observed, in the retention by the last molar of the two-lobed structure. In Artiodactyles, and especially the ruminant section, all the premolars have a simpler structure with the smaller size of crown.

From the figures by Bravard* we learn that *Macrauchenia*, like many other tertiary mammals, retained the type dentition, viz. $i\,\frac{3-3}{3-3}$, $c\,\frac{1-1}{1-1}$, $p\,\frac{4-4}{4-4}$, $m\,\frac{3-3}{3-3}=44$, and that, as in *Anoplotherium* and *Dichodon*, the series was unbroken by any notable interval, not any of the teeth having a crown much higher or longer than the rest.

Description of the Plate.

PLATE VIII.

Fig. 1. Portion of left mandibular ramus with the *p* 2–*m* 3 grinders: outside view.

Fig. 2. do. do. do. : inside view.

Fig. 3. Grinding-surface of the teeth. These figures are of the natural size.

Fig. 4. Entire mandible with the *p* 3–*m* 3 grinders, and a restoration of the missing teeth: outside view, reduced view: from Bravard, *op. cit.*

Fig. 5. *p* 3–*m* 3, inside view; similarly reduced: from Bravard, *op. cit.*

Fig. 6. Grinding-surface of second upper molar (*m* 2), right side, restored to the natural size: from Bravard's plate. [The upper molars (*m* 1, *m* 2) of *Macrauchenia*, as of *Nesodon*, are penetrated by three folds of enamel on their inner side, which, deepening as they extend, are soon interrupted by wear, and the ends converted into islands. In *m* 2 (Plate VIII. fig. 6) the mid fold is of unequal depth and becomes reduced to two islands of enamel (*e*, *h*). In *m* 1 the island (*h*) of Bravard's specimen is not shown, but this may be because the tooth is more worn. In *m* 3 the insular end of the fold (*k*) remains beyond the fold, which continues as in *m* 2: the mid fold is represented by a single island, and the third fold (*d*) is short. The outer lobe (*f'*, *b*) is turned, as usual, in *m* 3, so that the surface (*f'*) looks outward and backward, with concomitant contraction of the hind part of that molar.

The last premolar (*p* 4) is like a small molar, but shows only two enamel-islands on its inner half: *p* 3 has a mid indent on the outer side and indications of two shallow folds on the inner side; it has lost breadth, but retains fore-and-aft extent; *p* 2 is still more narrowed transversely, has an even outer surface, and a single fold of enamel on the inner one which runs forward. This fold is represented by a notch in *p* 1. The premolars decrease in breadth, and in a minor degree from before backward, as they approach the canine. —R. O., March 1870.]

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* *Op. cit.*

VII. *On a Group of Varieties of the Muscles of the Human Neck, Shoulder, and Chest, with their transitional Forms and Homologies in the Mammalia.* By John Wood, *F.R.C.S., Examiner in Anatomy at the University of London; Professor of Surgery in King's College, London, and Surgeon to King's College Hospital. Communicated by Dr.* Sharpey, *Sec. R.S.*

Received June 17,—Read June 17, 1869.

In the 'Proceedings of the Royal Society,' between the years 1864 and 1868 inclusive, were published five series of observations upon variations in Human Myology, made in the Anatomical Rooms of King's College, London. These observations extended over 202 dissected subjects; they were restricted chiefly to the muscles of the head and neck and those acting upon the extremities, and did not include the numerous irregularities which are usually found in those of the back.

From the extensive range of the subject, and the importance of ascertaining as far as possible the statistical frequency of the abnormal forms, there was little opportunity afforded of giving due prominence to many of the specimens which were entitled to special distinction, either from their first appearance in the records of anatomical science, or from their homological importance as transitional forms, or as representatives of muscles hitherto found only in certain classes of animals.

Many others had, indeed, been recorded by the older or more modern writers under various names, as irregularities of the muscle with which they were connected or contiguous. A great number of these, I believe, were mere varieties of the same transitional specimens, and were placed in this series for the first time in their proper relation and true homological significance to each other.

After some years of practical observation and bibliographical research, I have brought together some of the more interesting groups of these muscular varieties in illustration of the working of the law of variation in modifying muscular formations in the human subject; producing, in some instances, muscles which appear at first to be remarkably aberrant from the ordinary human construction, and identical in some with those which have usually been considered as peculiarly animal formations.

From this point of view one of the most interesting groups of muscles is that of the muscles which connect the *neck* and *shoulder*. From the variability of the bones of the shoulder-girdle and the dissimilar habits and requirements of the different classes of animals, it is not surprising to meet with a perplexing variety in the different portions of their muscular apparatus; but it is striking to find that a great tendency to the same kind of variability is to be found in the shoulder of Man, in which it exists to an extent which is only surpassed by that of the muscles of the fore arm. The reason for the latter can, from the variety of the uses and functions of the upper extremity, be better comprehended than in the case of the neck and shoulder, which would seem to demand much less various muscular appliances from a teleological point of view, and their varieties therefore might be considered to have a more decided morphological bearing.

I will first take the varieties of one of those groups which has for its function in the human subject to raise the shoulder-girdle behind and before, and to rotate it on the central axis of the shoulder-blade. In animals, the group draws these bones forwards and upwards, or downwards, and they are usually more movable in them than in Man. In both they also act conversely upon the cephalic extremity, either directly or through the neck-vertebræ. In the lower animals we shall find that the muscles of the group frequently obtain a longer leverage by becoming connected lengthwise, or are even blended with other muscles or portions of muscles acting in the same direction, and so obtain a greater play of motion and an increased power of a more direct kind upon the fore limbs and head.

We find in these muscles some of the best examples of what I have called in my former papers, *lateral* and *longitudinal fission*, and *fusion*, of adjacent muscles acting in the same general direction in regard to the axis of the limb.

The normal muscles constituting this group range from as far back, and as deep, as the *splenii* and *serrati*, through the *rhomboids*, the *trapezius*, and the *levator anguli scapulæ* to the *sterno-cleido mastoideus* of human anatomy.

The abnormal human muscles include those which I have described briefly in various previous publications in the 'Proceedings of the Royal Society' under the names of the *occipito-scapular*, the *levator claviculæ*, and the *cleido-occipital*, reaching as far back as that division of the *splenius* which has been called by WALTHER the *adjutor splenii*, and by MACALISTER the *rhombo-atloid*.

Occipito-scapular Muscle.—In the winter of 1866–67 I found in a muscular male subject, on both sides, a muscle extending from the occiput to the base of the scapula, under the trapezius, which I described in the 'Proceedings of the Royal Society' for May 1867, under the name of the *occipito-scapular* muscle. It was a distinct ribbon-shaped muscle, three-quarters of an inch wide, nearly a quarter of an inch thick, and 10 inches long (Plate IX. fig. 1, *d*), attached by a musculo-tendinous origin to the occipital bone on a level with the *splenius capitis* (*h*), and directly under the line of junction between the *trapezius* (T) and a variety of the *sterno-cleido-mastoid*, which was also present, and which I have named the *cleido-occipital* (*c*). Passing downwards and outwards, obliquely across the *splenii*, and between them and the *trapezius*, it became inserted by short tendinous fibres, superficial and opposite to the *rhomboideus minor* (*r*), into the vertebral border of the scapula opposite the base of the spine, its fibres being more or less blended with those of the rhomboids.

Since that time evidences of the muscle have been sought for in fifty subjects. The nearest formation to the complete muscle was again found, last session, in a muscular male. It existed on both sides. It was a muscular slip 7 inches long, and a quarter of an inch wide, on the right side, but rather shorter on the left. On both sides its lower attachment was identical with that first found, viz. to the base of the scapula opposite the spine (Plate IX. fig. 2, *d*, *d*). On the right side its upper attachment was to the transverse process of the atlas, along with and superficial to the first digitation of the *splenius colli* (*h′*); on the left side it ended above in the fascia covering the *splenius capitis* (*h*) opposite to the spines of the second and third cervical vertebræ, and was continued onward to the occiput by the intervention of this fascia only. On the right side there coexisted another peculiar muscular slip, having a similar action upon the scapula (*θ*); it lay beneath the rhomboids, arising by a fascial tendon from the spinous process of the seventh cervical vertebra, and passed downwards and outwards in a muscular band, 7 inches long by half an inch wide, crossing obliquely between the fibres of the *serratus posticus superior* (*s*) and the *rhomboideus major* (R), and was inserted by a short tendon into the lower angle of the scapula, opposite the origin of the *teres major*. It was absent on the left side. The muscle lay almost entirely parallel to the *occipito-scapular*, with which a simple upward displacement would bring it into exact coaptation, and the action of their fibres was identical, viz. that of rotation of the scapula upon its axis and elevation of its superior angle.

The next gradation was found in a muscular male subject, on the left side only (Plate IX. fig. 3). It was a fusiform slip of muscle (Δ) attached by a thin falciform aponeurosis to the fascia covering the splenius capitis (*h*), under cover of a *cleido-occipital* muscle (*c*), which coexisted in this subject also. The fusiform belly was 3½ inches long, and was placed parallel and superficial to the outer fibres of the splenius capitis, ending below in a spreading aponeurosis, which was implanted upon the superficial aspect of the *serratus posticus superior* (*s*), close to its tendon, and under the *rhomboideus major*. The rhomboids were also largely developed and united at their contiguous margins by another cross slip, passing from the spinal origin of the *major* to the scapular insertion of the *minor*. In the same subject and on the same side were found two other slips, given off from the lower margin of the *levator anguli scapulæ* (*f*) from the fourth cervical transverse process. The innermost one (*η*), the larger, was somewhat fan-shaped, spreading downwards to be inserted, by its inner fibres, into the fascia covering the *serratus posticus superior* (*s*) about the second or third rib, and by its outer fibres into the fascia covering the hinder surface of the *serratus magnus* (S), close to its insertion into the middle of the vertebral border of the scapula. The lesser or outermost slip (*ε*) came off from the border of the muscle like the other, and was inserted wholly into the upper fibres of insertion of the *serratus magnus*.

A somewhat different formation, having the same general character, was seen in another male subject, on the left side only (Plate IX. fig. 4). Two slips of muscle of about the same size as the foregoing (*ε* and *η*) were here found. The upper one (*η*) was attached above, in common with and between the atloid attachments of the *splenius colli* (*h′*) and the *levator anguli scapulæ* (*f*); it lay along the outer border of the is only surpassed by that of the muscles of the fore arm. The reason for the latter can, from the variety of the uses and functions of the upper extremity, be better comprehended than in the case of the neck and shoulder, which would seem to demand much less various muscular appliances from a teleological point of view, and their varieties therefore might be considered to have a more decided morphological bearing.

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A somewhat different formation, having the same general character, was seen in another male subject, on the left side only (Plate IX. fig. 4). Two slips of muscle of about the same size as the foregoing (*ε* and *η*) were here found. The upper one (*η*) was attached above, in common with and between the atloid attachments of the *splenius colli* (*h'*) and the *levator anguli scapulæ* (*f*); it lay along the outer border of the former, and ended below by dividing opposite the last cervical vertebra into two, which were both implanted by separate short tendons into the fascia covering the *serratus posticus superior* (*s*), the innermost near the junction of its tendon and fleshy belly. The other was exactly similar to the smaller one (ε) described in the last figure, and was inserted, like it, into the *serratus magnus* close to its insertion into the base of the scapula opposite the spine.

All the abnormal slips of muscle just described were apparently developed in the fascia covering the *hinder* surfaces of the *splenii*, *levator anguli scapulæ*, and *serrati* muscles. The one marked Δ in fig. 3 had an origin quite similar to that of the *occipito-scapular* found on the right side in the subject of fig. 2 (*d*); its insertion, however, took place upon the surface of the *serratus posticus superior* instead of upon the *rhomboid* or more superficial muscular layer, which it had, as it were, failed to reach. The other slips (ε and η) were directly connected with the scapula through the insertion of the *serratus magnus*, as was also the slip (ε) found in the subject of fig. 4.

In the subject of the next illustration (Plate IX. fig. 5), a muscular male, were found muscular slips (ε and η) of a precisely similar character, but connected rather with the *front surface* of the *levator anguli* than with the back. The upper and longer one (η) arose with the third digitation of the *levator anguli scapulæ*, but in front of it, and passed downwards, in *front also* of the fourth slip of origin of that muscle, as a fusiform muscle about 8 inches long, which was inserted by a falciform tendon into the lower part of the base of the scapula, between the insertions of the *rhomboideus major* (R) and the *serratus magnus* (S). The other slip (ε) was similar in its upper attachment to that marked with the same letter in the previous figures, but was inserted below into the *serratus posticus superior* (*s*), instead of into the *serratus magnus*, as was the case with the others.

In two other subjects, both females, were found also slips of muscle connected with and lying upon the *front surface* of the *levator anguli scapulæ*. In one it was found on the right side only, arising from the third cervical transverse process, as a fusiform muscle, which was inserted into the *hinder* surface of the upper digitation of the *serratus magnus* near its scapular attachment. In the other, the subject of fig. 6 (Plate IX.), the muscular slip (η) arose from the second cervical transverse process, was placed *in front of* the *levator anguli scapulæ*, and was inserted also in *front* of the upper digitation of the *serratus magnus*, upon the fascia covering which its tendon was lost in the axillary space; it was likewise fusiform in shape and about 6 inches in length.

I have now followed these curious transitional slips of muscle in the changes of their upper attachment from the superior curved line of the occipital bone, through the *splenius capitis* and *colli*, to the hinder surface, and, at last, to the front surface of the vertebral origin of the *levator anguli scapulæ*.

At their lower attachments we can trace them also from the base of the scapula, through the insertion of the *rhomboids* and the *serratus posticus superior*, to the back surface, and, at last, to the front surface of the *serratus magnus*. Here we link on a human muscular abnormality, also connected with the front surface of the *levator anguli* *scapulæ*, viz. the "*levator claviculæ*," which we shall find, when we discuss its homologues in the lower animals, to have something of a correlation with the *occipito-scapular*, placed at the opposite pole from which we started.

Before following this direction, however, I will notice a further indication of transmutation in the direction of the *splenius* muscle, *i. e.* backward, which may be considered to have some relation to the formation of the *occipito-scapular* in the human subject. In HALLER's 'Disputationum Anatomicarum Selectionum' (1733, vol. vi. p. 589), F. WALTHER gives a description of a muscle connected with the *splenii*, under the name of the "*musculus singularis splenii accessorius*" vel "*adjutor splenii.*" This muscle corresponds entirely with a variety I have found in four male subjects out of 41, and in three females out of 29, in a total of 70, in the years 1866, 1867, and 1868 (given in the 'Proceedings of the Royal Society' of these years respectively). One of the females was the subject of fig. 7 (Plate IX.), in whom the variety was found as a flat, ribbon-shaped muscle (*g*), lying upon and parallel with the fibres of the *splenius colli.* Its upper attachment was placed upon the transverse process of the atlas, between the tendons of the *levator anguli* (*f*) and *splenius colli* (*h'*). It was hardly to be distinguished at this place, without dissection, from the fibres of insertion of the latter muscle; but, as it descended, it became more distinct and differentiated, and was finally separated from both the *splenii* by the intervention of the tendon of the *serratus posticus superior* (*s*). It was about 6 to 7 inches long, by ½ an inch wide, passed under the *rhomboideus minor* muscle (*r*), and ended below in spreading tendinous fibres, which became blended, partly with that of the origin of the *rhomboideus major* (R), and partly with that of the *serratus.* The muscle has been described also by Mr. A. MACALISTER under the name of the "*rhombo-atloid*" (Notes on Muscular Anomalies in Human Anatomy, in the 'Proceedings of the Royal Irish Academy, April 23rd, 1866).

This formation, I believe, indicates the first loosening or differentiation of the *splenius* muscle in the direction of the formation of an *occipito-scapular* in the human subject. We shall find that, in the Mole, the whole of the *splenius* may become attached to the scapula (see Plate X. fig. 11, *h*).

In the subject of fig. 7 was also found a triple insertion and an extended origin of the *levator anguli scapulæ.* The cervical digitations of the muscle were continued down as low as the sixth transverse process. The three lowest digitations were inserted by two separate slips (F), the uppermost (larger) into the base of the scapula opposite the supraspinous fossa; and the lowest (smaller), conjointly with and superficial to the tendon of the *rhomboideus minor* (*r*), into the triangular commencement of the scapular spine. This formation indicates an approach to that found in some of the Carnivora, in whom the muscle corresponding to the human *levator anguli scapulæ* is inserted into the scapular spine close to the *occipito-scapular* (see Plate X. figs. 14 & 15, *f*, *f*).

I next proceed to describe a highly important human muscular variety, the *levator claviculæ*, to which I have before referred, the formation of which the slips previously described and shown in figs. 5 & 6 evidently foreshadow, and which, it may be said, constitutes the opposite pole of the line of departure here traced from the *occipito-scapular* muscle.

Levator claviculæ Muscle.—In my paper read before the Royal Society in 1864, I furnished a description and drawings of *levator claviculæ* muscle which I had found in two male subjects, on both sides.

In Man it usually assumes, as in animals, the form of a flat ribbon-shaped muscle. It is commonly about an inch to an inch and a half wide, arising fleshy by two or more digitations from the first and second, second and third, or third and fourth cervical transverse processes, in common with and in front of the origin of the *levator anguli scapulæ*.

Crossing the posterior triangle (Plate IX. fig. 8, *e*) obliquely downwards, outwards, and forwards, it is inserted tendino-fleshy into the middle or outer third of the clavicle to the median side of, or behind, the fibres of insertion of the *trapezius* (T). We have found this muscle in 4 males out of 131, and in 1 female out of 71, in a total number of 202 subjects. In three it was on both sides, and in two on the left side only. In one other male subject it was found to arise from the third cervical transverse process on the left side only, and, just failing to reach the clavicle, was inserted into the fascia immediately behind it, covering the axillary surface of the upper digitation of the *serratus magnus* muscle and the plexus of nerves lying upon it. This gives an average of about 3 per cent. in which this muscle has been found; in half on both sides, and in the other half on the left side only.

In looking up the bibliography of this subject, I have found that in 1813 W. G. Kelch described that he had seen, on the right side of a male subject, in whom both the *omohyoids* were wanting, a slip of muscle, a quarter of an inch wide, attached above to the sixth cervical transverse process, and below to the back edge of the clavicle near the acromial end (Beiträge zur pathologischen Anatomie, Berlin, 1813, xxiv. S. 32). This is quoted by F. G. Theile, who also describes that he saw on the right side of a male a muscle, 1 inch wide, attached above to the fourth and fifth cervical tranverse processes, between the *rectus capitis anticus major* and the posterior *scalenus*, and below to the middle of the clavicle at its upper border. It was considered by him as a variety of the *anterior scalenus* muscle (Traité de Myologie, in Sömmerring's Encyclopédie Anatomique, traduit par J. L. Jourdain, 1843, p. 153). In 1846 Macwhinnie described this muscle as a variety of the *levator anguli scapulæ*, arising from the first cervical transverse process, and inserted, in one case, into the outer end of the clavicle with the *trapezius*, and, in a second instance, into the middle of the clavicle close to the *sterno-cleido-mastoid* (London Medical Gazette, January 30th, 1846, p. 194).

In 1847 Professor W. Gruber, of St. Petersburg, described a muscle attached above to the second cervical transverse process, and below to the middle of the hinder border of the clavicle (in two subjects out of 40), as a variety of the *trapezius* (Vier Abhandlungen, S. 22). This observation is quoted by Henle in his last edition of the Handbuch (Muskellehre, S. 110) with the remark that it is doubtful whether it is to be considered as a variety of the *omo-hyoid*, *levator anguli scapulæ*, or *scalenus anticus*; and implying apparently that it may be referred to the *sterno-cleido-mastoideus*.

In 1866 Mr. A. MACALISTER describes it, in a spare female subject, as inserted under the *trapezius* into the outer third of the clavicle (*op. cit.* p. 7).

Besides the foregoing fully formed specimens of the muscle, we have repeatedly found imperfect slips from the cervical attachments of the *levator anguli* and *anterior* or *posterior scalenus*, which were inserted below into the axillary fascia behind the clavicle. Through these slips we travel towards and meet with those which I have before described in connexion with the *levator anguli* and *serratus magnus*, and thus we have a series of transitional forms intermediate between the *levator claviculæ* and the *occipito-scapular* muscles. Specimens of this kind have also been recorded by J. F. MECKEL, viz. a slip passing from the *levator anguli* to the second rib, where it was connected with the *serratus magnus*. He adduces it as an index of the more complete blending of these muscles in the lower animals (Archiv, viii. S. 585, and Muskellehre, 1816, Bd. ii. S. 402). He also quotes J. C. ROSENMÜLLER for a specimen in which a slip of muscle passed from the transverse process of the atlas to the *serratus magnus* (De nonnullis musculorum corp. hum. varietatibus, Leipzig, 1814, S. 5). KELCH saw, in a female, a tripartite division of the *levator anguli scapulæ*, the middle slip sending off from its hinder border an insertion into the *scapulo-thoracic* fascia (*op. cit.* xxv. S. 33).

More recently, Mr. FLOWER and Dr. MURIE found, in a Bushwoman, a slip proceeding from the *levator anguli scapulæ* to the axillary surface of the *serratus magnus*, which they considered to be an indication of the *levator claviculæ* muscle (Journal of Anatomy and Physiology, No. 2, May 1867, p. 199).

The first beginning of a differentiation of the fibres of the *levator anguli scapulæ* we have not unfrequently found as a double muscle. It is thus figured in CUVIER and LAURILLARD's plates of the muscles of the Negro, the anterior division being described as the homologue of the "*acromio-trachélien*" of the lower animals. MACALISTER has also found this muscle double (*op. cit.*).

Cleido-occipital Muscle.—Under this name I described, in a paper published in the 'Proceedings of the Royal Society' in June 1866, an abnormal human muscle placed along the hinder border of the *sterno-* and *cleido-mastoids*, from both of which it is separated by a more or less wide areolar interval, and sometimes joining, above or below, the fibres of the *trapezius*. Its width is from half an inch to an inch and a half, most commonly being about three quarters of an inch. It is made up of parallel muscular fibres, attached above to the superior curved line of the occipital bone by a thin muscular aponeurotic termination between the occipital attachments of the *sterno-mastoid* and *trapezius* muscles. Below it is connected with the back edge of the clavicle about its middle third, extending, when large, nearly or quite to the insertion of the *trapezius*, diminishing much the size of, and crossing or covering in the posterior triangular space (Plate IX. fig. 9, *c*). It is distinguished from the *cleido-mastoid* by its superficial position, by the more oblique and more backward direction of its fibres, and by its attachment to the superior curved line of the occipital bone, as well as by a more or less marked areolar interval between the two. The fibres of the latter muscle are directed from the clavicle upwards and forwards in such a manner as to become, in the upper part of the neck, entirely covered by those of the *sterno-mastoid*, which form here the entire posterior border of the compound human *sterno-cleido-mastoid* muscle, to the exclusion of those of the *cleido-mastoid*, which pass more deeply and are directed more forwards to the mastoid process.

We have found the *cleido-occipital* muscle more or less developed in 37 subjects out of 102, viz. in 27 out of 68 males, and in 10 out of 34 females. In 34 it was found on both sides, and in 3 on one side only. In two instances in which it was found on one side only, there existed on the opposite side a *levator claviculæ* muscle, showing a sort of correlation between the two formations. In two instances the muscle was double, or divided into two distinct portions. In two more male subjects the formation was so peculiar as to merit a more detailed description; one of them was the subject of fig. 29, Plate XI. Besides the ordinary clavicular attachment (*c*), which was connected with the bone near to that of the *trapezius* (T), so as almost entirely to cover the posterior triangle, there arose from the sternal end of the clavicle, external to and partially separate from the sternal origin of the *sterno-mastoid*, a strong flattish tendon giving origin to a considerable bundle of parallel muscular fibres (*c'*). These crossing upward and backward, superficial to the origin of the *cleido-mastoid* proper (*b*), joined, about the middle of the neck, the fibres from the ordinary origin (*c*), and were inserted with and anterior to them into the superior curved line of the occipital bone. The appearance of a completely double *sterno-cleido-mastoid* was thus given to the subject, bearing a striking resemblance to the formation of the homologous muscles in the striped Hyæna, Polecat, Genette, Coati, and Marmot, as figured in CUVIER and LAURILLARD's plates of these animals.

The muscular variety which I have distinguished by the name of *cleido-occipital* has been observed and described by various writers as an occasional variety of the *sterno-cleido-mastoid* or *trapezius* muscles.

It is mentioned by SÖMMERRING (De Corp. Hum. Fabric. p. 112), and was found by KELCH coexistent with a double *sterno-cleido-mastoid*, the whole possibly having been a formation similar to that last described (Beiträge, xxi. p. 31).

MECKEL describes it, both in the human subject and in animals. In the latter he states that it sometimes joins, below, the anterior fibres of the deltoid (*cephalo-humeral*); and in his description of *Ateles* and Magot, and some other animals, he seems to have looked upon it as a variety of the *acromio-trachélien* (*levator claviculæ*), and through this arrangement he appears, moreover, to have connected the latter with the *trapezius* rather than with the *levator anguli scapulæ*, as indicated by CUVIER (MECKEL, De Duplicitate Monstrosa, pp. 40, 41, and Muskellehre, p. 475).

THEILE describes an oblique slip of muscle, three lines wide and one thick, attached to the superior curved line of the occipital bone, partly under the *trapezius*, and spreading out below so as to cover the clavicular fibres of the *sterno-cleido-mastoid* and to join, finally, its sternal origin. This also may have been the double-headed variety of the *cleido-occipital* just described (Encyclop. Anatom. pp. 163, 164, 165).

MACWHINNIE mentions the clavicular origin of the *sterno-cleido-mastoid* as occasionally forming two distinct muscles, and sometimes blending with the fibres of the *trapezius* (*op. cit.* p. 186).

HALLETT found, in three subjects, a muscle arising from the clavicle, separate from the *sterno-cleido-mastoid* and the *trapezius* respectively. He also found a muscular slip joining the *trapezius* at the occiput, and the *sterno-cleido-mastoid* at the clavicle, and also more frequently (1 in 16 cases) meeting the clavicular insertion of the *trapezius* (Edinburgh Medical and Surgical Journal, 1846, p. 6). R. WAGNER discovered a similar slip, joining the *trapezius* at the clavicular insertion, on both sides of a female subject (HEUSINGER'S Zeitschrift, Bd. iii. S. 337). GRUBER found a slip of muscle arising from the clavicle, completely separate from the *sterno-cleido-mastoid* and *trapezius*, in 2 subjects out of 40, and with a separation less complete in 1 out of every 3 subjects. In 7 out of 70 subjects this observer found the same muscle joined above more or less with the *trapezius* (Vier Abhandlungen, S. 16, 17, 18). HENLE mentions it as an occasional abnormality of the *sterno-cleido-mastoid* (Muskellehre, S. 110), and QUAIN as a variety of the *trapezius* (Arteries, pl. 25). FLOWER and MURIE found a good specimen of the formation on both sides in the Bushwoman. It was best marked on the left side, and consisted of a long narrowish band of fibres attached above to the occiput half an inch from the *trapezius*. It crossed the posterior triangle, and was inserted into the clavicle an inch from its outer end (Journal of Anatomy and Physiology, May 1867, pp. 197, 198).

In connexion with this muscle we must place also, I believe, the occasional remarkable formation of the clavicular fibres of the *trapezius* described as an occasional human variety by QUAIN, MACWHINNIE, HALLETT, and GRUBER (*op. cit.*), and in my own paper in the Royal Society's 'Proceedings' for 1867 (No. 93, p. 522). In these cases, which have a considerable resemblance to each other, the anterior border and clavicular insertion of the *trapezius* appears to be prolonged over the posterior triangle, so as more or less completely to cover it, and to become connected with the origin of the *cleido-mastoid*. Opposite the middle of the clavicle a tendinous arch, thrown over the middle and sternal divisions of the descending cervical nerves and over the external jugular vein, affords attachment to the median portion of the muscular fibres in place of the clavicle itself. In the instances I have myself met with, an areolar interval more or less marked has usually existed between this irregular extension of the insertion and the normal fibres of the *trapezius*. The superficial insertion into the aponeurotic arch, formed in the deep fascia of the part, is connected below to the fascia of the arm, and indicates an homology with the brachial prolongation of the *levator humeri* of the lower animals. The intimate connexion of this abnormal part with the *trapezius* indicates its relation to those formations in which the so-called "*trapezius clavicularis*" of the Carnivora and Rodents is entirely absorbed in the formation of the great "*cephalo-humeral*" muscle.

In connexion with this abnormality I would also place, as a lateral displacement, the variety recorded by R. WAGNER as an irregularity of the *trapezius*, viz. an accessory muscle passing from near the mastoid process to the acromion process under the last-named muscle (*op. cit.* S. 337).

COMPARATIVE ANATOMY.—If we now turn to the homologies of the foregoing muscular varieties in the Mammalia, we shall find a correlation between these several developments sufficiently striking and suggestive.

It was asserted by MECKEL, against the opinion of CUVIER (Anat. Comp. vol. vi. p. 236), that in the lower mammalia the cervical portion of the *serratus magnus* was the most usual homological representative of the lower divisions of the *levator anguli scapulæ* of Man and the higher Quadrumana; extending usually as high only as the transverse process of the third cervical vertebra, it becomes, in most of these animals, continuous with the upper border of the thoracic *serratus*, at the last cervical transverse process. In the Badger and Weasel among the Carnivora, the Rabbit and Surmulot among the Rodents, and in the Bonnet-Monkey of the lower Quadrumana, I have found an areolar separation of the cervical from the thoracic part more decided than that between the other digitations, a division which becomes in the higher Quadrumana still more evident.

Occipito-scapular Muscle.—In 1775 Dr. JAMES DOUGLASS described in the Dog, under the name of the "*levator scapulæ minor vel posterior*," a muscle separate from but in the same layer as the rhomboids, arising from the occiput near its crest, superficial to the *splenius*, and inserted in connexion with and above the *rhomboideus minor* into the upper angle of the vertebral border of the scapula. Under the name of the "*levator scapulæ major vel anterior*," he also described, in the same animal, a ribbon-shaped muscle, arising from the first cervical transverse process, and inserted into the scapular spine near its outer end. In these two muscles we have evidently the homologues of the opposite extremes of the human varieties before described, and represented by the fully formed *occipito-scapular* and *levator claviculæ*. The homologue of the former was described by J. F. MECKEL, under the name of the *rhomboïde antérieur*, as a separate muscle, reaching from the angle of the scapula to the occiput, in the Magot and Lemurs, and forming one sheet with the *rhomboids* in the Coati. In the Insectivora he describes it in the Mole and Hedgehog as a separate muscle, and in the Armadillo as joined with the rhomboids. In the Carnivora he describes it in the Marten, Potto, Bear, Hyæna, Badger, Dog, and Cat, usually united to the other rhomboids and reaching as far as the occiput. In the Rodents he describes it in the Beaver, Porcupine, and Marmot, partially separated from the other *rhomboids* in the first, but united to them in the two latter; and he also describes it in the latter condition in the *Didelphis marsupialis* (Anat. Comp. vol. v.). In CUVIER, LAURILLARD, and MERCIER's magnificent 'Receuil de Planches de Myologie' (Anat. Comparée, Paris, 1855), this muscle is figured under the name of the "*rhomboïde de la tête*" as large and separate from the other *rhomboids* in *Callithrix*, Magot, *Papio Mormon*, Coati, Sajou, and Marmoset. In the Lion it is represented as continuous with the other rhomboids; but it constitutes a separate muscle in the Panther, Black Bear, Coati, Badger, Genette, Polecat, Dog, and Otter. It is also figured in the Hedgehog, Mole, and Armadillo. In the *Orycteropus* it is noted as the *rhomboïde du cou*. It figures also in the Hippopotamus, Peccary, and Pig; in the Hare, Beaver (large), Paca, Agouti, Capybara, Porcupine, Rabbit, and Squirrel; and in the great Kangaroo, Kangaroo-Rats, and Phalangers. Both Cuvier and Meckel also figure it in the *Ornithorhynchus*. In the *Orycteropus capensis* the *rhomboideus minor* is described as reaching to the occipital crest by Humphry (Journ. of Anat. and Phys. May 1868, p. 299) and by J. C. Galton (Trans. Linn. Soc. vol. xxvi. p. 590). The former author also describes the same arrangement in the Seal; and the latter states that, in the Six-banded Armadillo, the *occipito-scapular* muscle is enormously developed, arising from the whole of the occipital crest, and inserted into the *supraspinous* fascia, as well as into the upper angle of the scapula (*op. cit.* p. 525).

It is described by Krause in the Rabbit under the name of the *levator anguli scapulæ vel scapulæ minor* (Anat. des Kaninchens, Leipzig, 1865, S. 104), and also by Mivart and Murie in the same animal. The latter authors also describe it in the *Hyrax capensis* as blended with the *rhomboids*, and in the Hare and Guinea-pig under the name of the *occipito-scapular* or *rhomboideus capitis* (Proceedings of the Zoological Society, April 1865, p. 335, and June 1866, p. 393).

In the *Echidna hystrix* Mivart describes the *rhomboideus cervicalis* as reaching up to the occiput (Trans. Linn. Soc. vol. xxv. 1866). I have myself found this muscle in the Bonnet-Monkey, separate from the other rhomboids (Plate X. fig. 12, *d d*), and also in the Hedgehog (Plate XI. fig. 22, *d*), Mole (Plate X. fig. 11, *d*). In the last it constitutes a very distinct ribbon-shaped muscle (*d*), lying upon and parallel to the splenius (*h*), in no way connected with the other *rhomboids*, which are very feeble and almost wanting, nor with the *trapezius* (T), which overlies it. I have also found it in the Dog, Cat (Plate XI. fig. 23, *d*), Badger (Plate X. fig. 14, *d*), Weasel (fig. 15, *d*), Rabbit (fig. 16, *d*), Squirrel (fig. 18, *d*), and Norway Rat (fig. 19, *d*), in which last animal it is of a great size. This muscle has thus a very extensive range of existence in the Mammalia, and is represented by many specimens in most of the families. A very striking instance of modification in the attachment and uses of a muscle, having a suggestive resemblance to the differentiation of the *splenius* in the human variety before described (*rhombo-atloid*), is found in the lower attachment of this muscle in the Mole (see fig. 11, *h*). In this animal the *splenius* is a large, thick, and powerful muscle, lying immediately under the *occipito-scapular*, attached extensively in front to the occiput, and tapering off behind, where it is attached, not to the spinous processes of the vertebræ, but to the end of the *scapula*, which are united to each other across the spine by an interscapular ligament, upon the superficial surface of which are developed a few transverse fibres, feebly representing the *rhomboids*. The whole apparatus is freely moveable backwards and forwards upon the spine. According to Meckel, there is developed in the cervical and dorsal supraspinous ligaments in this situation an ossicle or cartilage, but this I was unable to find in the specimen which I examined and from which the drawing was taken.

The *splenius* is hereby transformed from a spinal muscle into one acting upon the combined fore legs and head, moving freely over the spine, and it becomes a notable accessory force in aid of the action of digging and burrowing with the snout and fore paws.

The muscle just described seems to correspond to that described by MECKEL as a very strong *rhomboid*, attached to the moveable ossicle. The muscle upon which it rests, however, seems to me to be clearly the *complexus*, with the large *trachelo-mastoid* muscle to its outer side, constituting the only muscular layer which intervenes between the muscle in question and the *semispinalis* and *multifidus* system of fibres, with the *obliqui* and *recti* of the occiput. The direction of the fibres of this muscular layer corresponds with that of the *complexus*, viz. from the transverse processes forwards and a little inwards to the median line of the occiput, where the two fellow muscles are closely placed together at their insertion.

Levator claviculæ Muscle.—Synonyms: The "*levator scapulæ major vel anterior*" of DOUGLASS and BURMEISTER; the "*omo-*" or "*acromio-trachélien*" of CUVIER and MECKEL; the *acromio-basilar* of VICQ D'AZYR; the *clavio-trachélien* of CHURCH; the *basio-humeralis* of KRAUSE; the *Kopf-Arm-Muskel* of PEYER; the *transverso-scapulaire* of STRAUSS-DÜRCKHEIM; the *omo-atlanticus* of HAUGHTON; and the *cervico-humeral* of HUMPHRY.

The homologies of this muscle in the Mammalia form too extensive a subject, and one presenting too many complex modifications, to be fully entered into in this paper. I will content myself with indicating the principal changes which occur in it, selecting such specimens as may throw light upon the developments I have found in the human subject.

In by far the greater majority of animals the muscle arises from the transverse process of the atlas singly. In some it extends also to that of the axis; and it seems to represent that which in Man and the higher *Simiadæ* are the two upper digitations of the *levator anguli scapulæ*. In the Rodents and Pachyderms, however, we shall find that, by becoming amalgamated by longitudinal and lateral fusion with the *recti capitis*, it may be attached to the lateral or basilar processes of the occipital bone.

In the Gorilla, Chimpanzee, and Orang it is always present, arising from one or two of the upper cervical transverse processes, and inserted into the clavicle external to its centre. This insertion into the clavicle, which is found, as we have seen, in the human subject, and has given to it the name of *levator claviculæ*, becomes in the lower Quadrumana, by external transposition, shifted to the upper border of the acromion process of the scapula (*omo-* or *acromio-trachélien*). Its clavicular attachment, however, reappears, according to MIVART and MURIE, in the *Nycticebus tardigradus* or Slow Loris (Proc. Zool. Soc. 1865, p. 243), and is found universally in the Bats, attached to the outer end of the clavicle close up to the insertion of the *trapezius*. In the former animal, as described by the above authors, the muscle arises singly from the transverse process of the atlas, the *levator anguli scapulæ* arising as far forwards as that of the axis, as well as from all the cervical transverse process behind that point. The single origin prevails also in most of the lower Quadrumana. In the *Papio Mormon* and Magot it arises from the axis as well as the atlas (CUVIER and LAURILLARD, plates 29 & 30). In these Apes, and in *Ateles* and *Callithrix* also, its scapular insertion is covered by the front fibres of the *trapezius*. I have found the same arrangement in the Bonnet-Monkey (Plate X. fig. 12, *e*). It is also the case, according to MIVART, in *Cercopithecus sabæus* (Proc. Zool. Soc. Jan. 10, 1865).

In *Ateles* and Magot MECKEL describes this muscle as double, having apparently connected it with the development of a *cleido-occipital* muscle, which is also found in these animals. This part of the supposed double muscle MECKEL refers to the *trapezius*, with which he thus connects the muscle under consideration rather than with the *levator anguli scapulæ*, as CUVIER does.

In the Coati, Marmoset, Slender Loris, and *Lemur macaco*, according to CUVIER, the *acromio-trachélien* passes superficially over and across the front fibres of insertion of the *trapezius* to be inserted into the superficial aspect of the acromion process, constituting apparently a form transitional to that of its insertion in the Carnivora and Rodents, where we usually find a *metacromial* process developed downwards from the outer end of the spine of the scapula (see Plate X. figs. 15 & 16, *z*). By this means is provided a longer leverage for the muscle, enabling it to rotate powerfully the shoulder and anterior extremity outwards from the trunk, around the long axis of the scapula. In the Hedgehog (fig. 18, *e*) the muscle is inserted superficial to the *trapezius* into the acromial process. It is totally absent in the Mole. In the Six-banded Armadillo (*Dasypus sexcinctus*) Mr. GALTON describes an *acromio-basilar* muscle taking origin from the lateral ridge of the supraoccipital bone and inserted into the metacromion process of the scapula (Trans. Linn. Soc. vol. xxvi. p. 527). This slip of muscle is, however, considered by CUVIER (plate 259. fig. 2) and by MECKEL (*op. cit.* p. 480) as the cervical portion of the *trapezius*, with which its origin from the supraoccipital bone would certainly ally it, while its insertion into a metacromial process would not be incompatible therewith. In the *Orycteropus capensis*, Professor HUMPHRY describes this muscle, under the name of the *cervico-humeral*, as arising from the transverse process of the atlas, and inserted superficial to the *trapezius* into the spine of the scapula. In the Tamandua it reaches over and far below the scapular spine and *trapezius*. It is not figured in CUVIER and LAURILLARD's plates of the Sloth, Anteater, or Armadillo. In those of the Lion and Panther it is represented as large, wide, and covered at its insertion by the *trapezius*; but in the Bear, Striped Hyæna, Coati, Badger, Polecat, Genette, Dog, and Cat it passes superficial to the insertion of the *trapezius* to the metacromion process. In the Cat I have found it to become at its origin partially blended with the *rectus capitis anticus major*, and thus to arise, as we find it also in the Rabbit, partly from the basilar process of the cranium (see Plate XI. fig. 23, *e e*, and Plate X. fig. 13, *e*). In the Otter, HAUGHTON describes the muscle as subdivided into two parts, one attached to the outer or lower, and the other to the inner or upper end of the scapular spine (Proc. Royal Irish Acad. vol. x. pt. iv.). In the Seal it is also, according to HUMPHRY, divided into two parts, one passing to the outer tuberosity of the humerus with the *trapezius* (thus extending still further its range of action as a swimming-muscle), while the other overlaps the *supraspinatus*, and is inserted into the angle of the scapula (*op. cit.* p. 299).

This remarkable bifurcation of the muscle I have myself found both in the Badger and the Weasel (Plate X. figs. 14 & 15, *e*, *f*); it bears significantly upon the connexion I have endeavoured to establish in this paper between the anterior and posterior levators of the scapula. In the latter of these animals the muscle arises by a single origin from the transverse process of the atlas; in the former it extends as far as that of the axis, as is sometimes the case in the Dog. Almost directly two flat, ribbon-like, diverging muscles are formed, one passing backwards and downwards to be inserted, superficial to the *trapezius*, into the metacromial process, and the other backwards and upwards, to join at insertion to that of the *occipito-scapular* at the angle of the scapula. Both the muscles are inserted also into the supraspinous fascia, and into the vertebral end of the scapular spine itself, under cover of the *trapezius*. The last-described division of this double muscle is evidently the homologue of the first or atloid digitation of the human *levator anguli*; it has the same origin, course, and insertion, and forms in the same manner a link of connexion between the origin of the anterior *levator scapulæ* or *acromio-trachélien* and the insertion of the posterior or *occipito-scapular* muscle. This link is rendered more continuous by the abnormal human varieties which I have described in the earlier part of this paper.

A perplexing resemblance to the bifurcation above described is seen in the arrangement of this muscle in the Rabbit (Plate X. fig. 16, *e b*); this will, however, be found, on closer inspection, to be essentially different in its nature. In this animal, as well as in the Guinea-pig, the origin of the muscle becomes blended (as we have seen to be the case in the Cat) with the fibres of the *rectus capitis anticus major*, contributing to form a thick muscular mass between the pharynx and the vertebræ (see fig. 13, *u*, *v*, *b*, and *e*). The outer part of the apparently double muscle is attached also to the transverse process of the atlas (1), while some appear to be connected with the *rectus minor* (*v*); but the greater and inner portion arises directly from the basilar process of the cranium, close and external to the insertion of the *rectus major* (*u*), and near the line of suture between it and the mastoid process and the large tympanic protuberance. The two parts composing the muscular mass are easily separated along a loose areolar interval extending quite up to the base of the skull; they pass together along the neck, crossed by the *sterno-mastoid* (fig. 16, *a*) and *cleido-occipital* (*c*) muscles, and then separate in the posterior triangle,—the upper or hinder one (*e*) to be inserted, superficial to the trapezius, into the prolonged metacromial process (z), and the other to join the tendinous intersection connected with front of the rudimentary clavicle (*x*) at its outer half, where it assists in forming the great *cephalo-humeral* muscle. The first of the two muscles is described by Krause (*op. cit.* S. 103) as the "*levator scapulæ major*," and the latter as the "*basio-humeralis*," referring this erroneously as a homologue to the "*transverso-scapulaire*" of Strauss-Dürckheim,—an homologue which applies more correctly to the former muscle.

This curious shifting of the origins and insertion of these muscles has given rise to the perplexing multiplication of names which meets us in the literature of this subject. It has also, as I shall endeavour to prove, led to the concealment of the real homologies of the last-described muscle in the Rabbit, as well as of those of the muscle marked *c* in the figure, which has been confounded with it. The muscle which is called by KRAUSE the *basio-humeralis*, presents, I believe, strong grounds for referring its homology to the *cleido-mastoid* of human anatomy. First, its lower attachment to the clavicle and its participation in the formation of the *levator humeri* is upon this supposition at once explained. Its deep position in relation to the muscles marked *a* and *c*, viz. the *sterno-mastoid* and *cleido-occipital*, is also more in accordance with this supposition. Its origin from the basilar process takes place close to the suture between it and the mastoid bone, some of its fibres even arising from the latter. Its deep displacement may in fact be referred to the great development of the tympanic element necessary to support the enormous ears of the animal, at the expense of the occipital and mastoid development, which are small and compressed. A comparison of the relative position of the three muscles marked *a*, *b*, and *c* in the figures of the Rabbit, with those of the Hedgehog (Plate XI. fig. 22), and especially with those of its congeners, the Squirrel (Plate X. fig. 18), and the Norway Rat (fig. 19 and fig. 26, Plate XI.), will render the resemblance of the muscles herein treated as homologous, more plain than any description, and will tend to remove the confusion into which, by want of precision, the names of these muscles have been plunged. The relation of the muscle to the true *levator claviculæ* (*e*) is thus one of juxtaposition merely, and not one of derivation. A further test of the accuracy of viewing this muscle as a *cleido-mastoid* is to be found in the arrangement of the group in the Ruminants as compared with the *Solidungulata*.

In the fawn of a Fallow Deer dissected by my friend and former pupil Mr. NETTLESHIP, from whose sketch figure 20 (Plate XI.) was taken, it will be seen that the *levator humeri* (*b e*) (*cephalo-humeral*) is made up of two parts (*b* and *e*) exactly corresponding to those of the Rabbit just described. They are marked with corresponding figures. The posterior or superior portion, or that corresponding to the *acromio-trachélien* or *levator claviculæ* (*e*), is the larger, and arises from the transverse process of the atlas. The anterior or inferior portion (*b*) joins the fibres of the *rectus capitis anticus major* (*u*), and is attached with it to the basilar process of the cranium. The two join about the middle of the neck to form one compound muscle (*b*, *e*), which passing over the shoulder without any clavicular "inscription," is inserted with the fibres of the *pectoralis transversus* (*major*) into the humerus just above the outer condyle. As the portion *e* corresponds clearly to the muscle which in the Rodents and Carnivora is attached to the metacromial process (and which MIVART and MURIE found in the Hare and Rabbit to send some fibres down to the humerus), so the portion *b* is clearly the homologue of that which I have considered in the Rabbit as the *cleido-mastoid*, blending in the same way with the *rectus capitis anticus major*. To enhance the value of this proof, the *sterno-mastoid* (maxillary) (*a*) in the fawn sends off a slip (a'') to join, and thus to claim connexion with, its usual colleague, the *cleido-mastoid*, in its new connexions with the *rectus capitis anticus major*.

A further light is thrown upon this arrangement by that found in the Donkey, of which the succeeding figure (Plate XI. fig. 21) was drawn from a sketch made also from nature by Mr. NETTLESHIP. In this animal the *acromio-trachélien* (*levator claviculæ*) (*e e*) arises by four digitations from the four upper cervical transverse processes, and is a far larger muscle than the *cleido-mastoid*, much resembling in appearance the origin of the human *levator anguli scapulæ*. The *cleido-mastoid* portion (*b*), however, arises, not in common with the *rectus capitis anticus* (*u*) from the basilar process, but with the *rectus lateralis* (*w*) from the paramastoid process just behind the ear, as in most of the Mammalia, and clearly asserting its homology with the human *cleido-mastoid*. The two muscles have exactly the same relation to each other as those of the Fallow Deer, and form, in the same manner, the compound *levator humeri* muscle (*b c*). As an evidence of the like tendency to transposition evinced by this group of muscles in the large heavy animals, both long- and short-necked, I may allude to the fact that in the Ass and Horse, as well as in the Camel, Elephant, Hyrax, and most of the Ruminants, the *sterno-mastoid* is inserted into the angle of the mandible (hence called *sterno-maxillaris* vel *mandibularis* by veterinary anatomists). In the Peccary, Hippopotamus, Pig, and Tapir it is, however, inserted into the mastoid process, with a slip to the transverse process of the atlas in the Hippopotamus.

The question now arises,—What, then, is the homologue of the muscle which has been usually considered by writers on the subject as the *cleido-mastoid*, viz. that marked *c* in the figure of the Rabbit (Plate X. fig. 16)? Its superficial position in relation to, and parallelism with the fibres of, the *sterno-mastoid* (*a*), its attachment to the ridge of the occipital bone as far as that of the *trapezius*, its intermediate position here between these two muscles, together with its superficial position at its attachment to the clavicle (*x*) in relation to the other muscle which I have affiliated to the *cleido-mastoid* (*b*), which it assists in forming the compound *levator humeri* muscle (*b c*), all lead me to consider it as the representative of the *cleido-occipital* muscle which I have described as a human variety. A comparison with the same muscle in the Guinea-pig (Plate X. fig. 17, *c*), and still more in the Squirrel (fig. 18, *c*) and Norway Rat (Plate XI. fig. 26, *c*), will render this more clear. In the Guinea-pig the muscle (*c*) has its cranial attachment to the ridge of the occipital bone, and not to the mastoid, reaching from the insertion of the *sterno-mastoid* to that of the *trapezius*. No other muscular fibres, except a few which join it high up from the last-named muscle, and which may represent the true *cleido-mastoid*, assist it to form the *levator humeri* (*c'*) in the arm. It is apparently the only muscle from the upper part of the neck or head which is connected with the clavicle (*x*).

In the crested Agouti (*Dasyprocta cristata*), MIVART and MURIE found a very similar blending of the two corresponding muscles on the right side (*op. cit.* p. 391). These authors recognized in this animal the homology of the *levator claviculæ* muscle, which they describe as arising by a tendon from the *basis crani*,—one part (the inner) being inserted into the outer end of the clavicle (the *cleido-mastoid* of this paper), and the other passing over the shoulder adherent to the *trapezius*, to help to form the *levator humeri* muscle at its insertion into the proximal end of the radius. They point out that this part probably led MECKEL to the false homology of considering this muscle to be represented by the anterior part of the *trapezius*. In the Hare they found that the outer part was inserted into the metacromial process. In the Porcupine, Capybara, and Paca the *acromio-basilar* is very large, and in them, as well as in the Agouti, Guinea-pig, Squirrel, Norway Rat, and Rat-mole of the Cape, it is placed superficial to the *trapezius* at its insertion into the acromion process. In the Guinea-pig I have found it to join the *rectus capitis anticus major*, and to be attached with it to the base of the cranium; in the Squirrel and Norway Rat, however, it arose, in the specimens I have dissected, from the transverse process of the atlas only. MECKEL did not seem to recognize it as a separate muscle in the Agouti, Paca, Squirrel, Hamster, or Guinea-pig, appearing to consider it as part of the *trapezius*. In the *Hyrax capensis*, according to MURIE and MIVART, it is strong, arising from the transverse process of the atlas, passing over the neck of the scapula, and inserted into the fascia covering the *teres minor* (Proc. Zool. Soc. April 1865, p. 334). In the Elephant, according to CUVIER and LAURILLARD's plates, it is large and superficial to the *trapezius* at its insertion; this is also the case with the Horse and Ass. In the Hippopotamus, Peccary, Pig, and Tapir it is small, and placed at its insertion superficial to, or on a level with, the *trapezius*.

In the Kangaroo HAUGHTON describes it as very broad, arising from the three upper cervical transverse processes, and inserted into the whole length of the clavicle as well as into the outer third of the scapular spine (*op. cit.* vol. ix. part 4). CUVIER also describes it as large in the Great Kangaroo, Sarigue, and Phalangers, and inserted under cover of the *trapezius*, a well-marked and distinct *levator anguli scapulæ* being also figured as coexistent in the 'Recueil de Planches.'

The *levator claviculæ* was found by MIVART in the *Iguana tuberculata*, arising from the transverse process of the atlas, and inserted into the acromial end of the clavicle and front margin of the scapula (Proc. Zool. Soc. June 1867, p. 780).

Cleido-occipital Muscle.—Second *cleido-mastoid* of MECKEL; *portio cervicalis trapezii* of CUVIER; *trapezius clavicularis* of HAUGHTON; *clavo-cucullaire* of STRAUSS-DÜRCKHEIM.

In the Mammalia this muscle approaches most closely to its occasional formation in the human subject among the higher *Simiadæ*. It is distinguished from the *sterno-* and *cleido-mastoids* on the one hand, and the *trapezius* on the other, in the Chimpanzee and Orang, where we find a simple areolar interval between it and these muscles. It is marked in the 'Recueil des Planches' of CUVIER and LAURILLARD, both in the text and illustrations of Callithrix, Sajou, and Marmoset. I have found it but slightly separated in the *Macacus radiatus* (Plate X. fig. 12, *c*). It is very broad in the Slender Loris and *Maki vari*. It is well marked and quite distinct in the Hedgehog (Plate XI. fig. 22, *c*), being attached below to the clavicle, outside of and deeper than the *cleido-mastoid* (*b*); and above to the curved ridge of the occipital bone, in connexion with the insertion of the *trapezius* (T). It is seen in the figure of the Tenrec given by CUVIER;

and LAURILLARD. In the Mole (Plate X. fig. 11, *c*) a broad and large muscle, parallel with the *sterno-mastoid* (*a*), seems to embody in itself both the *cleido-occipital* and *cleido-mastoid*; in this animal the origins of the *sterno-mastoids* cross each other over the median line. In the Bats, which have no *cleido-mastoids*, the *cleido-occipital* muscle seems to be represented by the occipital or neck portion of the long extensor of the wings, called by CUVIER the "*dorso-occipital*," which passes superficially over the clavicle to the thoracic limb, and is, apparently, the homologue of the *cephalo-humeral*. In the Armadillo Mr. GALTON describes, under the name of the *levator claviculæ*, a muscle arising from the occipital aponeurosis outside the *trapezius*; it is placed along the edge of, and parallel to, the *cleido-mastoid*, and is inserted close to its outer side into the clavicle. It is figured also by CUVIER and LAURILLARD in the same animal. It is clearly the homologue of the *cleido-occipital*, and not of the *levator claviculæ*. In the Great Anteater the muscle is very large and distinct, excluding the *trapezius* altogether from the occiput: it was considered by MECKEL as a second *cleido-mastoid* in this animal.

Through the claviculate and semiclaviculate Rodents the homologies of the *cleido-occipital* muscle can be clearly traced to the upper or cephalic element of the compound *cephalo-humeral* muscle which forms so important a part in the shoulder of the Carnivora. In the common Squirrel (Plate X. fig. 18, *c*) the muscle is connected at the occiput with the cranial attachment of the *trapezius* (T), which overlaps it superficially; below it is attached to the clavicle superficial to the *cleido-mastoid* (*b*). Intervening between it and the scapular attachment of the *trapezius* emerges the *acromio-trachélien* (*e*), to be attached to the acromial process superficial to the last-named muscle. This intervention of the *acromio-trachélien*, which I have before especially noted in this paper, is important as enabling us to discriminate between that portion of muscle which I take to form the homologue of the *cleido-occipital*, but which in the Carnivora and some Rodents is described by preceding writers as the clavicular portion of the *trapezius* itself. In the Flying Squirrel, the Beaver, and the Surmulot, this part forms equally a separate and distinct muscle, attached to the occiput near the trapezius, and separately to the clavicle below (see Plate XI. fig. 26, *c*). In the latter, as seen in the figure (*c*), the lower end is shifted outwards towards the acromial end of the clavicle, and with it is shifted the origin of the *cleido-mastoid* (*b*), so as still to preserve its relative superficial position: it is as if the clavicle had been elongated mainly at the inner or sternal end. In these animals the muscle has been described by anatomists as a second *cleido-mastoid*, but its invariable occipital attachment, the direction of its fibres, and its wide separation at its upper attachment from the real *cleido-mastoid* sufficiently distinguish it from that muscle. In the Marmot it is very large, and excludes the *trapezius* both from the occiput and the clavicle; anteriorly it passes forward superficial to the *cleido-mastoid*, and joins the hinder edge of the *sterno-mastoid*. In the Capybara and some other Rodents also, and in the Carnivora generally, it excludes the *trapezius* from the occiput, and is inserted into the movable clavicle, or the aponeurotic "inscription" which represents it, to which it is attached superficial to the *cleido-mastoid*, forming with it the cervical or cephalic portion of the *cephalo-humeral* muscle, the lower part of which is formed by the muscles which form, in Man, the clavicular fibres of the *deltoid* and *pectoralis major*. In some, as in the Cat, this compound muscle is inserted as far down as the coronoid process of the ulna by longitudinal fusion with the *brachialis anticus*; but in most, as in the Dog, Badger and Weasel, and in the Rabbit and Guinea-pig, it is inserted into the humerus close below the *pectoralis major*. It will be seen, then, that I find the homologue of the *cleido-occipital* in the muscle which Meckel and Cuvier described as a second *cleido-mastoid* in the Insectivora and semiclaviculate Rodents, as well as in that which they have described in the Carnivora as the *cervical portion* of the *trapezius*, or the *trapezius clavicularis* (the *clavo-cucullaris* of Strauss-Dürckheim). The intervention of the acromial insertion of the *acromio-trachélien* between the fibres of this muscle and the *trapezius* proper, before alluded to, and the gradual way in which it excludes the *trapezius* from the occipital bone in the lower Apes and Monkeys, its relation to the latter muscle at the occiput, and the occasional blending at their adjacent borders as an abnormal human variety, are all circumstances which favour this view, which has also the merit of simplifying the perplexing homologies of the cephalic portion of the compound *cephalo-humeral* muscle, and explaining, more fully than before, the abnormal human varieties so frequently found in connexion with the side of the neck and shoulder. The identity of these homologues is well seen in the transitional forms found in the Insectivora and in the claviculate and semiclaviculate Rodents, as I have shown in the case of the Rabbit and Hedgehog.

In many of the Carnivora the cephalic and cervical portion of the compound *cephalo-humeral* muscle is enormously developed, encroaching upon and excluding the *trapezius*, not only from the occiput, but also from the neck-vertebræ (see Plate XI. fig. 23, *c*). Forming a broad sheet of muscle, with the fibres directed downwards and backwards, its deeper fibres are, in the Cat, inserted upon a cartilaginous rod representing the clavicle (*x*). Into the deeper surface of this are also inserted the fibres of the distinct *cleido-mastoid* element (*b*); the superficial fibres are, however, continued over the surface of this rod uninterruptedly into the arm by longitudinal fusion with the clavicular fibres of the *pectoralis major* (as Strauss-Dürckheim thinks), and blending finally with the superficial fibres of the *brachialis anticus*, are inserted with it into the ulna. In most of the Carnivora the clavicle is represented simply by a tendinous intersection on the deeper surface of the *cephalo-humeral* muscle. In this family the most striking development of the *cleido-occipital* is found in the peculiar arrangement, before alluded to, in the striped Hyæna, Polecat, and Genette, and to a less marked degree in the Coati, as figured in Cuvier and Laurillard's 'Recueil.' A second clavicular head of the *cleido-occipital* crosses superficially over the *cleido-mastoid* to join the sternal origin of the *sterno-mastoid*, with the hinder border of which at the sterno-clavicular joint it is united, and not at all with the *cleido-mastoid*. Thus is produced an apparently double or second *sterno-cleido-mastoid* muscle, imbricated upon the true one, and strikingly resembling the human variety I have described in an earlier part of this paper. The Marmot shows a similar arrangement.

In the *Hyrax capensis*, which presents many points of connexion in its muscular system with the Rodents on the one hand, and the Pachyderms on the other, the arrangement of these muscles admits of the best reading, I believe, by the homologies herein indicated. As described by MURIE and MIVART, the *sterno-cleido-mastoid* apparatus is represented by three muscles:—One, attached to the occipital bone, and inserted with the *biceps* into the ulna, is the *cleido-occipital*, joining in the formation of a compound *cephalo-humeral* with some of the segregated fibres of the *pectoralis* and of the *brachialis anticus*. Another portion, the true *sterno-mastoid*, is connected above with the angle of the jaw and *masseter* (*sterno-maxillary* v. *mandibular* of the Ruminants and Pachyderms, see also MECKEL, Anat. Comp. t. vi. p. 163); below, it joins its fellow of the opposite side at and above the sternum. A third portion, a very slender muscle, somewhat resembling the *omo-hyoid*, is attached in front to the paramastoid process, and joins behind with the first on its deep surface: this third part, according to the homologies herein sustained, would be the true *cleido-mastoid* muscle, the name which is bestowed by MURIE and MIVART upon the first described portion (Proc. Zool. Soc. April 11, 1865, pp. 331, 332).

The limits of this paper will hardly permit me to follow further the various developments of the foregoing group of muscles in the Pachyderms, Ruminants, and the rest of the Mammalia. It will be enough for my purpose if I have succeeded in showing the more important forms which, when occurring as varieties in the human subject, tend to exhibit in a sufficiently marked manner what may be considered as proofs and examples of the Darwinian *principle of reversion*, or *law of inheritance*, in this department of anatomical science.

I will now proceed to consider another group of occasional varieties in the human shoulder, which I believe I was the first to connect with their homologies in the animal kingdom, and one of which, the *scapulo-clavicular*, I was the first to discover and name in the human subject.

Sterno-chondro-scapular and *Scapulo-clavicular Muscles.*—In my paper read before the Royal Society in 1864, I figured and described examples of an apparently double development of the *subclavius* muscle. The upper portion had the normal attachments of the *subclavius* muscle; the lower part, separated from it by a distinct areolar interval, somewhat wide externally, arose, in one case, by a distinct tendon from the sternum and first rib-cartilage, but, in another, in common with the *subclavius*. Passing outwards as a somewhat fusiform muscle over the axillary vessels and nerves, it was inserted fleshy into the tubercle of the coracoid process and conoid ligament. In my paper published in the 'Proceedings of the Royal Society' in 1865, I figured and described a similar muscle, found on the left side of a thin female subject (Plate IX. fig. 10, *i*). It was a roundish fusiform muscle, arising tendinous from the first rib-cartilage close to the sternum, and inserted into the suprascapular ligament and base of the coracoid process, where it was connected with the origin of the *omo-hyoid* muscle (*o*).

In the same shoulder a distinct band of muscular fibres, about an inch wide and an eighth thick (k), arose from the base of the coracoid and suprascapular ligament, and passing upwards and forwards was inserted into the outer third of the clavicle (x), close to the *subclavius* muscle ($m\ m$); it was quite distinct from the first-described *sterno-scapular* muscle, but was connected with it to the origin of the *omo-hyoid*, which was normal in all other respects. In my paper of 1868 I noted another specimen of this formation on the left shoulder of a male subject. These abnormal human varieties, the *sterno-scapular* and *scapulo-clavicular* muscles, have been found in comparatively few subjects, partly, I believe, on account of the difficulty of preserving them entire in the ordinary way of apportioning the subject to different students, and the liability of the latter especially to be removed along with the fat and tissues which fill up the interval between the scapula, clavicle, and first rib when the vessels and nerves are dissected. I have therefore considered that any estimate of their frequency in the human subject would necessarily be a fallacious one.

The *sterno-scapular* muscle corresponds in most respects with the following abnormal muscular slips described by their observers under various names.

Meckel described a double *subclavius* muscle, and compared it to the external and internal intercostals (Muskellehre, 1816). It had previously been described by Boehmer as a muscle connected with the origin of the *subclavius*, and inserted into the coracoid or acromion (!) process (Observ. Anat. rar. præfat. Halle, 1756, p. ix); and also by Haller as a double *subclavius* (De Corp. Hum. Fabr. t. v. part i. p. 95 *a*, and t. vi. p. 77, 1756); and by Sœmmerring as a variety of the *omo-hyoid*, arising from the first rib and inserted into the scapula (De Corp. Hum. Fabr. t. iii. p. 173, 1796); and by Rosenmüller, on the left side of a male subject arising from the rib-cartilage behind the *subclavius*, and inserted near the base of the coracoid process (Beiträge für die Zergliederungskunst, Bd. i. Heft 3, S. 375, Tab. ii., and 'De nonnullis Musculis,' 1814, p. 6).

R. Wagner described it as a variety of the *omo-hyoid*, arising from the first rib-cartilage, and inserted into the *incisura scapulæ* with the origin of the *omo-hyoid* (Heusinger's Zeitschrift, Bd. iii. S. 335).

Theile described a rounded muscle, arising from the first rib-cartilage, and inserted into the base of the coracoid and upper border of the scapula, in a male subject in whom the *omo-hyoid* was wanting. He considered it as a variety of the *serratus magnus* (Sœmmerring's Encyclop. Anat., Jourdan's Trans. p. 206, 1843).

Macwhinnie quotes Haller and Sœmmerring as above (*op. cit.* p. 187). Hallett describes as a variety of the *omo-hyoid* a considerable slip of muscle, connected by one tendon to the first rib-cartilage, and by another inserted into the upper border of the scapula with the origin of a normal omo-hyoid (*op. cit.* p. 4). Gruber also describes a similar abnormality as connected with the *omo-hyoid* (Vier Abhand. 1847, and Neue Anomalien, 1849).

The *Scapulo-clavicular* variety I believe to be homologically identical with the muscular slips described by the following writers.

Von Krause described, under the name of the *coraco-cervicalis*, a slip of muscle arising with the *omo-hyoid* from the base of the coracoid process and upper border of the scapula, and inserted into the cervical fascia close to the scapula (quoted by Quain, 'Arteries,' pl. 4. fig. 21).

Macwhinnie described a muscle as a variety additional to the *omo-hyoid*, arising from the scapula behind and internal to that muscle, forming a belly as thick as the little finger, and attached by a rounded tendon to the middle of the upper border of the clavicle (*op. cit.* p. 187).

Hallett described a muscle arising from the upper border of the scapula with the *omohyoid*, and inserted into the upper part of the sterno-clavicular articulation. He also found other slips of the same general character (*op. cit.* p. 4).

Luschka found a slip of muscle connected with the origin of the *omo-hyoid*, and inserted into the back part of the inner end of the clavicle, which he considered as a variety of the *omo-hyoid* (Müller's Archiv, 1856, S. 284). Similar slips are mentioned by Hyrtl (Lehrbuch, S. 344) and by Henle (*op. cit.* S. 116) as varieties of the *omo-hyoid.*

Sterno-clavicular Muscle.—In the 'Proceedings of the Royal Society' (June 21st, 1866, p. 238), I described under the above name (first given by Mr. Berkeley Hill) an abnormal muscle found on the left side of a male subject (see Plate IX. fig. 9, *l*). It was triangular in shape, arising by a thin tendon from the front of the *manubrium sterni* just below the origin of the *sterno-mastoid*, formed a distinct muscular layer spreading upwards and outwards in a fan-shape under the *pectoralis major* (P), and separated from the *subclavius* (*m*) by the costo-coracoid membrane, and was inserted into the lower border of the clavicle about its middle third, passing as far outwards as the origin of the *deltoid* muscle. A *cleido-occipital* muscle (*c*) coexisted. On both sides of another male described in the same paper (p. 231) I found the upper digitation of origin of the *pectoralis minor* somewhat separated from the rest, and arising as high as the first intercostal aponeurosis, passing upwards to be inserted into the costo-coracoid membrane and clavicle. This was specified as a formation similar to a *sterno-clavicular* muscle, produced by a differentiation of the fibres of the *pectoralis minor*, as found in the Rodents.

A similar variety was recorded by Haller (Elements of Physiology), and quoted by Henle. Theile also described as a variety of the *subclavius* a muscle arising from the first rib-cartilage, and inserted into the middle part of the front border of the clavicle (*op. cit.* p. 173). In May 1864 Mr. Berkeley Hill described a well-marked specimen of this muscle under the above name, quoting Haller, and pointing out its homology in the Bats and Birds (Proc. Royal Med.-Chirurg. Soc. vol. iv. No. 6, p. 351).

Comparative Anatomy *of the three foregoing varieties.*—In the Coati (*Simia paniscus*) Meckel describes a second insertion of the *subclavius* into the scapula (*sterno-scapular*), near the attachment of the *levator anguli scapulæ*. This is quoted by R. Wagner (*op. cit.* S. 336) as corresponding to the double *subclavius* in Man. In the Insectivora I have found a corresponding muscle distinctly marked in the Mole (Plate X. fig. 11, *i*), arising outside the true *subclavius* (*m*) from the sternum and first rib, separated from it by an areolar interval; it crossed the insertion of the *supraspinatus* (*q*), and was inserted into the acromion process and *acromio-clavicular* ligament, where it was still more distinctly separated from the *subclavius*. In the same animal Mr. Hill found, as I have myself subsequently also done, a remarkable development of the *sterno-clavicular* muscle, as a large triangular mass of muscle arising from the front half of the sternum, and inserted into the stunted but strong clavicle close to the origin of the *deltoid*. The same author also mentions the presence of this muscle in the Bats.

A *sterno-scapular* muscle is described by Mr. Galton in the *Dasypus sexcinctus* under the head of *subclavius*, arising from the first rib, and inserted by a flat tendon along the whole extent of the upper edge of the strong acromion process, and continuous with the supraspinous fascia. He refers to its similarity to the *sterno-scapular* muscle described by Mivart and Murie in the Agouti (Trans. Linn. Soc. vol. xxvi. p. 528). He also found the *subclavius* inserted into the acromion process of the scapula in the Two-toed Sloth. In the Cape Anteater, Humphry describes the *subclavius* as a large muscle arising from the first and second rib-cartilage and adjacent part of the sternum beneath the *pectorals*; it is attached by a few fibres to the clavicle, but the major part passes beneath that bone over to the coracoid and its ligaments to be inserted into the supraspinatus fascia, as well as to the margin of the acromion process (*op. cit.* p. 297, and plate 4). Galton found, in the same animal, a sesamoid bone at the insertion of the muscle, just under the acromio-clavicular joint (*op. cit.* p. 572).

In the Carnivora the homologies of the *sterno-scapular* muscle are but little decided. In the Weasel a deeper set of the pectoral muscular bundles, arising from the manubrium, pass upwards and outwards over the tuberosity of the humerus, and are continuous with a muscular layer lying upon the *supraspinatus*, and attached with it to the upper border of the scapula (Plate XI. fig. 24, *i*). In the Dog also, a few of the fibres of the *pectoralis* are differentiated and connected with the *supraspinatus*. In Cuvier and Laurillard's plates of the Lion, it seems to be represented by the muscle marked J, and in that of the Panther marked J+2. In the Hyæna it is very large, attached, on the one hand, to the sternum and first rib-cartilage, and on the other to the upper border of the scapula (*j*+). It seems also to be present in the Genette. In the Rodents it is better marked, blended, however, more or less, with the *subclavius*, the *scapulo-clavicular*, and the *sterno-clavicular* muscles. I have found it very large in the Rabbit (Plate X. fig. 16, B, *i i'*), arranged in two thick bundles, an upper (*i*) and a lower (*i'*) passing from the front of the *manubrium sterni* and *suprasternal* process, and continued uninterruptedly under the clavicle, forming a thick sheet of muscle upon the *supraspinatus* (*q*) to be inserted into the upper border of the scapula. Connected with it and covering its upper half above the clavicle is a layer of fibres (A, *k*), reaching only from the scapular spine to the clavicle (*x*), into the upper border of which they are inserted. This layer, more or less separable from the deeper or true *sterno-scapular* fibres, I have considered to represent the *scapulo-clavicular* muscle, which becomes a more distinct muscle in its congeners, the Guinea-pig and Norway Rat (Plate X. fig. 17, A & B, *k*, and Plate XI. fig. 25, *k*). On the opposite or lower border of the clavicle are attached the fibres of insertion of a broad triangular muscular layer (Plate X. fig. 16, *l*), arising from the sternum under the *pectorals*. This is clearly the homologue of the *sterno-clavicular* muscle, disguised by its connexion with the *sterno-scapular*.

The group of muscles just described have been taken together as one *m. pectoralis minor* by KRAUSE (*op. cit.* S. 104), and as the *subclavius* by CUVIER (Recueil, pl. 233), but the interposition of the clavicle between the upper and lower portions of the deeper layer points evidently to their correspondence, respectively, to the *scapulo-clavicular* and *sterno-clavicular* of other animals; and they are, at least, as worthy of separation as the component parts of the *cephalo-humeral*, as distinguished by the same bone or its tendinous representative. Another portion of the pectorals of the Rabbit answer more closely than these to the *pectoralis minor* of Man.

The *sterno-scapular* muscle in the Guinea-pig (Plate X. fig. 17, A & B, *i*) corresponds with a *subclavius* in being attached to the movable clavicle (*x*) by its more superficial fibres, while the deeper are continued to the outer part of the spinous process of the scapula, where it is inserted between the *supraspinatus* (*q*) and *infraspinatus*. In the same animal the *scapulo-clavicular* muscle (*k*) is very considerable in size and distinct from the *supraspinatus*, over which it moves freely; arising from the scapular spine and supraspinous fascia, internal to the last, and inserted into the movable clavicle (*x*) opposite to the *sterno-clavicular* (*l*). This last-named muscle arises from the middle of the sternum, between and partly covered by the upper and lower fibres of the pectorals, forming a triangular layer very distinct in its origin and insertion. In the Norway Rat (Plate X. fig. 19, & Plate XI. fig. 26) the *subclavius* proper (*m*) is represented by a small bundle of fibres arising to the inner side of the *sterno-scapular*, and inserted into the lower surface of the inner end of the clavicle (*x*). The *sterno-scapular* is a fusiform muscle (*i*) ending in two tendons of insertion, one with the *omo-hyoid* into the cervical border, and the other into the acromion process of the scapula. The *scapulo-clavicular* (*k*) in the same animal is very distinct, arising from the middle of the supraspinous fascia, and inserted into the outer end of the clavicle (*x*). This bone articulates directly both with the sternum and scapula, so that the action of the muscle upon it presents nearly the same conditions as in the human subject, and is necessarily more limited than in the Rabbit and other semiclaviculate animals.

The *sterno-scapular* muscle has been described by MURIE and MIVART in the *Hyrax capensis* as arising from the sternum in front of the origin of the *pectoralis minor*, and continued over the shoulder articulation to its insertion into the anterior angle of the scapula (*op. cit.* p. 338). They also found it in the Guinea pig, with a slip from the *supraspinatus* to the clavicle (*scapulo-clavicular*), and state that MECKEL describes it as a part of the *subclavius* in the Hare, Porcupine, and Agouti (pp. 259, 260), and that CUVIER considered it as part of the *trapezius*? (Leçons, vol. i. p. 373). In the 'Recueil de Planches' it is marked in the Rabbit as *subclavius* (pl. 233). In the Crested Agouti, MIVART and MURIE found the *sterno-scapular* to have a double origin, viz. a larger, from the sternum between the pectoral bundles; and a smaller, the manubrium and first rib. Some of the fibres of the larger origin are connected with the clavicle (*sterno-clavicular*), and the rest of them join the smaller origin under that bone, to be inserted near the anterior vertebral angle, and into the supraspinous fascia of the scapula. In the Hare they found one broad sternal origin and a very wide scapular insertion, extending along the whole cervical border, some of them adhering to the outer end of the rudimentary clavicle (*subclavius* or *sterno-clavicular*) (Proc. Zool. Soc. June 1866, p. 398). In CUVIER and LAURILLARD's plates of the Porcupine (pl. 229. fig. 2) the *sterno-scapular* muscle, marked as a *subclavius*, is very long, and passes outward to the scapular spine. MACALISTER has also found it in this animal (*op. cit.* p. 11). In the plates of the Capybara, Paca, and Squirrel, the same muscle, or the *sterno-clavicular*, is marked as a deeper portion of the pectorals. In the common Squirrel I have found it as a distinct muscle, passing under the clavicle to the scapular spine and suprascapular fascia (Plate X. fig. 18, *i*). The *sterno-scapular* muscle receives its highest development in the Pachyderms and Ruminants, and especially in the Elephant, Hippopotamus, Peccary, Pig, Horse, and Ass (Plate XI. figs. 20 & 21, *i*), to whose heavy bodies it forms a powerful, muscular, sling-like support, upon and between the fore legs, reversing its "*point d'appui*" as compared with its action in the animals before described, in whom its power is exercised chiefly in the direction of the fore limbs. In these heavier animals its arrangement scarcely calls for a more detailed description in this place. In his monograph upon the Hippopotamus, GRATIOLET describes the muscle as the *scapulo-sternal*, arising from the coracoid and acromion processes and the *supraspinatus* fascia, and inserted into the *manubrium sterni* and first costal cartilage. He considers it as probably the homologue of the *subclavius* (p. 256).

In the Marsupials, a muscle answering to it is represented by CUVIER and LAURILLARD in their plates of the Great Kangaroo, Kangaroo-Rat, and Sarigue, connected with the scapula and *supraspinatus* fascia. GALTON mentions that in the Wombat a portion of the *subclavius* muscle is carried on through the *supraspinatus* fascia to the scapular spine; and that the notes of a MS. work in the Oxford Museum describe in the same animal a second head of the *subclavius*, "a very delicate one arising from the lower ribs and passing vertically upwards to end in a fine tendon." MIVART describes a muscle passing from the sternum to the coracoid bone in the *Iguana tuberculata* (Proc. Zool. Soc. 1867, p. 779).

The *Scapulo-clavicular* muscle is described and figured in CUVIER and LAURILLARD's plates of the Rat-mole of the Cape (pl. 216+) with the following annotation:—"Dans ces notes marginales," M. CUVIER dit, "Il existe un muscle particulier allant de la portion moyen de l'omoplate à la clavicule, où il s'insère derrière la deuxième portion claviculaire du trapèze, ou pourra l'appeler 'sus-clavier' ou 'scapulo-clavien.'" They also describe and figure, under the same names and nearly the same terms, a like muscle in the Sarigue (*Didelphis marsupialis*).

On a review of this group of muscles in the foregoing animals, the *sterno-scapular* muscle seems, in many instances, to embody the fibres of the *subclavius*, and in others to be made up chiefly by the union, at the imperfect clavicle or its tendinous representative, of a *sterno-clavicular* and a *scapulo-clavicular* element.

In the heavier animals, in whom the clavicle is altogether wanting, it constitutes a continuous muscular support to the trunk upon and between the scapulæ, uninterrupted by the intervention of a clavicular representative, and embodying all the various elements. In animals possessed of a clavicle, and using the fore paws as hands, the distinction of the *sterno-clavicular*, *subclavian*, and *scapulo-clavicular* elements becomes more marked; while in some, like the Rabbit, a combination of all these is evident in the compound muscular mass. In the Guinea-pig the *sterno-clavicular* and *scapulo-clavicular* are large and distinct muscles, while the *sterno-scapular* is also very distinct and separate from them. In the Norway Rat the *subclavius* and *sterno-scapular* are distinct, and the *scapulo-clavicular* well marked, while the *sterno-clavicular* seems altogether wanting. The last-named element seems to be the most marked in Mammalia of burrowing and flying habits, and it draws powerfully the primary segment of the fore limbs, chiefly used in these motions, backwards against the resisting and reacting atmosphere. All these modifications are evidences of teleological adaptations of a common morphological structure, such as we find prevailing in other parts of the animal organization.

Supracostal Muscle.—An abnormal and infrequent variety in the human subject was first recorded and described by me in the 'Proceedings of the Royal Society' of 1865. It occurred, on both sides, in a muscular male subject, in whom a *levator-claviculæ* and thirteen other abnormalities were found (see Plate IX. fig. 8, *n*). It was a thin, flat, ribbon-shaped muscle, placed upon the upper four ribs, between the digitations of the *pectoralis minor* and those of the *serratus magnus.* It was attached above to the outer edge of the first rib near its cartilage, by a fleshy attachment about an inch broad, passing downwards and slightly forwards; the fibres gradually spread out in somewhat of a fan-shape, and dropped insertions into the outer surfaces of the second, third, and fourth ribs, close to the origin of the *pectoralis minor.* It was entirely distinct from the intercostals, from which a well-marked fascia separated it.

In the 'Proceedings of the Royal Society' of 1867 I recorded another specimen, found also in a male subject, and on both sides. It was attached above to the first rib, as in the first case, and was connected also externally to the cervical fascia covering the *scalenus anticus*, with which it seemed to be in part continuous. Below it was attached to the *third* rib only, in front of the *serratus magnus.* Since that time we have found another specimen of this muscle, also in a male, and on both sides.

Mr. Macalister has found this muscle several times, very large and muscular. In a male subject it existed on the left side only, attached above wholly to the cervical fascia, and below to the third and fourth ribs, about 6½ inches from the sternum, in the same situation as the foregoing. He has found it usually narrower and thicker on the right side than on the left. In one instance it measured 3¾ inches long, ¾ inch wide, and ⅛ inch thick, overlapping the upper digitations of the *serratus magnus*, and ascending behind the axillary vein, was attached on the right side to the first rib, and on the left into the cervical fascia only. It lay considerably external to the origin of the *lesser pectoral* muscle, and was inserted internal to the *scalenus anticus*. Mr. MACALISTER considered it as acting from below upon the cervical fascia above (Notes on Musc. Anomalies, p. 7). He states that the muscle occurs in the *Balænoptera rostrata*, in the Seal, and in several Monkeys.

Professor TURNER, of Edinburgh, has also met with this muscle in two male subjects; in one on both sides, and in the other on the right side only. In the former it consisted of a long, ribbon-shaped muscle attached by a thin expanded tendon to the upper border of the fifth rib, immediately internal to the *serratus magnus*. The innermost part of this attachment was continuous with the intercostal fascia, and was attached to the rib at its junction with the cartilage. From the anterior surface of the fourth rib, close to the origin of the *serratus magnus*, a second and smaller origin proceeded. The muscle passed over the third and second ribs to be attached to the first rib immediately external to the tendon of origin of the *subclavius*. It was 6 inches long and $\frac{5}{8}$ of an inch broad at its widest part. In the subject in which the muscle was found on the right side only, it was attached to the upper border of the fourth rib, 2 inches external to the junction with the cartilage, ascended over the third and fourth ribs, and was attached above to the first rib $\frac{3}{4}$ of an inch outside the origin of the *subclavius* (Journal of Anatomy and Physiology, May 1867, p. 251; and May 1868, p. 393, and figure). In the Liverpool Medical and Surgical Reports, October 1867, Dr. ROBERTS describes one of these muscles extending from the fourth to the first rib; and in VIRCHOW's 'Archiv,' November 18th, 1867, one is described by BOCHDALEK, Jun., under the name of "*supracostalis anterior*," and another by PYE-SMITH in No. XLIII. of the same Journal (p. 142).

COMPARATIVE ANATOMY.—Professor TURNER considers the *supracostal* muscle described by me to be the homologue of the thoracic prolongation of the mammalian *rectus*, which in the Cat, Otter, Beaver, Porcupine, and various other Mammalia reaches as far forwards as the first rib. But, as this accurate observer proceeds to say, "In these animals, however, the thoracic and abdominal parts of the rectus are directly continuous with each other, whilst in the human subject a break, corresponding in the first specimen to the fifth rib, and in the second to the fifth rib and fourth intercostal space, occurred; but this break may be regarded as comparable to one of those transverse tendinous intersections invariably found in the abdominal portion of the human *rectus abdominis*, and which exists also in the recti of the greater number of Mammalia." In his first paper "On the *Musculus sternalis*" (*op. cit.* 1867, p. 250) this author alludes also to a muscle described by BOERHAAVE and PORTAL, which was directly continuous with the thoracic attachment of the *rectus abdominis*, as undoubtedly to be regarded as the homologue of the anterior fibres of the mammalian *rectus*. In the case recorded by BOERHAAVE it reached behind the great pectoral muscle as high as the junction of the third rib with its cartilage, and in that described by PORTAL as high as the second rib. This latter muscle is, however, evidently a formation distinct from the *supracostalis*, which pertains rather to the upper part of the thorax than to its abdominal portion.

There is another muscle, almost universal in the Mammalia, which may be considered perhaps to be a part of the same apparatus as the *rectus* muscle; but which is confined, even in the most highly developed instances, to the upper part of the thorax. It is described by CUVIER under the name of the *sterno-costalis*. In many respects this muscle agrees better in its homological bearings with the abnormal human *supracostalis* than the *rectus thoracicus* does. Besides being confined to the upper part of the thorax, the upper attachment of the muscle is always in close relation to the insertion of the *anterior scalenus*, as in the human abnormality. This is well seen in the Dog (Plate XI. fig.27, *n*), where it is connected below, not to the sternum as in many Mammalia, but to the second and third costal cartilages and intercostal aponeurosis, and does not reach anywhere near the insertion of the *rectus abdominis*. The same arrangement is found also in the Badger. In the Rabbit the muscle is short, but broad, and reaches from the first rib and to the cartilage of the second rib, extending inwards as far as to the sternum (see Plate X. fig. 16, B, *n*); while the *rectus thoracicus* reaches only up to the second rib, the most common upward limit in the Mammalia generally. It is not mentioned by KRAUSE in his description of this animal. In cases in which the rectus extends to the first rib, the *sterno-costal* muscle usually overlaps it (see figs. 18 & 26, *n*), assuming an oblique position on the thorax. It is always attached to the first rib close to the inner side of the origin of the *scalenus anticus*, and thence passes obliquely downwards and inwards between the upper end of the *rectus thoracicus* and the pectoral system of fibres, the *pectoralis minor* and *sterno-clavicular* both lying superficial to it. The *rectus thoracicus* itself is attached to the first rib and its cartilage, always closer to the sternum, and much internal to the upper attachment of the *sterno-costal*, and never has so intimate a relation with the *scaleni* as the last-named muscle. In this respect especially the human *supracostal* muscle resembles the *sterno-costal* of animals much more than the *rectus thoracicus*. In the Mammalia generally the *sterno-costal* is a more or less triangular muscle, but sometimes tapers off below before it joins at the sternum with the deeper pectoral fibres, of which it always forms the deepest layer. It is covered by a fascia derived from that of the *rectus thoracicus* and *intercostals*. In the Quadrumana the *sterno-costal* is usually large. In the Bonnet-Monkey it reaches and crosses the upper part of the *rectus thoracicus* as low down as the second and third rib (see Plate X. fig. 12, *n*). In the Magot it is enormous, as figured by CUVIER and LAURILLARD, arising by digitations from the three upper ribs, and passing downwards and inwards to the three upper pieces of the sternum. In the *Papio Mormon* it is attached to the first rib above, and below to the second and third costal cartilages. In the Marmoset and *Lemur macaco* it arises from the first rib close to the *scalenus*, and crosses, as in the Carnivora generally, the upper fibres of the *rectus thoracicus*. It is found in the Hedgehog and Tenrec; and according to CUVIER, in the Black Bear, Coati, Panther, Lion, Hyæna (very large), Badger, Genette, Seal*, and Polecat. In the Panther a slip of muscle from the *scalenus* joins it. I have also found it in the Weasel and Badger, Dog (fig. 27), and Cat. In the Rodents Cuvier figures it in the Marmot, Beaver, Porcupine, and Agouti. It is not mentioned by Mivart and Murie in the latter animal. I have found it long, well marked, and overlapping the rectus in the Norway Rat and in the Squirrel, connected with the first rib between the *subclavius* and *scalenus*, crossing the upper part of the *rectus*, which reaches to the first rib, and reaching far down the sternum to join the pectoral fibres arising from that bone (Plate X. fig. 18, *n*). In Cuvier and Laurillard's plates it seems to be the muscle marked as the lower part of the *scalenus*, and figured as a very large muscle in the *Bradypus tridactylus* and Tamandua, reaching from the first rib to the eighth, between the pectorals and *serratus magnus*. In Meckel's description of the Two-toed Anteater, he describes a long muscle arising from the first rib, and inserted into the front part of the ninth, tenth, and eleventh, under the name of "der kleiner Brustmuskel," stating expressly that it is *not* attached to the scapula, but that it seems to be an elongation of the *external intercostal* (Archiv, Bd. v. Hft. i. p. 41, *c*). This is evidently a muscle of the same nature as our human *supracostalis*, and agrees more with the *sterno-costal* of the other Mammalia than with the *rectus thoracicus*, which is coexistent and well developed in the same animal. The same muscle is figured in this animal by Cuvier and Laurillard (pl. 257. figs. 2 & 6) as the *anterior scalenus*; but as it commences clearly at the first rib, and does not ascend into the neck, it cannot be admitted as entitled to this name.

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* In his paper on the Myology of the Cape Anteater and Seal, Professor HUMPHRY describes, in the latter animal, under the name of the *subclavius*, a muscle "thin from the margin of the sternum opposite the second,

In the Elephant the *sterno-costal* muscle seems to be represented, according to the same authorities, by a large muscle, attached on the one hand by three digitations to the first rib, and on the other to the side of the upper pieces of the sternum, reaching as low as, but not overlapping, the upper insertion of the *rectus abdominis*. By some this may be considered as a *rectus sternalis* with an interruption of the nature of a tendinous "inscription;" but the appearance of the muscle agrees best with that of the *sterno-costalis*, while a tendinous separation or interval between the *rectus thoracicus* and the *rectus abdominis* is not usually seen in the Mammalia. In the Peccary it is composed of two large digitations quite unlike a *rectus sternalis*; and in the Pig, the anterior fibres of the most anterior intercostal muscle cross superficially over the second rib-cartilage to be inserted into the sternum below it, showing a feeble development of the *sterno-costalis* or *supracostalis* muscle. In the Ass the muscle is represented by one digitation only.

Upon the whole it seems to me that the balance of facts is more in favour of the identity of the human abnormal *supracostalis* muscle with the *sterno-costalis* of the Mammalia, than with the *rectus thoracicus* of these animals, as in the homology proposed by Professor Turner. Finally, I would refer to the same *supracostal* or *sterno-costal*

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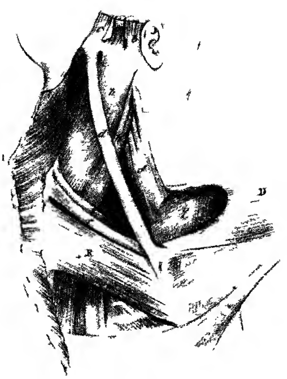
third, and fourth ribs, and inserted into the edge of the *first rib* near the point corresponding to the insertion of the *scalenus anticus* in Man. The artery of the fore limb crossed its insertion. It was in close contact with the *scalenus*, indeed some of its fibres joined that muscle" (*op. cit.* p. 297 and pl. 6). This description clearly answers to the *sterno-costal* muscle figured by Cuvier and Laurillard in the same animal.

homologue the muscular slip described and figured from the Crocodile by Professor ROLLESTON, in his valuable memoir on "The Homologies of certain Muscles connected with the Shoulder-joint" (p. 626 and fig. 3, *e*, *i*), as "a muscular fascicle which arises from the second *sterno-costal* cartilage in the same series as the external oblique and intercostals, but which may also be looked upon as homologous with an anterior segment of the *rectus abdominis*, which is often not distinguishable from the former of these muscles. It ends in a delicate tendon, which loses itself along the coracoid groove in the sternum and the origin of the *pectoralis major*."

EXPLANATION OF THE PLATES.

For the sake of convenient reference the letters of each figure point out the same or homologous muscles, &c.

a. *Sterno-mastoid*; *a′*, *sterno-mandibular* or *maxillary* of veterinary anatomists.
b. *Cleido-mastoid.*
c. *Cleido-occipital*, second *cleido-mastoid* of MECKEL and CUVIER; *trapezius clavicularis* of the latter and other authors.
d. *Occipito-scapular*. *Rhomboideus capitis*; *rhomboide de la tête* of CUVIER; *rhomboïde antérieur* of MECKEL; *levator scapulæ minor* vel *posterior* of DOUGLASS and BURMEISTER.
δ. Homologous slip of the last named.
D. *Deltoid.*
e. *Levator claviculæ* of HUXLEY; *acromio-trachélien* of CUVIER; *omo-trachélien*, *acromio-basilar* of VICQ D'AZYR; *levator scapulæ major* v. *anterior* of DOUGLASS and BURMEISTER; *transverso-scapulaire* of STRAUSS-DURCKHEIM; *clavio-trachélien* of CHURCH; *omo-atlanticus* of HAUGHTON.
ε and *η*. Homologous slips of the preceding connected with the *levator anguli scapulæ* and the *serrati* respectively.
f. *Levator anguli scapulæ.*
F. Abnormal slips of ditto.
g. *Adjutor splenii* of WALTHER; *rhombo-atloid* of MACALISTER.
h. *Splenius capitis.*
h′. *Splenius colli.*
i. *Sterno-scapular*; *subclavius* of MECKEL and CUVIER; *pectoralis anticus* of veterinary anatomists.
j. *Scalenus anticus.*
j′. *Scalenus posticus* or *medius.*
k. *Scapulo-clavicular*; *scapulo-clavien* of CUVIER; part of *sterno-scapular*, *subclavius* c *supraspinatus* of other writers.
l *Sterno-clavicular*; *pectoralis minor* of some authors.



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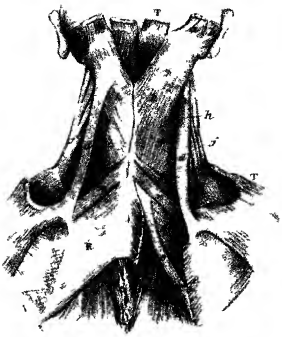
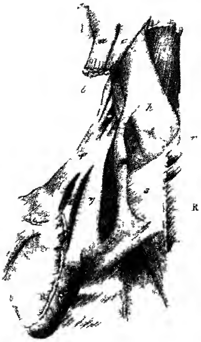
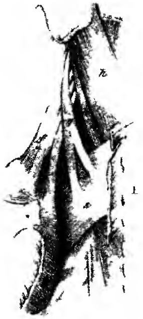
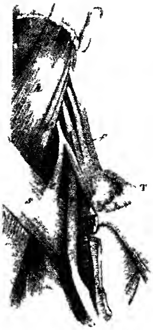
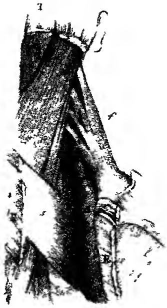
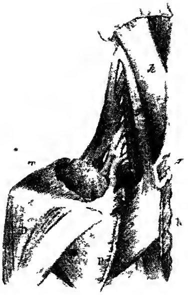


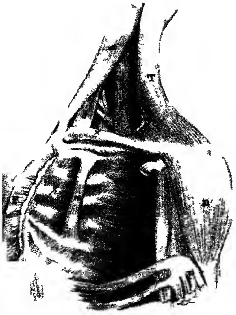


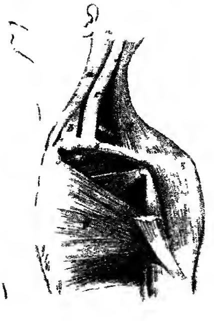


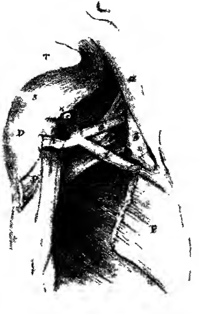












Bailey lith.

[illegible]

m. *Subclavius.*
n. *Supracostalis* or *sterno-costalis*; the *rectus thoracicus* of Turner.
o. *Omo-hyoid.*
P. *Pectoralis major.*
p. *Pectoralis minor.*
q. *Supraspinatus.*
R. *Rhomboideus major.*
r. *Rhomboideus minor.*
Rt. *Rectus thoracicus*; continuation of *rectus abdominis.*
S. *Serratus magnus.*
s. *Serratus posticus superior.*
T. *Trapezius.*
t. *Subscapularis.*
u. *Rectus capitis anticus major.*
v. *Rectus capitis anticus minor.*
w. *Rectus lateralis.*
x. *Clavicle*, or its rudimentary representative, or tendinous "inscription."
y. Tuberosity of the *humerus.*
ʒ. Acromion process, or spine of scapula.
z. Metacromion process of scapula.
1. Transverse process of the atlas.
b c. *Levator humeri*, lower part of *cephalo-humeral* of some writers, when formed by the *cleido-mastoid* (*b*) and *cleido-occipital* (*c*) above, with the clavicular fibres of the *deltoids* (D) or *pectoralis major* (P) below.
b e. The same compound muscle, when formed above by the *cleido-mastoid* (*b*) and the *acromio-trachélien* or *levator claviculæ* (*e*), and by the same muscles below.
θ. An abnornal anomalous muscular slip connected with the *rhomboideus major*, found only in one subject (male).

PLATE IX.

Fig. 1. Back view of the muscles of the human shoulder in a male subject, showing the *occipito-scapular* variety (*d*) on the right side.
Fig. 2. Back view on both sides, showing varieties homologous with the *occipito-scapular* (*d*, *d*), and one other anomalous slip (*θ*) of rhomboideus major (R).
Fig. 3. Back view on left side, showing homologous *occipito-scapular* (*δ*) and two slips (*ε* and *η*) from *levator anguli scapulæ* (*f*) to *serratus magnus* (S) and *serratus posticus superior* (*s*).
Fig. 4. Back view on left side, showing slips (*ε* and *η*) of *levator anguli scapulæ* (*f*) to *serratus posticus superior* (*s*) and *serratus magnus* (S).
Fig. 5. Back view on right side, showing homologous slips (*ε* and *η*) to the same muscles from the front surface of *levator anguli scapulæ* (*f*).

Fig. 6. Back view on right side, showing homologous slip (*η*) in front of *levator anguli scapulæ* (*f*) to *serratus magnus* (S).

Fig. 7. Back view on left side, showing slips (F) of *levator anguli scapulæ* (*f*) to *serratus magnus* (S) and *rhomboideus minor* (*r*).

Fig. 8. View of dissection of the left side of the neck and thorax of male subject, showing *levator claviculæ* (*e*) and *supracostalis* (*n*) muscular varieties.

Fig. 9. View of dissection of the left side of the neck and thorax of male subject, showing the *cleido-occipital* (*c*) and *sterno-clavicular* (*l*) muscular varieties.

Fig. 10. View of dissection of the right side of the neck and thorax of a female subject, showing the *sterno-scapular* (*i*) and *scapulo-clavicular* (*k*) muscular varieties.

PLATE X.

Fig. 11. View of the dissection of the right side of the neck- and shoulder-muscles of the common Mole (*Talpa europæa*), showing *cleido-occipital* (*c*), *occipito-scapular* (*d*), *splenius capitis* (*h*), and *sterno-scapular* (*i*).

Fig. 12. View of the dissection of the left side of the Bonnet-Monkey (*Macacus radiatus*), to show *cleido-mastoid* (*b*), *cleido-occipital* (*c*), *levator clavicalæ* or *acromio-trachélien* (*e*), and *supracostalis* or *sterno-costalis* (*n*). The clavicle (*x*) is disarticulated from the sternum and turned back with the scapula and muscles.

Fig. 13. Front view of the dissection of the muscles of the prævertebral muscles of the Rabbit (*Lepus cuniculatus*), to show the cranial attachments of the *cleido-mastoid* (*b*), *levator claviculæ* (*e*), and their relation to the *rectus capitis anticus major* (*u*) and *minor* (*v*).

Fig. 14. View of the dissection of the neck- and shoulder-muscles of the right side of the Badger (*Meles taxus*), to show *cleido-mastoid* (*b*), *cleido-occipital* (*c*, *c'*), *occipito-scapular* (*d*), *acromio-trachélien* (*e*), *levator humeri* (*b c*), and the slip homologous to the upper digitation of the human *levator anguli scapulæ* (*f*).

Fig. 15. View of the dissection of the same muscles on the left side of the Weasel (*Mustela vulgaris*), to show the same muscles indicated by the same letters.

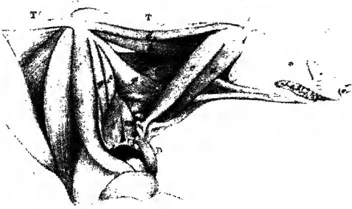
Fig. 16. Dissection of the neck- and shoulder-muscles of the right side of the Rabbit.

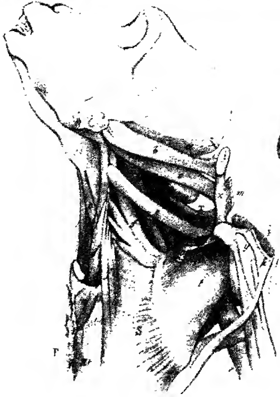
A. The muscles "*in situ*" to show those homologous to the two foregoing, marked by the same letters, with the addition of the *scapulo-clavicular* (*k*) and the *sterno-clavicular* (*l*) attached each to the clavicle (*x*).

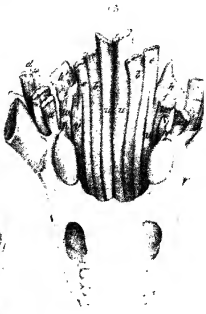
B. The scapula of the same side turned over with its muscles attached, to show the double *sterno-scapular* muscle (*i i i'*) as seen from the deep surface, passing under the clavicle (*x*). It also shows the *sterno-costalis* muscle (*n*).

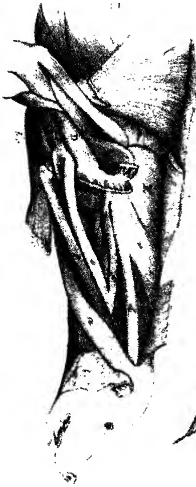
C. View of the deep or under surface of the clavicle (*x*), and *sterno-clavicular* ligament (*x'*), showing the attachment of the *sterno-clavicular* muscle (*l*) below, and the *levator humeri* (*b c*) turned up.

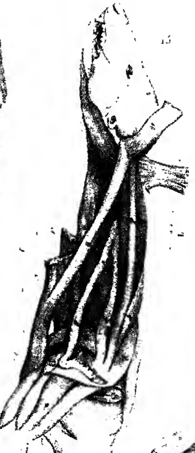
D. View from the superficial aspect of the same, showing the relative

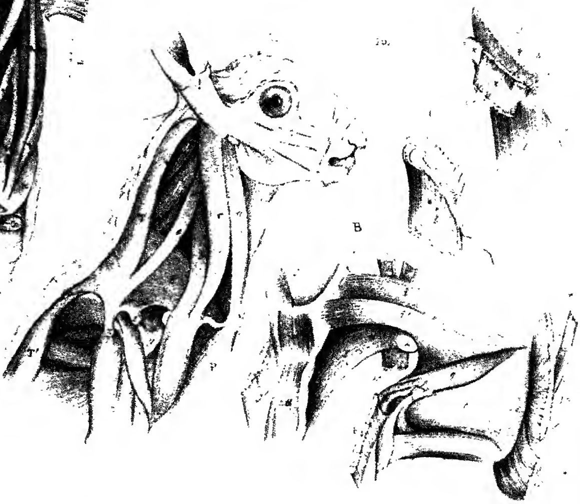


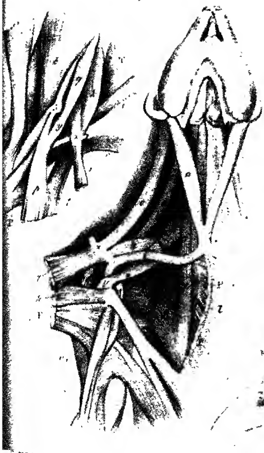


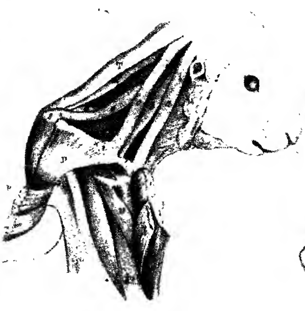


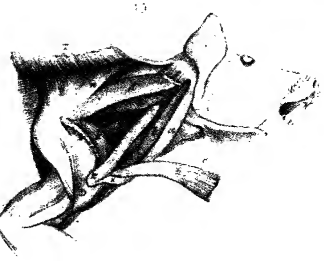












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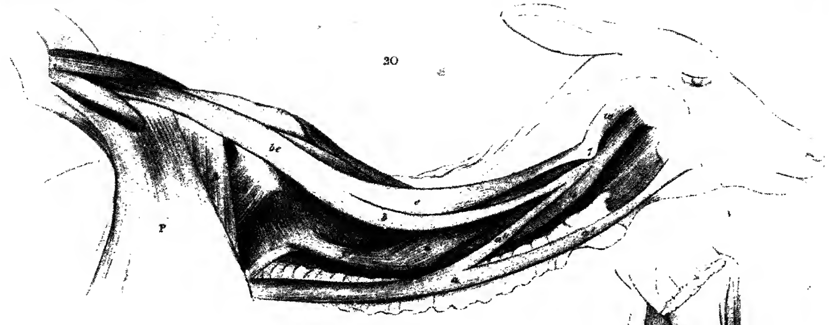

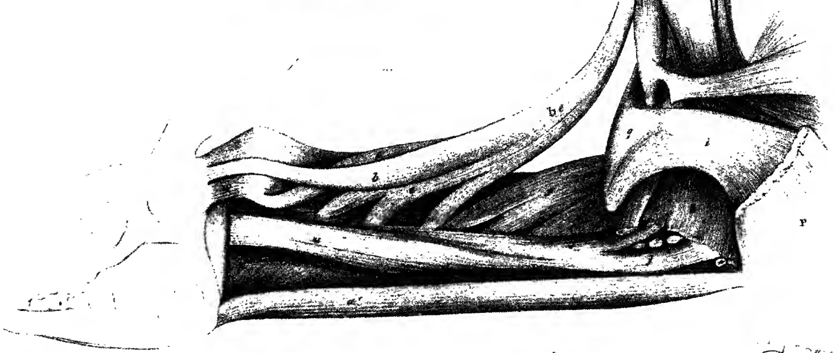

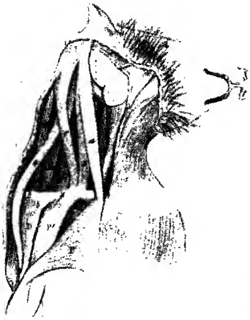

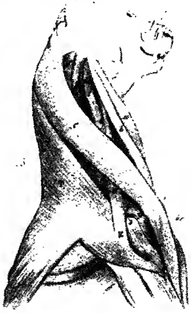

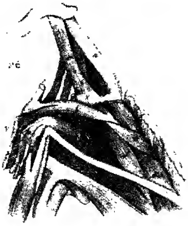

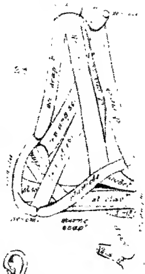
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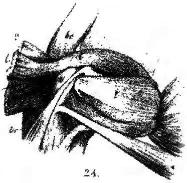

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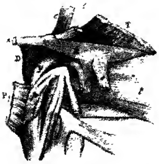

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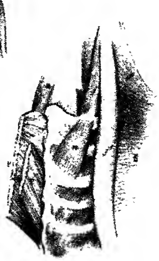

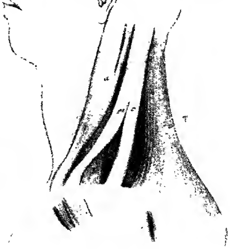

J. B. Wesley lith.

attachments to the clavicle (*x*) and sterno-clavicular ligament (*x'*) of the following muscles, viz. *scapulo-clavicular* (*k*), the *cleido-mastoid* (*b*), the *cleido-occipital* (*c*), the *levator humeri* (*b c*) (clavicular fibres of the *pectoralis major* of the human subject), and the *sterno-clavicular* (*l*), including the *subclavius.*

Fig. 17. Front view of the dissection of the muscles of the neck and shoulder of the Guinea-pig (*Cavia vulgaris*). A, the muscles "*in situ*," the lettering corresponding to the foregoing. B, the right scapula and its muscles with the rudimentary clavicle (*x*), detached from the trunk, dissected out and turned back.

Fig. 18. Right side view of the same muscles of the Squirrel (*Sciurus vulgaris*) with corresponding lettering, showing also the *sterno-costalis* muscle (*n*).

Fig. 19. Right side view of the corresponding muscles, with the same lettering, of the Norway Rat or Surmulot (*Mus decumanus*).

PLATE XI.

Fig. 20. View of the left side muscles, taken from a dissection of a fawn of the *Fallow Deer*. *a*, *sterno-mandibular* (v. *maxillaris*) dividing into two slips, one (*a'*) to the angle of the lower jaw, and the other (*a'*) to join the homologue of the *cleido-mastoid* (*b*) and the *rectus capitis anticus major* (*u*), and to be attached with the latter to the basilar of the occipital, as seen in the Rabbit (in fig. 13); *e*, the *acromio-trachélien* attached to the transverse process of the atlas (1), and forming, with *b*, the *levator humeri* (*b e*). The figure also shows the *sterno-scapular* muscle (*i*).

Fig. 21. View of the right side muscles of the neck and shoulder of a *Donkey*. The lettering refers to muscles corresponding to those of the last figure. The *cleido-mastoid* (*b*) passes, in this animal, to the mastoid process with the fibres of the *rectus lateralis* (*w*), instead of joining those of the *rectus anticus major* (*u*).

Fig. 22. Front view of the muscles of the right side of the neck and shoulder of the Hedgehog (*Erinaceus europæus*). The lettering refers to the homologous muscles of the preceding. The *acromio-trachélien* (*e*) is here seen to form the *levator humeri* alone, and by its relation to the clavicular fibres of the *deltoid* (D) to foreshadow the closer union of these muscles which occurs in the two preceding figures. 1, the transverse process of the atlas.

P. Sternal fibres of *pectoralis major*. P'. Clavicular fibres of the same muscle, showing at their origin a close homology with the *levator humeri* (*b c*) in the next figure of the domestic Cat.

Fig. 23. Right side view of the corresponding muscles of the domestic *Cat*, with the same lettering. X shows the position of the rudimentary clavicle, covered by the fibres of *b* and *c*, which form below the *levator humeri* (*b c*).

surface-peculiarities which meet the eye, were not sufficient to determine the real nature of the differences existing between the crania of different nations or individuals; that it was necessary to consider the arch and the base of the skull in their connexion one with the other, and to measure the relations of parts by means of distances and angles more systematically than had been done; and that if this were done it would appear that there were far more important variations in the antero-posterior direction in skulls than were suspected, or than existed in their breadth. The various forms of forehead, vertex, and occiput are noted by anatomists without sufficient knowledge how these local appearances are related to the structure of the cranium as a whole. Even such generally used words as dolichocephalous, brachycephalous, orthognathous, and prognathous, though efforts have been made to render them perfectly explicit, refer to varieties of form which have not been properly understood.

Mode of measurement.—It may be frankly admitted that probably the system of "geometrical drawing" recommended and described by LUCAE* would have been preferable in some respects to the mode of craniometry employed by the writer, but most of the measurements were made before LUCAE's method was published. Also it may be admitted that vertical sections, which afford the most accurate of all bases for profile views, might have been used to a greater extent than they were; but there was a difficulty in asking that a number of skulls in Museums should be bisected for examination by a private individual. Still some bisections have been obtained, sufficient to illustrate the substantial accuracy of the system in most instances followed; and while mentioning this, it is right to say how much indebted the writer has been to the late Professor GOODSIR and Professor ALLMAN of Edinburgh, and to Professor ALLEN THOMSON, for their kindness in placing specimens at his disposal. The craniometer which the writer has employed is not without its advantages, being an instrument fitted to determine the exact relation of any point in space to a given starting-point. The skull is suspended in a horizontal frame by means of two pointed screws, one on each side, which work in fixed supports; and by other screws moving on slides it may be set with any two points on a level. A vertical bar, which can be slipped up and down, slides along the side of the frame, and bears a sliding horizontal bar directed inwards, to which a needle may be attached at right angles if necessary, in either a vertical or longitudinal direction. The frame, the bars, and the needle are all marked off in inches and tenths, and by this means the vertical and horizontal distance of any point on the skull from the place of suspension is easily determined and marked on paper, so that by a series of such points a diagram may be constructed. With the assistance of a sheet of ruled paper such a diagram may be constructed in a few minutes from a series of figures not occupying more than a couple of lines. It is convenient to register the number indicating the vertical position of a point with that indicating the horizontal position placed immediately below it, like the denominator of a vulgar fraction; while backward and downward directions may be respectively distinguished from forward and upward directions by placing — before the figure. Thus the following formula is sufficient for the construction of a diagram of the Irish skull 54:—

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* Zur Morphologie der Rassenschädel, 1861, p. 16.

$$\frac{-\cdot4}{\cdot05}\ \frac{-\cdot7}{-1\cdot35}\ \frac{\cdot1}{1\cdot05}\ \frac{-\cdot65}{1\cdot9}\ \frac{-1\cdot2}{3\cdot5}\ \frac{-\cdot6}{3\cdot65}\ \frac{-\cdot3}{-3\cdot}\ \frac{\cdot75}{-3\cdot65}\ \frac{1\cdot0}{-3\cdot65}\ \frac{4\cdot4}{-2\cdot3}\ \frac{\cdot5\cdot}{\cdot55}\ \frac{3\cdot85}{3\cdot05}\ \frac{1\cdot8}{3\cdot65}\ \frac{1\cdot3}{3\cdot55}\ \frac{1\cdot1}{1\cdot5}$$

If to this formula there be added the breadth at as many points as may be desired, and the positions of those points, the utmost completeness may be given to it. By this system of notation the outline of the profile of every skull in every Museum might be recorded with the greatest accuracy, either from measurements taken with the craniometer described, or from geometrical drawings, or tracings of vertical sections.

Close to the upper and posterior margin of the external auditory meatus there is in almost every skull a slight pit, which may be termed the postauricular depression, left between the margin of the meatus and the main part of the pars squamosa. This pit, from its minute size and its centrical and constant position, is well fitted for receiving the points of the screws by which the skull is kept in the craniometer, and for being the starting-point from which the horizontal and vertical distance of other parts may be computed; and even when the screws have to be fixed to some other part, as happens occasionally when the pits are ill marked, it is easy to correct the record of measurements so as to calculate the distances from the usual place. In making the craniometric measurements on which this communication is founded, the skull in each instance was first placed in the frame with the base upwards, and, while so fixed, the position was taken of the fore and hinder limit of the foramen magnum, the occipital tuberosity and the point midway between the tuberosity and upper angle of the occipital bone which is distinguished as the midoccipital point, also the spheno-occipital suture, the posterior limit of the hard palate in the middle line, the lowest point of the alveolar process between the middle incisors and the tip of the nasal spine. The skull was then turned round and fixed with the arch upwards, the sliding screws being replaced to support it at exactly the same level as when it was reversed. The positions of the midoccipital point and nasal spine were again taken, to secure against error from strain, and the accurate adjustment of the skull having been thus tested, one could proceed to take the position of the upper angle of the occipital bone, the middle point of the sagittal suture, its anterior extremity, the fronto-nasal suture, the point on the frontal bone midway between these two last points, and also the most prominent point of the glabella.

Sometimes the exact point which was to be considered as the one wanted for measurement had to be determined a little arbitrarily, but this did not occur to any great extent. Thus a difficulty of more than a line would often occur as to the exact point to be marked as occipital tuberosity; in infants a point in the anterior fontanelle had to be chosen as representing the junction of the frontal and parietal bones in the middle line; and often a difficulty would occur at the upper angle of the occipital bone from the presence of an os triquetrum; but this was usually solved by taking the point at which the limbs of the lambdoidal suture would have met had they passed up uninterruptedly. With regard to the spheno-occipital suture, while in young subjects the anterior margin of the somewhat open suture was the point taken for measurement, in adult skulls the rough line was chosen which seems to form the limit between the two bones. In fact the writer supposed, in common with anatomists generally, that this line was the mark of the obliterated suture, but afterwards, suspecting the accuracy of this view, made a special examination and found that it was really the mark of attachment of the pharynx to the sphenoid bone, and that the position of the spheno-occipital suture was slightly further back, and disappeared without leaving any trace.

Besides the positions of the points on the skull already mentioned, it was necessary that another should be registered to indicate the extent of the anterior cranial fossa. For this purpose the outer edge of the foramen opticum was chosen, the needle being rested on it as far up as possible. It is quite true that a point in the mesial section would have been somewhat preferable to this, especially as the position of the foramen opticum varies a little in height in its relation to the floor of the anterior cranial fossa, but the point chosen has suited sufficiently well, and was the best which could be got in the circumstances.

The various points now indicated being marked on ruled paper and joined by means of straight lines, a diagram is produced on which it is easy to measure a great variety of lines and angles. The measurements on which the present communication is founded have been made from such diagrams, and the lines and angles measured are indicated by descriptive names.

In making any series of craniographic observations, it is desirable not merely to select carefully the measurements to be registered, but to determine in what position the skulls shall be placed for the sake of comparing their outlines; and in the present instance, as it was proposed to measure by vertical and horizontal distances, it was natural to attempt establishing a criterion by which a skull might be placed precisely as it had been during life when the person stood in the erect posture. On looking, however, at the numerous methods of placing the skull adopted by different observers, it is impossible not to see that we now touch on a most unsatisfactory part of the subject, and a fruitful source of error. This will appear more evidently in the sequel; meanwhile it is sufficient to state that all the plans proposed are arbitrary, and if any one of them be true, it has at least not been proved to be so. In the Anatomical Museum of the Queen's College, Galway, are placed the skulls of two criminals (skulls 49 and 50) with casts of the features taken immediately after death by Dr. Croker King, at that time the Professor of Anatomy and Physiology; and on comparing the skulls with the casts, it is clearly noticeable that they are not placed in the position which they occupied in the erect posture, and when looking directly forwards during life, by following any of the plans which have been recommended, or by using the rule which has been adopted in the present inquiry. In all probability the skull does not possess any two points which in every instance lie in one vertical plane, or one in front of the other in a horizontal plane; and it is quite likely that it will always remain impossible to determine from the characters of a skull what was its precise position in the erect posture of the body. But the only modes by which accurate information can be arrived at appear to be by extensive examination of the living subject, and by comparison of the skulls of the dead with carefully taken casts of the features. Early in these inquiries it occurred to the writer that doubtless the same principle of balance came into play in the support of the head as in the rest of the body, but that as the proportions of the head are very different at different ages, the position of balance on the vertebral column must be different at different periods of life. Proof will be adduced in support of this proposition further on; but it is necessary now to mention that it was with the view of collecting evidence on this point that the arbitrary horizontal line was chosen which has been used throughout the present investigations, and which was obtained as follows. The skull to be examined having been placed in the frame with the base upwards, a flat slip of wood was rested on the condyles and posterior boundaries of the foramen magnum, and allowed to project backwards; the skull was then rotated till the wooden slip was in the horizontal position. Now, probably in most cases the posterior limit of the foramen magnum is close to or rests on the arch of the atlas, and in elderly subjects it is not uncommon to find a flat facet at the back of the foramen magnum indicating this contact. If, therefore, the deepest parts of the upper articular surfaces of the atlas and the posterior arch of that bone lay in a horizontal plane, the skull would be correctly placed by the means now described. Unfortunately, however, this is not always the case; and all that can be maintained is that some approximation to accuracy is thus reached, which will probably be admitted on examination of the majority of the accompanying diagrams. To eliminate, therefore, as much as possible an element in which uncertainty is inherent, the measurements in the present inquiry have been all made so as to be independent of the position of the skull, except in the section in which the question of position is itself considered.

To prevent the possibility of any mistake, it may be proper to explain at the outset that all statements which will be made with regard to national forms of crania are to be understood as referring simply to the results of the measurements of the skulls enumerated in the General Table; and as it will be observed that in the case of some nationalities the specimens are too few in number, and in many instances the history is less complete than would be desirable, the statements founded on those specimens are not put forward dogmatically as of general application, but rather as suggesting probable laws which must be left for other observers to investigate. When the form of the Greek skull is spoken of, it is of course only the Modern Greek which is alluded to; and the peculiarities exhibited by each of the five Greek skulls examined makes the writer particularly regret in this instance the paucity of specimens and the incompleteness of their histories. So also it is much to be regretted that there is no record of the particular part of Germany from which any of the eight German skulls examined have been obtained, and that therefore it has been impossible to distinguish between North and South German.

I. CRANIUM PROPER.

Extent of arch and base-line (General Table, columns 6, 7, 8).—When it is considered that one of the most marked peculiarities of the human skull is the great elongation of the arch and shortening of the base, it becomes interesting to know what relation the length of arch has to the length of base at different ages and in different nations. The length of arch has been measured along the middle line of the roof, from the fronto-nasal suture to the back of the foramen magnum, while the direct distance between the same points has been taken as base-line. The distance from the occipital tuberosity to the foramen magnum is best included along with the arch, because, whether its variations or its morphological constitution are regarded, it appears to be closely associated with the arch, and because it has been found in making the measurements that the tuberosity is not a good land-mark, but varies in position according to its prominence. The foramen magnum is best considered as part of the base; for although, when we take into account the development of the medullary canal, and the appearance of parts in the lower animals, the strict base of the skull must be looked on as commencing at the front of the foramen magnum, or opposite the condyloid margins of the basilar ossification of the occipital bone, still we shall find that such a close connexion exists between the angles at which the foramen magnum and the floor of the anterior cranial fossa respectively lie to the intermediate portion of the cranium, that it is convenient to consider all three as belonging to the base.

The general average proportion which the length of the arch bears to the base-line in the adult may probably be estimated at about 2·70 : 1; but it varies considerably both in individuals and nations.

So far as one may judge from the fœtal skull examined, it would appear that before the middle period of fœtal life the arch is considerably less developed in proportion to the base than it is in the adult, but that afterwards the proportion is altered by the great growth of the arch, so that in the later months it is about three to one. In new-born infants and in children it more frequently exceeds than falls short of this proportion; the average found in five skulls of new-born infants being 3·06 : 1, and the average in seven children of ages varying from one to ten years being 3·07 : 1. The ten-years-old skull is the only one of these seven in which the base-line has acquired a length which might be permanent, while in four of them the arch is such as might be found in the adult. We may judge therefore that the base-line continues to elongate after the arch has acquired its permanent dimensions.

On examining the proportion of arch to base-line in adults we find, as we shall also find in all other measurements, that the variations in individuals of one nation are so great that the minimum figure in a nation in which the proportion is high is always within the limits of variation found in nations in which it is low; and that therefore it is very necessary to compare different specimens of one nation together. Much the smallest proportion of arch to base in the series of skulls examined is in the Esquimaux skull 77, in which it is 2·28 : 1; but in the other Esquimaux it is 2·67, as high as the average of the French skull. The Kafir, the Negro, the Australian*, the Greek, and the French skulls show little difference in this proportion, their averages varying from 2·61 : 1 in the first to 2·67 in the last. The Irish have the proportion highest, 2·89 : 1; and next them come the Chinese*, closely followed by the German, 2·80, and these by the Hindoo, 2·78. The highest proportion found among the Irish skulls, it will be observed, exceeds the average proportion in infants.

Proceeding to analyze the effect of sex on the proportion of the arch to the base, it appears that in five out of seven nationalities, in which several skulls have been examined, one or more of which have been females, either the highest or lowest extreme of proportion is in a female; and the cause of this is that in the female the base-line is almost always short, while the extent of the arch is in some instances as great as in the male, and in others diminished to a greater proportional extent than is the base. To clear the averages from the disturbing effects of the introduction of a varying number of female skulls in different nationalities, and to exhibit the peculiarities of sex now stated, a Table has been made exhibiting the average proportion of arch to base-line in males and females separately.

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* In order to obtain a sufficient number of measurements by which to judge of the proportions of arch to base, and of the parts of the arch, in Chinese and Australian skulls, the writer has availed himself of the "geometrically drawn" figures given by Lucae in his work 'Zur Morphologie der Rassenschädel,' and made on them measurements the results of which have been combined with the measurements given in the General Table, in estimating averages, and are therefore here given. The columns are numbered the same as the corresponding columns in the General Table.

	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
Lucae, XXI. 7, Chinese from Java, with Javan blood	4·4	5·5	5·	14·9	5·7	2·61	80	90	3·9	5·	4·4	78	88
„ XXI. 5, „ „ „	4·3	5·9	5·	15·2	5·4	2·81	72	84	3·7	5·1	4·4	72	86
„ XXI. 3, „ „ „	4·1	5·6	4·9	14·6	5·1	2·86	73	87	3·7	4·8	4·4	77	91
„ XXI. 6, „ „ with Javan mother	4·	5·4	5·2	14·6	5·2	2·80	74	96	3·5	4·7	4·5	74	95
„ XXI. 8, Pure Chinese	4·3	5·3	5·3	14·9	5·2	2·86	81	100	3·8	4·7	4·7	80	100
Chinese, 85 of the General Table	4·8	4·8	4·8	14·4	4·8	3·00	100	100	3·95	4·3	4·2	91	97
Average	4·31	5·41	5·03	14·76	5·23	2·82	80	92	3·75	4·76	4·43	78	92
Lucae, XXII. 10, Australian male	4·3	5·1	5·2	14·6	5·4	2·70	84	101	3·5	4·6	4·5	76	97
„ XXII. 11, „ „	4·6	5·	5·5	15·1	5·5	2·74	92	110	3·8	4·6	4·7	82	102
„ XXII. 12, „ „	4·	5·3	4·3	13·6	5·4	2·51	75	81	3·5	4·6	3·9	76	84
„ XXII. 9, „ „	4·6	5·2	4·7	14·5	5·5	2·63	88	90	3·6	4·6	4·2	78	91
„ I. a 322, „ female	3·8	5·3	4·3	13·4	4·9	2·73	71	81	3·4	4·7	3·8	72	80
Australian, 72 of the General Table	4·4	5·2	4·9	14·5	5·5	2·63	84	94	3·7	5·	4·3	74	86
„ 73 „ „	4·25	5·1	4·9	14·25	5·4	2·63	83	98	3·5	4·7	4·5	74	95
Average	4·27	5·17	4·82	14·27	5·37	2·65	82	93	3·57	4·68	4·27	76	90

It must not be forgotten that four of Lucae's Chinese skulls are from persons with Javan blood by the mother's side.

	Numbers measured.	Males.			Numbers measured.	Females.		
		Arch.	Base.	Proportion.		Arch.	Base.	Proportion.
Esquimaux	2	13·85	5·6	2·47				
French	6	14·	5·36	2·61	3	13·96	5·	2·79
Hindoo	3	14·2	5·1	2·78				
Greek	5	14·2	5·3	2·69				
Negro	5	14·3	5·52	2·58	2	14·3	5·15	2·77
Scotch	4	14·36	5·3	2·70	2	14·85	5·3	2·78
Australian	6	14·42	5·45	2·64	1	13·4	4·9	2·73
Kanaka	1	14·6	5·5	2·65	1	14·7	5·	2·94
Chinese	6	14·77	5·23	2·82				
German	5	14·82	5·24	2·82	3	13·73	4·96	2·76
Kafir	3	15·06	5·66	2·65	1	13·5	5·3	2·54
Irish	6	15·23	5·23	2·91	3	14·56	5·1	2·85

This Table shows constant shortness of base-line in the female, while the arch is only sometimes shorter than in the male. The proportion of arch to base-line is usually larger in the female skulls examined than in the male, as is held to be the rule by WELCKER and ECKER; but in the German, Irish, and Kafir skulls, in which this is not the case, the reason is that though the base-line is shorter than in the male, the arch differs still more from the male arch.

The Table also brings out more distinctly than before the very large proportion of arch to base in the Irish, which will probably be found to be a marked and general national character. It further shows that the proportion of arch to base in a nation may vary from either arch or base-line deviating from the average, or from deviation of both; and therefore probably the actual length of these measurements is of more value than the proportion of one to the other. The most striking and altogether remarkable fact is that in uncivilized nations, while the length of the arch is very variable, the length of the base-line is always great. Thus in length of arch the three Kafir skulls exceed all other skulls in the foregoing Table, with the exception of the Irish, but the Kafir base-line exceeds the Irish by not much less than half an inch. In the Esquimaux skull 77, and the Carib skull 88, both of them skulls of most savage type, the base-line reaches the extraordinary length of 5·9. In only one European skull, the French 24, has a base-line of 5·7 occurred, while in only two others, the French skull 22, and the Scotch skull 38, it has amounted to 5·6. The low proportion of arch to base in the French skulls depends on a concurrence of a long base-line with a short arch, three out of the six males having the base-line 5·5 or more, and four of them having the arch 13·8 or less. The Irish, German, and Chinese skulls very closely agree in average length of base-line; but the Irish have greater proportionate extent of arch, because in them the arch is absolutely greater, reaching as it does to a greater length than has been obtained in any other skulls in the series. The shortest arch and shortest base are found united in the Peruvian skull 78; the Scotch skull 37 has as short an arch, but its base-line is longer.

In only one out of the fifteen female skulls examined does the base-line reach to 5·4, and in two to 5·3; in the remainder it varies from 4·9 to 5·2, while we have already seen that in the child ten years old it reaches 5·1. It appears from this that the additional growth by which the base-line in the male comes to exceed that found in the female takes place after the age mentioned. It might therefore be naturally supposed, as occurred to the writer, that the facts which have now been detailed are mere results of greater development of the frontal sinus in the adult male than in the female and child, and in the savage than in civilized races; but this is not the case; for we shall find that though development of the frontal sinus does add to the base-line, growth of the base of the skull from the front of the foramen magnum forwards to the level of the foramina optica plays a much greater part in the addition which takes place.

Regions of the base.—According to the system of measurement pursued in the present inquiry, the base of the skull is represented by three lines, the middle one of which, the foramino-optic line, corresponds pretty nearly with the 'basicranial axis' of Huxley; while the hindmost displays the length of the foramen magnum, and the foremost the length of the orbit; the length of the base-line varying according to the length of these distances, and the angles at which they are placed one to another.

Foramen magnum (General Table, column 25).—This is the region of the base which presents least characteristic variation in length. It usually measures from 1·4 to 1·5. The extremes met with are 1·25 and 1·70. Little or no distinction of a national character is discoverable in its variations; and although skulls which have a markedly long base-line sometimes owe the peculiarity in a small degree to the length of this foramen, as Esquimaux 77 and Kafir 69, there are other skulls, such as 23 and 56, and the infant skull 11, which combine a long foramen magnum with a short base-line. The proportion borne by the foramen magnum to the base-line does not appear to undergo any constant alteration in the passage from infancy to adult age.

Orbital length (General Table, column 27).—This measurement likewise has the same proportion to the base-line at birth as in the adult. In the six-years-old as well as in the twelve-years-old child it is of a length which might be permanent; in the adult male it is slightly but distinctly greater than in the female. The national variations are well worthy of remark. If influenced by the belief that large development of the frontal part of the skull accompanies intellectual capacity, we might expect to find a greater orbital length in civilized than in savage nations. But this is not the case. Lucae, in his comparison of European and Australian skulls, pointed out that the floor of the anterior fossa of the skull was quite as well developed in the latter as in the former*; and this observation is quite in harmony with the present measurements, although unfortunately he has accompanied it with the hasty remark, not borne out by his drawings, that the Australian development is defective in the upper part of the frontal region.

In the meantime, however, let us consider simply the orbital length. The range of its variation in the great majority of skulls is from 1·8 to 2·1. In the Australian, Negro, and Kafir the average is high, while in the French and German it is shorter than in any but the brachycephalous Americans. But the amount of orbital length in the Australian, Negro, and Kafir is not sufficient to account for the great length of base-line of these skulls; the Greek, the Hindoo, and the Irish skulls agree with them in the average orbital length, while in these, especially in the Hindoo and Irish, the base-line is much shorter. At the same time the orbital length is to be recognized as one element on which length of base-line depends; and examination of the different skulls of each nation shows that the extent of the orbital length usually varies in harmony with that of the base-line.

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* Lucae, *op. cit.* p. 40.

The adult skulls in which the orbital length is less than 1·8 are the male and female French skulls 23 and 26, the female German 34, and the Chinese 85, all of them well-proportioned skulls. The anomalous male French skull 89 cannot be taken into account, since in it the shortness of the orbit (1·6) is obviously the direct result of early synostosis of the lower part of the coronal suture on each side, and has been compensated for by undue height of the cranial arch, the result being a very peculiar deformity.

In the following skulls the orbital length is above 2·1, viz.:—in BURKE, in the Greek 48, in the male Kanaka, in the Australian 73, and the Negro 62 it measures 2·15; in Hindoo 60 and in the Hottentot Chief and the Idiot it measures 2·2; and in the Carib 88 it reaches the extraordinary depth of 2·4. The list thus contains some of the very ugliest skulls examined; and of the nine only three, namely, the Greek, the Hindoo, and the aberrant skull of BURKE, belong to civilized nations.

So far, then, as these observations extend, it would appear that shortening of the orbital length never occurs as a feature of degraded national type, but that elongated orbital length does so occur. Where it does so, the probability is that the elongation is in connexion rather with development of the frontal sinus or diploë than of the cranial cavity. It produces antero-posterior elongation of the nasal fossæ in their upper part, which may perhaps be favourable for the expansion of the olfactory nerve and production of a keen sense of smell.

Foramino-optic line (General Table, column 26).—This line, extending from the front of the foramen magnum to the point midway between the foramina optica, and thus indicating the middle portion of the base, corresponds, as has been said, pretty closely with Mr. HUXLEY'S "basicranial axis"*, which, starting from the same point, passes to the junction of the sphenoid and ethmoid bones in the anterior cranial fossa. Probably the one line is as well chosen as the other; and it may be frankly admitted that a line drawn to the angle of separation of the middle and anterior cranial fossæ in front of the optic commissure might be preferable to either.

In new-born infants and in children the foramino-optic line is about equal in extent to the orbital length, whereas in the adult it usually exceeds it. In the ten-years-old skull it is of such length as might possibly be permanent, being of a length not uncommon in the adult female. The extent of this line in the female is less than in the male, the difference between the sexes in this respect being still more distinct than in the extent of orbital length. The Table shows an apparent exception to this statement in the case of the Scotch skulls; which arises from one of the two female skulls having the exceptionally long foramino-optic line of 2·4, a mere individual aberrancy. Occurring, however, in a skull known beyond all possibility of question to be female, and in other respects having female characteristics, it affords a good illustration that the characters of sex, although more constant than those of nationality (HUSCHKE*), are, like them, not all present in every instance. Examining the length of the foramino-optic line in different nations, it is found to be shortest in the Irish, Peruvian, and Hindoo, partly accounting for the shortness of base-line in those skulls, and to be longest in the Kafir and Negro skulls, which have the base-line long. Also, an inspection of those skulls in which the foramino-optic line amounts to or exceeds 2·4 shows that length of this line, like great orbital length, is a character liable to be found in skulls of low type. But it is likewise long in others which cannot be so classed. Thus the female Scotch skull 43, already alluded to, and the French skull 24, in both which the foramino-optic line measures 2·4, and the Kafir skull 45, in which it is 2·45, are not skulls of a low type; but the French skull 20, in which this line measures 2·45, is the worst proportioned of the six French males, and has a depression at the top of the coronal suture which has probably been the direct result of spheno-parietal synostosis. In the Negro 61 and the Carib 88, the foramino-optic line measures 2·4; in Esquimaux 77 and Kafir 68, it measures 2·5; in the Maori 2·55; while it reaches the unwonted length of 2·6 in the Spanish pirate, a skull which in various respects may be described as being as peculiar in character as the man to whom it belonged is said to have been savage.

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* Journal of Anatomy and Physiology, November 1866, p. 67.

	Males.			Females.		
	Number measured.	Orbital length.	Foramino-optic line.	Number measured.	Orbital length.	Foramino-optic line.
German ……	5	1·93	2·13	3	1·8	2·03
French ……	6	1·95	2·24	3	1·83	2·03
Scotch ……	5	2·	2·22	2	1·95	2·3
Irish ………	6	2·05	2·09	3	1·91	1·93
Greek………	5	2·05	2·18			
Kafir ………	3	2·05	2·36	1	2·	1·95
Hindoo ……	3	2·05	2·1			
Negro………	5	2·07	2·32	2	1·92	2·12
Australian …	2	2·1	2·2			
Kanaka ……	1	2·15	2·3	1	2·05	2·1

Angles at which the three divisions of the base lie one to another (General Table, columns 28, 29, 30).—The angles which the foramen magnum and the orbital length respectively form with the foramino-optic line may be termed the *foramino-basilar* and *orbito-basilar angles*. The number of degrees between the line of the foramen magnum and the line of orbital length on the upper side may be termed the *cranial curve*; for if we look on the cranial cavity as the continuation of the cylinder of the spinal canal curving rapidly forwards, and consider the incisura of the frontal bone as being morphologicaly its distal extremity, then the angle mentioned indicates approximately the amount of curve which exists. When the foramino-basilar and orbito-basilar angles are equal, the foramen magnum and the orbital length lie at an angle of 180°, or, in other words, are parallel. According as the orbito-basilar angle is greater or less than the foramino-basilar, the cranial curve is less or greater than 180°.

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* Schädel, Hirn, und Seele, p. 2.

The foramino-basilar and orbito-basilar angle and the cranial curve require to be all considered together; and this is more particularly the case, since it is a remarkable fact that, while the foramino-basilar and orbito-basilar angles have a wide range of variation, the adult cranial curve varies within much more restricted limits; or in other words, the difference between the foramino-basilar and orbito-basilar angles in any skull being limited, these angles are to a considerable extent interdependent. Thus in the Kafir skull 68, the foramino-basilar angle measures 128°, and in the female Kafir it is 150°; the orbito-basilar angle of the skull of the Hottentot measures 129°, and that of the Greek skull 46 measures 150°; but a cranial curve 21° more or less than 180° is probably never to be met with in any undeformed skull. One female French skull, 25, has a foramino-basilar angle of 162°, while another, 26, has an orbito-basilar angle of 128°; but these two angles could not coexist in any skull except with very great deformity; were they to do so they would give a cranial curve of 214°; and the only skull in which so great a curve is approached is 91, the skull with the base driven in, in which the curve is 212°.

While in adult European skulls the cranial curve seldom falls short of 180° by more than three or four degrees, and more frequently exceeds that amount, in the six fœtal skulls examined it varies from 150° to 176°. As, however, those in which the amount of curve is lowest are skulls of fœtuses of the 4th, 7th, and 8th month, while the highest amounts occur in a fœtus of the 5th and another of the 8th month, it would appear that the increase of curve proceeds more rapidly in one skull than in another. A smaller degree of the same irregularity is seen on comparing the five skulls of new-born infants; these seem, however, clearly to show that the cranial curve is not quite completed at birth, while an examination of the skulls of older children shows that at least in some instances it is completed before the third year.

	Males.				Females.			
	Number measured.	Foramino-basilar angle.	Orbito-basilar angle.	Cranial curve.	Number measured.	Foramino-basilar angle.	Orbito-basilar angle.	Cranial curve.
Kafir	3	$133^{\circ}\frac{2}{3}$	144°	$169^{\circ}\frac{2}{3}$	1	150°	144°	186°
Esquimaux...	2	$132\frac{1}{2}$	$141\frac{1}{2}$	171				
Negro.........	5	136	139	177	2	137	138	179
Kanaka	1	129	132	177	1	138	129	189
Greek.........	5	138	$140\frac{3}{4}$	$177\frac{1}{4}$				
Hindoo	3	128	$127\frac{1}{3}$	180				
Australian ...	2	$139\frac{1}{2}$	$138\frac{1}{2}$	181				
French	6	$141\frac{2}{3}$	$139\frac{5}{6}$	$181\frac{5}{6}$	3	$145\frac{2}{3}$	138	$187\frac{2}{3}$
Scotch	5	$135\frac{3}{5}$	$133\frac{2}{5}$	$182\frac{1}{5}$	2	$139\frac{1}{2}$	$136\frac{1}{2}$	183
Irish	6	$143\frac{2}{3}$	139	$184\frac{2}{3}$	3	$147\frac{1}{3}$	140	$187\frac{1}{3}$
German	5	$146\frac{2}{5}$	$139\frac{1}{5}$	$187\frac{1}{5}$	3	146	$141\frac{1}{3}$	$184\frac{2}{3}$

A perceptible difference exists in respect of cranial curve between the skulls of different nations, and this difference displays a distinct relation between the amount of curve and the length of base-line. Taking males only into account, the greatest cranial curve is found in the German skulls, in which the average amounts to 187°, and next to them come the Irish; while in the Kafirs the average is lowest, amounting to 169°; and next them come the Esquimaux and Negro. Thus the German and Irish skulls, which have the base-line short, are highly curved, while the Kafir and Negro skulls, which have the base-line long, are less curved. So also, excepting in the case of the Germans, the average curve of the female skulls is greater than the curve of the males of the nation to which they belong. These differences recall to mind Virchow's theory of kyphosis in the skulls of Cretins, according to which, in cases of premature synostosis of the base, increased curvature supplies a means of enlargement of the cranial cavity, which to some extent compensates for the arrested basal growth. Without venturing to agree with that theory in its details, as an explanation of the pathological form of the skull in Cretins, to which further reference will be made, it may be allowed to allude to it as the first recognition of the principle that increased curving of the base of the human skull is a means by which the cranial cavity may be enlarged*. In the estimate, however, of the cranial curve now being made there is included an important element not taken into account in Virchow's angula sellæ (sattelwinkel) or any modification of it, viz. the foramino-basilar angle, which indicates the degree in which the commencement of the inferior wall of the cranial cylinder is bent forwards from the spinal inlet; and since, as has been seen, this angle compensates and to a great extent varies with the curve estimated in this paper by the orbito-basilar angle, and which Virchow estimated by means of his angula sellæ, it is quite evident that the increase of cranial curve now spoken of as a means of expansion of the cranial cavity is entirely different in detail from Virchow's kyphosis. The size of the foramino-basilar and orbito-basilar angles individually varies according to a totally different law from that which regulates the cranial curve; and seeing that accordingly as the foramino-basilar angle is large or small, so also to a certain degree is the orbito-basilar, the base of a skull may be termed *level* when both angles are large, and *steep* when both are small.

The base of the skull at birth is more level than afterwards, and it is still more level before birth. In the fœtus of the fourth month, in consequence of the want of development of the arch as compared with the base, a remarkable steepness or want of opening of the foramino-basilar angle is combined, contrary to the rule in the adult, with a very large orbito-basilar angle. In all the other skulls of fœtuses and infants examined the orbito-basilar angle is above the adult average, and in most of them the foramino-basilar angle is likewise large, although in the eight months' fœtal skull 6 and the infant skull 10 it is small, and makes the cranial curve likewise exceptionally small. Scarcely any national distinction can be determined from the present measurements in respect of levelness and steepness of base, both conditions being met with in most of the national lists; but it may be noted that all the three Hindoo skulls have marked steepness of base, and this is one of the elements which make those skulls shorter and higher than most of the others.

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* It will appear in the sequel that diminished curving of the base may likewise become a means of enlarging the cranial cavity. Increased curving is only a means by which the base accommodates itself to the growth of the roof-bones when its own growth is limited; but if the arch and base be each of a given length the cranial capacity will be increased by diminishing the curve of the base.

The averages of the basal angles in females show in the Table in each case a somewhat greater levelness of base than the males of the same nation. Some of the most level bases belong to females, and are accompanied with great cranial curve. As illustrations of this, the French and German female skulls 25 and 34 may be mentioned; and it may be remarked that in this, as in many other matters, a specially feminine character may be seen to be given to a skull by the persistence of the form of childhood.

The following list of skulls in which the cranial curve is 174°, or a smaller amount, will serve to show that very frequently a slight cranial curve is combined with more than average steepness of base. The most marked instances in which this is not the case can be somewhat explained away. Thus in the Greek skull 46 the slightness of the cranial curve, as well as the length of base, depends on a peculiar idiosyncrasy which will be made the subject of comment hereafter (p. 165); the compressed and flattened American skulls 95 and 96 owe their want of curvature, as will be subsequently shown (p. 167), to the mode in which they have been deformed; and in the synostotic skull 89 the same arrest of the forward growth of the cranium which has caused the brain to push the roof of the skull upwards, has caused it to push the base downwards in the region of the orbito-basilar angle.

Numbers in General Table.	Description.	Base-line.	Cranial curve.	Foramino-basilar angle.
46	Greek	5·6	172	142
63	Negro	5·7	170	139
64	Negro	5·4	174	138
68	Kafir	5·8	165	128
69	Kafir	5·7	168	137
77	Esquimaux	5·9	164	129
81	Spaniard	5·5	172	126
83	Turk	5·1	174	133
86	Maori	5·7	167	126
88	Carib	5·9	171	125
89	Synostotic French	5·	172	140
93	Hunchback	5·35	169	126
95	Compressed Chinook ...	5·3	173	141
96	Flattened American ...	4·95	166	144

This list also shows that slightness of cranial curve is usually an accompaniment of long base-line; the Turkish skull is, however, a good example of slightness of curve and short base-line going together; and it is fair to note that in the French skull 22 there is a marked instance of a long base-line and a more than average cranial curve going together.

Length of arc of the different portions of the arch (General Table, columns 3, 4, 5, 9, 10). *Young skulls.*—It is well known that in early life the frontal and occipital parts of the skull are smaller as compared with the parietal than in the adult*; and in accordance with this an examination of the Table shows that in fœtal life, infancy, and childhood, the length of the frontal and of the occipital portion of the arch both bear, as a general rule, a smaller proportion to the length of the parietal portion than they do in the adult. It is not, however, to be supposed that the parietal has a predominance over the frontal and occipital region from the first, and that these grow proportionally larger in an equable manner; for setting aside the consideration that in the young embryo the wall covering the posterior cerebral vesicle is much larger than what is afterwards to become the parietal and frontal part of the cranium, and confining attention to the measurements in the Table, it appears that from the fourth month of fœtal life till birth, while the disparity between the parietal and occipital regions is diminishing, the disparity between the parietal and frontal is increasing. Five of the six fœtal skulls examined have the occipital smaller in proportion to the parietal than it is in the skulls of the new-born infants, and four out of the five skulls of infants have the frontal smaller in proportion to the parietal than it is in the fœtal skull. The conclusion from this is that the parietal has grown more than the frontal, and the occipital more than the parietal in the later months of fœtal life. But after birth the frontal, which has been for a time the slowest growing region, begins to expand most rapidly of the three, while the occipital region still continues to expand more rapidly than the parietal. This is indicated in the General Table by the children's and adult skulls having the proportionate length of the frontal. as compared with the parietal region, much higher than the new-born infants, and likewise having the proportion of the occipital to the parietal region higher than the infant skulls, though not greatly so.

It further appears, on estimating the percentage of the whole arch formed by each region respectively, that already in the youngest fœtus examined the frontal region forms as high a proportion of the whole arch as it does in the adult, but that at birth the proportion is temporarily diminished; while the percentage belonging to the parietal region goes on diminishing, and the percentage belonging to the occipital goes on slightly increasing, from the youngest fœtus examined till the adult skulls are reached†. No doubt the result is obscured by the large amount of individual variation which exists in different skulls, and to arrive at a precise estimate of the average extent of each region at different periods of development would probably require the examination of a large number of skulls; but a careful review of the figures justifies the statement now made.

Taking a survey of the growth of the cranium from the earliest period, the following account is probably correct. At first, in the early embryo, the occipital region is much the longest; then, when the cerebral hemispheres begin to expand, growth passing forwards, the parietal grows so rapidly as to exceed its proportion in the adult, and the frontal so rapidly as to attain to the adult proportion: in the latter half of fœtal life, when the hemispheres push backwards over the cerebellum, growth goes on again more rapidly in the back than in the fore parts of the cranium, so that while the parietal region maintains its proportional length to the whole arch, the proportion of the occipital region increases, and that of the frontal diminishes: lastly, after birth, the proportional length of the occipital region increases slightly, and that of the frontal region much more markedly till the adult proportions are attained, which appears to be at a variable period within a few years after birth.

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* Huschke, 'Schädel, Hirn, und Seele,' p. 46.

† The percentage which each portion of the arch forms of the whole, though calculated, is omitted from the General Table, to prevent unnecessary multiplication of figures.

Adult skulls.—In the adult skull the individual variation in the proportional length of the different regions of the arch is very considerable, and the national variation for the most part only slight, while no sexual variation can be safely deduced*. Such national variation as exists, however, is of a definite kind. The proportional length of the frontal region as compared with the parietal varies pretty nearly *pari passu* with the proportional length of the occipital region, or, in other words, the variation in proportion arises almost entirely from lengthening and shortening of the parietal region. Thus the French and German skulls have both the largest proportion of frontal region and the largest proportion of occipital region to parietal; and more markedly the Chinese and Australian skulls have both the frontal and occipital region of considerably smaller length as compared with the parietal than is usual. This is the more remarkable in the case of the Chinese, since it has been already noted that they have the additional childlike peculiarity of shortness of the base as compared with the arch†.

Length of chord of different portions of the arch‡ (General Table, columns 11, 12, 13, 14, 15).—In estimating the length of the different regions of the arch, it seemed advisable to measure not merely the arc of each portion, but likewise its direct length or chord; for it seemed possible that local bulgings and flattenings might produce variations in the length of arc while the chord remained unaffected. The measurement has not proved necessary for the purpose for which it was undertaken, but it has served to lay bare some points of interest. In most of the nationalities examined, including the German, Scotch, Irish, Greek, Negro, Kafir, Maori, and Australian, the ratio of the

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* It may indeed be noted that the highest proportions of occipital to frontal are nowhere to be found in the list of female skulls measured; but the writer is inclined to impute this to the uniform absence of that degree of muscularity which leads to great prominence of the occipital tuberosity and consequent increase of the surface length of the occipital bone. It would not be surprising, however, if an examination of a large number of female skulls were to show on the average a diminished proportion of both occipital and frontal arc to the parietal when compared with male skulls; such a result would harmonize with HUSCHKE's statement, that in the female the capacity of both the occipital and the frontal segment is smaller in proportion to the parietal than in the male; and with WEISBACH's account of the low and small forehead of the German female.

† The measurements of the three portions of the arch in different nations given by Dr. BARNARD DAVIS in his 'Thesaurus Craniorum,' the elaborate description of his magnificent collection do not altogether corroborate these statements, especially with regard to the Chinese. But the object of the present paper being tentative, written as it has been rather in the hope of pointing out explicit methods of comparison than to dogmatize on a slender basis with regard to characteristics of particular nations, it has been deemed advisable rather to add this caution than to alter the statement in the text.

‡ These are the measurements termed by CARUS height of the anterior, middle, and posterior cranial vertebra CARUS, Grundzüge einer neuen Cranioscopie, p. 16.

	Proportion to parietal arc of				Proportion to parietal chord of			
	Occipital arc.		Frontal arc.		Occipital chord.		Frontal chord.	
	Average.	Extremes.	Average.	Extremes.	Average.	Extremes.	Average.	Extremes.
2 Esquimaux	98	100 97	108	109 108	88	90 87	107	110 104
8 German	94	102 83	104	126 91	82	90 75	100	120 90
9 French	94	118 85	102	118 93	87	97 80	103	115 97
8 Scotch..............	93	102 86	100	109 92	85	94 81	99	107 90
9 Irish	91	110 71	100	107 94	80	91 67	97	102 91
4 Kafir	91	97 85	100	104 94	84	90 77	97	102 91
7 Negro	88	108 73	100	110 87	82	100 68	97	107 86
5 Greek	88	102 78	98	108 92	81	88 73	95	104 91
3 Hindoo	87	88 37	100	110 94	84	88 81	101	114 95
2 Peruvian...........	85	87 83	100	110 90	90	95 85	107	117 97
2 Kanaka	84	85 84	94	102 87	81	83 80	94	102 86
7 Australian	82	92 71	93	110 81	76	82 72	90	102 80
1 Maori..............	81	......	90		74	......	88	
6 Chinese	80	100 72	92	100 84	78	91 72	92	100 86

occipital to the parietal chord is considerably less than that of the occipital to the parietal arc; and the ratio of the frontal chord to the parietal is also a little less than that of the frontal to the parietal arc. This means that in those nations the frontal part of the arch is more curved than the parietal, and the occipital more curved than either. In the French, according to the average of the nine skulls examined, the ratio of the frontal to the parietal chord is slightly greater than that of the frontal to the parietal arc; and on looking at the measurements of the individual skulls, it is seen that this is the case in four instances, that in two instances the proportions of the chords and arcs are the same, and that in the remaining three instances the figures indicating the proportionate lengths of the frontal chords are only very slightly smaller than those of the frontal arcs. Therefore, so far as these nine skulls bear evidence, the French differ from the nations above mentioned in having the parietal region of the arch as much curved as the frontal, or more so. In this the French skulls agree with the Hindoo, the Chinese, and the Kanaka; but these three nations present the additional peculiarity that in the occipital region the proportional length of the chord approaches very near to that of the arc, so that the amount of curvature in the three regions in these nations is nearly equal. The Chinese skull from the Edinburgh Collection is an exception to this statement; but the calculations made from LUCAE'S drawings agree completely one with another (p. 123). This same phenomenon of near correspondence of the proportional lengths of the chords with the lengths of the arcs of the different parts of the arch is a marked characteristic of the skulls of infants; we have therefore here another interesting child-like peculiarity of the Chinese skulls. Both the Peruvian skulls present the peculiarity of having the parietal region distinctly more curved than either the frontal or occipital, and in this they agree with the compressed and flattened American skulls; but how far the peculiarity is referable to the slight compression which the Peruvian skulls have both to a certain extent undergone is an open question. It may be interesting to note that measurements taken from the cast of the Tartar skull described by Professor HUXLEY* as an example of extreme brachycephalism give 93 and 111 as the proportions of the occipital and frontal arcs to the parietal estimated at 100, and 92 and 112 as the proportions of the occipital and frontal chords, thus indicating an equally distributed curvature. Dolichocephalism and brachycephalism will be treated of in a subsequent page; meanwhile the writer may be allowed to state, without further explanation, that he believes this equally distributed curvature to be a brachycephalic characteristic.

A further and more detailed acquaintance with the curves of the arch may be sought by examining the angles formed by lines passing from point to point; and this will be now attempted.

Angles expressive of the form of different parts of the arch (General Table, columns 16 to 24). *Young skulls.*—In the fœtus and in infants the forehead springs at a considerably greater angle from the roof of the orbit than in the adult. Whereas the average orbito-frontal angle varies in different nationalities from 77° to 83°, and only one adult skull (46) has an orbito-frontal angle exceeding 90°, in the fœtal skulls and those of infants the same angle exceeds 90° in all except two instances, and in one infant it even reaches 104°. In childhood it suffers a little diminution, but principally it gets smaller at a later age, when also the orbit is deepened and the frontal sinus enlarged by the growth forwards of the upper part of the face, as will be subsequently shown. Thus also the orbito-frontal angle is generally larger in females than in males.

The frontal bone, in the progress of development, changes its curve not only where its orbital and frontal plates meet, but also in the course of its frontal plate; for the midfrontal angle appears to get smaller in the passage from fœtal or infantile life to childhood, and again enlarges in the passage to the adult state; or, in other words, the bone becomes more curved in childhood and is again flattened in subsequent growth. In the four-months' fœtus the midfrontal angle is of an average adult size, in the two five-months' fœtuses it is decidedly smaller, in the seven-months' fœtus it is again larger, and in the eight-months' fœtuses it is of a size which would be very flat in the adult. These variations may be to some extent the result of mere individual peculiarities of the few skulls examined, but they agree with what is observed with the unassisted eye; for it is clear enough, when attention is attracted to the circumstance, that a fœtus eight months old has a flatter forehead than a fœtus five months old, or a child a few months after birth. The skulls of infants present great differences in this respect, but the six skulls of children from 2½ to 10 years old, have all got the midfrontal angle of a size which would be accounted small in the adult, and therefore indicating a greater curvature than usual.

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* *Loc. cit.*

As is the case with the frontal region, so also with the parietal and the part of the occipital above the tuberosity; the curve does not go on uniformly increasing or diminishing, but at one period of growth is flattened and at another more convex. The parietal angle, indicating the curve of the mesial edges of the parietal bones, is as large in the three fœtuses of the fifth and seventh month as it is in the adult, whereas in the fœtus of the fourth, and in the two of the eighth month it is remarkably small; at birth it has begun again to enlarge, and in childhood apparently it reaches the condition which remains in the adult. On an average the parietal angle is smaller in women than in men, the feminine form in this respect resembling that of the young skull. The midoccipital angle, which indicates the curve of the subcutaneous part of the occipital bone, is flatter in five of the six fœtal skulls than in any of the six infant skulls, and of a size very common in the adult, whereas both in infancy and childhood it is of an average size much smaller or more prominent than in the adult. It is to be observed, however, that the flattening of this part of the skull in the passage from childhood to adult life is no doubt due, in part at least, to the laying on of additional substance in the neighbourhood of the tuberosity.

In perfect keeping with the changeful development of the curves just mentioned is the variation at different ages of the transverse curve of the calvarium between the parietal eminences; for, as is palpable to the most careless observer, that curve rises rapidly towards the middle line in the fœtus, becomes in childhood remarkably flat, and again rises in the middle line as growth proceeds. This will be again referred to.

Of the remaining angles illustrating the curvature of the arch, the most important to be noticed in connexion with the form of the young skull are the fronto-parietal angle and the angle of the tuberosity, both of which are decidedly flatter in the infant than in the adult. The fronto-parietal angle is also flatter in women than in men. The occipito-parietal angle is rather more prominent in the infant skull than in the adult. The postforaminal angle is very variable, both larger and smaller numbers of degrees occurring in the fœtuses, infants, and children than in any of the adult averages. The postforaminal angle of the female is on an average smaller than that of the male; but this is to be accounted for by the lighter weight of the female skull, making it less liable to be affected by the gravitation changes described in the next paragraph. It may be stated generally with regard to the infant skull, that the flatness of the fronto-parietal angle and of the tuberosity, and the rapid curvature of the parts of the arch between these two angles, together with the great development of the parietal region of the arch, and the shortness of the base, are the circumstances which give the characteristic outline to its profile.

Elderly skulls. Gravitation changes.—It is a well recognized fact that the skull continues to undergo change of form after adult life is reached. LAVATER in the general appearance of the head, and FRORIEP* in the skull, depict the retreat of the forehead characteristic of old age, but the precise nature of the change and the causes on which it depends have not been recognized. The changes referred to result entirely from the operation of mechanical causes, and consist in a yielding of the skull in consequence of its own weight. Precisely as the apparently solid glacier flows down its valley at a rate too slow to be appreciated by direct observation, so the skull falls gradually down by its own weight and that of the contained brain. The condyles of the skull are supported on the vertebral column, and by the process of gradual yielding the basal part of the skull is driven in, from the occipital tuberosity behind, to the fronto-nasal suture before. Thus the postforaminal angle is flattened out, while the angle of the tuberosity and the orbito-frontal angle are made smaller. At the same time the skull is increased in breadth, being made to bulge out at the squamous suture; this bulging being partly produced by a forcing open of the angle between the squamous and petrous parts of the temporal bone, and partly by the depression of the outer end of the petrous part to a lower level than its inner end. This lateral bulging is very characteristic, and ought not to be lost sight of by the artist in representing old age. In the production of these alterations of form it is plain that the cooperating causes are weight of the skull and its contents, softness of the bone, and lapse of time; therefore the larger the skull the more likely they are to be developed in a marked manner; and if the bone be more than usually yielding they may be developed at an earlier age than usual, even though the skull be not remarkably large. This is probably the explanation of the very considerable gravitation changes in the skull of BURKE the murderer, who, although somewhat past middle life, was by no means an old man at the date of his execution. Most probably these changes begin in a slight degree in all skulls at an early period of adult life; and it may be the lot of some one who has better opportunities of investigating the subject than the present writer, by a comparison of a number of adult skulls of different ages, to demonstrate the changes of form which the skull undergoes in the passage from twenty to thirty, forty, or fifty years of age. The accompanying Table gives a list of the skulls from the study of which the above observations have been made; and if the angles given be compared with the corresponding angles of other skulls, they will illustrate the way in which by gravitation changes the base is driven in. In the German 29 the forehead appears to have escaped being bent back, and instead of its being so the parietal region has become flattened out.

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* FRORIEP, Die Charakteristik des Kopfes nach dem Entwicklungsgesetz desselben, 1845.

	Angle of tuberosity.	Postforaminal angle.	Orbito-frontal angle.
French 24	124°	163°	77°
German 28	122	158	76
German 29	125	154	80
Old Officer	120	166	74
Corfu 45	120	153	75
Irish 51	125	153	78
Irish 52	130	150	75
Burke	120	166	76

A much more astonishing instance of driving-in of the base of the skull is seen in the anomalous skull (91) belonging to Professor THOMSON. In it the part of the occipital bone between the tuberosity and foramen magnum has not yielded, while the parts further forwards have given way to an astonishing extent. This may be accounted for by the great thickness of the occipital bone in this instance; or, if the theory be true that it is a baker's skull driven in by the weight of heavy trays carried habitually on it, it is likely to have happened that the pressure began to be applied after the superior and lateral ossifications of the occipital bone were united, and before synostosis of the elements of the base had been completed. These observations on this remarkable skull are, however, put forward subject to the criticisms of Professor THOMSON, in whose possession it is.

National differences in angles connected with the arch.—These will be most rapidly noticed by grouping the peculiarities of each nationality together. The writer has studied them with the aid of averages in which the sexes have been kept separate, and regard has been had to the skulls in which gravitation changes have taken place; but it will be sufficient to state the conclusions at which he has arrived, leaving the reader to verify them from the data in the General Table.

In the Scotch skulls the orbito-frontal angle is decidedly below average, that is to say, the forehead slopes more than usually back on the floor of the skull; also the skull rises more rapidly than usual behind the foramen magnum, and the angle at the occipital tuberosity is unusually flat.

The Irish, on the contrary, have the occipital bone extending very horizontally backwards from the foramen magnum, as indicated by the large postforaminal angle. They have the curve of the forehead unusually prominent, as indicated by the smallness of the midfrontal angle.

The Germans, like the Irish, have the midfrontal angle prominent; they have great curvature at the occipital tuberosity.

The French have the orbito-frontal angle decidedly smaller than even the Scotch; they have coronal flatness as indicated by large fronto-parietal angle, and at the back of the head bend rapidly at the tuberosity.

While in the different European nations the midparietal angle retains an average size of about 133°, the Esquimaux, Kafir, Negro, and Australian agree in having it larger, a circumstance possibly connected with the length of base-line in these nations; that is to say, that the length of the arch being completed before the length of the base-line, the extremities of the arch are possibly separated by continued opening-out of the midparietal angle in the latest growth of the base. The Kafir, Negro, and Australian all have the orbito-frontal angle large, the Kafir having it remarkably so. The Kafir, further, has the midfrontal angle flatter than the Negro, otherwise the curves of these skulls, as exhibited by angles, are very similar.

In the Australians the curvature of the roof is very evenly distributed, as is indicated by the smaller than usual fronto-parietal angle in conjunction with the large midparietal angle; the midoccipital angle is prominent.

The two Kanaka skulls agree in presenting remarkable curvature both at the midparietal and occipito-parietal angles; and in both, but especially in the male, a slope backwards is given to the whole skull by the small orbito-frontal angle and the flat midfrontal, and the unusually flat angle at the occipital tuberosity.

The Peruvian skulls are remarkable for the extreme flatness of the midfrontal and midoccipital angles, and extreme smallness of parietal and postforaminal angles. They have the orbito-frontal angle small.

The Hindoo skulls have the fronto-parietal and midparietal angles small, the orbito-frontal rather small, and the midoccipital angle flat.

The Greeks have the orbito-frontal angle large, the fronto-parietal small, the midoccipital flat, and the postforaminal angle small.

The size of the orbito-frontal angle in different nations appears to be a matter of sufficient importance to demand some further consideration. Among the loose notions which are popularly current about the form of the skull, and which have been with too little care incorporated among the beliefs of scientific men, and been allowed to assume among them a more definite and erroneous shape than they have in the unscientific mind, is one to which allusion has already been made, that amplitude of forehead is a criterion of high development both in individuals and nations, and that nations of inferior intellectual development have low or retreating foreheads. The unscientific man in expressing such a notion considers nothing but external appearances, and his statements are not without a certain foundation of truth, for increased height and breadth of the whole cranium, and large proportion of arch to base are among the circumstances which may make the forehead well developed; but when ethnologists go the length of imagining that in the lower types of humanity there is a local deficiency in the frontal part of the skull, giving room for only a small development of the anterior lobes of the brain, they fall into an anatomical error, as the measurements of most characteristic skulls will serve to show. It has been already shown that the Kafirs, Negroes, and Australians have a great development of the orbital length, and now we have occasion to observe that in these same skulls, but especially in the Kafir, the forehead springs very erectly from the orbit. This may be seen by a comparison with European skulls.

In three Kafir males the average number of degrees of the orbito-frontal angle amounts to 84¾; in four Irish males, omitting from consideration the two which have undergone gravitation changes, it amounts to 84¼; in three Greek, omitting both the skull which has gravitated and another (46) in which the orbito-frontal angle is anomalously large, it amounts to 83; in three German, omitting the two which have gravitated, it amounts to 82⅓; in two Australian, 82; in five Negro skulls, 81; in five Scotch males, omitting the Old Officer, it amounts to 78⅘; and in five French, omitting the Old Knight, it amounts to 76⅗. Thus there are Kafirs and Irish at one end of the series, French and Scotch at the other, and Germans and Negroes in the middle.

Whatever of inaccuracy has been imported into the measurement of this angle by the choice of the optic foramen as the posterior limit of the frontal floor, instead of a point in the mesial plane, makes the conclusion with regard to the Kafir, Negro, and Australian all the more trustworthy; for in these races the tendency of the lesser wings of the sphenoid as they pass outwards is to rise more than in European skulls, and thus to raise the optic foramina more than usual above the level of their origin, which makes the estimate of the orbito-frontal angle less than if it had been measured on a mesial section. But, further, an examination of some of the skulls, tracings of the mesial sections of which have been preserved, will be of itself sufficient to show how little a large orbito-frontal angle is to be trusted as an index of a superior type of skull. The Negro skull 63, a skull of a very inferior type, and the Australian skull 73, which, though rather well developed for an Australian skull, is yet a thoroughly characteristic specimen of that race, have the orbito-frontal angle respectively 83° and 82°; while the German skull 29, an extremely well-developed skull, has an orbito-frontal angle of 80°, and the young Scotch female skull 43, which, notwithstanding its defects, is a much more finely proportioned skull than Negro 63, has the same angle measuring only 74°.

Deep frontal angle (General Table, column 32).—The testimony obtained by measurement of the angle formed at the foramen opticum by lines from the fronto-nasal and fronto-parietal sutures, and which may be termed the deep frontal angle, corroborates the conclusion that a forehead retreating on the floor of the anterior cranial fossa is by no means a character of the least advanced races. Indeed a glance over the list of diagrams will show that a certain proportion exists between the deep frontal and orbito-frontal angles, so that it may be considered the normal condition at different ages and in different races for these angles to be together nearly equal to two right angles. The most extreme instances of the sum of the two angles falling short of 180° is the Carib skull 88, in which it is only 167°, the deviation being dependent in great measure on the largeness of the frontal sinus. The most extreme instances of the sum of the two angles exceeding 180° are the large British skull 92, in which it amounts to 188°, and the five-year-old skull 16, in which it reaches 186°; and in both these it is the great length of the frontal part of the arch as compared with the orbital length which causes the deviation. The average sum of the orbito-frontal and deep frontal angles in adult male skulls, omitting, as before, those which have undergone gravitation changes, comes to about 176° alike in the Kafir, Negro, Australian, Hindoo, Greek, Scotch, French, and

Esquimaux skulls; but it rises to about 180° in the German and Irish, which have a high proportion of arch to base-line; and in the Kanaka, Maori, and Peruvian skulls it sinks to 173° and 174°.

The deep frontal angle gets regularly larger from fœtal up to adult life, as the frontal bone gets larger in proportion to the rest of the arch; or, to put it differently, it may be said that the increasing length of the frontal bone pushes down the fore part of the ethmoid and bends it on the sphenoid.

Area of the frontal, parietal, and occipital parts of the diagrammatized profile (General Table, columns 40 to 46).—So far as the examination of angles and of measurements of arch and base has gone, the present inquiry has shown no reason for the belief that frontal development is more important or nobler than development of the other cranial regions; nor is any evidence to that effect to be got by comparing the areas of the different regions. The calculation of the areas has for the sake of simplicity been made on the angular figures produced by joining by means of straight lines the series of points already enumerated as selected for measurement. Thus the area of a trapezium has been measured in the case of the frontal region, a pentagon in the parietal, and a hexagon in the occipital region. The results are therefore only approximate, but they are sufficiently accurate for practical purposes.

It is remarkable that in the relative proportions of these three areas, no national nor sexual differences whatever are exhibited. Unfortunately the writer is unable to say how far this statement is applicable to the Chinese and Australian skulls, in which the parietal part of the arch was found habitually to predominate over the frontal and occipital parts; for LUCAE's figures do not furnish sufficient details for making the calculation in the same way as it has been done in other skulls. But on comparing the average area in those nationalities of which three or more specimens have been measured, viz. Scotch, Irish, French, German, Greek, Negro, Kafir, and Hindoo, it is found that the variation in the occipital area is only 1·56 per cent., in the parietal area 1·19, and in the frontal 1·74 per cent., a result equivalent to no variation at all. This is the more remarkable as in the series of skulls examined, the variation among individuals in the occipital area reaches 11·98 per cent., in the parietal area 10·99 per cent., and in the frontal area 6·36 per cent. The Chinese skull 85 has the smallest proportion of frontal area in the whole series, on account of the shallowness of its orbits, and on the same account the synostotically deformed skull 89 has it nearly as small. Also among those which have the proportion of the frontal area smallest are the two female skulls, Irish and Scotch, 57 & 42, in which there is no spheno-parietal contact.

The same variability of the proportions of the three areas in individuals which is seen in the adult is observed also in young skulls. The figures expressing those proportions in the fœtal and infantile skulls are almost all such as might be found in the adult; but the averages show a somewhat larger proportion of parietal, and a smaller proportion of occipital and frontal area. The difference, however, does not seem to be such as to account for the obvious predominance in bulk of the parietal region in young skulls, which probably depends most on breadth.

Distance of the arch at various points from the base (General Table, columns 33 to 39).—To estimate the distance from arch to base in different parts of the mesial extent of the skull four diameters have been chosen, namely, a line from the level of the foramen opticum to the fronto-parietal suture, which may be termed the frontal depth, and three lines converging to the spheno-occipital suture from the midparietal point, occipito-parietal suture, and the occipital tuberosity, which may be termed respectively the parietal, occipito-parietal, and occipital depths. In comparing them the parietal depth has been chosen as the standard, not merely because it is the longest, but because, as may have been observed from what has been already stated with respect to the angles connected with the arch, it may be regarded as an axis behind and in front of which the arch of the skull expands in growth from birth onwards.

In the first four fœtal skulls the frontal depth is such as would be esteemed high in the adult, but in the two skulls of the eighth month it is low, and in the skulls of infants it is still lower; in childhood, however, it rises and appears to gain the adult proportion. The occipito-parietal and occipital depths are on an average slightly smaller in the fœtus than in the infant, and both, but more especially the occipital, increase in childhood.

These measurements, like those which have gone before, show that the sugar-loaf-like form of the infant skull is no mere mechanical result of compression during birth, but is prepared beforehand, and subsequently lost in the proper process of growth.

As in the case of other measurements, so also with those under consideration, the results in the adults of different nations are not altogether distinct at first sight; and it must be owned that on them alone a speculation could not be based with regard to the nationality of a particular skull; but by taking averages in those instances in which several skulls have been examined, national differences are indicated of a not altogether uninstructive kind. Comparing fourteen different races together (some of them represented, however, by only one or two specimens), and marking as high, moderate, or low the proportionate lengths of the frontal, occipito-parietal, and occipital depths as compared with the parietal depth, the following results are obtained:—

Proportion of the occipital, occipito-parietal, and frontal depths to the parietal depth. High proportions are marked *a*, intermediate *b*, and the lowest *c*.

	Occipital depth.			Occipito-parietal depth.			Frontal depth.		
	Average.	Extremes.		Average.	Extremes.		Average.	Extremes.	
3 Hindoo	71	69 72	*c*	90	88 92	*c*	71	70 73	*b*
2 Peruvian	67	66 69	*c*	87	87 88	*c*	67	66 69	*c*
2 Kanaka	70	68 72	*c*	92	91 94	*c*	67	66 68	*c*
5 Greek	74	71 78	*b*	91	87 95	*c*	72	70 75	*a*
1 Maori	74		*b*	92		*c*	72		*a*
2 Australian	76	76 77	*b*	94	94 94	*b*	72	71 74	*a*
8 German	76	73 82	*b*	93	90 97	*b*	72	69 77	*a*
4 Kafir	76	70 82	*b*	95	93 98	*b*	73	70 78	*a*
7 Negro	77	68 85	*a*	95	92 100	*b*	70	66 74	*b*
9 French	76	71 80	*b*	93	89 98	*b*	71	67 78	*b*
8 Scotch	75	68 81	*b*	94	90 97	*b*	70	68 75	*b*
9 Irish	79	75 86	*a*	97	92 100	*a*	71	65 76	*b*
1 Chinese	77		*a*	98		*a*	68		*c*
2 Esquimaux	77	76 78	*a*	96	96 96	*a*	72	72 73	*a*

The Peruvian and Kanaka have all the three lines in small proportion to the parietal depth, and the Hindoo skulls only differ in having a greater frontal depth. The Greek skulls and the Maori resemble these only in the short occipito-parietal depth, but have the forehead high and the occipital tuberosity moderately projecting. The single Chinese skull is deficient in frontal depth, but is full behind. The Esquimaux have all three lines well developed as compared with the parietal depth, which means that the parietal depth is deficient, a consequence of the flatness of the parietal part of the arch formerly alluded to. The French and Scotch skulls have the occipital, occipito-parietal, and frontal depths all moderately developed in proportion to the parietal depth; the German, Kafir, and Australian skulls differ from them in having the frontal depth comparatively great, and the Negroes in having the occipital depth greater, while the Irish have not only the occipital, but likewise the occipito-parietal depth great. Probably the most important part of these results is that which relates to the Peruvians, Kanaka, Hindoos, and Greeks, as it bears on what will fall to be advanced in considering the proportion of height to length.

Height (column 72).—The height of a skull is estimated by different writers in various ways. Von Baer measures it from the *plane of the foramen magnum* to the most distant point of the vertex*. Mr. Huxley has preferred a line with *definite termini*, and measures from the fronto-parietal suture in the mesial plane to the front of the foramen magnum*, thus making the line of height agree as nearly as possible with that which he has adopted from Mr. BUSK† as expressing vertical direction, namely, a line from the fronto-parietal suture to between the openings of the ears. The present writer, however, appreciating, as Mr. HUXLEY has also done, that some skulls appear to stand vertically over the base while others slope upwards and backwards, has sought for a line the inclination of which will vary with the slope of the skull; and believing that the best criterion of that slope is the direction of the line of frontal depth, he has measured the height of the skull by a line passing upwards from the front of the foramen magnum parallel to the frontal depth. This line has the disadvantage in the present inquiry that its upper extremity does not exactly correspond with any of the measured points laid down in the diagrams, but the possibility of error from this source has been found to be very slight.

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* Crania Selecta, p. 4.

Proportion of height to frontal depth (column 74).—The proportion which the height measured as now stated bears to the frontal depth varies very much in different individuals; but, like steepness and levelness of the base, on which to some extent it is dependent, it is not a matter apparently of national distinction. It may, however, be remarked that those nationalities in which the parietal depth was great in proportion to the occipital, occipito-parietal, and frontal depths, have also a high proportion of height to frontal depth.

The question arises, how far excess of the line of height over the line of frontal depth depends on rise of the roof of the skull, and how far on sinking of the base; and this may be determined by examining the relation borne to the line of orbital length by the line uniting the midparietal and fronto-parietal points. This line, which is always cut by the line of height, sometimes, as compared with the line of orbital length, rises five or six degrees as it passes backwards, sometimes falls as much, but on an average and much more frequently is parallel with it in the adult. In the infant it rises greatly and most characteristically as it passes backwards; when the gravitation changes set in, they tend to make it fall; and in the compressed American skulls 95 and 96, and even in the French skull 27, which has probably been accidentally compressed by a head-dress in the manner described by GOSSE‡, it rises enormously behind, as it does in no natural adult form. Thus it appears that in the normal adult skull the excess of the line of height over the frontal depth is dependent almost entirely on sinking of the base, but that in the infant and in artificially deformed skulls it depends in great part on rise of the roof. The conditions in the base of the skull which increase the excess of the line of height over the frontal depth are length of the foramino-optic line and greatness of the angle between that line and the line of frontal depth, which of course involves steepness of the base. Also, the proportional excess is increased by absolute shortness

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[*] Journal of Anatomy and Physiology, November 1866.

[†] Natural-History Review, October 1862.

[‡] Déformations artificielles du Crâne: Paris, 1855.

of the frontal depth, since the absolute excess of height produced by sinking of the base then forms a greater proportion of the whole frontal depth. Thus in the skull of the Idiot, 94, the greatest proportion in the whole collection is found, and in the Carib, 88, it is nearly as great.

Length (column 71).—As in the estimation of height, so in that of length, it seems advisable to make use of a somewhat different measurement from that usually adopted; for it is scarcely accurate to compare, as is generally done, diameters which do not pass between corresponding points, and which have nothing necessarily in common except that they happen in differently shaped skulls to be the points furthest asunder. The measurement of length in the present Memoir has therefore been made from the mid-occipital point to the fronto-nasal suture, the midoccipital point being usually the most prominent part of the back of the head, and the fronto-nasal suture being chosen in preference to the glabellar prominence, because the glabellar prominence is exceedingly variable, and its development is of no importance as regards the general shape of the skull.

Proportions of height and breadth to length (columns 73 & 76).—By assorting the skulls of different nations according to the proportion which the height bears to the length, a highly natural arrangement results, those skulls being brought together for the most part which are similar in general form. Thus the Peruvian, Hindoo, and Kanaka skulls, which were associated in respect of the comparative distance of different points of the arch from the base, are again placed together, and a just and well-marked distinction is made between the Kafir and Negro, the French and German, and the Scotch and Irish. When the same groups of skulls are arranged according to the proportion of breadth to length, the result appears to be less satisfactory.

Proportion of Height to Length.			Proportion of Breadth to Length.		
	Average.	Extremes.		Average.	Extremes.
2 Peruvian	85	83 88	1 Peruvian	98	
3 Hindoo	83	82 84	8 German	86	84 92
1 Maori	81		1 Chinese	83	
2 Kanaka	78	78 78	9 French	82	80 90
1 Chinese	77		3 Hindoo	81	79 84
5 Greek	77	75 82	7 Scotch	80	77 85
4 Kafir	76	74 81	5 Greek	80	76 89
7 Scotch	76	72 78	1 Maori	77	
9 French	75	68 84	2 Kanaka	77	77 78
2 Esquimaux	74	74 75	2 Esquimaux	77	75 79
7 Negro	74	71 78	9 Irish	77	74 82
2 Australian	73	73 74	4 Kafir	73	70 78
8 German	72	69 76	7 Negro	72	68 75
9 Irish	71	65 75	2 Australian	71	71 72

We find certainly the Peruvian at one extremity of the series, and the Kafir, Negro, and Australian at the other, which seems a natural arrangement; but the German skulls, placed next to the Peruvian, have no resemblance whatever to that form beyond the mere matter of proportional breadth, while on the other hand they have much resemblance to the Irish skulls which they are far separated from. The Kanaka, also, should come near the Peruvian rather than be associated with the Irish skulls, with which they have not the slightest affinity. The tendency of recent writers has been greatly to exaggerate the importance of breadth of skull as a distinguishing race character; and while Aeby* would divide skulls according to their breadth into two great groups, the stenocephalous and the eurycephalous, other writers have likewise given an enormous importance to breadth by estimating dolichocephalism and brachycephalism by nothing else than the "cephalic index." An inquiry into the origin of these terms may serve to show how objectionable this is.

The point which mainly struck the attention of Retzius was that certain skulls had less development and posterior projection of the occipital bone than others, and on that account were shorter from before backwards than they; he therefore termed those skulls brachycephalic, and others dolichocephalic. But to get a criterion of proportional length or shortness, it was necessary to select some measure with which to compare the length, and for this purpose Retzius selected the breadth; but he does not appear to have based his statements with regard to the dolichocephalism or brachycephalism of different nations on detailed calculations of the proportion of breadth to length in individual skulls; there is no evidence that he did so, and in his letter to Professor Duvernoy, in 1852, he expressly states that he does "not as yet wish to determine fixed measurements to distinguish them, but that ordinarily the longitudinal diameter of the dolichocephalous surpasses the breadth about a fourth, while in the brachycephalous the difference varies between a fifth and an eighth. But *the most distinctive characters* are:" he proceeds (the italics being his); and forthwith he lays down seven distinctions, not one of which is founded on the proportion of breadth to length, but of which the fifth consists in the height as compared with the length†.

In the end of the same year, in his letter to Dr. Nicolucci, he enumerates nine distinctions, the first of which is that in the dolichocephalous skulls the longitudinal diameter surpasses the transverse by about one-fourth, while in brachycephalous skulls it surpasses it by about one-fifth to one-eighth. Thus it appears plain that while M. Duvernoy's French taste for precision led Retzius into fixing certain proportions of length and breadth as characteristic of his two great classes of crania, Retzius never allowed himself to forget that the importance of his division lay in the classes being distinguished by a number of different characters, a circumstance well appreciated by the editor of his works, in his note on the letter to Dr. Nicolucci. But the effect of the subdivisions introduced by BROCA and others* has been to convert, in the attempt at precision, a natural classification into one as artificial as the Linnean system in botany.

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* Schädelformen des Menschen und der Affen, noticed in Henle's Bericht, 1867.

† Ethnologische Schriften von Anders Retzius, p. 118.

The superiority of RETZIUS's point of view over the more recent one is well illustrated by what has taken place with regard to German skulls. WELCKER, writing from Halle, has pointed out the prevalence of a very high cephalic index in Germans, and ECKER has shown that in the South Germans it is still more remarkable; and on that account they throw those skulls into the brachycephalic group, and in this are followed by others; while GUSTAVE RETZIUS in defending the opinion of ANDERS RETZIUS, whose works he edits, labours to show that WELCKER's estimate of the cephalic index in the German skull is too high. The measurements of the eight German skulls given in this memoir support WELCKER's view that the German cephalic index is high, some of them, perhaps South Germans, being extremely broad; but they are all of them in their antero-posterior mechanism completely dolichocephalic in character. Their height compared with their length is low, and this is true of the broadest of them; they have the occipital squama prominent, and the tuberosity particularly so; and the arch of the profile, instead of sinking "precipitously" in the region between the parietal eminences, "forms an oval curve from the forehead to the occipital protuberance," all which are mentioned by RETZIUS in his letter to DUVERNOY as characteristics of the dolichocephalic skull, and are circumstances which, apart from any such historical consideration, point out the propriety of associating the German with dolichocephalic forms.

Sandwich-Islanders and New-Zealanders were both considered by RETZIUS as brachycephalic†. The description which he gives of a Sandwich-Island skull applies perfectly to the two skulls of the Kanaka race presented by Dr. BARNARD DAVIS to Mr. GOODSIR, and used in the preparation of this Memoir. Had RETZIUS classified according to a numerical rule, he would have had no difficulty in settling the place of any specimen; but he sought a natural classification which would take into account the whole character of the skull, and therefore he was "at first somewhat doubtful about the right place" of his Sandwich-Islander, and the thoroughly accurate decision to which he came was arrived at notwithstanding "the considerable length compared with the small intermastoid distance"‡. The comparison which he makes between the Sandwich-Islander and the New-Zealander is quite borne out by the diagrams of the two Kanaka skulls and the Maori skull now before us. His words are, "compared with a New-Zealander's skull, this skull (the Sandwich-Islander's) shows much agreement therewith, but is distinguished from it especially by the compression referred to in the lower part of the occipital bone. The occipital bone in the New-Zealander is almost quite flat and more sloping forwards than perpendicular." This is brought out in the diagrams by the large size of the angle of the tuberosity in the Kanaka skulls and its small size in the Maori. In addition to this difference and connected with it, must be noticed the large orbito-frontal angle and small midfrontal angle of the Maori, contrasting with the small orbito-frontal and large midfrontal angle of the Kanaka; for all these differences are summed up in this, that the arch in the Kanaka is pushed back over the base-line, and in the Maori is pushed forward.

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* THURNAM, Ancient British and Gaulish Skulls, p. 50; and LAING and HUXLEY, Prehistoric Remains of Caithness, p. 84.

† Ethnologische Schriften, pp. 65 and 66.

‡ Dr. BARNARD DAVIS, in his 'Thesaurus Craniorum,' vindicates the decision of RETZIUS on the ground of the proportion of breadth to length, the average of 116 Kanaka skulls of both sexes having given the proportional breadth as ·80: it is noticeable, however, that the 64 males gave the proportion ·79, which falls short of the arbitrary limit of brachycephalism.

It is to be regretted, however, that the same considerations which led RETZIUS to include the Sandwich-Islanders in the brachycephalic group did not lead him to admit the Hindoos also. For although the Hindoo skull is certainly dolichocephalic, according to the criterion derived from proportion of breadth to length, it is nevertheless short and high, and possesses the peculiarities of occipital bone characteristic of brachycephalic skulls, namely, commencement of the upward slope immediately behind the foramen magnum, flatness of the subcutaneous portion, and indistinctness of tuberosity, to which may be added almost constant want of symmetry in the occipital plate. It has greater height of forehead and greater proportion of total height compared with length than the Kanaka, and has more of the characters of the brachycephalic skull than the New-Zealander. The affinity of the Hindoo skull with the brachycephalic group was well brought out by the comparison of the parietal with the occipital, occipito-parietal, and frontal depths. Reverting to that comparison, it may be said that the small proportion of the occipital, occipito-parietal, and frontal depths to the parietal depth, together with the flatness of the midfrontal angle, in the short-headed Americans and Sandwich-Islanders shows a brachycephalism dependent on a natural antero-posterior compression of all the regions of the skull, and not of the occipital region only: from these the Hindoos differ only, as regards profile, in having a slightly higher forehead, while the Maori and the Greeks have the forehead both high and prominent, and have the occiput more prominent at the tuberosity. The Maori and Greek forms may be looked on as links between complete brachycephalism and the dolichocephalism of the negritic races and the west of Europe respectively.

Of course, this allusion to the Greek skull is to be held as merely referring to them as they are illustrated in the few examined. While the occurrence of brachycephalic skulls among the Greeks is indubitable, it is not to be forgotten that the dolichocephalic form has been suggested as the normal one in that nation; but it may be allowable to suggest the possibility that while the proportion of breadth to length may be variable, the profile view may perhaps adhere to the brachycephalic type. Of the five Greek skulls measured, only one has a dolichocephalic profile, and in that instance it is, as will be shown, the result of idiosyncrasy.

If it be too late now to restore the terms dolichocephalic and brachycephalic to a broader meaning than has latterly been given to them, and if skulls must needs be grouped according to the indications of some single proportion, the proportion of height to length will probably be a better basis on which to proceed than the proportion of breadth to length. The short-headed Americans, the Polynesians, the Hindoos, the New-Zealanders, and the Greeks may well be contrasted with the Germans, the Irish, the Australians, and the Negroes, the one set being high and the other low in proportion to their length: and if the division cannot be made distinct without the use of Greek nomenclature, the one group may be termed *hypselocephalic*, and the other *tapeinocephalic*. But the truest expression of the facts will be obtained by instituting a subdivision of the brachycephali of RETZIUS under the name of *angustiores*, to which shall belong the Hindoo, Sandwich-Islander, and New-Zealander, probably the Greek, and possibly the Chinese. The mere establishment of such a division would set on permanent record that the brachycephali have more than one character. But whatever system of classification and nomenclature be determined on, it will always be artificial to associate with proportionally lofty skulls, like the brachycephalic Americans, low-lying skulls with dolichocephalic profile, such as the Germans, because they happen to have a cephalic index above 80°. These might be termed dolichocephali *latiores*.

Position of greatest breadth (see the diagrams).—The position of greatest breadth in well-developed skulls is always near the squamous suture, usually towards the place where it descends posteriorly. In ill-filled savage skulls it lies a good way up the parietal bone. By an ill-filled skull it is meant to express the condition of a mesial and two lateral ridges on the roof, with flatness of the adjacent surfaces. This ridged condition is a reversion, so far, to the infantile form, in which also the point of greatest breadth is placed high up; but the infant soon loses the ridged condition, and in childhood the roof of the skull becomes flatter than it is in the adult*. Probably the point of greatest breadth descends at this period (as in skulls 14, 15, & 16), and afterwards reascends temporarily (as in 17 and 18) when the mesial part of the roof begins again to rise, then lastly descends a second time as the bones become more uniformly convex in the latest expansion of the brain; for, as has been pointed out by SCHAAFFHAUSEN†, breadth of skull goes on increasing up to adult age, although the permanent length is reached already at the seventh or tenth year. Possibly the rise in the roof after its flat condition in childhood, as well as whatever temporary reascent there may be in the point of greatest breadth, may be explained thus, that the closure of the frontal suture, while the sagittal suture remains open, imposes a limit to the lateral expansion of the skull, and the inner margins of the parietal bones continuing to grow, at the same time that they are kept together by the connexions of the parietals with the frontal and occipital, and being pressed on by the growing brain below, are forced upwards and produce the mesial elevation, partly by rotation of the parietal bones on their inferior margins, and partly also by unbending them, in which they are assisted by the continuing increase in the breadth of the base of the skull‡. While these changes are taking place, and after they have ceased, the surfaces above and below the lateral ridges are rounded out in well-filled skulls; but in ill-filled savage skulls that rounding-out fails to take place, and thus there is the more prominent mesial ridge and the higher position of the point of greatest breadth. The rounding-out of the sides of the skull which occurs from the latest expansion of the brain is followed by that which is brought about by gravitation, as already described, and thus the increase in breadth does not necessarily cease on arrival at adult age, but may continue through life. Another position of the point of greatest breadth, indicating a more degraded form than that in which it is placed high on the parietal, has only been met with in one skull in the list, that of the Idiot. In that skull the greatest breadth is between the mastoid processes, as it is in the Gorilla and Chimpanzee, a condition resulting from the poor development of the arch of the skull.

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* The persistence of this flatness in the female is referred to at p. 164.

† HENLE, KEFFERSTEIN, und MEISSNER'S Bericht for 1865, p. 73.

‡ Professor WYMAN in his recent "Observations on Crania," republished from the Proceedings of the Boston Society of Natural History, vol. xi., in writing about the much discussed variety of synostotic skull called sca-

Radial measurements and position of the ear (columns 47 to 63, and 69 and 70).—As it is impossible on the living subject to obtain measurements extending from points in the arch to points in the base, it would often be convenient to obtain information as to cranial configuration by means of measurements radiating from the external auditory meatus, like those proposed by Mr. BUSK*. In the present instance the postauricular depression has been chosen for convenience instead of the centre of the auditory meatus as the starting-point, and the radius to the midparietal point has been chosen as a standard with which the other radii are compared.

The radial measurements serve well to display the differences between the infantile and the adult form of skull. In the infant the radii in front of the midparietal radius bear a smaller proportion to that radius than they do in the adult, and the difference is greatest in those furthest forward. Thus the proportional distance of the fronto-nasal suture from the postauricular depression, counting the midparietal radius as 100, is 70, and in the adult European about 83. To understand the significance of this difference, let the midparietal radius be estimated at 5 inches in the adult, and let the infant's head be magnified sufficiently to bring that radius to the same length in it, then the distance to the fronto-nasal suture in the adult will be 4·15, and in the infant 3·5, or ·65 of an inch less than in the adult. Turning to the parts behind the midparietal point, it may be observed that the proportional distance of the occipito-parietal point from the ear is as great as in the Irish and Scotch skulls, and greater than in the French and German; the distance of the midoccipital point is slightly less than in the Irish

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phocephalic, of which he describes various examples, the youngest of them being the cranium of a fœtus, remarks on the deficiency of height in the scaphocephalic skull, and asks why the compensatory growth required in consequence of the want of breadth does not take place upwards as well as in the longitudinal direction. The answer to that question may be gathered from the text. The height of the parietal region of the skull above the parietal eminences is obtained by the growth of the inner margins of the parietals, and when synostosis of those margins occurs, or when, as may be the case in some instances, there is but a single centre of ossification from which one 'biparietal' bone takes origin, not only is this source of upward growth lost, but the parts corresponding to the portions below the eminences of normal parietals are placed in a sloped instead of a vertical position.

* Natural-History Review, October 1862.

and Scotch, equal to what it is in the French, and slightly greater than in the German; the average distance of the occipital tuberosity is less than the average obtained in any instance in which two or more adult skulls are compared, with the exception of the Peruvian and Kanaka. If the adult proportional distance of the occipital tuberosity from the ear be taken as 62, then if the midparietal radius in both adult and infantile form be again estimated at 5 inches, the distance of the occipital tuberosity from the ear will be 3·1 in the adult and 2·8 in the infantile form, giving a difference of ·3 of an inch. It would appear, therefore, comparing the infant skulls with the Scotch and Irish adults, that the shape of the arch is elongated in the passage to adult life by its anterior limb and, to a less extent, its posterior limb opening out round a part extending from in front of the midparietal point to behind the occipito-parietal suture. But if it be assumed that in all nations the infantile form is nearly the same (and on this point there is no accurate information), it would appear that the opening out of the posterior part of the arch is sometimes confined to the neighbourhood of the occipital tuberosity, for example, in the French, German, Hindoo and Greeks, while sometimes the whole arch may open up more and more from the occipital tuberosity forwards, for example, in the Kanaka and Peruvians.

National skulls, compared in respect of the radial distances, may be conveniently arranged in four groups.

The first group, with the radii both in front and behind the midparietal point short, includes the brachycephalic Americans, the Kanaka, and the Hindoos; and the French incline to that group.

The second group, with the radii both in front and behind the midparietal point long, includes the Australians, Esquimaux, and Negroes; and the Kafir lies on the borders between this and the next group.

The third group, with the radii behind long and those in front short, includes the Scotch and Irish, Chinese and Maori.

The fourth group, with long radii in front and short radii behind, includes the Greeks and Germans.

		Proportional length of radii from the ear, the midparietal radius being reckoned 100.							Actual length of mid-parietal radius.
		Radii behind the midparietal			Radii in front of the midparietal				
		To occipital tuberosity.	Mid-occipital.	Occipito-parietal.	Fronto-parietal.	Mid-frontal.	Glabellar.	Fronto-nasal.	
	4 Infants	56	75	86	95	91	83	70	3·05
First group	2 Peruvians	52	67	79	97	88	80	75	4·8
	2 Kanaka	54	70	82	100	96	88	81	4·8
	3 Hindoo	58	71	81	102	99	86	82	4·53
	9 French	60	75	83	98	98	89	84	4·51
Second group	2 Australian	65	79	86	104	104	96	90	4·3
	2 Esquimaux	63	79	87	103	103	95	89	4·4
	7 Negro	62	77	86	101	102	97	87	4·51
	4 Kafir	63	77	86	101	100	93	85	4·65
Third group	8 Scotch	63	79	86	99	96	85	81	4·6
	9 Irish	64	79	86	99	100	90	82	4·56
	1 Chinese	63	79	88	97	95	87	81	4·4
	1 Maori	62	77	85	104	100	87	81	4·8
Fourth group	5 Greek	59	71	82	103	102	91	86	4·56
	8 German	59	73	83	101	102	91	84	4·56

To the first of these groups there is scarcely any objection; for it must be admitted that the French skulls examined, with the exception of skull 24, exhibit a certain approach to the brachycephalic profile. But in the other groups there are great misplacements, sufficient to show that the radial measurements are not calculated by themselves to exhibit altitude and comparative development of the frontal and occipital regions. The key to this may be found on examination of the deviations in position of the postauricular depression as compared with the front of the foramen magnum. Thus the Kafir skulls exceed the French in height both actual and as compared with length, yet the radial measurements would lead to a contrary supposition; and the explanation is that in the Kafir the postauricular depression is one-fifth of an inch more elevated above the foramen magnum than it is in the French. So also the low-skulled Germans are associated with the rather high-skulled Greeks by the ear being a quarter of an inch lower in the German. The Maori and Greek are placed in opposition, the Maori seeming to have the anterior part of the arch weakly developed, and the Greek the posterior part weakly developed, but the postauricular depression is placed more than a fifth of an inch further forwards in the Maori than in the Greeks.

The circumstances which regulate the level of the postauricular depression as compared with the front of the foramen magnum are various. Yielding of the base, so as to make it transversely flat or concave as viewed from the exterior, obviously causes descent of the ear. Thus the gravitation changes of advancing years depress it, and the European skulls, with the exception of the Scotch, have it low, apparently because the base is not so massive and resisting as in races which have the base-line longer. Levelness of base-line is accompanied with a low ear, as in the French and German female skulls 25 and 34. Partly from this cause, and probably partly from slenderness of base, the ear is lower in the female than the male. The Germans have the ear particularly low, partly from slenderness of base, and partly, as may be supposed, from breadth; for growth of the lateral part of the brain will not only press outwards the lateral wall of the cranium but will depress its inferior limit. In all this there is evidence of what has been already suggested, that yielding from pressure is not a phenomenon confined to elderly skulls. Pressure is continually acting on the cranium, and produces most effect when the bones are least capable of resistance, precisely as has been shown by ENGEL* to be the case with the bones of the face.

The variations of the position of the postforaminal depression backwards and forwards are more difficult to explain; but it is evident that this point corresponds very nearly with the outer extremity of the upper border of the pars petrosa, which limits posteriorly the middle fossa of the base of the skull; therefore the further back it is it indicates the greater elongation of the temporo-sphenoidal lobe and the portion of the brain above it, with corresponding shortening of the occipital lobe and cerebellum.

Seeing that radial measurements extending to the roof of the skull are vitiated as an indication of height by the variable elevation of the ear above the base, probably the radial measurements most likely to be useful to the craniologist, in conjunction with measurements of other kinds, are those extending to the occipital tuberosity and fronto-nasal suture, indicating the comparative development of the anterior and posterior parts of the brain.

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* Das Knochengerüste des menschlichen Antlitzes: Vienna, 1850.

II. THE FACE.

The most important points to be attended to in the measurements of the face are those affecting orthognathism and prognathism. The appearances so termed form the basis of a fundamental part of the classification of crania generally recognized, and in recent years it has been sought more explicitly to divide them by adding a third term and distinguishing opisthognathous skulls; yet it will not be hard to show that the opinions entertained by anatomists as to the causes which concur to produce these appearances are both vague and inaccurate.

So far as prognathism depends on the forward direction of the incisor teeth, or what may be designated prognathous dentition, it is simple enough and affords a stable foundation for classification; but besides that this is only one of the characters of prognathism originally enunciated by RETZIUS, the term is always considered as indicating projection of the face from underneath the cranium.

The broad contrast between the straight European face and the prominent muzzle of many savage tribes was evidently present alike to CAMPER in suggesting the facial angle, to BLUMENBACH in laying down the advantages of the norma verticalis, and to RETZIUS in distinguishing orthognathous and prognathous skulls; but these methods leave unexplained the nature of the anatomical peculiarities producing the appearances sought to be registered. VIRCHOW* stepped forward, and estimating prognathism by the size of an angle, the "nasal angle," situated at the fronto-nasal suture and contained between two lines, one passing down to the nasal spine, and the other back to the posterior limit of the sphenoid bone, he laid down the rule that the shorter the base of the skull, and the more curved on itself, the more the face projected from underneath it. LUCAE†, taking the zygomatic arch as a horizontal line, and drawing a perpendicular to it from the fronto-nasal suture, estimated the retreat of the forehead and the projection of the face by horizontal lines drawn from points in them and cutting the perpendicular at different heights, and concluded that there was no relation whatever between the form or extent of the base of the skull and the projection of the face. Then WELCKER‡, choosing a nasal angle which differed from that of VIRCHOW in that the upper of its containing lines passed from the fronto-nasal suture back to the foramen magnum, put forward the statement that the base of the skull and the projection of the face were indeed in relation, but that the relation was precisely the reverse of that stated by VIRCHOW. LANDZERT§ agrees with VIRCHOW that the nasal angle and the angula sellæ exhibit an inverse ratio one to the other; but, appreciating that the nasal angle is no true measure of prognathism, he recommends for that purpose LUCAE's system of ordinates.

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* Entwicklung des Schädelgrunde, 1867. † *Op. cit.* p. 40.

‡ Wachsthum und Bau des menschlichen Schädels, 1862, pp. 48 & 140.

§ Der Sattelwinkel und sein Verhältniss zur Pro- und Orthognathie, 1867.

Both VIRCHOW's and WELCKER's nasal angles are objectionable, because the upper of the containing lines is a mere compromise between the directions of two portions of the base which lie at a very variable angle one to the other; while LUCAE's method fails by being dependent on a line which is neither horizontal, as he and WELCKER suppose, nor gives any indication of the position of any part of the base; and the same objection holds good against setting the skull, for the determination of its prognathism according to Mr. BUSK's method, in such a manner that a line from the ear to the fronto-nasal suture shall be vertical.

VIRCHOW has been misled by his method. The skull of the Cretin 53 years old and that of the new-born Cretin figured in his work, do not get their peculiar characters of base, as shown in his plates, accounted for by the theory of "kyphosis" or increased curvature. The basilar process in both those skulls certainly lies at a less obtuse angle with the body of the postsphenoid, to which it is synostotically joined, than it does in the respective healthy skulls with which he compares them; but that circumstance is nearly made up for in the Cretin child, and in the adult Cretin is more than made up for, by the longitudinal axis of the sphenoid lying almost in a line with the cribriform plate of the ethmoid, instead of making the angle with it which is usual; so that a line from the fronto-nasal suture laid on the floor of the anterior cranial fossa and continued backwards, in the young Cretin touches, and in the old Cretin cuts the dorsum sellæ, while in the healthy skulls with which they are compared it lies far above that process. This altered relation of the sphenoid to the ethmoid accounts for VIRCHOW finding the "nasal angle" in the Cretins larger than in healthy skulls*. Had he taken the floor of the anterior cranial fossa as the upper of the two lines containing the nasal angle, he would have found that the angle was smaller in the Cretin skulls than in the healthy, and that however prognathous the appearance which the Cretin skulls may have presented during life, that appearance was not to be accounted for by projection of the face from under the floor of the cranium, but in some other way.

WELCKER's statement that the greater the curvature of the base as estimated by means of the angula sellæ (Sattelwinkel) the smaller the nasal angle as he measures it, is undoubtedly true, but is uninstructive; for it is self-evident that the more the basilar process is bent down from the direction of the floor of the anterior cranial fossa the more will the upper of the two containing lines of WELCKER's nasal angle be depressed, and that if in any skull the inclination of the basilar process were changed while the face was left untouched, the alteration in the angula sellæ and the nasal angle would only be different expressions of the one anatomical change. Seeing, then, that this nasal angle is subject to such variations altogether independent of the position of the face, it is no wonder that, as WELCKER has observed, "the unprejudiced consideration of the whole skull in many instances exhibits a very different degree of prognathism from what the size of the nasal angle would indicate."

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* This combination of what may be termed curving downwards of the presphenoid with curving upwards of the ethmoid is often met with in skulls of low type (see p. 169). It has occurred in the Australian skull figured by LANDZERT; and had that writer caused the upper limb of his angula sellæ to pass from the upper surface of the presphenoid forwards to the fronto-ethmoidal or fronto-nasal suture, instead of following the plane of the presphenoidal surface, his conclusions must have been materially modified.

LUCAE is in one sense right in keeping altogether out of view the position of the base of the skull in seeking to estimate the amount of what is generally understood by prognathism and orthognathism; for it is evident that the relation of the face-bones to the base was never examined by any of the writers who attracted attention to the degree of prominence of the face; but, on the other hand, his method throws no light on the concurrent causes of the general appearances which they have noticed, nor does it furnish an accurate index of the extent to which either of the appearances is present. For example, it might not be difficult to find two skulls tolerably similar in appearance as seen in vertical mesial section, and which might be considered to have as thus seen the same degree of prognathism, but one of which would have the auditory meatus placed on a much higher level than the other. In such a case the difference in position of the auditory meatus would be accompanied with a difference in the disposition of the zygomatic arch such as would materially affect both CAMPER'S facial angle and the degree of prognathism as indexed by LUCAE'S method.

The foregoing remarks may serve to illustrate the uncertainty which invests the whole question of the varying relations of the face to the cranium, and may prepare us to investigate those relations in detail.

The orbito-nasal angle (column 24).—By this name may be designated the angle contained between two lines extending from the fronto-nasal suture, one to the optic foramen and the other to the tip of the nasal spine of the maxilla. The tip of the nasal spine is chosen rather than its root, because it is a more definite point, and because the nasal spine is a characteristic portion of the maxilla independent of the teeth. The orbito-nasal angle in the fœtal skulls is of sizes such as are found in adults; but in the seventh and eighth month fœtuses it is smaller than in those of the fourth and fifth months. In the infants at birth it has attained a greater size than it has either before or afterwards, but as childhood advances it becomes rather smaller than the adult average. Thus, if prognathism were to be taken as meaning projection of the face from underneath the floor of the anterior cranial fossa, infants would have to be considered as exhibiting the maximum of prognathism, notwithstanding that in them the face is far smaller in proportion to the cranium than it is in the adult. Observation on the living subject will fully verify in this respect the results of the measurements here given; for it will be readily seen that in infants and young children the sum of the orbito-frontal and orbito-nasal angles is much greater than in the adult. After birth the orbito-nasal angle diminishes apparently *pari passu* with the increase in the orbital length, dependent on the growth of the anterior lobes of the brain; but the orbital length is further increased at puberty by the enlargement of the frontal sinuses; and it is a remarkable and important fact that this is not accompanied with a further diminution of the orbito-nasal angle, but, there being about that time apparently a general growth of the face-bones, the whole mesial extent of the face is brought forwards, and the nasal spine often even more than the root of the nose.

In the adult the range of variation of this angle is great, but is principally dependent on individual peculiarity, not at all on sex, and only to a limited extent on race. The extremes are 79° in the German skull 33, and 103° in the Hottentot skull 87. In almost all the nationalities of which several specimens have been examined, there is a range of variation of 10° or more, while the difference between the lowest and the highest national average is only 7°. The range of variation appears to be as great in the female as in the male; thus, for example, the largest and the smallest orbito-nasal angles found among the nine Irish skulls are in females. Welcker's[*] statement that women are more prognathous than men, like his other statement that opisthognathism specially accompanies brachycephalism, and prognathism dolichocephalism, is merely a geometrical result of his unfortunate choice of an upper bounding line for his nasal angle. He states that the female skull has "a greater tendency to prognathism as well as a less strongly bent tribasilar bone than the male;" thus the really valuable part of the statement resolves itself into finding the base more level in the female than in the male. This may be illustrated by reference to the French female skull 25, which in consequence of the levelness of its base would give a high figure for Welcker's nasal angle, and yet has the orbito-nasal angle small, and is altogether in general appearance rather opisthognathous.

Considering the great individual variation and comparatively limited national variation of the orbito-nasal angle, and the exceedingly small number of skulls of any one nation which the writer has been able to measure, the results in the accompanying Table can-

	Average.	Extremes.	Range.	Average male.	Average female.
5 Greeks	$87^{\circ}\frac{3}{5}$	80° 94	14°		
9 Irish	$88\frac{1}{3}$	82 93	11	$88^{\circ}\frac{1}{3}$	$88^{\circ}\frac{1}{3}$
2 Esquimaux ...	$88\frac{1}{2}$	84 93	9		
8 German	$88\frac{3}{4}$	79 94	15	$89\frac{3}{5}$	$87\frac{1}{3}$
4 Kafir	$88\frac{1}{4}$	87 92	5	89	88
8 Scotch	$90\frac{3}{4}$	84 99	15	$90\frac{5}{8}$	$90\frac{1}{2}$
2 Kanaka	91	85 97	12	85	97
9 French	$91\frac{1}{9}$	86 97	11	$91\frac{1}{2}$	$92\frac{1}{3}$
3 Hindoo	$93\frac{1}{3}$	88 98	10		
7 Negro	$93\frac{4}{7}$	85 102	17	93	95
2 Australian ...	$94\frac{1}{2}$	94 95	1		
2 Peruvian	$94\frac{1}{2}$	91 98	7		

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[*] *Op. cit.* pp. 51 & 66.

not be considered as a sufficiently trustworthy exposition of the national averages of this angle; still it is curious to note that, such as they are, these results make the angle smallest in the Greek, Irish, and German, decidedly larger in the Scotch, and still larger in the French, which agrees well with the appearance of the features in those nations. Thus it will be generally admitted that prominence of face is a well-marked national peculiarity in the French; and probably no one will deny that the reverse condition is eminently characteristic of the Irish. There is enough in these results to lead to the anticipation that in an extended series of observations the orbito-nasal angle will furnish a marked character of distinction between the different European nations. But enlargement of this angle is certainly not the only source of the concrete phenomenon called prognathism. Any one handling the Kafir skull 68, the Esquimaux 77, or the Carib 88, would probably pronounce them decidedly prognathous; yet their orbito-nasal angles are respectively 88°, 84°, and 85°. Another example is furnished by the diagrams of the Negro skulls 61 and 63, which are as similar as possible in the amount of their prognathism, as well as in many other particulars; and yet the one has an orbito-nasal angle of 102°, and the other an angle of only 89°; but the sum of the orbito-frontal and orbito-nasal angles is in the one skull 177°, and in the other 176°, giving a similarity of external appearance notwithstanding great difference of structure.

Length of face from fronto-nasal suture to nasal spine (column 66).—The distance from the fronto-nasal suture to the nasal spine is not subject to much variation, the differences in length of face from the root of the nose to the mouth being chiefly dependent on variations in size of the incisor teeth and the depth of their sockets, while the amount allotted to nose, and the amount to upper lip, depend greatly on the extent to which the alar cartilages descend below the level of the nasal spine. It does not, however, do so altogether; and in the shortness of the distance from fronto-nasal suture to nasal spine (1·75) in the skull of GEORGE BUCHANAN may be seen an indication of the shortness of his nose. This distance does not extend beyond 1·65 in any of the children's skulls examined; it would appear, therefore, that the growth of the upper jaw in length is completed at a later period, which agrees well with the evidence afforded by the orbito-nasal angle that the growth of the jaw forwards is not completed till the full development of the frontal sinus. This distance is shorter in the female than in the male; its shortness in the Australian skull 73 (1·65) is probably an exceptional idiosyncrasy, for in five figures of Australian skulls by LUCAE it varies from 1·85 to 2·05.

Naso-basilar angle and naso-basilar line (columns 64 & 65).—It is only necessary under this head to record, for the sake of saving other observers needless trouble, that the distance from the spheno-occipital suture to the nasal spine, and the size of the angle between a line joining these points and the line along the under surface of the basilar process of the occipital bone, have been carefully measured in the series of skulls examined in the hope that they might throw some light on the causes of prognathism. But the size of the angle is dependent in great measure on the steepness or levelness of the base of the skull as well as on the form of the face; and the length of the line is affected by the size of the whole skull as well as by the proportionate size of the face and its projection forwards; and thus neither measurement yields results of much importance.

Causes of prognathism.—If the reader take into consideration the various points bearing on prognathism which have already been considered, he will easily convince himself that the causes of that appearance are not, as is usually supposed, confined to the form of the face. The statement of the case may not unfairly be put thus:—that a retreating forehead and projecting face are regarded as characters of a savage or degraded form of the human skull; but it has been shown that the result on the profile produced by that combination is exactly the same as what is produced by the combination of a vertical forehead with a non-prominent face, or, in other words, a large orbito-frontal angle plus a small orbito-nasal angle is equal to a small orbito-frontal plus a large orbito-nasal angle; and by studying the direction of the floor of the anterior cranial fossa which separates those two angles, it has been further seen that it is far from being true that the forehead in the ruder races of men slopes more back on the floor than in cultivated nations, and that it is by no means always the case in admittedly prognathous races that the orbito-nasal angle is larger than in orthognathous races. Therefore, although prognathism sometimes is the accompaniment of a large orbito-nasal angle, it remains to be explained how a skull with a small orbito-nasal angle may yet be prognathous, and why a skull with that angle large is not necessarily so.

The first thing to be noted in explanation is that in prognathous races skulls often are found which are not prognathous except in their dentition. But besides this there is another matter to be taken into account. The division of skulls into orthognathous and prognathous is simply an application of the norma verticalis, and a skull is judged of according to the apparent prominence of the face when it is laid on a flat table. Such apparent prominence may be made to disappear by placing a support beneath the occipital bone; and thus it is apparent that the form of the back of the head affects prognathism as estimated by the norma verticalis. The principal circumstance which acts in this way is the degree of cranial curvature; for it is evident that if the fore part of the skull be taken as fixed in position, the greater the curvature the lower will the back part of the base be brought. Two other causes, namely, shortness and levelness of the base, conspire when present along with deficient curvature to increase apparent prognathism: they are not necessarily or even usually present in prognathous races; but the flattened American skull 96 affords a good example of the combination alluded to producing great apparent prognathism with a rather small orbito-nasal angle. In the Esquimaux shull 77 an illustration is given of a prognathous appearance dependent on deficient curve, notwithstanding the great length of the foramino-optic line and the smallness of the orbito-nasal angle.

It ought further to be noticed that length of face, including dental sockets, will tilt up the fore part of a cranium laid on a table and produce apparent prognathism similar to what deficient curvature would; and thus it happens that projection forwards and elongation downwards of the face are liable to be confused together in judging of prognathism. The Kafir skull 68 and the Australian 73 may be taken as illustrations of these two kinds of facial prominence. Not only in judging of dry skulls, but also with respect to skulls clothed with the integuments and during life, the elongation of the face downwards may be mistaken for projection forwards; for this seems to have been done by VIRCHOW in the instance of the adult Cretin head already referred to. So also the appearance simulating prognathism, extremely common in the rustic population of the west of Ireland, is certainly not the result either of prognathous dentition or large orbito-nasal angle, but reaches its maximum when there is a heavy dentition in the upper jaw, and the lower jaw is so small that the chin fails to come forward into a straight line below the upper incisors.

In column 67 of the General Table it has been sought to express by a single measurement, termed index-angle of prognathism, the gross amount of prognathism, whether dependent on projection or length of face, or on deficient cranial curvature. It is the angle between two lines, both starting from the alveolar process between the middle incisors, and one of them passing to the fronto-nasal suture, while the other touches the lowest part of the base in the diagram, which in most instances is the back of the foramen magnum, but in some fœtuses is the front of that foramen. The angle exhibits a great deal of individual variation in the nations in which several specimens have been examined, the variation in the German skulls amounting to 14°; still the order in which it arranges the nations is worth noting. The most orthognathous skulls are the Greek, and after them come Scotch, French, Irish, German, Kanaka, Hindoo, Esquimaux, Peruvian, Kafir, Australian, Negro, and lastly, most prognathous of all, the compressed and flattened Americans.

A much more precise comparison, however, than can be made either by this index-angle or by division of skulls into prognathous, orthognathous, and opisthognathous, may be made by distinguishing the different points which together combine to constitute prognathism.

First. Prognathous dentition can be easily detected with the unassisted eye.

Secondly. The size of the orbito-nasal angle should be distinguished.

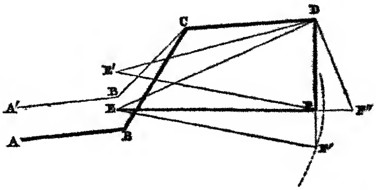
Thirdly. The cranial curvature ought to be considered quite apart from the preceding characters, although the following may be taken in lieu of it.

Fourthly. As the curvature of the cranium is continued in the face, and in most characteristically prognathous skulls the facial part of the curve is unusually slight even when the proper cranial curve appears fully developed, it is well to take account of the whole curve which is expressed by the retreating angle contained between the long diameter of the foramen magnum and a line from the back of the palate to the alveolar process between the middle incisors. This may be termed the foramino-palatal angle.

The foramino-palatal angle (column 31) gives results not very dissimilar from those given by the angle of cranial curve. It differs from that angle, however, in that apparently it is not affected by sex. The cause of this seems to be that while the cranial curve is greatest in the female sex, females have also shorter faces than males, and the one difference counterbalances the other. The arrangement according to the size of the foramino-palatal angle separates the European from other skulls more completely than the cranial curve does.

	Average.	Extremes.	
Esquimaux	198°	189° and	207°
Peruvian	199½?	191?	208
Kafir	201½	193	216
Negro	202¾	197	212
Kanaka	202½	195	210
Australian	204	202	206
Hindoo	206	201	212
Greek	206⅔	202	213
Scotch	208¼	198	216
French	210 8/9	202	216
German	212½	201	219
Irish	214¼	210	220

Relations of the upper jaw to the ear (columns 54, 55, 62, 63, & 68).—The line from the postauricular depression to the nasal spine is in many instances exactly equal to the line from the same point to the fronto-nasal suture; much more frequently it exceeds it, doing so to the greatest extent when the ear is high above the level of the foramen magnum and the elements of prognathism are present; while occasionally in the Irish, German, and Scotch skulls it falls considerably short of it. The worst fault of CAMPER'S facial angle is that the anterior line is rested on the glabella instead of the fronto-nasal suture, and thus an element is made to operate on its size which has really nothing to do with the conformation of either face or cranium, but is a mere local accident. If it be modified by drawing one line from the postauricular depression, and the other from the fronto-nasal suture, to meet at the nasal spine, an angle will be obtained which will vary according to the same circumstances as regulate the proportionate length of the lines from the postauricular depression to the fronto-nasal suture and nasal spine. On the whole, the absolute difference between these two lines is the simplest register of the information. Four circumstances influence the comparative length of these lines, as also the size of the facial angle now described; they are:—first, the height of the ear above the level of the foramen magnum; secondly, levelness or steepness of base, and the extent of the foramino-optic line; thirdly, the length of the face from the fronto-nasal suture to the nasal spine; and fourthly, the size of the orbito-nasal angle. The accompanying diagram (p. 160) makes these points clear, and also illustrates that a certain amount of projection forwards of the nasal spine has a much greater effect than the same amount of elongation downwards, both in diminishing the facial angle and in increasing the distance from the ear to the nasal spine.



A B C D represents the base, E the post-auricular depression, and F the nasal spine; E' represents the position of the postforaminal depression when the ear is more elevated above the foramen magnum A B, or with a more level base A' B' C D. The lines drawn to F' and F'' show the effects of removal of the nasal spine downwards or forwards. The diagram suffices for the geometrical proof of the statements which have been made.

III. POSITION OF THE SKULL ON THE VERTEBRAL COLUMN.

We are now in a better position to consider the way in which the skull is set on the vertebral column than we were at the commencement of the investigation, as various points which have emerged with regard to form bear on the question of position. This question is the more difficult as it cannot be altogether settled by means of precise methods, as the position of the body is continually changing. It may, however, be pretty safely assumed that in the erect posture the head in the human subject is nearly balanced. There can be no doubt, and it is well known, that in the ordinary standing positions the whole body is supported by balance, while muscular action is only required to preserve that balance. This is seen most distinctly with regard to the lower limbs, for the gastrocnemius and soleus are flaccid in standing, as may be easily proved by observing how they become rigid and change their form as soon as an attempt is made to rise on tiptoe; the flaccidity of the quadriceps extensor cruris is proved by the moveability of the patella so long as the knee is straight and the foot on the ground; and that the glutei muscles are entirely relaxed in standing is shown by observing how the gluteus maximus at once becomes rigid on bending the body forwards, and the gluteus medius and minimus when the opposite foot is lifted up. It is not so easy to prove the relaxation of the muscles of the back in standing, although the curves of the vertebral column speak distinctly enough of balance; but if, with the head held erect and loosely, the fingers of both hands be rested on the upper part of the complexus muscles, and flexion be made from the hips, without changing the relative positions of head, neck, and trunk one to another, those muscles will be felt to swell out and harden, again to relax on resumption of the erect posture. It appears to be a general principle of animal statics that the body can be supported without muscular action; and in the human subject the absence of an elastic ligamentum nuchæ shows that it is not by the principle of suspension that the head is preserved erect.

But if the head be balanced on the vertebral column, it must change its position with growth, and be gradually tilted up more and more from childhood to adult life, to throw more weight behind the condyles as the frontal and temporo-sphenoidal lobes of the brain increase in size, and the face-bones become heavier; and the greater the growth of these parts the greater must be the tilting up. This tilting or rotation backwards must be accomplished either by movement of the vertebral column so as to make the upper surface of the atlas look more and more backwards, or by gradually increasing prominence of the anterior extremities of the occipital condyles. There seems to be no evidence that the change is in the position of the atlas; on the contrary, the arch of that vertebra appears to preserve a horizontal position throughout life; but the increasing prominence of the anterior extremities of the condyles is sufficiently great to catch the eye, and is what will now be more explicitly proved.

A flat surface laid on the occipital condyles, and on the skull behind the foramen magnum, having been assumed as horizontal, the degree of elevation of the condyles is easily estimated by measuring the angle which the long axis of the foramen magnum forms with that plane. But the tilting up of the skull is complicated by the increase in cranial curvature which proceeds with growth. Were there no alteration of position of the back part of the skull in the progress from infancy to adult life, the increase in cranial curvature would turn the face more and more downwards; it may be necessary therefore to distinguish tilting up which involves only the back part of the skull from such as involves the whole cranium. With a view to this distinction, as fair an index of the position of the face as can be found is the line of orbital length, and therefore not only the angle at which the horizontal line lies to the long axis of the foramen magnum has been measured (column 77), but also the angle at which it lies to the line of orbital length (column 78).

In infants' skulls the foramen magnum forms the smallest angle with the horizontal line, while the cranial curve being as yet imperfect, the line of orbital length forms a larger angle with the horizontal than it does in many adults. But it is to be recollected that in infants there is at first little attempt at balance, the erect posture being not yet assumed. In the children's skulls the average angle of the foramen magnum with the horizontal remains smaller than that of any of the adults except the Hindoos, while the angle of the line of orbital length has sunk lower than the average of adult males in any nation.

Among adults there seems to be least tilting round in the Hindoos, and next to them in the Australians. In the Esquimaux and Kafirs the angle of the foramen magnum with the horizontal line is not very great; but owing to the small cranial curvature, that of the line of orbital depth is greater than in any of the other nations. In the Irish, French, and Scotch, on the other hand, the angle of the foramen magnum with the horizontal is high, the curvature being great. In the Germans, where the curvature is greatest, the tilting up of the foramen magnum is not quite so great; but this may be accounted for by lightness of the face-bones and by the greatness of the parietal breadth as compared with the frontal breadth in some of them, a circumstance which throws more weight behind the condyles.

It appears very distinctly from the accompanying Table that the female skull is much less tilted back on the condyles than the male, being in this, as in various other respects, more child-like than the male skull; and in the two apparent exceptions to this rule, the Kanaka and Kafir, a full explanation is found in the high cranial curvature.

Deviation from the assumed horizontal plane					
Of the foramen magnum.			Of the line of orbital length.		
4 Infants	6°·2			11°·5	
7 Children	7·8			5·5	
	Males.	Females.		Males.	Females.
Hindoo	7°·6		German	5°·8	4°
Esquimaux	8·5		Hindoo	7	
Australian	9		Australian	10	
Kafir	10	20°	Irish	10·8	4·3
Greek	10·7		Scotch	11·4	8
German	13	8·6	French	12·8	3·6
Negro	13·4	8·5	Greek	13·5	
Scotch	13·6	11	Negro	16·4	9·5
French	14·6	11·3	Esquimaux	17·5	
Irish	15·5	11·6	Kanaka	19	8
Kanaka	16	17	Kafir	20·3	14

After the greatest adult male rotation backwards of the skull has been accomplished, a reverse change may take place as a feature of old-age degeneration. The condyles in that case become flatter, and the head, in consequence of this, is necessarily rotated forwards; for it may be safely assumed that no elevation of the floor of the facets of the atlas takes place, by which alone such rotation could be prevented. That such an old-age degeneration actually occurs can scarcely be denied, but among the skulls measured there are only three well-marked examples of it, namely, the old Scotch skull 40, the German 28, and the Greek 47; they are, however, really good examples. The first two exhibit in a marked manner all the gravitation changes of form; the third does not exhibit those changes in a typical manner, as it has a nearly vertical forehead which can scarcely be imagined to have fallen much backwards; but probably it has undergone a partial change like the German 29, which appears to have enlarged its foramino-basilar angle by the weighing down of the back part of the cranium and its contents. The tendency of the gravitation changes is to throw more of the weight of the brain and skull behind the condyles; and this, together with absorption of the substance of the jaws, and that enlargement of the air-sinuses at the expense of osseous tissue which goes on in old age even after the entrances into the sinuses are blocked up, makes the fore part of the head lighter; and thus the rotation forwards by flattening of the condyles tends to preserve the balance. This flattening of the condyles is in harmony with the general tendency in old age to absorption of osseous matter, especially cancellated tissue—a tendency which is shown in the skull, not only in the extension of the air-sinuses, but in thinning of the bone opposite the inferior occipital fossæ, and in the squamous portions of the temporals. Indeed the much remarked thickening of the skull in old age seems to be confined to those parts which are subjected to increased exposure by baldness.

Applications of the foregoing remarks to artistic anatomy.—The foregoing remarks tend to explain the superficial appearances seen in the form of the head at different ages and in the two sexes, as well as in individuals; but, in applying them, it must be kept in view that the rotations of the skull on the column at different ages involve change in the relations of the flexible soft parts to the bony framework of the features. Hence, while there is no ambiguity as to what is meant by placing the face of either a living or dead person so as to look directly forwards, the greatest of all difficulties in examining skulls is to know how the features lay to the face-bones.

When a young infant is supported with its face directed straight forwards, the most characteristic point in the appearance of its head is the height of the parietal region, which we now know depends partly on the small proportional development of the frontal region, and partly on the position of the skull on the vertebral column; while the prominence of the face in the region of the nostrils depends on the large orbito-nasal angle and the slope upwards and forwards of the line of orbital length, the result of incomplete cranial curvature.

In childhood the parietal height becomes less marked as the forehead fills up; and there being as yet little rotation on the vertebral column, and the orbits remaining comparatively shallow, while the cranial curvature has increased, the frontal eminences appear more prominent than in either the infant or the adult. Thus the frontal region, although it has by no means gained the proportion to the parietal which it bears in the adult, makes a great show; but its actual smallness is revealed by the low position and closeness one to the other of the frontal eminences. Both in infancy and childhood, the midoccipital point, owing to the head being not yet tilted up, is placed very high, which, with the non-development of the tuberosity and the want of muscularity of the neck, gives a slender appearance to the lower and back part of the head.

In the adult male the rotation backwards on the vertebral column combines with the development of the frontal sinuses and face to give a retreating appearance to the forehead, and, together with the increased prominence of the occipital tuberosity, gives a rounded fulness to the lower back part of the skull. It is a grave mistake to predicate deficient development of the anterior lobes of the brain from a retreating forehead, or great development from a vertical forehead, without reference to the rest of the form of the head. Large anterior cerebral lobes may be the cause of a retreating forehead, by causing rotation backwards, for they will add to the weight of the fore part of the head as much as heavy jaws would: but if the cranial curvature be not deficient, the rotation consequent on the weight in front will give a rounded appearance to the lower back part of the skull, by lowering the position of the occipital tuberosity. The physiognomical effect of this is very important. Much is said of the physiognomic value of a high forehead, and little store is placed on fulness of the lower and back part of the head; but the artistic value of apparent occipital development is easily demonstrated by drawing a profile and varying the limiting line of the lower part of the skull behind. The change of one line at this part may be made to convert an intelligent-looking head into that of a weak-minded person, without alteration of a single feature of the face. The anatomical fact which the change of line indicates is an arrest of development of the whole brain, whether large or small, leaving the cranial curvature incomplete.

The question of the beauty of a high or low forehead in the representation of feminine form receives elucidation from the principles laid down. The question is less one of taste than one of anatomy, for the just proportions of the head in the two sexes are as subject to law as those of the trunk. The child-like position of the feminine skull on the vertebral column, dependent on the lightness of the face, and on that deficient frontal capacity demonstrated by HUSCHKE, combines with smallness of the postforaminal angle to cause the continuance of other peculiarities of the young head, viz. an appearance of slenderness at the lower back part, and a rise of the roof as it passes back from the forehead. The kind of height of head required in the representation of feminine beauty is this rise of the roof, a result not of real height but of the position of the head on the vertebral column; the kind of height which is objectionable is height of the frontal eminences. There is probably no objection to the whole arch of the skull being represented high in the female; but it is inconsistent with the greatest feminine grace to place the frontal eminences as high up and as far asunder as they are in the male, and to make the arch retreat horizontally backwards. It is established by WELCKER, WEISBACH, and ECKER* that the average height of the female cranium is less than that of the male; and although the present measurements do not prove this statement, it is in all likelihood correct. Among the present measurements are several feminine skulls of markedly depressed form, such as do not occur in the male, and others which are not in the least depressed. Probably it may be correct to say that in consequence of the persistence in the female, as has been pointed out by ECKER, of the flatness of the roof found in childhood, the latest accession of height in the male skull is wanting in the female (p. 148), but that the mould of the female head in childhood may vary like the male.

Further characteristics of a femininely shaped head have their origin in peculiarities pointed out in considering the angles in connexion with the arch, viz. the largeness of the orbito-frontal and fronto-parietal angles and the angle of the tuberosity, and the smallness of the frontal, parietal, and postforaminal angles. Among these characteristics those which affect the forehead and roof have been appreciated by ECKER. The difference in the general contour of the profile in male and female may be expressed by drawing a line to represent the male profile round a line representing the female, and making it touch the female profile at the midfrontal point, and in the region extending from the midparietal to the midoccipital point (see Plate XXI.).

IV. ANOMALOUS FORMS OF SKULL.

Unusual magnitude, Kephalon of VIRCHOW.—The large skull 92, in the possession of Professor THOMSON, appears to be a fair specimen of individual enlargement with preservation of shapeliness, and uncomplicated with hydrocephalus or any other pathological condition; and the peculiarities about to be mentioned are probably most of them common to the majority of cases of individual regular enlargement. The base-line has undergone no increase; but the elongation of the inferior part of the skull is due to the length of the occipital bone from the foramen magnum to the tuberosity. Hence the proportion of the arch to the base-line exceeds even what is usually found in infancy or childhood, being 3·3. The orbito-frontal angle is large and the midfrontal small, the postforaminal angle large, and the angle of the tuberosity small, so that the combination is produced, not otherwise occurring, of a vertical forehead with prominent eminences going together with the rounded and full appearance given behind by a low-placed tuberosity.

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* ECKER, Anthropological Review, October 1868, p. 350, translated from the Archiv für Anthropologie.

It may further be observed that the enlargement of the arch has taken place principally in the frontal and occipital regions, the parietal remaining of a normal length. Now in the growth from infancy, the frontal and occipital regions normally increase in length as compared with the parietal. If, then, further researches should show that in unusually large skulls the parietal is generally less enlarged than the frontal and occipital regions, we shall have evidence that the excessive development of the arch takes place by a continuance of the method of growth by which the latest steps of its ordinary growth are effected.

The Greek skull 46, already alluded to as having some peculiarities, appears to be an instance of Kephalon less marked and of a different kind. It is true that this skull is not singular in its size, when compared with all the other skulls in the list; it is, however, the largest of the five Greek skulls, and bears evidence of its shape being altered from the national model by continued enlargement. The profile diagram has a much more dolichocephalous appearance than those of the other Greek skulls, and one might be disposed to doubt the alleged nationality; but such a doubt does not explain the peculiarities, namely, the small cranial curvature, the long base-line, and the large orbito-frontal angle. All the peculiarities, however, are removed and the shape of the cranium assimilated to that of the other Greek skulls if we suppose the frontal portion of the arch to be rotated downwards round the fronto-parietal point, so as to lessen the fronto-parietal angle, shorten the base-line, push back the line of orbital length, and throw the foramino-optic line into a more nearly vertical position. This seems to show that this skull presents an instance of individual enlargement accomplished by flattening out of the fronto-parietal angle, the result of elongation of the base-line in consequence of continued levelness of base, the growing brain having pressed too much downwards to admit of the usual contraction of the orbito-basilar angle in process of growth. The enlargement of the skull in this instance is produced less by any unusual increase of size of the bones than by their being placed at unusual angles one to another.

Hunchback (skull 93).—This skull exhibits arrest of development in various respects, while in others it is fully grown. There is no deficiency in the face nor in the regions of the base; but the frontal part of the arch is short, and, consequent on this, the deep frontal angle is small and the cranial curvature deficient. Also the midparietal angle is more acute than is usual in adults. These peculiarities, so far as one may judge from external appearances, seem to be very usual among rachitic dwarfs, and not the mere idiosyncrasies of this one skull.

Cretins and Idiots.—The Cretin form of skull described by VIRCHOW has been already referred to in treating of the relation of the face to the cranium (p. 153). Reverting again to VIRCHOW'S figures of the skull of the Cretin aged fifty-three, and of the new-born Cretin's skull, there seems little difficulty, after all that has been said, in determining the nature of the cranial degeneration, and accounting for the appearance of prognathism which led VIRCHOW to expect a projection of the face from underneath the cranium. The line of attachment of the tentorium, instead of beginning at a considerably lower level than the floor of the anterior fossa of the skull, and passing directly backwards, begins on a level with that part, and appears to rise as it passes backwards; so that, instead of there being increased curvature to make up for shortness of base, as VIRCHOW supposed, there seems to be a marked deficiency in cerebral curvature, and, as it were, a thick slice taken off the lower part of the middle and posterior lobes. This deficiency of curvature does not show itself in the skull by decrease of the angle between the plane of the foramen magnum and the floor of the anterior cranial fossa; but although that angle is the index which we have been using as the measure of curvature, it in this instance misleads in consequence of a drawing up of the front of the foramen magnum, which is not the result of curvature, but of the mere shortness of the basilar process from synostosis, and the crushing up of the dorsum sellæ to the level of the anterior cranial fossa. The want of development backwards of the cerebral hemispheres, indicated by the line of attachment of the tentorium in VIRCHOW'S drawings, must, according to the principle of balance, have led to a great tilting backwards of the skull during life, and consequent projection of the jaws. The crushing up of the dorsum sellæ, in these Cretins, to the level of the anterior cranial fossa, is a phenomenon not unlike what has occurred in skull 91 (Professor THOMSON'S "baker's skull"), only that in that instance the basilar process has become more horizontal as it has been pushed up, while in the Cretin skulls it has not been so; but in the Cretins, as well as in the "baker's skull," there is the appearance of deformity by gravitation. Probably, as happens in the bones of rachitic skeletons, they had first yielded too freely, and then ossified too rapidly; nor is the circumstance that one of the skulls is that of a new-born infant adverse to this theory, seeing that *in utero* the whole body presses down on the head.

In the skull of the Idiot, 94, there is a totally different state of matters, probably a typical idiot form. Evidently the idiot's skull figured by CARUS* is of the same description, although still more degenerated. Here there is no defect in the development of the base, no evidence of gravitation changes. The foramino-optic line and orbital length are both long, the base is steep, the cranial curvature great, the face largely developed, and, in connexion with this, there is a frontal sinus projecting far forwards in front of the brain. *The base and face have gone through all the stages of a complete and full development; the vault alone is diminished, and that diminution is in height as well as breadth.* The deficiency of height is best illustrated by the shortness of the frontal depth; the deficiency of breadth is such that, as already mentioned, the greatest width is between the mastoid processes. Plainly, in this case, the deficient cerebral development is the cause and not the consequence of the form of the skull.

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* C. G. CARUS, Neuer Atlas der Cranioscopie, taf. xix.

Artificial deformities.—Among the American skulls examined three have been subjected to considerable artificial pressure. Of these, the Peruvian, 79, is the least deformed, and the writer has deemed it not unfair to use it in drawing up tables of national peculiarities. One of the other two (95) has been tightly compressed with bandages, while the other (96) has been flattened by boards not interfering with supplemental lateral projection. A study of these two skulls shows some important points. In the flattened skull the proportion of the arch to the base is smaller than in any other skull examined, which, taken in conjunction with the circumstance that the base is a pretty short one, renders it probable that the growth of the roof-bones has been interfered with; but in the other skull, in which the transverse and longitudinal diameters have been equally compressed, there is no evidence of any obstruction to the growth in length of the roof-bones. It does not appear that any arrest in the growth of the base has taken place in either skull. No doubt the orbits are shallow, but they are not unnaturally so; and if it had happened that the development of the frontal sinuses at the proper period had been prevented by pressure in infancy, a thing in itself unlikely, there would have resulted from this, and indeed from any restraint of the normal growth forwards of the frontal bone, a large orbito-nasal angle consequent on the unrestrained natural growth forwards of the lower part of the jaw; but the orbito-nasal angle does not surpass 90° in either skull. The base in both skulls is very level and deficient in cranial curvature; and both these peculiarities are best marked in the flattened skull, in which also the arch is most deformed. The deformity of the arch longitudinally consists in flattening of the frontal and occipital bones and bending of the parietals, as exhibited by the large midfrontal and midoccipital angles and small midparietal angle. Now this deformity of arch results from the cerebral mass being pressed out from the grasp of the compressing agent, and this grasp presses as much downwards as upwards; and although it meets with more resistance in the downward than the upward direction, the effect of the downward pressure is exhibited in the levelness of the base and the deficient cranial curvature,—the action being, in fact, precisely of the same description as has been already mentioned as occurring in the large Greek skull 46.

It is not, however, to be forgotten that, as has been pointed out by Professor TURNER*, such skulls as these are not subjected to pressure during the whole period of growth, but owe their deformity to pressure in infancy; and the question remains, how it happens that the deformed shape given to the infantile skull is preserved. Professor TURNER suggests arrest of development from premature synostosis as the explanation, and his own observations, together with those of Dr. DANIEL WILSON, show that in such skulls there is great tendency to obliteration of sutures; while the flattened skull 96 gives an instance of arrest of development of the roof-bones. But it is difficult to see how synostosis, although it may occur, accounts for the phenomena sought to be explained. To the writer the explanation seems rather to be found in the consideration that the process of growth in the frontal and parietal bones and occipital squama consists normally of two parts,—the addition of new bone to the margins, extending the planes in which those margins lie, and the change of shape of each bone, by bending and unbending, from the form of a cone with truncated apex at the centre of ossification to a more uniformly curved but, on the whole, flatter shape. In the artificially compressed skulls the frontal and occipital bones are flattened out, while the parietals are forcibly bent; there is no original tendency of the flattened central parts of the frontal and occipital bones to become elevated, and the natural tendency of the parietals to become flatter is insufficient to undo the enormous bending to which they have been subjected; while, in addition, the directions of the planes of marginal growth are entirely changed.

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* Natural-History Review, 1864, p. 106.

V. COMPARISON OF THE HUMAN SKULL WITH THE SKULLS OF VARIOUS ANIMALS.

On comparing the human skull with that of the Chimpanzee or Orang, the most interesting points noticeable in the latter, in connexion with the facts brought forward in this paper, are the smallness of the cranial curvature, and the length of the base as compared with that of the arch. There is also a complete absence of balance of the head on the vertebral column, which is in harmony with the fact that no Ape is capable of supporting itself by balance on its hind limbs, but requires persistent muscular action to prevent it from falling.

To compare the human cranium properly with crania of animals, the cranial cavity must be regarded as a dilated and curved continuation of the spinal canal. The advance in the form of the cranial cavity of Man, as compared with that of the Chimpanzee, consists in increased dilatation both in height and breadth and in increased curvature, whereby not only is the vault expanded, but the bones of the base are crowded together, the postsphenoid and presphenoid being fused, the ethmoid depressed, and the vomer pushed back in the way more fully described by the writer on a former occasion*. It is curious to note that while the dilatation of the cavity by height is greater in the Orang than in the Chimpanzee, the curvature is much greatest in the Chimpanzee (Plate XXI.).

It is impossible, however, in examining the curvature in the lower animals, to use any longer the means of estimating it which have served us hitherto; for as we pass to lower forms we find the frontal bone coming further down on the face and the optic foramen varying in direction, so that the line joining the optic foramen and fronto-nasal suture no longer indicates any thing with regard to the cranial cavity. Another difficulty, when it is sought to compare the forms of brains, is that the roofs of the orbits, which form the floor of the anterior lobes of the brain, project to a variable and often considerable extent above the level of the cribriform plate. The writer has therefore, with the view of comparing the curvature of the brain in Man and Animals, availed himself of casts of the interiors of the crania of Dean Swift, an Australian, and a Gorilla, prepared under the direction of Professor WRIGHT of Dublin, together with the casts of the interiors of the Tartar and Negritic skulls described by Professor Huxley; and has taken as an expression of the cerebral curvature the angle between a line laid along the anterior lobe of the brain in the groove formed by the roof of the orbit, and another extending from the torcular Herophili to the most depending part of the middle lobe. The results are:—

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* Philosophical Transactions, 1862, p. 296.

Dean Swift	164°
Australian	164
Tartar	155
Negritic skull	156
Gorilla	130

In the lower Mammalia the direction of the orbit is so variable that it was found necessary to try still another method of comparison. The following results are obtained from vertical sections by measuring the angle between the middle line of the upper surface of the presphenoid in front of the optic commissure, and a line drawn so as to express as nearly as possible the direction of the tentorium.

German, No. 29	184°	Negro, 63	175°	Cat	133°
Scotch Female, 43	182	Kafir, 70	191	Dog	150
Greek, 44	186	Australian, 73	193	Pig	150
Irish Male, 53	196	Synostotic, 89	175	Deer	131
„ „ 54	182	Chimpanzee	172	Rabbit	116
„ Female, 55	177	Orang	168	Squirrel	99
„ „ 56	182			Turkey	103

It is noticeable that this angle varies much in different human skulls, and not according to the variation of the angle of cranial curvature made use of in the former part of this paper. This is probably due partly to the impossibility of estimating the angle with perfect precision, partly to different arrangement of the cerebellum in different skulls, and partly to a circumstance seen best in the Australian, Kafir, and Negro. In those skulls, while the upper surface of the presphenoid is directed to a marked degree downwards as well as forwards, the cribriform plate of the ethmoid is directed again upwards and forwards, so that a mean between the directions of the two parts would most justly express the direction of the floor of the anterior cranial fossa in the middle line. The angle under consideration scarcely exceeds in some human brains the size found in the Chimpanzee and Orang, but in these animals the ethmoid is considerably turned up. Taking this into consideration, together with the comparison of the casts of the interior of the cranium in the human subject and the Gorilla, it may be held that the advance in form of the human brain as compared with the brains of the higher Apes consists partly in an increase of cerebral curvature, dependent on depression of the sphenoid and ethmoid, and on descent of the orbital roofs towards the level of the ethmoid, but to a greater degree consists in increased expansion both in height and breadth of the cranial dilatation of the cerebro-spinal canal.

DESCRIPTION OF THE SKULLS MENTIONED IN THE GENERAL TABLE.

1 to 6, Fœtal skulls.—1 & 3 are specimens preserved in spirit, the others are dried. The first five were bisected and the section traced with ink on oiled paper; the sixth was measured with the craniometer.

7 to 11, Skulls of new-born infants.—7 was a recent specimen, which was bisected and the section traced. The others are specimens in the Anatomical Museum of the University of Edinburgh.

12 to 18, Children of various ages mentioned in the Table.—The ages of the Edinburgh specimens were taken from the Catalogue and from the dentition of the skulls. 13 is a specimen prepared under the writer's observation.

19 to 27, French.—19 to 24, male; 25 to 27, female. 24 bears the inscription "Soldier of Napoleon and Knight of the Legion of Honour." It is a large and heavy skull, with the face large as well as the cranium, and with the base-line singularly long for a European; probably it belonged to a tall man. 20 has a clino-cephalic depression at the top of the coronal suture, which may have been the result of spheno-parietal synostosis; the spheno-parietal and spheno-frontal sutures are obliterated, and the sagittal begun to disappear. The arch is much thrown backwards. 25 is a marked instance of retention in the adult female of certain infantile characters—levelness of base, unelevated condyles leaving the skull unrotated on the vertebral column, and, in harmony with this, smallness of face and anterior lobes of the brain; the malar and frontal breadth both being very small as compared with the greatest breadth. 26 contrasts with 25 in being high in the arch and steep in the base: both skulls may have exhibited, during life, prominence of forehead, 25 from non-rotation on the column, 26 from shortness of orbital length. 27, the skull apparently of a female about thirty, has undergone a certain amount of deformation in the upper and back part from accidental pressure. The occipital squama is flat, the parietals are curved, and a hollow is formed at the top of the coronal suture, not by restriction of growth from synostosis, but by the rise of the parietals.

28 to 35, German.—Unfortunately there is no information from what part of Germany any of these skulls were obtained. 28 to 32 are male, 33 to 35 female. 28 & 29 are skulls of aged persons. 28 is toothless, very unsymmetrical, with the foramen magnum much sunk between the mastoid processes, and the base fractured, apparently posthumously; the skull is anomalously broad behind. 33 is remarkable for the smallness of the fronto-nasal angle. 34 has the frontal suture open, the base level, the orbit shallow, the face short, and the condyles non-elevated.

36 to 43, Scotch.—36, the skull of the historian GEORGE BUCHANAN. The base is injured, a portion of the occipital bone in front and behind the foramen magnum being destroyed. The injured portion is restored in the diagram in dotted lines, and the possibility of error is extremely slight. This skull was obtained when some alterations were being made in the Grey Friars Church, Edinburgh, and was deposited in the Natural-History Museum of the University. 37, a small skull with a large orbito-nasal

angle, is thus described in Sir GEORGE BALLINGALL's Catalogue:—"Skull found on the field of Kilsyth, blackened by lying in a moss, and showing several extensive wounds. Supposed to be the head of a Covenanter." 38, skull of a Fife man named EDMUNDS, executed for the murder of his wife, under circumstances, however, of provocation. 39, skull of HAGGART, a noted thief, who was executed. 40 is described in Sir GEORGE BALLINGALL's Catalogue:—"Skull showing extensive ulceration of the frontal bone, from a gunshot wound received at Waterloo, which the patient survived for many years. Some parts both of the frontal and the parietal bones are thickened to more than half an inch. Both tables of the skull had been involved in the ulcerative process, and an irregular opening is left of nearly 6 inches in circumference, the margins of which have been rounded off. The nasal portions of the frontal bone, as well as the nasal bones and nasal processes of the superior maxillary bones, present a curious appearance. The ethmoid, lachrymal, and other small bones are quite destroyed. As most of the teeth of the upper and the whole of those of the lower jaw are wanting, and the alveolar processes absorbed, it is seen that the subject of this extensive injury, a field-officer in the army, must have lived to an advanced age." 41 is a skull of a young man aged twenty. 42 was not a Museum specimen, and may not have been preserved; it appeared to be the skull of a middle-aged female, and was remarkable in this, that the sphenoidal wings did not reach up to the parietal bones. 43 is the skull of a female aged eighteen. Both 41 and 43 were prepared under the writer's observation.

44 to 48, Modern Greek.—44 and 45 are both marked "from Corfu." 45 is injured at the back of the foramen magnum; it is very unsymmetrical, and remarkable for the length of the foramino-optic line. The slope of the forehead may be fairly attributed to gravitation change. 46, 47, & 48 were preserved to show varieties of sabre cuts, and are described in Sir GEORGE BALLINGALL's Catalogue as "the skulls of patriot Greeks who fell in the actions between the Turkish and Greek forces, under General CHURCH, in 1827. They were brought from the plain between the Piræus and city of Athens, and presented by Dr. M^cWILLIAM, R.N." 46 is referred to in the text as an instance in which enlargement of the brain probably prevented the base from increasing in steepness, and thus elongated the base-line, throwing forwards the frontal bone. The position of parts which might have existed if this supposed action had not taken place is illustrated on the diagram by means of dotted lines; it is the skull of a young man. 47 appears to be the skull of an old person, and has a considerable articular surface at the back of the foramen magnum for the arch of the atlas.

49 to 57, Irish.—49 is the skull of a man named HURLEY, from the neighbourhood of Galway, executed for a brutal murder. 50 is the skull of a young man named LYDON, from the neighbourhood of Galway, executed for the murder of his wife who had been unfaithful. 51 is the skull of a tall old man from Ballinasloe. The remaining six Irish skulls are all from the Abbey of Claregalway, where, as in some other neighbouring places, a peculiar custom with respect to burials prevails. There is no sexton; but when a peasant dies, the friends dig a grave within or around the old ruins, displacing the

bones met with, which they think have been long enough in possession of the soil. The exhumed bones are collected in great heaps, some consisting entirely of skulls, others of limb-bones. There is no reason to believe that any of these skulls have much antiquity. Though 52 has been selected as exhibiting most distinctly some of the signs of gravitation, 54 is also apparently the skull of an old person; it is toothless, the sutures begun to be obliterated, and the condyloid surfaces small and flat. 53 has markedly prognathous incisor sockets. 55, 56, & 57 are obviously female skulls. 55 is toothless, and has the condyloid surfaces small and flat. It has no contact of the sphenoid with the parietal on the left side. 57, in the possession of Dr. BRERETON, of Oughterard, has no spheno-parietal suture on either side, but on each side a temporo-frontal suture about $\frac{3}{4}$ inch long.

58, 59, 60, Hindoo.—60, much more massive than the other two, is marked "Brought from the banks of the Hoogley by the Marquis of Hastings." All three skulls, together with others in the Anatomical Museum of the Edinburgh University, present unsymmetrical flattening of the occipital region, such as is described by Dr. BARNARD DAVIS as occurring in Siamese skulls*. Seeking an explanation of the same sort as Dr. DAVIS sought, the writer once asked the wife of an Indian officer if the natives had any peculiar way of cradling their children, and got for answer that they had not, but that they were very fond of laying them on their backs, and that when the Indian nurses placed the English children in that position their mothers turned them on the side, as they fancied lying on the back would make their heads the same round shape as those of the natives. Possibly when the occipital arch is flat, and therefore of a weak form, it more readily yields to accidental pressure than in other cases.

61 to 67, Negro.—61 is the skull of a Negro drummer of a French regiment; the whole skeleton is preserved. 62, the skull of an old person, has a superficial resemblance to an Australian skull, but the arch lacks the characteristic curve of the roof, and the supraorbital ridge is owing to enormous frontal sinuses. 63 has the dorsum sellæ reaching above the level of the floor of the anterior cranial fossa, a sign of a badly developed skull. 64 has the upper part of the occipital bone replaced by a large Wormian bone: this skull and 65 are well developed. 66 is catalogued as a Negress, and has some feminine characters. 67 is marked "Negress, aged fifteen, from West Africa."

68 to 71, Kafir.—68 has the cranial curvature remarkably deficient, and the posterior nares remarkably low. 71 is female, and, as compared with the males, has a level base and high cranial curvature.

72 & 73, Australian.—73 has the dorsum sellæ on a level with the floor of the anterior cranial fossa.

74 & 75, Kanaka.—74, male; 75, female.

76 & 77, Esquimaux.—76 is of unascertained sex; it presents various feminine characters. 77 is remarkable for its great length of base-line and deficient cranial curvature.

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* Thesaurus Craniorum, p. 176.

78 & 79, Peruvian.—78, skull of supposed Inca, from Temple of the Sun. The roof is partially covered with fine brown hair, 3 or 4 inches long. The only evidences of compression are a slight depression and want of symmetry above the occipital tuberosity and a slight flatness in the midfrontal region, which might be the accidental result of wearing some kind of head-dress. 79, also a supposed Inca, "dug up near Arica, in a burial-ground which had been disused since the conquest by Pizarro in 1532, by R. T. C. SCOTT, Surgeon, H.M.S. Talbot." The results of compression by bandages are very obvious.

80, skull of BURKE the murderer.—The peculiarities of this singular skull are probably chiefly referable to gravitation changes prematurely setting in, either from softness of the bones or carrying weights on the head.

81, "skull of PEPE, a Spaniard, captain of a piratical crew, who was captured at the Isle of Pines, by Captain GRAHAM, R.N."—From accounts of this man, which do not seem to have passed through many hands, he appears to have been a monster of brutality. The skull is also a singular one. The foramino-optic line is extremely long, and the cranial curvature deficient, while the posterior half of the skull appears to be too slender from above downwards for the fore part. The condyles are unusually prominent; and by this circumstance, together with the deficient cranial curvature, the head is thrown in the diagram into a position which it could scarcely have occupied in life.

82, Swiss.—This is probably a female skull.

83, Turk.

84, Tartar.

85, Chinese.—The skull of a young man.

86, Maori.—For the opportunity of examining this skull, in the possession of Dr. MURRAY, who brought it from New Zealand, I am indebted to the kindness of Dr. HENRY S. WILSON.

87, Hottentot Chief.—This is an uncommonly large skull with a very large orbito-nasal angle.

88, Carib.—This skull is remarkable for its flat retreating forehead, great orbital length, and long base-line. The orbito-nasal angle is rather small.

89, French acrocephalus.—The deformity is the result apparently of synostosis of the lower parts of the coronal suture, whereby the growth of the orbital length has been checked, while the loss of growth forwards has been supplemented by exaggerated height at the vertex. The dorsum sellæ reaches above the level of the anterior cranial fossa; and the levelness of base indicates that the brain has exercised pressure downwards as well as upwards.

90, a low and elongated skull illustrating aberrant form not caused by synostosis.—It is toothless, the frontal suture open, as also the others, save that the lower parts of the coronal and the fronto-sphenoidal sutures are beginning to fade.

91, skull with the base curiously driven in, as if by carrying great weights, such as a baker's tray.

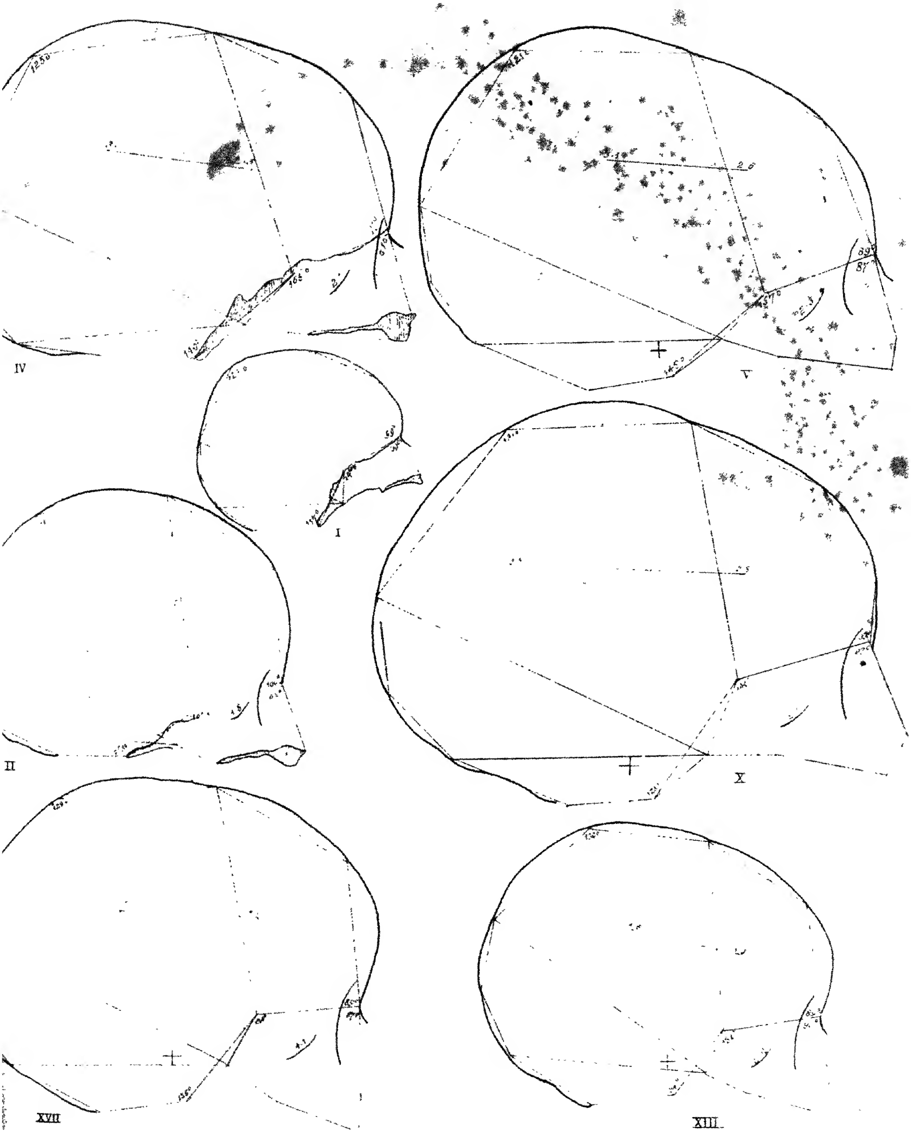
92, anomalously large skull.
93, skull of a hunchback; dissecting-room specimen.
94, skull of an idiot, who died in Morningside Asylum.
95, compressed Chinook skull from Columbia River.
96, flattened Carib or other skull, allowed to grow unlimitedly in breadth.

EXPLANATION OF PLATES XII.–XXI.

Fifty-seven of the ninety-six skulls whose measurements are given in the General Table are represented in the Plates, and are there distinguished by the same numbers as in the Table; the letter *f* being added in the case of female skulls. The first five, namely, four fœtal skulls and the skull of an infant at birth, are represented full size; all the rest are reduced to half-size. The cranial area in each case is divided by lines into frontal, parietal, upper occipital, and lower occipital parts; and the number of degrees in a few of the more important angles is expressed in figures to assist the eye in comparing different forms. Also the positions of the upper and lower borders of the orbit are in most cases expressed by a line uniting those two points; the position of the upper extremity of the temporo-malar suture is indicated by the lower end of a short curved line, above which the maximum breadth between the zygomatic arches is stated; and a straight line is drawn from the point where the greatest depth of the cranium occurs to the position of the greatest breadth in the course of the coronal suture, termed the frontal breadth, the amount of breadth at these two points being indicated at the extremities of the line.

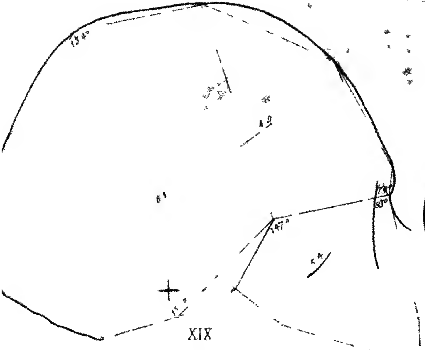
To give the diagrams framed from measurements greater completeness, the curve of the arch was obtained by means of a lead wire pressed against the skull, and then carefully laid on the diagram; and the curves so obtained are reproduced in the Plates. This method, together with other useful hints, was recommended to the writer by Dr. HECTOR, now of New Zealand, and it is right to state that the recommendation was given previous to the appearance of HUSCHKE's work, in which the same method is described. Used with care the lead wire gives details of the arch curves not easily otherwise obtained. Those instances, however, in which tracings of vertical sections have been secured show that the points settled by measurement are more accurate than the indications of the lead wire.

In Plate XXI. the figure of skull 91 has been obtained entirely by means of measured points and the lead wire, without the assistance of a vertical section. The views of the skulls of animals in this Plate are taken from tracings of vertical sections, and the names of the animals are mentioned in the Plate.

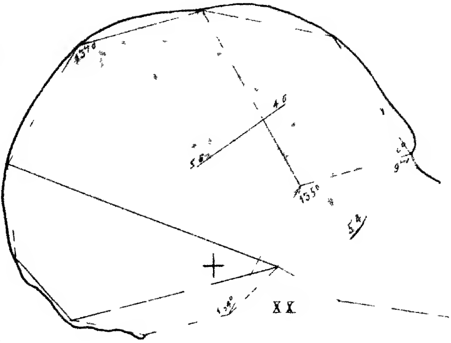

IV
V
I
II
X
XVII
XIII
125°
89°
87°
157°
145°

W West

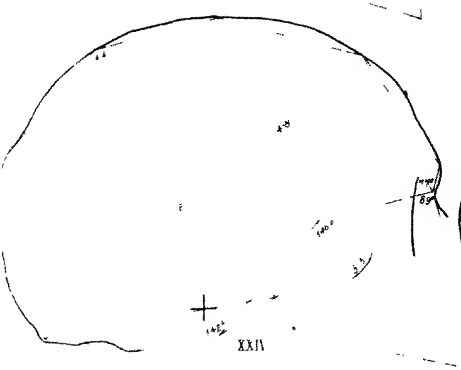
Skulls of Children before & after birth
(The first five are full size)


134°
49
61
147°
54
15
93°

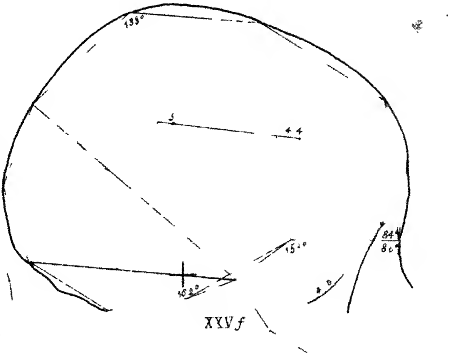
XIX


137°
46
58
135°
54
134°

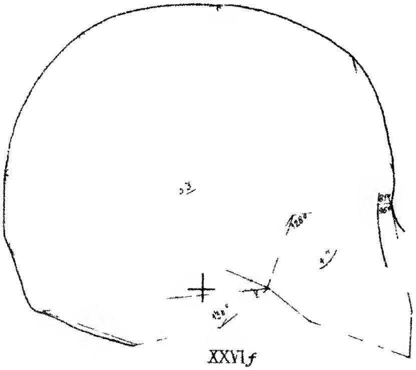
XX


44
48
140°
53
142°
89°

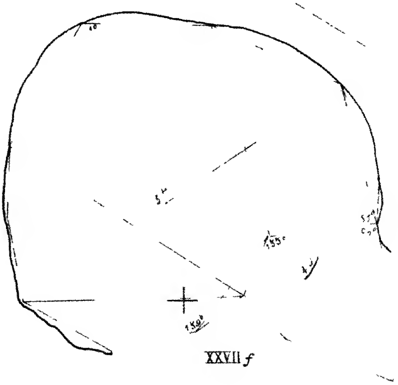
XXII


135°
5
44
84
151°
160°
40

XXV *f*

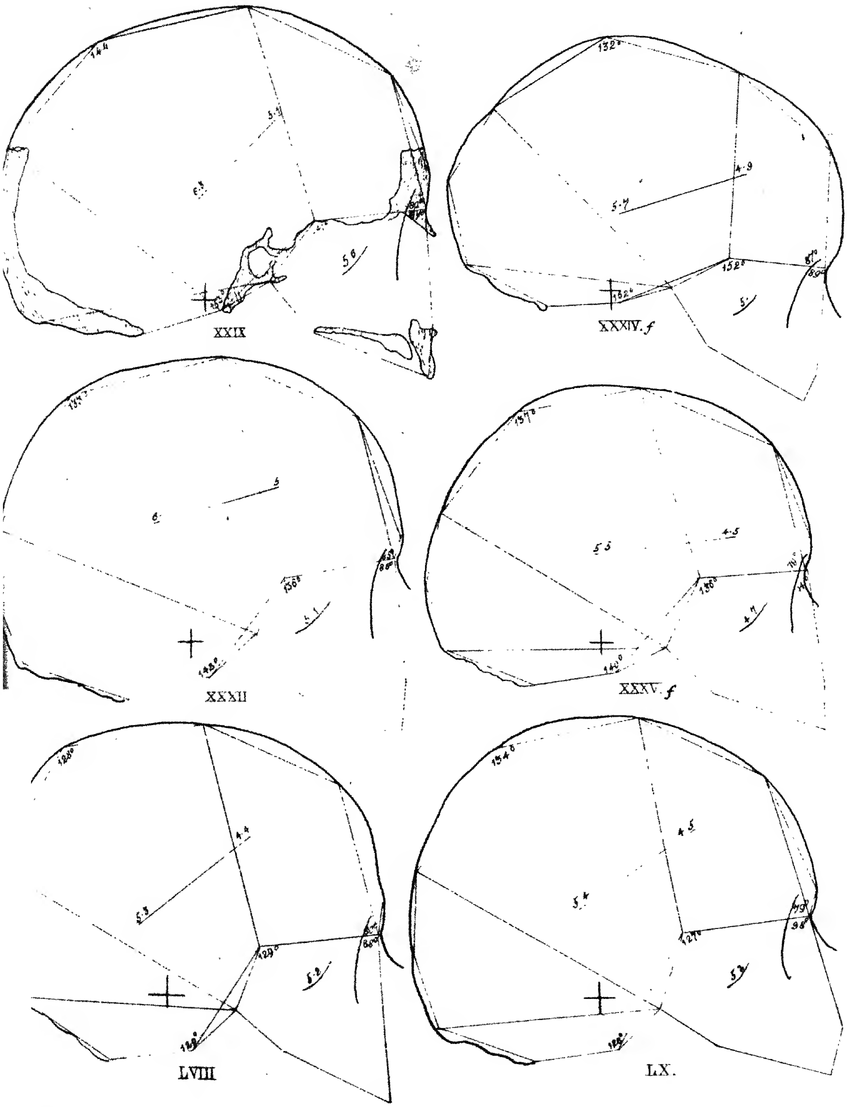

53
128°
47
128°
96°

XXVI *f*



XXVII *f*

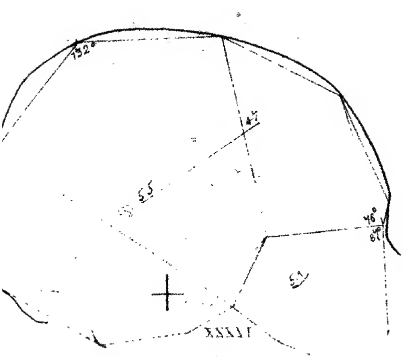
French Skulls.


144
5.4
6.3
5.6
XXIX
132°
4.9
5.7
152°
81°
89°
XXXIV. f
137°
5
6
88°
136°
5.1
148°
XXXII
137°
4.5
5.5
136°
76°
4.7
140°
XXXV. f
128°
4.4
5.3
129°
5.2
80°
128°
LVIII
134°
4.5
5.4
127°
79°
96°
5.3
128°
LX.

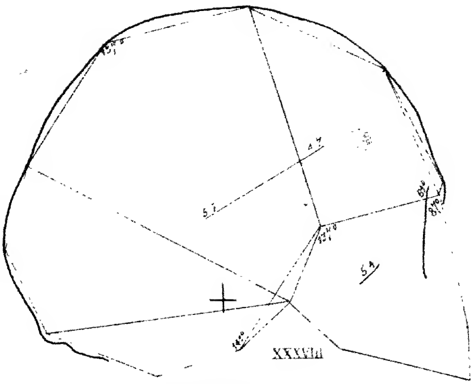
eley lith.

W. West

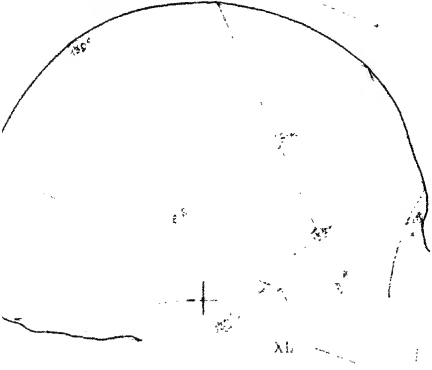
XXIX, XXXII, XXXIV f, XXXV f *German*
LVIII, LX, *Hindoo*


132°
4.7
5.5
46°
87°
5.1

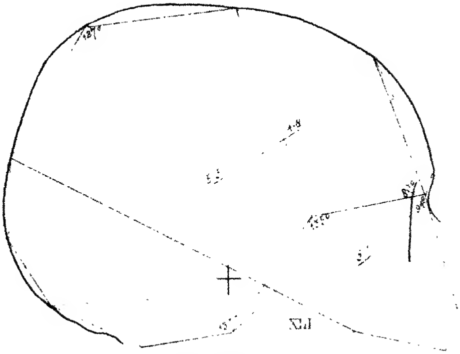
XXXVI


130°
4.7
5.7
84°
84°
127°
5.4
14°

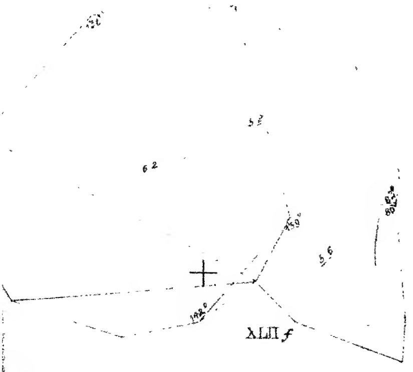
XXXVIII



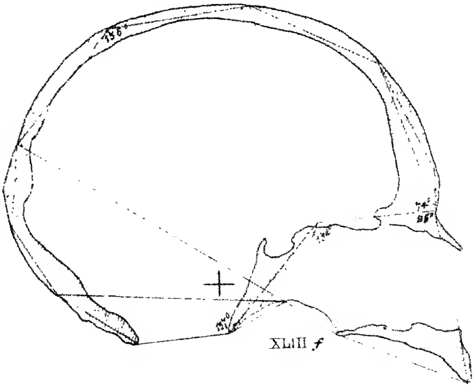
XL


127°
4.8
5.5
84°
135°
5.1

XLI

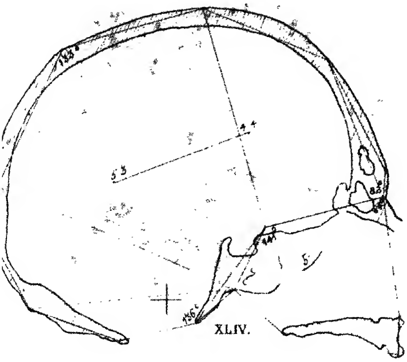

130°
5.3
6.2
130°
5.6
142°

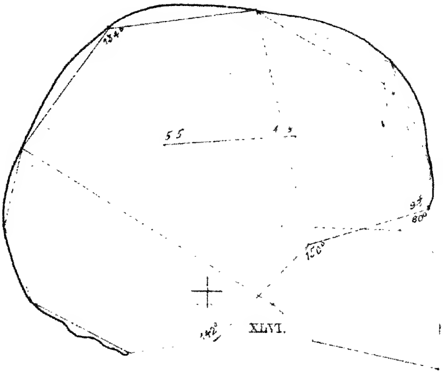
XLII *f*

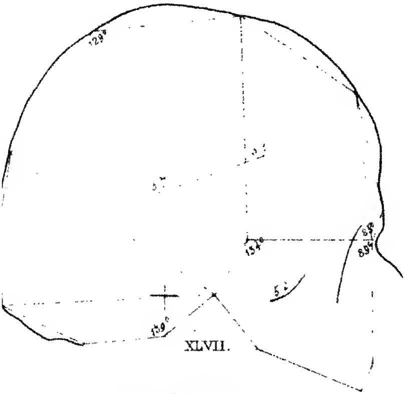

136°
74°
88°
134°

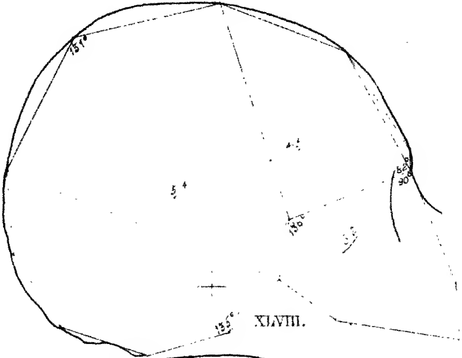
XLIII *f*

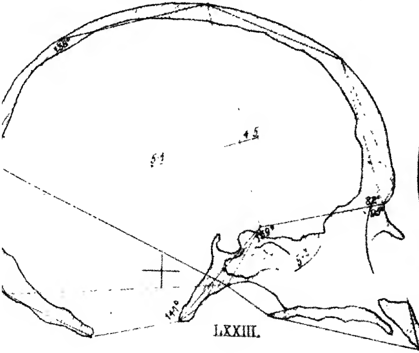
Scotch Skulls

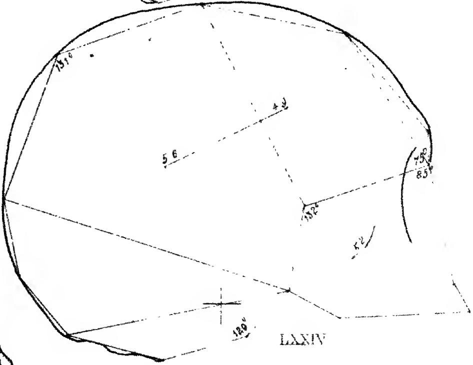

133°
4.4
5.3
88°
85°
144°
5.
136°
XLIV.


134°
5.5
4.3
94°
80°
100°
142°
XLVI.


129°
5.3
88°
89°
134°
5.2
139°
XLVII.


151°
4.5
5.4
82°
90°
130°
5.2
135°
XLVIII.

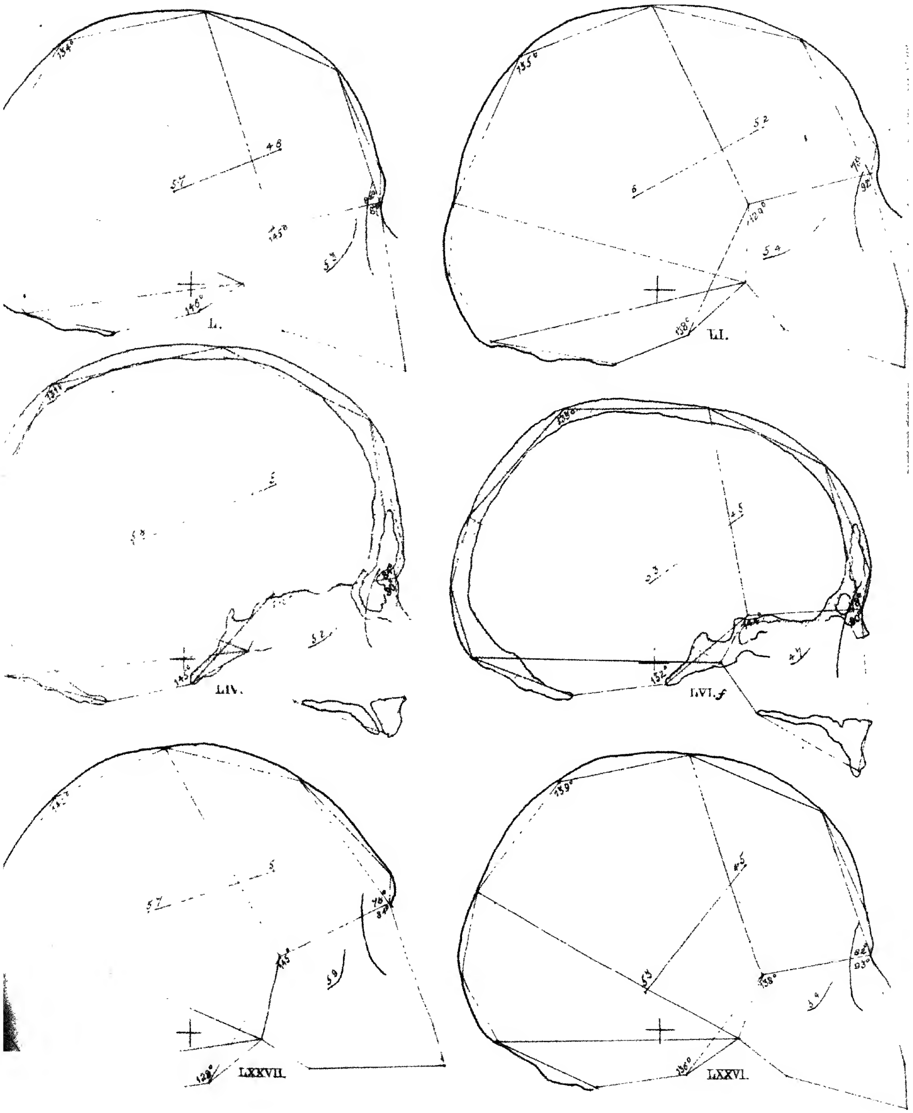

138°
4.5
5.1
82°
80°
130°
5.3
141°
LXXIII.


131°
4.9
5.6
75°
85°
132°
5.2
129°
LXXIV.

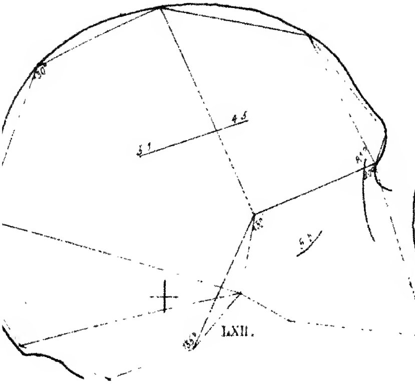
Wesley lith.

XLIV, XLVI, XLVII, XLVIII Greek
LXXIII, Australian, LXXIV Kanaka

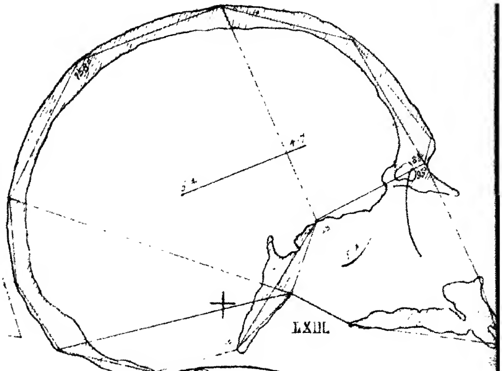
W. We.


134°
48
57
145°
146°
53
L.
135°
52
6
120°
54
138°
LI.
131°
5
58
52
145°
LIV.
138°
45
53
132°
47
LVI. f
5
57
145°
59
128°
LXXVII.
139°
45
53
138°
136°
LXXVI.

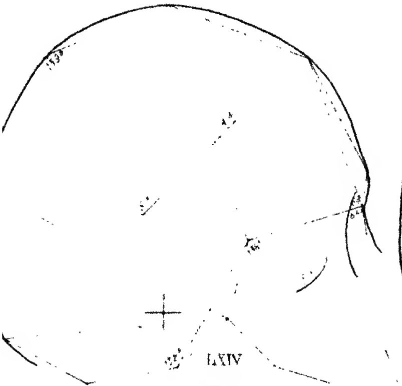
L. LI. LIV. LVI. f Irish.
LXXVI. LXXVII. Esquimaux.



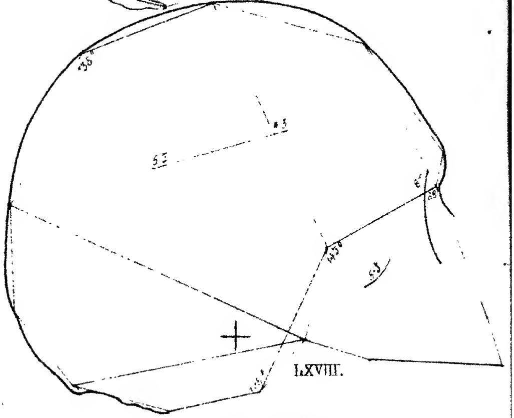
LXII.

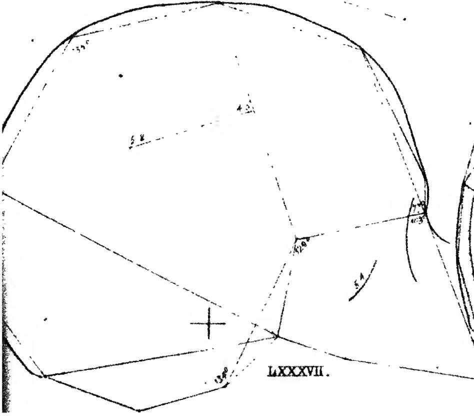


LXIII.

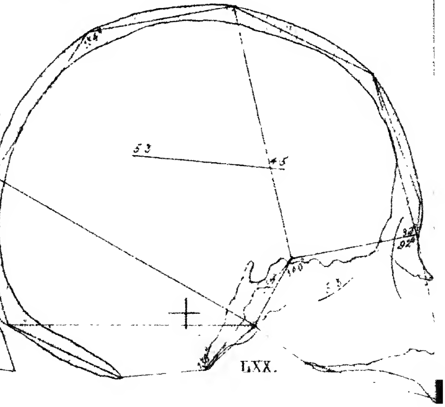


LXIV.

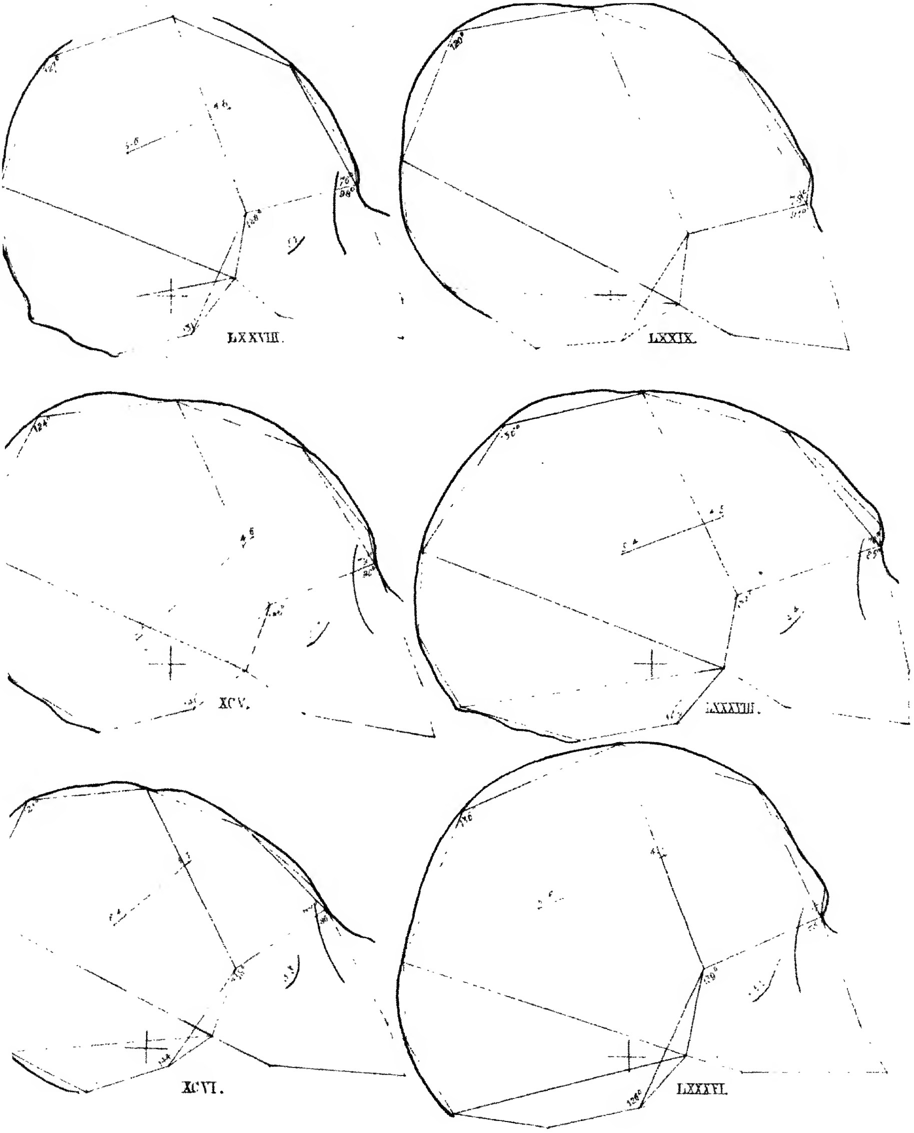

5.3
4.5
5.3
145.0
LXVIII.



LXXXVII.

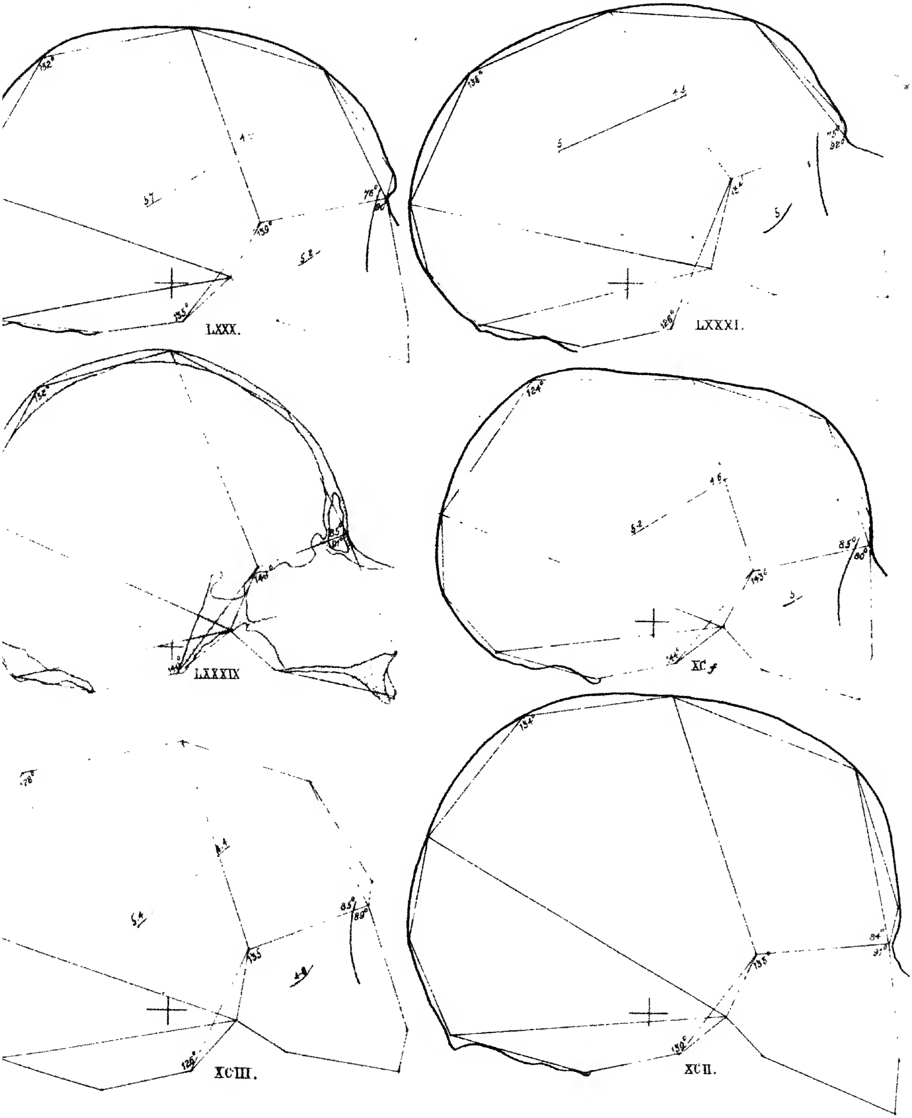


LXX.


LXXVIII.
LXXIX.
XCV.
LXXXVIII.
XCVI.
LXXXVI.
127°
120°
124°
130°
126°

LXXVIII. LXXIX, *Peruvian (Inca)*, LXXXVI *Moor* LXXXVIII *Arab*
XCV *Compressed Chinook* XCVI *Flattened American*

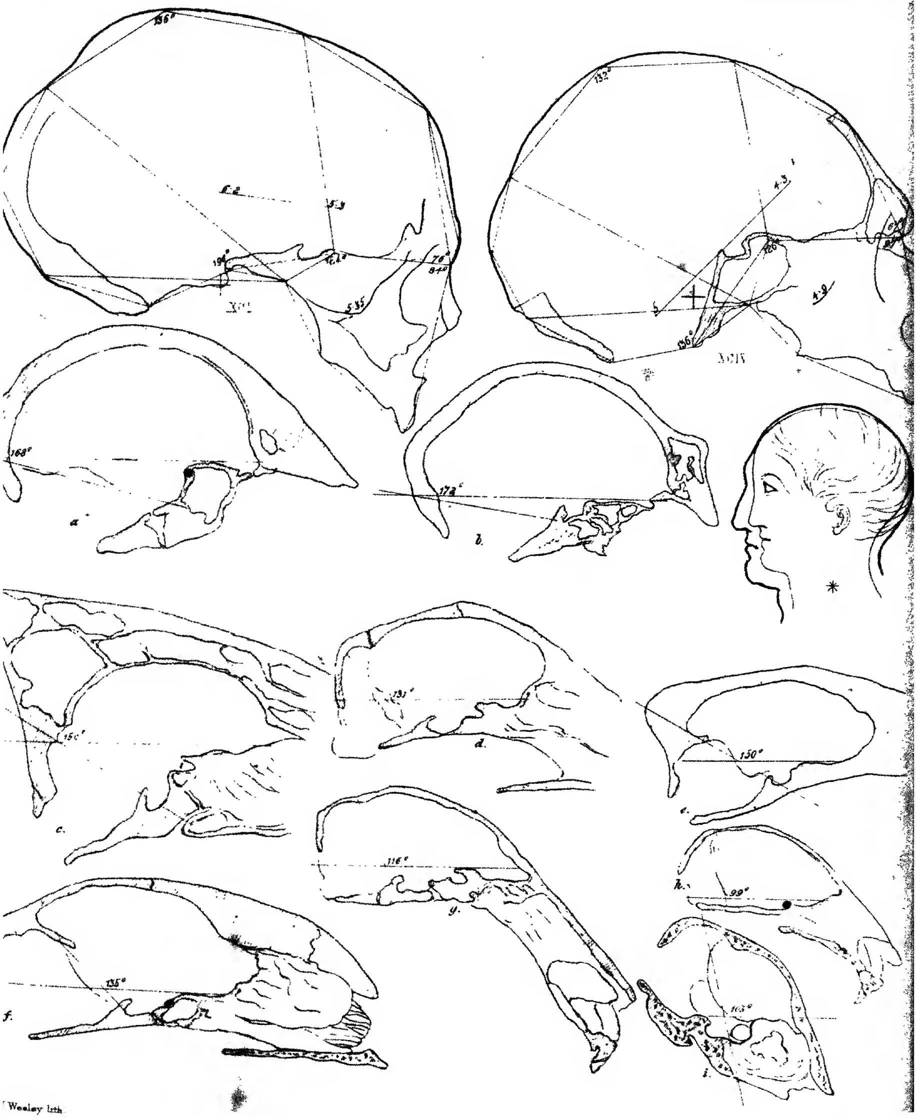
nd


132°
76°
90°
139°
135°
LXXX.
136°
92°
126°
LXXXI.
132°
143°
LXXXIX
124°
85°
90°
143°
144°
XC f
128°
85°
89°
135
126°
XCIII.
134°
84°
97°
135°
130°
XCII.

ıth

W. West imp.

LXXX. Burke. LXXXI Spanish pirate LXXXIX Synostotic French.
XC f Elongated form XCIII. Hunchback XCII Anomalously large


136°
6.2
5.9
194°
6.1°
76°
84°
5.35
XCI
132°
4.3
126°
4.9
156°
XCIV
168°
a
172°
b
*
150°
c
131°
d
150°
e
116°
g
h
99°
f
135°
103°
i

Wesley lith.

W West imp.

XCI. *Base driven in.* XCIV *Idiot* — *a Orang. b. Chimpanzee c. Pig d. Deer e Dog*
*f Cat g Rabbit h Squirrel i. Turkey * Male & Female profiles* —

GENERAL TABLE OF MEASUREMENTS.

[To face page 174.

List of Skulls measured (E stands for Anatomical Museum, University of Edinburgh; E N for Natural-History Museum of that University; T for Professor Thomson's Collection; and Q C G for Queen's College, Galway.)	The arch and its portions.										Chords of portions of the arch					Angles connected with the arch									Portions of the base			Angles connected with the base					Distance of various points in the arch from the base.						
	From foramen magnum to occipital tuberosity	From occipital tuberosity to upper angle of occipital bone	Occipital arc from foramen magnum upwards	Parietal arc along the sagittal suture	Frontal arc to fronto-nasal suture	Total arch from foramen magnum to fronto-nasal suture	Base-line from back of foramen magnum to fronto-nasal suture (chord of total arch)	Proportion of arch to the base reckoned as 1	Proportion of occipital arc to parietal arc reckoned as 100	Proportion of frontal arc to parietal arc reckoned as 100	Occipital chord	Parietal chord	Frontal chord	Proportion of occipital chord to parietal chord reckoned as 100	Proportion of frontal chord to parietal chord reckoned as 100	Postforaminal angle	Angle at occipital tuberosity	Midoccipital angle	Occipito-parietal angle	Midparietal angle	Fronto-parietal angle	Midfrontal angle	Orbito-frontal angle	Orbito-nasal angle	Length of foramen magnum	Foramino-optic line from foramen magnum to level of optic foramen	Orbital depth from foramen opticum to fronto-nasal suture	Foramino-basilar angle	Orbito-basilar angle	Angle of cranial curve.	Foramino-palatal angle.	Deep frontal angle	Occipital depth from spheno-occipital suture to occipital tuberosity.	Occipito-parietal depth from spheno-occipital suture to top of occipital	Parietal depth from spheno-occipital suture to midparietal point.	Frontal depth from optic foramen to fronto-parietal suture	Proportion of occipital depth to parietal depth reckoned as 100.	Proportion of occipito-parietal depth to parietal depth reckoned as 100.	Proportion of frontal depth to parietal depth reckoned as 100.
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
Fœtal Skulls																																							
1. Four months old, wet	·3	·5	·8	1·1	1·15	3·35	1·45	2·31	57	82	·75	1·2	1·	62	83	140°	150°	151	156	125	113	132*	99	90°	·55	·55	·55	117°	147°	150°	168°	75°	1	1·35	1·4	1·05	71	86	[illegible]
2. Five months old, dry, Q C G	·6	1·	1·6	2·3	2·1	6	2	3	70	91	1·4	2	1·8	70	90	158	145	150	137	136	157	128	100	93	·6	·75	·7	153	167	166	185	85	1·5	2·05	[illegible]	1·75	68	98	[illegible]
3. Five months old, wet	·6	1·1	1·7	2·4	2	6·1	2·4	2·54	78	83	1·5	2·15	1·6	70	71	137	135	119	148	132	150	121	97	93	·8	·9	·9	139	143	176	201	81	1·6	2·1	2·1	1·55	76	100	72
4. Seven months old, dry, Q C G	·8	1·6	2·4	2·8	2·5	7·7	2·6	2·96	85	89	1·9	2·5	2·2	76	88	170	141	137	155	125	116	132	97	81	·75	1·05	·9	140	165	155	184	83	1·9	2·7	2·8	2·1	67	96	76
5. Eight months old, dry, E	1	1·3	2·3	3·1	2·5	7·9	2·6	3·03	74	80	2·05	2·6	2·25	78	83	152	147	151	140	121	173	138	88	87	·7	1·05	·95	116	157	169	199	87	2·05	2·7	2·95	2·05	69	91	69
6. Eight months old, dry, Q.C.G	1	1·3	2·3	3·2	2·6	8·1	2·9	2·8	71	81	2·05	2·7	2·1	76	88	153	140	140	118	120	151	137	92	86	1	1·15	1·05	129	152	157	193	85	2·3	3·	3·25	2·25	70	92	69
Average								2·77	74	84				72	81	151	116	118	117	126	117	131	95	88				137	155	162	188	82	....	....	....	....	70	94	[illegible]
At Birth																																							
7 Recent section	1·1	1·8	2·5	3·5	2·4	8·8	2·9	3·03	82	68	2·6	2·9	2·2	90	75	112	118	145	134	125	182	131	88	102	·7	1·1	1·1	148	117	181	207	85	2·2	3·2	3·15	2	69	101	63
8 E (Monro's) 1	1·2	1·8	3	3·5	2·7	9·2	3·	3·06	85	77	2·5	2·95	2·3	81	77	140	155	135	113	127	160	120	104	91	·8	1·1	1·2	118	157	173	198	78	2·2	3·1	3·5	2·2	67	95	67
9 E (Monro's) 2	1·3	1·6	2·4	3·5	2·7	9·1	2·9	3·14	82	77	2·45	2·95	2·4	83	81	146	112	112	118	121	152	131	91	103	·8	1·1	1·15	113	112	181	206	86	2·15	3·1	3	2·2	74	93	66
10 E 498	1·1	1·55	2·65	3·15	2·7	8·8	2·8	3·14	76	78	2·35	3	2·4	76	80	136	160	159	135	131	172	133	93	97	·7	1·2	1·15	128	140	168	191	85	2·15	3	3·5	2·2	78	95	69
11 E 123 c	1·2	1·4	2·6	3·2	3	8·8	3	2·93	81	93	2·25	2·8	2·55	80	91	143	113	118	119	120	150	126	92	96	1	1·1	1·1	139	142	177	201	91	2·5	3	3	2·3	78	93	71
Average						8·94	2·92	3·06	81	78				81	80	135	119	141	141	127	135	128	93	98	·8	1·12	1·11	141	145	176	200	85					71	95	67
After Birth																																							
12. 1 year old, E. 123 b	1·6	2	3·6	4·1	3·4	11·4	3·6	3·16	81	77	2·95	3·75	3	78	80	150	134	112	152	124	150	131	88	93	1·05	1·35	1·1	134	139	175	198	88	3·2	3·9	4·	2·7	80	97	67
13. 2½ years, girl, T.	1·6	2·2	3·8	4·5	3·9	12·2	4·1	2·97	84	86	3·4	3·85	3·1	88	88	140	145	141	145	130	153	128	89	90	1	1·3	1·65	141	132	182	205	87	3·15	4·3	4·4	3	78	97	68
14. 3 years, E 133 a	2·1	2·1	4·2	4·5	4·1	12·8	4·4	2·98	93	91	3·5	3·95	3·6	88	91	131	145	140	151	138	151	127	93	88	1·4	1·5	1·7	159	147	183	212	86	3·8	4·4	4·5	3·25	84	97	72
15. 3 years, E 122 b	1·6	2·3	3·9	5·1	4·6	13·9	4·4	3·13	72	85	3·35	4·7	3·95	71	84	141	153	139	143	132	140	126	91	86	1·5	1·6	1·6	147	131	180	201	93	3·5	4·6	4·8	3·6	72	95	75
16. 5 years, E 133	2·	2·8	4·8	4·8	5	14·6	4·3	3·39	100	104	3·8	4·25	4·15	89	97	151	140	137	158	135	150	122	89	80	1·2	1·5	1·7	145	138	187	215	97	3·8	4·8	4·8	3·6	73	100	75
17. 6 years, E 115	1·8	2·6	4·1	5·1	5	14·8	4·7	3·15	81	92	3·7	4·9	4·25	75	86	148	151	131	148	129	117	125	89	87	1·1	1·75	1·85	136	134	182	212	93	3·8	4·9	5·5	3·8	73	95	73
18. 10 years, boy, E 6	2·4	2·4	4·8	5·4	5·2	15·4	5·1	3·02	88	98	3·95	4·7	4·5	81	97	111	137	117	155	133	118	128	85	89	1·6	1·65	1·8	148	127	181	210	96	4·4	5·15	5·4	3·85	81	95	71
Average								3·07	85	98				81	88	136	113	119	119	131	118	127	89	88				139	130	182	208	91					78	96	71
French.																																							
19. Male, E No 21	1·9	2·6	4·5	4·5	4·8	13·8	5·3	2·60	100	106	3·7	4	4·5	92	112	139	127	153	160	131	118	133	79	93	1·25	2·2	2	152	117	185	211	97	3·9	4·55	5·	3·7	78	91	74
20. ,, E. No 24	1·3	3·4	4·4	4·5	4·7	13·6	5·5	2·47	97	101	3·7	4·2	4·1	88	104	158	110	112	116	137	151	136	78	97	1·5	2·15	1·05	131	135	179	203	105	3·65	4·9	4·95	3·4	73	98	68
21 ,, E. No 25	2·2	2·2	4·4	5	4·9	14·3	5·1	2·80	88	98	3·6	4·5	4·5	80	100	117	138	111	161	133	111	135	77	93	1·1	2·3	1·85	117	131	183	213	101	4·1	4·75	5·1	3·7	80	93	73
22. ,, E. No 26	1·6	2·6	4·2	4·9	4·6	13·7	5·6	2·44	85	93	3·5	4·3	4·2	81	97	141	129	158	135	129	151	138	79	86	1·5	2·2	2·05	150	111	180	216	94	3·7	4·4	4·9	3·5	75	89	70
23. ,, E No 28	1·7	2·6	4·3	4·6	4·9	13·8	5	2·76	93	106	3·7	4·3	4·3	86	100	152	138	118	145	132	154	131	77	91	1·6	2	1·75	155	133	182	200	103	3·6	4·7	4·9	3·5	73	98	70
24. ,, E Old Soldier	1·8	3·4	5·2	4·4	5·2	14·8	5·7	2·59	118	118	3·9	4·	4·8	97	115	163	121	111	113	114	153	132	77	80	1·4	2·1	2·1	142	146	176	202	101	4	5	5	3·65	80	100	73
25. Female, E. No 27.	1·6	2·7	4·3	4·7	4·8	13·8	5·1	2·70	91	102	3·6	4·2	4·3	85	102	116	111	110	152	131	154	128	81	86	1·4	1·85	1·9	162	153	189	218	98	3·5	4·5	4·7	3·7	74	95	78
26. ,, E No 29	1·8	2·5	4·3	5·05	4·95	14·3	4·9	2·91	85	98	3·6	4·5	4·4	80	97	118	137	153	151	131	147	120	81	90	1·5	2·15	1·7	136	128	188	211	104	3·8	4·7	5·2	3·4	73	90	69
27. ,, E. No 23	1·9	2·5	4·4	4·7	4·7	13·8	5	2·76	93	100	3·8	4	4·3	95	107	140	133	161	150	121	157	128	81	95	1·1	2	1·9	139	133	186	213	99	3·7	4·7	5·2	3·5	71	90	67
Average						13·98	5·21	2·67	94	102				87	103	147	131	118	152	132	150	131	78	91	1·13	2·17	1·9	143	139	183	210	100					76	93	71
German																																							
28. Male, E. No. 37	1·8	2·8	4·6	4·5	5·7	14·8	5·1	2·90	102	126	3·7	4·1	4·9	90	120	158	122	158	146	137	116	128	76	94	1·1	2·1	1·9	147	138	189	212	110	3·8	4·8	5·1	3·9	74	94	76
29. ,, E No 37 a	1·8	3·2	5	5	5·3	15·3	5·3	2·88	100	106	3·8	4·7	4·7	80	100	154	125	145	147	111	111	138	80	87	1·45	2·2	1·9	154	111	192	219	102	3·85	4·85	5	3·85	77	97	77
30. ,, E. No. 41	2	2·7	4·7	4·7	5·1	14·5	5·3	2·73	100	108	3·7	4·3	4·1	86	102	151	130	141	156	133	153	127	78	89	1·35	2·3	1·95	141	138	183	201	102	4·1	4·8	5	3·5	82	96	70
31. ,, E No 42	1·7	2·8	4·5	5	5·2	14·7	5·3	2·77	90	104	3·7	4·5	4·5	82	100	143	138	117	152	133	149	128	81	92	1·5	2	2	149	113	186	211	95	3·75	4·7	5·1	3·85	73	92	75
32. ,, E No. 43	1·9	2·5	4·1	5·3	5·1	14·8	5·2	2·84	83	96	3·6	4·8	4·5	75	94	135	133	151	151	133	142	128	85	86	1·6	2·05	1·9	142	136	186	217	96	4	4·7	5·2	3·85	76	90	74
33. Female, E. No 36	1·9	2·4	4·3	5	4·9	14·2	5	2·84	86	98	3·6	4·5	4·1	80	97	151	138	149	152	130	140	134	79	79	1·4	2	1·8	136	130	180	218	98	3·95	4·8	5·2	3·6	75	92	69
34. ,, E No. 38	1·5	3·1	4·6	4·7	4·3	13·6	4·8	2·77	97	91	3·5	4·2	3·8	83	90	117	131	132	156	132	155	127	87	89	1·35	1·95	1·7	162	132	190	214	92	3·5	4·25	4·5	3·25	77	94	72
35. ,, E No. 40	1·6	2·6	4·2	4·4	4·8	13·4	5	2·68	95	100	3·1	4·1	4·2	83	102	151	130	118	153	137	147	128	79	94	1·35	2·15	1·8	140	130	181	208	102	3·65	4·45	4·6	3·35	76	92	69
Average						14·41	5·13	2·80	94	104				82	100	150	131	147	151	134	117	128	81	88	1·42	2·09	1·88	146	140	186	212	99·6					76	93	72
Scotch																																							
36. George Buchanan, E	2·?	2·1	4·1?	4·8	4·6	13·8?	5·1?	2·70?	87?	95	3·6?	4·2	4·2	85?	100			155	171	132	150	147	76	87	1·4	2·1?	2			180	207	97	3·85	4·75	5	3·4	79	95	68
37. From Kilsyth, E (Ballingall's) B. VII.*	1·7	2·6	4·3	4·2	4·6	13·1	5	2·62	102	100	3·7	3·3	4·2	91	107	149	137	150	119	112	112	137	77	99	1·45	2·1	2·	127	130	177	198	96	3·8	4·6	4·7	3·5	80	97	74
38. Edmunds, E.	1·9	3·1	5	4·9	4·85	14·75	5·8	2·60	102	99	4·15	4·5	4·4	92	97	116	140	141	157	137	113	135	81	87	1·5	2·35	2·1	141	137	184	213	93	4·05	4·85	5·3	3·7	76	93	69
39. Haggart, E	2·1	2·5	4·6	5	5·2	14·8	5·2	2·84	92	104	3·9	4·6	4·65	81	101	116	111	144	159	130	117	129	80	91	1·5	2·1	2	140	132	188	212	100	4·05	4·9	5·35	3·8	75	91	71
40. Old Officer, E (Ballingall's) B. VI	2·1	2·75	4·85	5·2	5·5	15·55	5	3·11	93	105	3·9	4·8	5	81	101	166	120	113	152	139	148	133	71	84	1·35	2·25	1·8	137	133	184	206	111	4·4	5·25	5·4	4·1	81	97	75
41. Male, æt 20, T.	1·7	2·75	4·45	5·15	5·2	14·8	5·1	2·74	86	101	3·9	4·5	4·5	86	100	116	119	113	151	127	152	130	80	91	1·5	2·3	2·1	133	135	178	202	98	3·7	4·9	5·1	3·8	68	90	70
42. Female, middle aged, E.	1·9	3·2	5·1	5·2	4·8	15·1	5·2	2·86	98	92	4	4·7	4·25	85	90	152	163	135	150	136	158	132	82	86	1·45	2·2	1·85	142	139	181	210	95	4·05	5·05	5·15	3·6	78	98	69
43. ,, æt 18, T.	1·7	2·8	4·5	5	[illegible]	14·6	5·4	2·70	90	102	[illegible]	4·5	4·6	86	102	142	156	139	148	136	145	137	71	95	1·4	2·1	2·05	137	131	183	209	100	3·75	5·1	5·4	3·7	69	94	68
Average						14·52	5·3	2·72	93	100				86	100	117	139	114	153	131	116	133	78	90	1·44	2·24	1·98	136	134	182	207	98·8					75	94	70
Greek.																																							
44. From Corfu, E No 45	1·8	2·2	4	5·1	4·8	13·9	5·	2·78	76	94	3·3	4·5	4·3	73	95	119	119	153	119	133	143	135	82	81	1·25	2	2	136	111	175	203	93	3·5	4·4	4·9	3·7	71	89	75
45. ,, E (Monro's)	1·9?	3	4·9?	4·8	5·2	14·9?	5·3?	2·81?	102?	108	3·9?	4·4	4·8	88	101	?		159	150	138	140	138	71	91		2·15	1·9		(129)	188?	205?	101	4	4·8	5·1	3·7	78	94	72
46. E (Ballingall's) B III	1·8	2·8	4·6	5	5	14·6	5·6	2·60	92	100	3·85	4·5	4·25	85	94	115	158	115	151	141	135	141	92	80	1·5	2·2	2·1	112	150	172	204	85	4·1	5	5·3	3·8	77	94	71
47. ,, B IV.	1·6	2·5	4·1	5·	4·8	13·9	5·2	2·67	82	96	3·4	4·5	4·2	75	93	117	121	161	157	129	145	129	85	80	1·4	2·1	2·1	133	131	185	210	91	3·7	4·2	4·8	3·6	77	87	75
48. E. ,, B. V	1·5	2·9	4·4	5·	4·6	14·	5·4	2·59	88	92	3·95	4·6	4·2	85	91	111	116	117	145	131	115	140	82	90	1·1	2·15	2·15	135	138	172	202	87	3·7	4·05	5·2	3·65	71	95	70
Average						14·2	5·3	2·89	88	98				81	95	116	136	153	150	133	115	133	82	87	1·38	2·18	2·05	138	140	177	204	93·2					74	91	72
Irish.																																							
49. Male, Hurley, Q.C.G.	1·75	2·95	4·7	5·4	5·6	15·7	5·5	2·85	87	103	3·6	4·9	4·8	77	98	156	137	118	150	146	119	125	87	81	1·6	2·05	2·1	146	118	178	215	94	4·2	5·3	5·5	4·2	76	98	76
50. ,, Lydon, Q.C.G	1·7	2·7	4·4	5·1	4·9	14·1	5·	2·88	86	96	3·6	4·5	4·2	80	93	153	111	118	148	131	119	130	84	87	1·5	1·9	1·9	116	115	181	212	95	3·8	4·9	5	3·8	76	98	72
51. ,, From Ballinasloe, old, Q C G	2·3	2·4	4·7	5·1	5·5	15·3	5·3	2·88	92	107	3·8	4·7	4·7	88	100	150	130	116	157	135	148	130	75	92	1·3	2·3	2·1	138	129	189	217	104	4·5	5·2	5·3	3·7	84	98	69
52. ,, From Claregalway, Q C G.	2	2·8	4·9	4·9	5	14·8	5·4	2·74	100	102	3·8	4·4	4·5	86	102	153	125	115	155	139	145	131	78	92	1·5	2·2	2·1	138	134	181	210	97	4·3	4·9	5	3·6	88	98	72
53. ,, ,, ,,	1·8	3·7	5·5	5	5·	15·5	5	3·10	110	100	4·1	4·5	4·4	91	97	151	137	141	153	131	146	135	84	85	1·3	2	2	140	148	189	212	92	3·9	5	5·1	3·6	76	98	74
54. ,, ,, ,,	1·8	2·3	4·2	5·9	5·6	15·7	5·2	3·91	71	91	3·5	5·2	4·8	67	92	155	135	150	151	131	141	126	84	90	1·1	2·1	2·1	135	158	187	210	98	4·1	5	5·4	4	75	92	74
55. Female ,, ,,	1·9	2·8	4·7	5	5·2	14·9	5·2	2·86	94	101	3·8	4·6	4·6	82	100	152	121	118	135	148	150	128	80	82	1·5	1·9	2	119	141	188	217	100	4·1	5	5·1	3·8	80	96	74
56. ,, ,, ,,	1·9	2·5	4·4	4·7	4·9	14	5·1	2·74	93	104	3·5	4·5	4·1	77	91	148	131	115	151	133	155	129	78	90	1·6	1·8	1·85	152	144	188	220	99	4·2	4·9	4·8	3·4	85	100	69
57. ,, ,, ,,	2·	2·9	4·9	5	4·9	14·8	5	2·96	98	98	3·8	4·3	4·3	88	100	154	139	145	155	126	158	118	76	93	1·4	2·1	1·8	111	135	180	219	105	4·1	5·	5·2	3·4	78	96	65
Average						15·01	5·16	2·89	91	100				80	97	152	131	112	152	131	118	129	80	88	1·45	2·03	2	144	139	185	211	98·2					79	97	71
Hindoo																																							
58 E No 48	1·9	2·3	4·2	4·8	5·3	14·3	5·1	2·80	67	110	3·7	4·2	4·8	88	114	139	146	150	153	128	118	128	80	88	1·45	2·1	1·95	129	129	180	212	102	3·7	4·7	5·3	3·9	69	88	73
59. E No 49	1·8	2·7	4·3	4·9	4·7	13·9	4·9	2·83	87	97	3·7	4·4	4·2	81	93	148	139	150	149	124	114	135	80	91	1·4	2	2	127	126	181	201	91	3·7	4·7	5·1	3·6	72	92	70
60 E N	1·8	2·7	4·5	5·1	4·8	14·4	5·3	2·71	88	94	3·75	4·6	4·4	81	95	151	128	150	151	134	113	134	70	98	1·4	2·2	2·2	128	127	181	205	94	3·75	4·7	5·2	3·65	73	90	70
Average						14·2	5·1	2·78	87	100				81	101	117	141	152	152	130	111	132	79	93	1·41	2·1	2·05	128	127	180	206	96·6					71	90	71
Negro																																							
61 Male, E No 3	2·	2·1	4·1	5·6	4·9	14·6	5·5	2·65	73	87	3·5	5·1	4·1	68	86	155	137	149	116	136	117	135	75	102	1·4	2·1	2·1	135	137	178	197	100	4·4	5·15	5·15	3·45	83	100	66
62. ,, E No 52	1·7	2·3	4·	5·	4·7	13·7	5·6	2·44	80	91	3·4	4·5	4·5	75	95	145	142	149	156	131	149	130	81	89	1·45	2·3	2·45	135	134	176	201	94	3·85	4·6	5	3·45	77	92	69
63 ,, E No 53	1·8	2·5	4·3	4·9	4·8	14	5·7	2·45	87	98	3·7	4·63	4·2	70	90	151	130	149	146	138	118	125	83	91	1·55	2·35	2	138	143	176	200	92	4·2	5·1	5·1	3·6	82	100	70
64. ,, E No 54	1·7	2·6	4·3	4·6	5·1	14·	5·1	2·59	93	110	3·6	4·2	4·5	85	107	117	138	139	150	143	142	130	81	85	1·5	2·25	2·05	132	138	174	205	94	3·9	4·7	5·1	3·8	76	92	74
65. ,, E No 55	1·1	3·8	5·2	4·8	5·2	15·2	5·4	2·81	108	108	4·3	4·3	4·6	100	107	117	141	147	113	137	151	127	82	96	1·5	2·3	2·05	140	134	187	212	97	3·7	5·2	5·4	3·8	68	96	70
66 Female, E. No 56	1·8	2·5	4·3	4·9	5·2	14·4	5·3	2·71	87	106	3·7	4·5	4·6	82	102	117	131	155	153	138	111	127	81	96	1·5	2·2	2·05	137	135	182	201	96	4	4·8	5·1	3·8	78	94	74
67 ,, æt 15, E (Monro's)	2·	2·4	4·4	4·9	4·9	14·2	5·	2·61	99	100	3·7	4·3	4·2	86	97	119	113	141	152	131	154	125	86	94	1·4	2·05	1·8	137	147	176	198	99	4·1	5	5·15	3·5	79	97	67
Average						14·30	5·41	2·64	88	100				82	97	118	138	147	151	135	147	129	82	93	1·17	2·26	2·17	136	138	177	202	95·5					77	95	70
Kafir.																																							
68. Male, E No 59	1·8	3·2	5	5·3	5·3	15·6	5·8	2·69	94	100	4·2	4·8	4·7	87	97	157	111	144	148	138	112	140	81	88	1·6	2·5	2·05	128	143	165	193	90	4·1	5·5	5·8	4·2	70	93	72
69 ,, E (Monro's)	1·9	2·6	4·5	4·9	5·1	14·5	5·7	2·51	91	104	3·7	4·5	4·6	82	102	137	111	116	153	142	145	129	88	87	1·7	2·3	2	147	149	168	197	91	4·3	5·1	5·2	4·1	82	98	78
70. ,, E (Dr Black's)	2·1	2·5	4·6	5·4	5·1	15·1	5·5	2·74	85	94	3·8	4·9	4·5	77	91	152	131	156	147	134	147	132	82	92	1·4	2·3	2·1	136	140	170	200	94	4·1	5	5·2	3·8	78	96	73
71 Female, E	1·9	2·5	4·4	4·5	4·8	13·5	5·3	2·54	97	102	3·8	4·2	4·2	90	100	128	115	149	151	136	117	137	81	88	1·5	1·95	2	150	141	186	217	92	3·9	4·8	5·1	3·6	76	94	70
Average						14·67	5·37	2·62	91	100				84	97	116	139	148	149	137	145	134	83	88	1·55	2·26	2·03	137	141	173	201	91·7					76	95	73
Australian																																							
72. E.N.	1·7	2·7	4·1	5·2	4·9	14·5	5·5	2·63	81	[illegible]	[illegible]	5	4·3	71	86	149	139	143	153	137	112	135	82	91	1·45	2·35	2·05	138	138	180	202	93	3·95	4·8	5·1	3·65	77	84	71
73. E. No. 66	1·7	2·55	4·25	5·1	4·9	14·25	5·4	2·63	83	[illegible]	3·5	4·7	4·5	74	95	113	111	139	153	138	111	131	82	95	1·5	2·05	2·15	111	138	182	200	95	3·8	4·7	5·	3·7	78	94	74
Average						14·37	5·45	2·63	83	96				74	90	116	110	141	153	137	113	133	82	94	1·17	2·2	2·1	139	138	181	201	94		....	...	.	......	94	72
Kanaka.																																							
74. Male, E.	1·6	2·7	4·3	5·1	5·2	14·6	5·5	2·65	84	102	3·85	4·6	4·7	83	102	151	116	119	116	131	116	136	75	85	1·5	2·3	2·15	129	132	177	195	98	3·75	5·05	5·5	3·75	68	91	68
75. Female, E.	1·9	2·7	4·6	5·4	4·7	14·7	5	2·94	85	87	3·9	4·85	4·2	80	86	111	112	150	142	131	116	136	79	97	1·25	2·1	2·05	128	129	189	210	91	3·85	5	5·3	3·5	72	94	66
Average						14·65	5·25	2·79	81	94				81	94	117	111	149	111	131	116	116	77	91	1·37	2·2	2·1	133	130	183	202	96					70	92	67
Esquimaux.																																							
76. E. (Monro's)	1·8	2·8	4·6	4·6	5	14·2	5·3	2·67	100	108	3·8	4·2	4·1	90	104	117	110	111	151	139	118	129	82	93	1·6	2·15	1·9	136	138	178	207	97	4	4·9	5·1	3·7	78	96	72
77. E. 68 a	1·3	3	4·3	4·4	4·8	13·5	5·9	2·28	97	109	3·6	4·1	4·5	87	110	133	139	140	142	145	113	129	78	81	1·7	2·5	2·	129	145	164	189	95	4	5	5·2	3·8	76	96	73
Average						13·85	5·6	2·47	98	108				88	107	150	139	140	153	142	145	131	80	88	1·65	2·32	1·95	132	141	171	198	96		. .	..	.	77	96	72
Peruvian																																							
78. E.N.	1·7	2·3	4	4·8	4·3	13·1	4·8	2·72	83	90	3·5	4·1	4	85	97	139	137	148	117	127	140	112	76	98	1·3	2·1	1·8	131	128	183	208	95	3·1	4·3	4·9	3·4	69	87	69
79 E.N.	1·8?	2·3	4·1?	4·7	5·2	14·?	5·1?	2·74?	87?	110	3·8?	4	4·7	95?	117	?	?	[illegible]	152	120	153	[illegible]	75	91		2·4?	1·85					100	3·95	5·2	5·9	3·8	66	88	66
Average									85	100				90	107			160	119	123	116	130	77	91		2·1	1·87					97·5	3·67	4·75	5·4	3·65	67	87	67
Miscellaneous																																							
80. Burke, E	1·9	2·4	4·3	5·2	4·8	14·3	5·1	2·64	82	92	3·3	4·55	4·35	72	95	166	120	155	140	142	154	130	76	90	1·5	2·1	2·15	135	139	176	201	98	4·	4·7	4·8	3·4	83	97	70
81. Spanish pirate, E. No 33	1·9	2·1	4·3	5	5	14·3	5·5	2·60	86	100	3·8	4·6	4·5	82	97	155	113	151	141	138	152	130	75	92	1·5	2·6	2·	126	131	172	195	103	4	5·1	5·2	3·5	76	98	67
82 Swiss, E. No 34	1·8	2·9	4·7	4·5	5	14·2	5·3	2·67	101	111	3·9	4·2	4·5	92	107	143	141	145	142	138	154	140	81	88	1·4	2·1	1·95	152	149	183	208	98	3·9	5	4·95	3·7	78	101	74
83. Turk, E. (Monro's)	1·6	2·8	4·1	4·7	4·7	14·8	5·1	2·70	93	100	3·7	4·3	4·2	86	97	156	112	141	148	132	151	132	79	98	1·3	2·2	2	133	139	174	203	97	3·7	4·9	5	3·4	74	98	68
84 Tartar, E. No 46	1·7	3·4	4·8	4·4	4·8	14	5·4	2·59	109	109	4·1	4·1	4·2	100	102	118	136	154	144	143	153	151	78	91	1·6	2·2	2·1	128	130	178	207	97	4·	5·1	5	3·4	80	102	68
85. Chinese, E	1·9	2·9	4·8	4·8	4·8	14·4	4·8	3·00	100	100	3·95	4·3	4·2	91	97	137	136	145	147	135	149	131	79	91	1·3	2·2	1·7	132	134	178	209	101	3·95	5·	5·1	3·5	77	88	66
86. Maori	1·7	2·8	4·5	5·5	5	15	5·7	2·64	81	90	3·7	5	4·1	71	88	160	131	147	154	136	140	133	87	85	1·67	2·55	2·05	126	139	167	190	87	4·	5	5·4	3·9	74	92	72
87. Hottentot Chief E	1·7	3·3	5·	5·4	5·5	15·9	5·8	2·74	92	101	4·2	4·9	4·8	83	98	148	117	137	151	135	148	131	77	103	1·5	2·5	2·2	134	129	185	205	99	4	5·4	5·7	3·9	70	94	68
88. Carib, E No 62	2·4	2·8	4·9	5	4·9	14·8	5·9	2·50	98	98	4	4·5	4·6	88	102	151	137	145	150	136	152	141	70	85	1·6	2·1	2·4	125	134	171	205	107	4·3	5·3	5·3	3·6	83	100	67
89 Synostotic French, E No 32	1·4	3	4·4	4·7	4·4	13·5	5·	2·78	93	93	3·7	4·2	4·2	88	100	153	131	141	154	132	138	145	85	91	1·5	2·1	1·6	140	148	172	209	90	3·7	4·75	5·1	3·8	72	93	74
90. Elongated skull, E N	1·8	2·6	4·4	5·5	4·5	14·4	5	2·88	80	81	3·8	4·7	4·	80	85	152	145	142	143	121	162	125	85	80	1·3	2	1·9	114	148	173	207	96	3·75	5	5·2	3·3	72	96	63
91. So-called Baker's skull, T	2	3·7	5·7	4·9	5·3	15·8	5·2	3·05	116	108	4·1	4·5	4·7	81	101	137	138	128	150	136	156	131	76	84	1·4	1·9	2	196	161	212	252	104	4·1	5·2	5·1	3·8	80	101	74
92 Large British T	2·4	3·4	5·8	5·2	6·5	17·5	5·3	3·30	111	125	4·7	4·6	5·4	102	117	151	126	148	152	134	152	120	81	91	1·6	2·1	2·1	139	133	186	215	104	4·5	5·6	5·9	4·45	76	95	75
93. Hunchback Dwarf, E	1·9	2·7	4·6	5·5	4·6	14·7	5·35	2·71	83	83	3·8	4·8	4·1	79	83	171	113	142	151	128	151	133	85	80	1·5	2·2	2·1	126	125	169	200	88	4·	5·05	5·35	3·65	74	94	66
94 Idiot, E.	1·7	2·7	4·1	4·8	4·5	13·7	5·3	2·58	91	103	3·6	4·3	4·1	83	95	142	141	140	154	132	147	141	67	97	1·4	2·2	2·3	136	126	190	215	98	3·85	4·55	4·7	3·05	81	96	64
95. Compressed Chinook, E N	1·8	2·4	4·2	4·9	4·5	13·6	5·3	2·57	85	91	3·8	4·2	4·2	90	100	137	116	160	141	124	153	142	70	90	1·55	2·15	1·8	141	148	173	201	94	3·9	5	5·4	3·65	72	92	67
96. Flattened American, E (Monro's)	1·5	2·4	3·9	4·1	3·8	11·8	4·95	2·38	95	92	3·5	3·5	3·6	100	102	136	140	160	151	121	140	130	77	86	1·35	1·85	1·85	144	158	166	203	84	3·3	4·4	4·9	3·3	67	89	67

List of Skulls measured (E stands for Anatomical Museum, University of Edinburgh; E N for Natural-History Museum of that University, T for Professor Thomson's Collection, and Q C G for Queen's College, Galway)	Area							Radial measurements from postauricular depression								
	Occipital area	Parietal area	Frontal area	Total area	Proportion of occipital area to total area reckoned as 100	Proportion of parietal area to total area reckoned as 100	Proportion of frontal area to total area reckoned as 100	Radius to occipital tuberosity	To midoccipital point	To occipito parietal suture	To midparietal point	To fronto-parietal suture	To midfrontal point	To glabellar prominence	To fronto nasal suture	To nasal spine
	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
Fœtal Skulls																
1. Four months old, wet	·400	·870	·350	1·620	24·69	53·7	21·6									
2. Five months old, dry, Q C G	·900	2·456	·990	4·346	20·70	56·52	22·77									
3. Five months old, wet	1·428	2·442	1·000	4·870	29·32	50·14	20·53									
4. Seven months old, dry, Q C G	1·797	3·910	1·485	7·192	24·98	54·37	20·64									
5. Eight months old, dry, E	1·733	4·375	1·462	7·570	22·89	57·79	19·31	1·6	2	2·3	2·8	2·5	2·4	2·2	1·9	2
6. Eight months old, dry, Q C G	2·202	4·696	1·740	8·638	25·49	54·36	20·14									
Average					24·67	54·48	20·83									
At Birth																
7. Recent Section	2·667	4·772	1·650	9·089	29·34	52·50	18·15									
8. E (Monro's) 1	2·479	5·112	2·012	9·603	25·81	53·23	20·95	1·7	2·4	2·8	3·1	2·9	2·9	2·7	2·2	2·3
9. E (Monro's) 2	2·526	5·210	1·920	9·656	26·15	53·96	19·88	1·8	2·4	2·7	3·1	2·9	2·7	2·5	2·2	2·3
10. E 498	2·110	5·147	1·920	9·167	23·01	56·04	20·94	1·5	2·1	2·5	2·9	2·9	2·8	2·5	2·2	2·3
11. E 123 c	2·617	4·817	2·040	9·474	27·63	50·84	21·53	1·9	2·3	2·6	3·1	3	2·8	2·5	2·1	2·1
Average					26·38	53·31	20·29				3·05					
After Birth																
12. 1 year old, E 123 b	4·399	7·880	2·850	15·129	29·07	52·09	18·83	2·5	3·1	3·4	3·9	3·5	3·2	3	2·5	2·5
13. 2½ years, girl, T.	5·330	8·828	3·995	18·153	29·36	48·61	22·00	2·5	3·2	3·6	4	3·7	3·6	3·3	2·8	2·8
14. 3 years, E 131 a	5·832	9·667	4·500	19·999	29·16	48·33	22·50	2·9	3·4	3·6	4	3·9	3·8	3·6	3·1	3·2
15. 3 years, E 122 b	5·311	11·925	4·868	22·107	24·17	53·94	21·88	2·6	3·4	3·8	4·3	4·3	4	3·7	3	2·9
16. 5 years, E 133	6·475	11·002	5·395	22·872	28·31	48·02	23·58	2·8	3·7	3·9	4·3	4·1	4·1	4	3·2	3·1
17. 6 years, E 115	6·520	13·388	5·950	25·858	25·21	51·77	23·01	2·8	3·7	4	4·6	4·5	4·3	3·7	3·2	3·2
18. 10 years, boy, E 6	7·712	13·712	5·850	27·304	28·35	50·22	21·42	3·3	3·9	4·2	4·9	4·7	4·8	3·9	3·6	3·6
Average					27·67	50·13	21·57				4·28					
French																
19. Male, E No 21	6·662	11·270	5·962	23·894	27·88	47·16	24·95	2·7	3·3	3·7	4·5	4·7	4·7	4·3	4·1	4·3
20. ,, E No 24	7·280	11·976	5·060	24·316	29·93	49·26	20·80	2·6	3·1	3·9	4·3	4·2	4·2	4	3·8	4·2
21. ,, E No 25	6·880	12·592	5·400	24·872	27·66	50·62	21·71	2·9	3·5	3·7	4·5	4·6	4·1	3·9	3·8	3·9
22. ,, E No 26	6·650	12·075	5·460	24·185	27·49	49·94	22·57	2·8	3·1	3·5	4·4	4·4	4·5	4	3·9	3·9
23. ,, E No. 28	6·732	11·902	5·052	23·686	28·42	50·25	21·32	2·8	3·5	3·9	4·5	4·3	4·3	3·7	3·5	3·6
24. ,, E Old Soldier	8·428	12·070	6·095	26·593	31·69	45·30	22·91	2·8	3·6	4·1	4·6	4·7	4·9	4·6	4·4	4·1
25. Female, E No 27	6·537	11·432	5·697	23·666	27·61	48·31	24·07	2·6	3·4	3·8	4·4	4·3	4·4	4·2	3·7	3·8
26. ,, E. No 29	6·602	12·980	5·390	24·972	26·43	52·38	21·18	2·8	3·4	3·8	4·6	4·6	4·5	3·8	3·5	3·6
27. ,, E. No 23	6·827	11·252	5·482	23·561	28·97	47·75	23·27	2·7	3·3	3·9	4·8	4·5	4·4	3·9	3·6	3·7
Average	6·953	11·949	5·510	24·416	28·45	48·96	22·57				4·51					
German																
28. Male, E No 37	6·677	12·697	6·370	25·744	25·94	49·32	24·74	2·6	3·2	3·9	4·6	4·8	4·8	4·3	4	4·1
29. ,, E No 37 a	7·590	13·665	6·109	27·364	27·73	49·94	22·32	2·8	3·6	4·2	4·7	5	5	4·4	4·1	4
30. ,, E No 41	7·342	12·312	5·720	25·374	28·94	48·53	22·53	3	3·6	3·9	4·5	4·1	4·6	4·3	3·9	3·9
31. ,, E No 42	6·944	12·085	6·300	25·329	27·41	47·71	24·87	2·8	3·5	4	4·8	4·8	4·8	4·1	3·9	4·1
32. ,, E No 43	7·027	13·775	6·075	26·877	26·18	51·31	22·60	2·9	3·5	3·8	4·7	4·8	4·7	4	3·7	3·7
33. Female, E No 38	6·407	12·565	5·500	24·472	26·18	50·93	22·88	2·6	3·4	3·7	4·7	4·7	4·7	4·2	4	3·6
34. ,, E. No 39	6·297	10·897	4·655	21·849	28·81	49·88	21·30	2·5	3·3	3·7	4·3	4·3	4·5	4·1	3·7	3·8
35. ,, E. No 40	6·203	11·260	5·100	22·563	27·48	49·91	22·60	2·8	3·2	3·5	4·3	4·3	4·1	4	3·7	3·9
Average	6·810	12·407	5·728	24·016	27·22	49·78	22·98				4·56					
Scotch																
36. George Buchanan, E	6·616	10·500	5·145	22·261	29·72	47·16	23·11	2·8	3·4	3·7	4·1	4·3	4·3	4	3·8	3·7
37. From Kilsyth, E (Ballingall's) B VII *	6·073	9·927	5·115	21·115	28·72	46·94	24·33	2·7	3·3	3·7	4·2	4·4	4·2	3·8	3·5	3·9
38. Edmunds, E.	8·705	13·600	5·830	28·135	30·94	48·33	20·72	2·95	3·65	3·95	4·6	4·75	4·65	4·15	4	4·1
39. Haggart, E	7·482	13·709	6·322	27·513	27·16	49·77	23·06	3	3·7	4	4·8	4·7	4·7	4·1	3·8	4
40. Old Officer, E (Ballingall's) B. VI.	7·018	14·797	6·900	28·715	27·57	51·53	20·89	3·1	3·9	4·2	4·8	5	4·7	4·1	3·8	3·5
41. Male, æt. 20, T.	7·140	13·777	6·135	27·052	26·39	50·93	22·67	2·9	3·7	4·1	4·8	4·5	4·5	3·9	3·7	4
42. Female, middle aged, E	8·394	13·712	5·348	27·454	32·59	47·93	19·47	3·4	4·1	4·3	4·6	4·1	4·3	3·7	3·5	3·5
43. ,, æt. 18, T	6·980	13·830	5·750	26·560	26·48	51·89	21·64	2·7	3·6	4	4·9	4·6	4·5	4	3·8	4·1
Average	7·413	12·981	5·713	26·108	28·60	49·31	21·98				4·6					
Greek																
44. From Corfu, E. No 46	5·360	12·617	5·590	23·567	22·74	53·54	23·71	2·4	3	3·5	4·4	4·7	4·7	4·2	4	4
45. ,, E (Monro's)	7·969	13·260	5·405	26·634	29·92	49·78	20·29	2·7	3·1	3·7	4·5	4·9	4·8	4·1	4·2	4·2
46. E. (Ballingall's) B III	7·600	13·318	6·300	27·218	27·92	48·93	23·14	3	3·7	4	4·6	4·6	4·8	4·3	3·9	3·9
47. ,, B. IV.	6·485	11·812	5·880	24·177	26·82	48·85	24·32	2·8	3·1	3·5	4·5	4·7	4·6	3·9	3·6	3·7
48. E ,, B. V	7·262	12·900	5·565	25·727	28·26	50·10	21·63	2·6	3·4	4	4·6	4·6	4·4	4	3·9	4
Average	6·935	12·781	5·750	25·464	27·13	50·25	22·61				4·56					
Irish																
49. Male, Hurley, Q C G	7·565	15·116	7·320	30·001	25·21	50·39	24·39	2·6	3·3	3·7	4·5	4·9	5·2	4·7	4·4	4·5
50. ,, Lydon, Q C G	6·492	12·555	5·355	24·402	26·60	51·45	21·94	2·9	3·7	4·1	4·5	4·3	4·3	3·7	3·5	3·5
51. ,, From Ballinasloe, old, Q.C.G.	8·005	13·365	6·227	27·597	29·00	48·43	22·56	3	3·6	3·8	4·4	4·5	4·7	4·4	4·1	4·2
52. ,, From Claregalway, Q.C.G	8·290	12·137	5·850	26·288	31·60	46·14	22·25	3	3·5	3·8	4·4	4·7	4·7	4·3	4	4·1
53. ,, ,, ,,	6·166	15·387	6·960	28·813	22·16	53·38	24·15	2·8	3·7	4·1	4·7	4·6	4·5	4·4	3·7	3·7
54. ,, ,, ,,	8·942	12·497	5·830	27·269	32·82	45·80	21·37	3	3·7	4·1	5	5	4·9	4·1	3·8	3·7
55. Female ,, ,,	7·175	13·064	6·210	26·449	27·12	49·40	23·47	3	3·7	4·2	4·7	4·7	4·7	4·2	3·9	3·7
56. ,, ,, ,,	6·674	11·805	5·227	23·706	28·15	49·80	22·04	3·1	3·7	3·9	4·3	4·2	4·3	4·0	3·6	3·7
57. ,, ,, ,,	7·465	12·555	5·160	25·180	29·64	49·86	20·49	3·1	3·9	4·1	4·6	4·2	4·2	3·7	3·5	3·7
Average	7·133	13·464	6·015	26·633	28·06	49·40	22·51				4·58					
Hindoo																
58. F. No 48	6·705	12·585	6·600	25·890	25·89	48·61	25·49	2·6	3·2	3·6	4·5	4·6	4·6	4	3·8	3·8
59. E No. 49	6·927	11·767	5·355	24·049	28·80	48·93	22·26	2·6	3·2	3·8	4·6	4·6	4·4	3·7	3·5	3·7
60. E.N	7·077	13·035	6·180	26·292	26·91	49·58	23·49	2·8	3·3	3·7	4·5	4·7	4·6	4·1	3·9	4·2
Average	6·903	12·462	6·038	25·410	27·20	49·06	23·72				4·53					
Negro																
61. Male, E No. 3	6·557	14·121	5·610	26·288	24·94	53·71	21·34	3·1	3·6	3·9	4·4	4·5	4·6	4·4	4·1	4·5
62. ,, E No 52	6·157	12·362	5·912	24·731	26·18	49·91	23·90	2·6	3·1	3·4	4·3	4·4	4·7	4·6	4·1	4·3
63. ,, E No 53	7·399	13·305	5·460	26·161	28·27	50·86	20·86	3	3·6	4	4·4	4·5	4·5	4·4	4·1	4·4
64. ,, E No 54	6·964	12·180	6·300	25·444	27·36	47·87	24·73	2·8	3·4	3·8	4·4	4·7	4·6	4·4	3·8	3·8
65. ,, E No 55	9·115	12·065	6·555	27·735	32·86	43·50	23·63	2·6	3·8	4·4	5·1	5·1	5	4·3	4	4·1
66. Female, E No 56	7·199	13·802	6·555	26·556	27·18	51·13	21·68	2·8	3·3	3·7	4·4	4·7	4·8	4·2	3·9	4·2
67. ,, æt 15, E (Monro's)	6·737	12·282	5·460	24·479	27·52	50·17	22·30	3·0	3·7	4·1	4·6	4·4	4·4	4·1	3·5	3·7
Average	7·204	12·731	5·978	25·913	27·75	49·16	23·06				4·51					
Kafir																
68. Male, E No 59	8·603	15·945	6·462	31·010	27·74	51·42	20·83	2·9	3·8	4·4	5·1	5·2	4·9	4·5	4·1	4·3
69. ,, E (Monro's)	8·006	13·464	6·555	28·025	28·56	48·04	23·39	3·1	3·7	4	4·5	4·7	4·7	4·2	3·9	4·2
70. ,, E (Dr Black's)	7·153	14·420	6·187	27·760	25·76	51·97	22·28	3·1	3·5	3·9	4·6	4·7	4·8	4·3	4·1	4·1
71. Female, E	7·000	11·962	5·460	24·422	28·66	48·98	22·35	2·8	3·4	3·8	4·4	4·4	4·3	4	3·8	4
Average	7·690	13·947	6·166	27·804	27·68	50·09	22·21				4·65					
Australian																
72. E N	6·875	13·005	6·300	26·180	26·26	49·67	24·06	2·9	3·5	3·8	4·4	4·6	4·5	4·2	3·9	4·2
73. E No 66	7·347	13·522	5·797	26·666	27·55	50·71	21·73	2·8	3·1	3·6	4·2	4·4	4·5	4·1	3·9	4·2
Average	7·111	13·263	6·048	26·423	26·90	50·19	22·89				4·3					
Kanaka																
74. Male, E	6·654	14·242	6·110	27·006	24·63	52·74	22·62	2·6	3·4	4·1	4·9	4·9	4·8	4·4	4·1	3·9
75. Female, E	6·832	13·660	5·355	25·847	26·43	52·85	20·71	2·6	3·4	3·9	4·7	4·7	4·55	4·1	3·8	4
Average	6·743	13·951	5·732	26·426	25·53	52·79	21·66				4·8					
Esquimaux																
76. E (Monro's)	7·634	12·354	5·720	25·708	29·69	48·06	22·24	2·8	3·5	3·8	4·4	4·5	4·6	4·1	3·9	4·2
77. E 68 a	6·905	12·590	5·737	25·232	27·36	49·91	22·73	2·8	3·5	3·9	4·4	4·6	4·5	4·3	4	4·1
Average	7·221	12·472	5·728	25·470	28·52	48·98	22·48				4·4					
Peruvian																
78. E N	6·065	11·145	4·500	21·710	27·93	51·34	20·72	2·3	2·9	3·5	4·5	4·6	4·2	3·8	3·6	3·8
79. E N	6·260	13·077	5·942	25·279	24·76	51·73	23·50	2·8	3·6	4·2	5·1	4·7	4·3	3·9	3·6	3·7
Average	6·162	12·111	5·221	23·494	26·34	51·53	22·21				4·8					
Miscellaneous																
80. Burke, E	6·537	12·148	5·847	24·532	26·65	49·50	23·84	3·1	3·5	3·8	4·3	4·2	4·4	4·1	3·9	4
81. Spanish pirate, E No 33	7·042	13·785	5·625	26·442	26·59	52·13	21·27	2·6	3·3	3·8	4·4	4·6	4·9	4·5	4·3	4·5
82. Swiss, E No 34	7·335	12·006	5·850	25·191	29·11	47·66	23·22	3	3·7	4·2	4·4	4·2	4·2	3·9	3·7	3·9
83. Turk, E (Monro's)	6·695	11·805	5·250	23·750	27·13	51·65	21·21	2·5	3·3	3·7	4·3	4·2	4·1	4	3·8	4
84. Tartar, E No 46	8·585	10·876	5·460	24·921	34·41	43·65	21·90	2·8	3·6	4·2	4·5	4·5	4·5	4·4	3·8	3·9
85. Chinese, E	7·574	12·394	4·725	24·693	30·67	50·19	19·13	2·8	3·5	3·9	4·4	4·3	4·2	3·8	3·6	3·8
86. Maori	6·529	15·190	6·160	27·879	23·41	54·49	22·09	3	3·7	4·1	4·8	5	4·8	4·2	3·9	3·8
87. Hottentot Chief, E	8·502	15·520	6·060	30·082	27·41	50·09	22·46	3	3·9	4·3	4·9	4·9	4·9	4·3	4·1	4·5
88. Carib, E No 62	8·830	13·287	5·980	28·097	31·42	47·29	21·28	3·3	3·9	4·2	4·6	4·4	4·4	4·4	4·2	4·2
89. Synostotic French, E No 32	7·243	12·237	4·620	24·100	30·05	50·77	19·17	2·7	3·5	4·3	4·9	4·9	4·3	3·7	3·5	3·8
90. Elongated skull, E N	6·505	13·190	5·200	24·895	26·12	52·99	20·88	2·7	3·5	3·9	4·5	4	4·4	4	3·8	3·6
91. So-called Baker's skull, T	8·345	12·920	6·110	27·375	30·48	47·30	22·21	3	4·2	4·5	4·8	4·6	4·6	4·2	4	3·9
92. Large British, T	10·260	15·690	8·505	34·455	29·77	45·54	24·68	3·4	4·2	4·7	5·3	5·2	5·2	4·4	4·1	4·1
93. Hunchback Dwarf, E	7·377	13·886	5·535	26·798	27·52	51·82	20·65	3	3·7	4	4·6	4·4	4·4	4	3·7	4
94. Idiot, E	7·232	10·701	4·715	22·651	31·92	47·26	20·81	3·9	4·5	3·7	4·2	4·1	3·9	4	3·7	4
95. Compressed Chinook, E N	6·720	12·880	4·830	24·430	27·50	52·72	19·77	2·7	3·3	3·9	4·6	4·5	4·2	4	3·8	4·2
96. Flattened American, E (Monro's)	5·537	9·784	3·690	19·011	29·12	51·47	19·40	2·3	3	3·6	4·4	4·2	3·9	3·8	3·7	4

List of Skulls measured	Radial measurements from postauricular depression: Proportion to midparietal radius reckoned as 100																						
	Radius to tuberosity	Midoccipital radius	Occipito-parietal radius	Fronto-parietal radius	Midfrontal radius	Glabellar radius	Fronto-nasal radius	Radius to nasal spine	Naso-basilar angle	Naso-basilar line from spheno-occipital suture to nasal spine	Facial length from fronto-nasal suture to nasal spine	Index angle of prognathism	Facial angle, modification of Camper's	Vertical height of postauricular depression above foramen magnum	Distance forwards and backwards of postauricular depression from front of foramen magnum (distance backwards marked −)	Length of skull from midoccipital point to fronto-nasal suture	Height of skull from front of foramen magnum parallel to line of frontal depth	Proportion of height to length reckoned as 100	Proportion of height to frontal depth reckoned as 100	Greatest breadth	Proportion of breadth to length reckoned as 100	Angle between the plane of the foramen magnum and a plane touching the condyles and the part of the skull behind the foramen	Angle between the line of orbital depth and the plane touching the condyles and the skull behind the foramen magnum
	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
Fœtal Skulls																							
1. Four months old, wet									134	·65	·35	90				1·6	1·5	93	142				
2. Five months old, dry, Q C G									163	1·1	·6	79				2·7	2·1	77	121	2·2	81		
3. Five months old, wet									131	1·2	·65	88				2·95	2·3	77	118				
4. Seven months old, dry, Q C G									153	1·4	·7	87				3·6	2·8	77	133	3	83		
5. Eight months old, dry, E	57	71	82	89	85	78	67	71	141	1·5	·7	76	77	2	−1	3·7	3	81	141	3·1	83	8	19
6. Eight months old, dry, Q C G									113	1·5	·8	88				3·9	3·3	84	140				
Average									117			86						81	138				
At Birth																							
7. Recent Section									149	1·7	·8	81				4·1	3·2	78	160				
8. E (Monro's) 1	54	77	90	93	93	87	70	71	150	1·8	·85	88	73	15	−·05	4·2	3·3	78	150	3·6	85	9	16
9. E (Monro's) 2	58	77	87	94	87	80	70	71	147	1·7	·9	85	72	2	−·1	4·2	3·3	78	150	3·5	83	8	7
10. E 498	51	72	86	100	96	86	77	79	141	1·65	·85	88	73	3	−·2	4	3·3	82	150	3·3	82	2	14
11. E 123 c	61	74	84	96	90	80	67	67	133	1·7	·8	85	77	1	−·15	4·1	3·3	80	143	3·5	85	6	9
Average	56	75	86	95	91	83	70	73	145	1·67	·81	84	73	18	−·12	4·12	3·28	79	150	3·47	83	6	11·5
After Birth																							
12. 1 year old, E 123 b	61	79	87	89	82	76	64	64	129	1·9	1·05	85	75	3	−·05	5·3	4	75	148	4·3	81	8	13
13. 2½ years, girl, T.	62	80	90	92	90	82	70	70	150	2	1·1	91	74	55	−·2	5·7	4·3	75	143	4·6	80	8	6
14. 3 years, E 131 a	72	85	90	97	95	90	77	80	144	2·3	1·4	84	71	5	−·1	6·1	4·5	73	138	4·9	80	12	9
15. 3 years, E 122 b	60	79	88	100	93	86	69	67	140	2	1·25	90	81	6	−·15	6·1	4·9	80	136	5·2	85	4	2
16. 5 years, E 133	65	86	90	95	95	93	74	72	131	2·2	1·4	83	78	·5	−·25	6·8	4·8	72	134	5·5	83	5	2
17. 6 years, E 115	60	80	87	97	93	80	69	69	126	2·3	1·5	86	77	7	−·1	6·7	5·2	77	156	5·7	85	9	7
18. 10 years, boy, E 6	67	79	85	95	93	79	73	73	131	2·5	1·65	85	76	65	−·25	7·1	5·5	77	142	5·7	80	9	8
Average	64	81	88	95	91	83	70	70	132			86	76	54	−·15			75	139		82	7	5·5
French																							
19. Male, E No 21	60	73	82	104	104	95	91	95	117	3·1	2·05	75	71	45	−·1	6·8	5	74	135	6·1	89	18	13
20. ,, E No 24	60	72	90	97	97	93	88	97	130	3·1	2·2	78	66	85	−·05	7	5·1	72	150	5·6	80	14	15
21. ,, E No 25	64	77	82	102	91	86	84	86	123	2·75	1·95	84	71	8	−·2	7	5·3	75	113	5·8	82	17	14
22. ,, E No 26	63	70	79	100	102	90	88	88	113	2·65	2·05	88	73	7	·1	6·75	5·1	75	145	5·5	81	15	9
23. ,, E No. 28	62	77	86	95	95	82	77	80	111	2·75	2	80	70	7	−·25	6·65	5·2	78	148	5·4	89	13	11
24. ,, E Old Soldier	60	78	87	102	106	100	95	95	141	3·2	2·1	75	74	15	−·25	7·5	5·1	68	139	6	80	11	15
25. Female, E No 27	59	77	86	97	100	95	82	86	145	2·9	1·8	82	75	4	0	6·8	4·8	70	130	5·5	80	9	0
26. ,, E. No 29	60	73	82	100	97	82	76	78	120	2·55	1·8	81	74	7	−·15	6·55	5·3	80	147	5·3	80	13	5
27. ,, E. No 23	56	68	81	93	91	81	75	77	130	2·75	1·95	79	71	6	−·1	6·3	5·3	84	151	5·7	90	12	6
Average	60	75	84	98	98	89	84	85	131	2·80	1·98	80	71	62	−·11	6·81	5·13	75	143	5·85	82	13	9
German																							
28. Male, E No 37	57	69	84	101	101	97	86	87	132	2·05	2	83	72	4	−·4	6·9	5·1	73	137	6·4	92	4	5
29. ,, E No 37 a	59	70	89	106	106	93	87	85	131	2·6	2·05	84	80	25	−·2	7·3	5·1	70	132	6·3	86	16	6
30. ,, E No 41	65	80	86	97	102	95	86	86	133	2·8	1·9	79	75	7	−·1	7·15	5·15	72	147	6·1	85	11	8
31. ,, E No 42	58	72	81	100	100	85	81	85	132	3·1	2	71	71	4	−·1	7	5·2	74	135	6·1	87	16	10
32. ,, E No 43	61	74	80	102	100	85	78	78	120	2·65	2·05	80	73	65	−·2	7	5·4	76	140	6	88	16	19
33. Female, E No 38	55	68	78	100	100	89	85	76	124	2·3	1·95	85	85	·45	−·5	6·8	5·1	75	141	5·8	85	12	12
34. ,, E. No 39	58	78	86	100	101	95	86	88	132	2·8	1·8	79	75	2	−·15	6·6	4·6	69	141	5·7	86	4	−6
35. ,, E. No 40	61	76	83	102	104	85	88	92	138	2·8	2	75	70	6	−·2	6·5	4·8	73	143	5·5	84	10	6
Average	59	73	83	101	102	91	81	81	130	2·78	1·96	79	75	15	−·23	6·89	5·06	72	130	5·95	86	11	6
Scotch																							
36. George Buchanan, E	63	77	81	97	97	90	80	84		2·6	1·73	81	78			6·7	5·1			5·5	89		
37. From Kilsyth, E (Ballingall's) B VII *	64	78	88	104	100	90	84	92	121	2·8	1·9	75	65	8	−·2	6·4	5·1	79	145	5·5	85	12	15
38. Edmunds, E.	64	79	85	103	101	90	83	80	128	3	2·2	79	72	9	−·05	7·3	5·8	76	150	5·7	78	19	15
39. Haggart, E	62	77	84	97	97	85	79	84	127	2·9	1·95	83	71	7	−·2	7·1	5·5	77	144	5·5	77	20	12
40. Old Officer, E (Ballingall's) B. VI.	66	81	87	101	97	85	79	72	119	2·4	2·1	87	80	6	−·3	7·4	5·5	74	134	6·9	83	7	8
41. Male, æt. 20, T.	60	77	85	103	93	81	77	83	129	3·15	2	81	68	9	·05	7·1	5·6	78	147	5·5	77	10	12
42. Female, middle aged, E	71	89	93	95	93	80	76	76	128	2·7	1·95	84	73	8	·2	7·3	5·3	72	147	8·2	85	13	10
43. ,, æt. 18, T	58	73	86	100	97	86	84	89	141	3	2·1	75	68	85	0	7·1	5·5	77	148			9	6
Average	61	79	86	99	96	85	81	83	127	2·81	1·99	81	71	78	−·07	7·07	5·41	76	145	5·72	80	12	10
Greek																							
44. From Corfu, E. No 46	54	68	79	106	106	95	90	90	126	2·83	2	81	77	·5	−·5	6·9	5	75	135	5·8	80	11	16
45. ,, E (Monro's)	60	68	82	108	107	97	93	93	125	2·85	1·85	85	76			6·9	5·4			5·5	79		
46. E. (Ballingall's) B III	65	80	86	100	104	94	84	84	113	2·85	2	83	75	8	0	7·2	5·5	75	142	5·5	78	8	16
47. ,, B. IV.	62	68	77	104	102	86	80	82	119	2·83	2·1	86	73	6	·05	6·4	5·3	82	147	5·7	89	7	9
48. E ,, B. V	57	73	86	100	95	85	84	86	130	2·9	1·95	75	70	·8	−·1	6·9	5·4	79	150	5·4	76	17	20
Average	50	71	82	103	102	91	86	87	123	2·86	1·98	82	74	7	−·22	6·8	5·32	77	143	5·48	80	10	13
Irish																							
49. Male, Hurley, Q C G	57	74	82	108	115	101	97	100	144	2·8	2·1	80	75	8	−·6	7·5	5·4	72	128	5·6	74	19	21
50. ,, Lydon, Q C G	64	82	91	95	95	84	77	77	138	2·65	1·85	73	73	5	0	6·95	4·9	70	136	5·7	82	14	18
51. ,, From Ballinasloe, old, Q.C.G.	68	81	86	102	106	100	83	95	128	2·75	2·2	79	71	·9	−·2	7·4	5·35	71	144	6	81	21	12
52. ,, From Claregalway, Q.C.G	68	79	86	106	106	97	90	92	131	2·85	1·85	80	71	·8	−·25	7·2	5·3	73	147	5·5	76	13	9
53. ,, ,, ,,	59	78	87	97	95	87	78	78	131	2·6	1·8	80	78	65	−·05	7·2	5·3	73	139	5·4	75	14	5
54. ,, ,, ,,	60	74	82	100	98	82	76	74	137	2·7	1·9	85	77	4	0	7·2	5·4	75	135	5·8	80	19	5
55. Female ,, ,,	63	78	89	100	100	89	82	78	129	2·65	1·95	83	81	3	−·25	7·2	5	69	131	5·7	79	11	3
56. ,, ,, ,,	72	86	90	97	100	90	83	86	128	2·7	1·9	80	72	4	−·15	7	4·6	65	135	5·3	76	11	3
57. ,, ,, ,,	67	84	83	91	91	80	76	80	128	2·7	1·95	79	70	·7	0	7·1	5·2	73	152	5·5	77	13	7
Average	61	79	86	99	100	90	82	84	134	2·71	1·94	79·8	74	60	−·16	7·19	5·19	71	138	5·61	77	14	8
Hindoo																							
58. F. No 48	57	71	80	102	102	88	84	84	120	2·6	1·9	78	74	1	−·4	6·6	5·6	84	143	5·3	80	7	7
59. E No. 49	56	69	82	100	95	80	76	80	125	2·75	1·8	81	72	7	−·25	6·4	5·4	81	150	5·4	84	7	6
60. E.N	62	73	82	101	102	91	86	93	130	3·15	2	79	67	9	−·2	6·8	5·9	82	153	5·4	79	9	8
Average	58	71	81	102	99	86	82	85	125	2·83	1·9	79·3	71	86	−·28	6·6	5·53	83	148	5·36	81	7·6	7
Negro																							
61. Male, E No. 3	70	81	88	102	101	100	93	102	143	3·2	1·95	68	65	7	−·15	7·4	5·2	71	150	5·1	68	9	11
62. ,, E No 52	60	72	79	102	109	106	95	100	136	2·95	1·95	78	74	9	−·25	6·95	5·2	74	150	5·1	73	19	23
63. ,, E No 53	68	81	90	102	102	100	91	100	143	3·2	1·95	68	65	8	−·15	7·25	5·3	71	144	5·4	74	16	26
64. ,, E No 54	63	77	86	106	104	93	86	80	131	2·75	2·25	73	72	85	−·1	6·8	5·3	77	139	5·1	75	6	12
65. ,, E No 55	50	74	87	100	98	81	78	80	130	3	2·1	77	71	5	−·1	7·2	5·6	77	147	5·2	72	17	10
66. Female, E No 56	63	75	84	106	109	95	88	95	137	2·85	1·9	73	70	85	−·25	6·9	5·4	78	143	5·1	73	6	15
67. ,, æt 15, E (Monro's)	65	80	89	90	90	89	76	80	141	2·6	1·7	69	72	6	−·1	6·8	5·1	73	147	5	73	11	15
Average	62	77	86	101	102	97	87	91	137	2·95	1·97	73·1	69	75	−·15	7·05	5·28	74	145	5·14	72	12	16
Kafir																							
68. Male, E No 59	56	74	86	101	96	88	80	84	140	3·1	2·15	76	72	9	−·25	7·5	6·1	81	145	5·3	70	11	26
69. ,, E (Monro's)	69	82	88	101	104	93	86	93	144	3	2	70	70	9	−·2	7·3	5·5	75	134	5·4	73	13	25
70. ,, E (Dr Black's)	65	76	84	102	104	93	89	95	134	3·2	1·95	74	68	85	−·2	7·4	5·5	74	144	5·3	71	6	10
71. Female, E	64	77	86	100	97	90	86	90	141	2·8	2·2	78	70	65	−·05	6·75	5·1	75	141	5·3	78	20	14
Average	63	77	86	101	100	93	85	90	138	3·02	2·02	74·5	70	83	−·17	7·21	5·55	76	141	5·32	73	12	18
Australian																							
72. E N	67	79	87	101	102	95	88	95	133	3	1·9	72	69	95	0	7·2	5·35	74	146	5·2	72	9	9
73. E No 66	66	80	85	104	107	97	92	100	138	3·05	1·65	75	70	9	−·15	7·1	5·2	73	140	5·1	71	9	11
Average	65	79	86	104	104	96	90	97	135	3·02	1·77	73·5	69	92	−·07	7·15	5·27	73	143	5·15	71	9	10
Kanaka																							
74. Male, E	53	69	83	100	97	89	84	79	135	2·75	1·95	80	82	65	−·3	7·1	5·6	78	149	5·6	78	16	19
75. Female, E	55	72	82	100	96	87	80	85	138	2·75	1·85	79	71	7	−·2	6·8	5·35	78	152	5·3	77	17	8
Average	54	70	82	100	96	88	81	82	136	2·75	1·90	79·5	76	·67	−·25	6·95	5·47	78	150	5·45	77	16·5	13·5
Esquimaux																							
76. E (Monro's)	63	79	87	102	104	93	88	95	137	2·85	2·	75	68	9	−·35	7·	5·3	75	143	5·3	75	8	10
77. E 68 a	63	79	88	104	102	97	90	93	141	2·9	2·1	78	74	95	−·2	7·15	5·3	74	139	5·7	79	9	25
Average	63	79	87	103	103	95	89	91	138	2·87	2·05	76·5	71	92	−·27	7·07	5·3	74	141	5·5	77	8·5	17·5
Peruvian																							
78. E N	51	61	77	102	93	84	80	81	118	2·8	2·1	75	67	65	−·2	5·9	5·2	88	152	5·6	98	18	15
79. E N	54	70	82	92	81	76	70	72		2·55	1·75	75	73			6·7	5·6						
Average	52	67	79	97	88	80	75	78		2·67	1·92	75	70			6·3	5·4						
Miscellaneous																							
80. Burke, E	72	81	88	97	102	95	90	93	126	3	1·85	78	74	65	−·15	7·2	5·05	70	148	5·7	79	7	11
81. Spanish pirate, E No 33	59	75	87	104	111	102	97	102	128	3·1	2·1	85	73	·9	−·6	7·3	5·4	74	154	5	68	14	22
82. Swiss, E No 34	68	84	95	95	95	88	84	88	143	2·95	1·9	79	71	65	·1	7·1	5·05	71	136	5·4	76	13	10
83. Turk, E (Monro's)	58	76	86	97	102	93	88	93	131	2·8	2·1	78	70	85	−·25	6·75	5·1	75	150	5·2	77	15	21
84. Tartar, E No 46	63	80	93	100	100	91	84	87	125	2·75	2·1	80	73	7	−·2	6·9	5·3	76	137	5·4	78	10	12
85. Chinese, E	63	79	88	97	95	87	81	86	121	2·6	2·05	76	68	9	−·3	6·8	5·3	77	151	5·7	83	13	13
86. Maori	62	77	85	101	100	87	81	79	141	2·8	1·85	75	79	95	0	7·2	5·9	81	151	5·6	77	10	23
87. Hottentot Chief, E	61	79	87	100	100	87	84	91	135	3·3	2	71	64	1	0	7·7	5·9	76	151	5·2	67	16	11
88. Carib, E No 62	71	84	91	95	95	95	91	91	127	3	2·1	84	76	1·05	−·25	7·8	5·7	73	158	5·4	69	9	18
89. Synostotic French, E No 32	55	71	87	100	87	75	71	77	131	2·75	2·1	73	68	15	−·1	6·4	5·3	82	139			15	23
90. Elongated skull, E N	66	77	86	88	97	88	84	80	132	2·45	1·75	88	81	7	−·2	7	5	71	151	5·2	74	13	14
91. So-called Baker's skull, T	62	87	93	95	95	87	83	81	156	2·95	2	80	77	·2	−·15	7·8	4·5	57	118	6·2	78	11	5
92. Large British, T	64	79	88	98	98	101	77	77	122	2·9	1·9	79	74	8	−·35	7·8	6·1	78	137				
93. Hunchback Dwarf, E	65	80	86	95	95	86	80	86	126	2·9	2·15	80	67	1·05	−·2	7·15	5·5	76	134	5·4	75	11	22
94. Idiot, E	92	107	88	97	92	95	88	93	124	2·95	2·05	77	68	85	1	6·9	5	72	163	5	72	14	4
95. Compressed Chinook, E N	58	71	84	93	91	85	82	91	136	2·9	2·1	70	65	8	−·25	6·67	5·1	81	147	5·2	78	14	21
96. Flattened American, E (Monro's)	52	68	81	95	88	84	84	90	118	2·95	2·1	69	67	1	−·25	5·9	4·8	81	145	6·1	108	18	32

IX. *On the Proof of the Law of Errors of Observations.*

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1. So much has been published upon the Theory of Errors, that some apology seems to be required from a new writer who does not profess to have arrived at any results which were unknown to his predecessors. Nevertheless, so great, as is well known, are the difficulties of the theory, whether we seek to form a correct estimate of the principles on which it rests, or to follow the subtle mathematical analysis which has been found indispensable in reasoning upon them, that any contribution which tends to simplify the processes, without weakening their logical exactness, will probably be considered of some value. My object in this paper is to give the mathematical proof, in its most general form, of the law of single errors of observations, on the hypothesis that an error in practice arises from the joint operation of a large number of independent sources of error, each of which, did it exist alone, would produce errors of extremely small amount as compared generally with those arising from all the other sources combined. Now this proof is contained in a process given for a different object, namely, POISSON'S generalization of LAPLACE'S investigation of the law of the mean results of a large number of observations, to be found in his 'Recherches sur la Probabilité des jugements,' and which is reproduced in Mr. TODHUNTER'S valuable 'History of the Theory of Probability.'

It is obvious that we should altogether restrict the generality of the proof, confining it merely to a few artificial and conventional cases, if we were to suppose each source of error to give positive and negative errors with equal facility, or to assume the law of error (even supposing it unknown) to be the same for all the sources. None of the processes, therefore, contained in the 4th chapter of the 'Théorie Analytique des Probabilités' are of sufficient generality for our purpose, though some writers have so employed them; nor will the method apply here which LESLIE ELLIS has given in his memoir "On the Method of Least Squares" (Camb. Phil. Trans. 1844), based upon FOURIER'S theorem, on account of the assumption of equal facility for positive and negative errors. The proof which follows will be found, I think, of full generality,—the only cases excluded being incompatible with the existence of the exponential law (see art. 7), and at the same time greatly simpler than POISSON'S, dispensing with his refined and difficult analysis[*].

2. It is remarkable that the well-known exponential function which is now pretty

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[*] The *length* of this communication may seem at variance with the statement that the proof here given is a simpler one than those of former writers. Still I think it will be found to be so on examination; the length of the paper arises from fuller explanations being given than is usually the case. I am persuaded that the doubts and misconceptions which have prevailed so extensively with relation to this subject have been in great part occasioned by the extreme brevity and scanty explanation of the great writers who have treated it.

generally received among mathematicians as expressing the law of frequency of single errors of observation, does not seem to have been distinctly given by any one of the three great philosophers Laplace, Gauss, and Poisson (who may be called the founders of the Theory of Errors) as being, in their opinion, the expression of that law. It has been erroneously supposed, as Leslie Ellis points out, that Gauss's and Laplace's proofs of the method of Least Squares depend upon that assumption. It is true that Gauss's first method, in the 'Theoria Motûs,' does require it; but he does not present that method as other than tentative and hypothetical: and later, in the 'Theoria Combinationis Observationum,' he says, speaking of the law of single errors, "plerumque incognita est."

As, however, this law of error seems in our day to have been adopted by general consent, some inquiry into the grounds on which its validity rests will be appropriate here. And first I would remark that it can scarcely be maintained that any attempt hitherto made to establish this law independently of the hypothesis I have named in art. 1 has been successful. We may pass by Gauss's proof in the 'Theoria Motûs,' which shows that the law must hold *if we take as an axiom* that the arithmetical mean of several observations is the most probable result. Now this really is not an axiom, but only a convenient rule which is generally near the truth: this we see by considering any case in which we are certain that the errors do not follow the exponential law; does the mind see here *à priori* that the rule does not give the most probable result? It seems certain that we should have just the same confidence in it here as in any case; yet Gauss's proof shows that it does not give the most probable result[*]. It should indeed be stated that Gauss himself (as might have been expected from that acute and accurate mind) is very far from asserting the above assumption to be an axiom; consequently he does not give his proof as more than hypothetical. He only states that the rule is generally accepted—"axiomatis loco haberi solet hypothesis." A method of remarkable simplicity was given by Sir J. Herschel in a very interesting review of Quetelet's 'Letters on Probability[†],' which conducts to the same law of error by means of one or two bold assumptions; but striking as the coincidence is, it can hardly be seriously viewed as a *demonstration*; nor is it formally so presented by its distinguished author. However, the methods both of Gauss and Sir J. Herschel are of great interest to the natural philosopher, as showing that certain *à priori* mathematical assumptions of a very simple kind lead to the same law of error which reasoning based on a study of the facts which surround us also points out as expressing, at least approximately, what generally does occur *in rerum naturâ*: though we can see no necessity that the facts should be so, it being quite easy to *conceive* a different economy of nature in which no such accordance would subsist*.

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[*] See Ellis, *loc. cit.* p. 207.

[†] Edinburgh Review, July 1850. See a criticism by Leslie Ellis in the Philosophical Magazine, vol. xxxvii. Also Boole (Edinb. Trans. vol. xxi.) and Thomson and Tait (Natural Philosophy), who speak more favourably. M. Quetelet's 'Lettres' will amply repay a perusal; in connexion with our present inquiry, he points out that not only errors of observations, but the variations of many other fluctuating magnitudes, such as the stature of men, the temperature of the weather, &c. from their mean values, seem to follow the same law. If this be so, the inference seems legitimate that these divergences from the mean types, or *errors of Nature* herself, as they may be called, are produced in each case, not by one or two, but by a vast number of hidden coexisting causes.

It is possible *à priori* to conceive that the law of single errors of observation might be of any form whatever, varying with each kind of observation: how far it is true that in practice one general law will be found to prevail, is essentially a question of facts—an inquiry, not into what *might be*, but what *is*. Now the hypothesis above mentioned,—namely, that errors *in rerum naturâ* result from the superposition of a large number of minuter errors arising from a number of independent sources,—when submitted to mathematical analysis, leads to the law which is generally received; as far therefore as this hypothesis is in accordance with fact, so far is the law practically true. Fully to decide how far this hypothesis does agree with facts is an extremely subtle question in philosophy, which would embrace not only an extended inquiry into the laws of the material universe, but an examination of the senses and faculties of man, which form an important element in the generation of error. Still, without pretending to enter on a demonstration of the truth of this hypothesis, a few reflections upon the facts, especially in the case of Astronomy (which is *par excellence* the science of observation, and where accordingly the lessons of experience are the clearest and most complete), will, I think, at least convince us of its *reasonableness* in certain large classes of errors of observations. Now if we attend to what has taken place in the history of astronomical observation, we find that the gross errors of the earlier observers proceeded mainly from three or four principal causes—for instance, refraction, imperfect measurement of time, and the use of the naked eye in pointing to objects. When these few capital occasions of error were removed (at least approximately), refraction being discovered and allowed for, and the pendulum and telescopic sights introduced, it was found that observations at once attained a high order of accuracy, showing that the principal sources of error had been eliminated. It would seem, in fact, that in coarse and rude observations the errors proceed from *a very few* principal causes; and in this case, consequently, our hypothesis will probably represent the facts only imperfectly, and the frequency of the errors will only approximate roughly and vaguely to the law which follows from it†. But when astronomers, not content with the degree of accuracy they had reached, prosecuted their researches into the remaining sources of error, they found that, not three or four, but a *great number* of minor sources of error, of nearly coordinate importance, began to reveal themselves, having been till then masked and overshadowed by the graver errors which had been now approximately removed*. It was as if a small number of forest trees had been cut down, leaving an innumerable growth of shrubs and brushwood at their feet, remaining to be cleared. There were errors of graduation, and many others, in the construction of instruments; other errors of their adjustments; errors (technically so called) *of observation*; errors from changes of temperature, of weather, from slight irregular motions and vibrations; in short, the thousand minute disturbing influences with which modern astronomers are familiar, and which it is superfluous to recapitulate here. Many of these are known and allowed for, or eliminated, at least approximately, in practical astronomy; still we seem to be justified in considering the error which remains as the result of a great number of yet minuter errors, each inconsiderable in itself. Thus a cursory view of the nature of astronomical errors, and the light which this throws on various cognate classes of observations, seem to lead to the conclusion that the above hypothesis will be found to hold, generally, in the case of refined and delicate observations. No doubt much more would be necessary to justify us in asserting

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* The extreme simplicity of the exponential relation itself, whether considered as expressing the law of single errors, or that of the mean results of a large number of observations, as contrasted with the long and difficult methods by which it was established, has naturally led to several attempts to dispense with or simplify the latter; in some the hypothesis we here adopt is taken as a basis; but, so far as the present writer is aware, every process given, except Poisson's, fails in generality. In a recent Memoir on the Law of Frequency of Error by Professor Tait (Edin. Trans. vol. xxiv.) (where, it should be stated, the learned author speaks with some hesitation, and only gives his method as an attempt), it is assumed that each of the elementary errors which are combined can be assimilated to the deviation from its most probable value of the number of white balls among a given large number of balls drawn from an urn, which contains white and black in a given proportion. It is then shown (as indeed is done in Laplace's 3rd chapter) that this error follows the exponential law. Thus the proof only applies to the combination of a number of elementary errors, each of which follows that law. But it is quite certain that many simple errors do not follow that law; hence the method is altogether deficient in generality.

† We cannot, however, assert this positively, if there is reason to believe that the error which arises from each principal cause is itself a composite error, which certainly is often the case. The "error in time," for

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instance, is certainly not a simple error, but one resulting from the joint action of several causes, one or more of which we can conceive detected and allowed for, leaving the others in operation. An error may thus arise from the superposition of only three or four component errors, which at first sight are of simple origin, but in reality represent each a group of minor errors; and the hypothesis would then hold. It is questionable whether, among the causes which in practice vitiate any observation, any *simple* error ever does enter, of considerable magnitude and importance as compared with the others combined; such, for instance, as would be the error produced in the time (or through the time on some astronomical magnitude) by the pendulum being $\frac{1}{10}$ of an inch too long or too short, every thing else being pretty accurate. If it be said that ignorance or negligence might produce such a result, we may answer that such negligence or ignorance would make itself felt in other ways also: one such error would not stand alone. Isolated acts of neglect by a careful observer would come under the head of *occasional* errors, as explained further on.

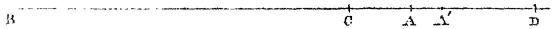
It seems very difficult to discern, *à priori*, the nature of the errors incurred in *estimating* magnitudes by the eye, or of errors arising from the imperfection of our senses, such as those incurred in pointing to a star with the naked eye. It is quite possible that such errors may arise each from *several* sources, though their nature be hidden from our view.

* A similar law to that mentioned above seems to prevail in many kindred cases. Thus in the successive improvements in artillery, machinery, &c., in proportion as the greater sources of imperfection and inaccuracy are understood and remedied, the *number* of minor disturbing influences which are thus rendered perceptible, and still vitiate the results, though to a less extent, increases rapidly. We may even trace a sort of analogy here in various phenomena both of the moral and material universe, which apparently have no bearing on the point we are considering. Thus the principal wants of human nature, the necessaries of life in fact, are very few; and so long as these are supplied with difficulty, minor wants are scarcely *felt*, as we see in uncivilized communities: but when the greater wants are satisfied, the number and variety of the *secondary* requirements of our nature are visible in the multitudinous productions of civilized life. The diseases which mainly operate in shortening human existence are very few in number; but could they be extirpated, the number of minor causes, of nearly coordinate importance, which still would influence the rate of mortality would be very large. The statistics of crime, and many other phenomena, would give rise to remarks of a similar nature.

this absolutely; thus it is not enough for our purpose to show, could we do so conclusively, that each error in practice is compounded of a large number of smaller errors; we must also show that they are *independent*, at least for the most part. Thus we may conceive one of the minute errors affecting an astronomical magnitude to be an error in the refraction proceeding from a rise in the general temperature, and another affecting the same observation to be an error of time arising from the expansion of the pendulum through the same cause; now these two minute errors are not independent, and would have to be mathematically combined in quite a different way from two that were independent; and, indeed, such a change of temperature would influence the actual error of the observation in other ways also. However, we may at least safely conclude that the hypothesis in question is not a mere arbitrary assumption, but a reasonable and probable account of what does in fact take place in the case of careful and refined observations.

3. In proceeding to submit this hypothesis to mathematical analysis, the minute simple errors which go to form the observed compound error will be assumed to follow each its own unknown law, expressed by different unknown functions of the utmost generality[*]: positive and negative values of each error will not be assumed equally possible; on the contrary, the cases will be included, as obviously ought to be done, of minute disturbing influences which always cause the observed magnitude to err in excess, and of others which cause it to err only in defect. I will exclude all mention of the term *probability*, and will consider solely the *frequency* or *density* of the error, viewed as a function of its magnitude.

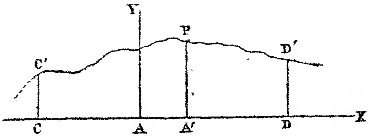
Let any magnitude which has to be determined by observations affected with some one cause of error (simple or compound) be represented by the line BA;



let a large number of such observations be made, and let the observed values be represented by a number of lengths BA′, measured from B: it will be found in general that in the neighbourhood of A the line will be dotted over with a multitude of points A′, the distance AA′ being the error in each case. These dots will begin at some point C, and end at some point D, which generally are on opposite sides of A, but may both be at the same side. Between C and D the dots will be distributed over CD with a variable density: this density, at any point A′, will represent the *frequency* or *density* of errors of magnitude AA′.

If at every point A′ we erect an ordinate A′P representing the density at A′, we shall thus trace out a locus or curve C′D′, whose equation we may call, taking A as origin,

$$y=\phi(x). \tag{1}$$



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[*] With regard to the limits or amplitudes of the errors, see note on art. 7.

This we may call the *curve* or *function* of Error*. It is of course generally discontinuous, as it is only to include values of x between the points C, D. The function $\phi(x)$ strictly speaking should vanish for all values of x beyond C and D; however, we shall not require any consideration of the analytical methods of expressing such functions. If N be the number of observations taken, and if we put $AD=a$, $AC=b$, then as ydx denotes the number of errors lying between x and $x+dx$,

$$N=\int_{-b}^{a}\phi(x)dx. \quad \ldots\ldots\ldots\ldots \quad (2)$$

It is well to notice that, if C be any constant, the equation

$$y=C\phi(x)$$

really is the same function of Error as (1), the number of observations only being altered.

4. In order not unduly to limit the generality of the investigation, it is necessary further to study the nature of the possible ways in which the dots we have spoken of as representing the observations may be scattered along the line CD, in the case of various unknown *simple* causes of error; noting also what becomes of the *function* $\phi(x)$ and the curve C′D′ in each case. And first, in many cases the dots will be distributed *continuously* along CD, thus giving a *curve* without gaps or intervals. It is by no means necessary that this curve should descend towards CD at its two extremities more than in the middle; in other words, the extreme values of a simple Error are not always less probable than the intermediate ones. There may be cases where the extreme values are the most probable; for instance, the Error occasioned by supposing a point fixed, which is in reality performing extremely minute and slow oscillations about its mean position. But besides the cases of continuous distribution, there are others, not only conceivable, but which we may be sure do actually occur, in which a function or curve does not assist our conceptions, and we shall do better merely to consider the points or dots themselves. There may be what is called a *constant* Error; that is, some cause which gives the observation always too great (or too small) by the same fixed minute amount: the distribution here is simply a group of N coincident points somewhere on CD. Or a certain cause may only admit of two or more definite values for the error; the distribution will be two or more groups of coincident points, the numbers in each group being equal or unequal. Again, an important class of Errors are those which may be called *occasional Errors*, that is, produced by intermittent causes not always in operation. In such a case, if N observations be made, a certain number of them (say n) are unaffected by the Error; the remaining $N-n$, made when the cause is in operation, we may suppose represented by dots continuously or discontinuously distributed; we have then a group of n coincident points at A, besides a number $N-n$ distributed in some way over CD. Errors of mistake or forgetfulness, and many others also, are of this description.

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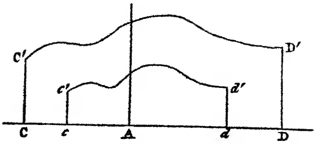
* The word "error" is sometimes used for shortness to express a source of error. To avoid confusion we may write it with a capital E, when used in this sense. Thus "an Error" will mean a source of error, or the assemblage of actual errors (or the curve or function symbolizing them) which that source produces in a large number of trials, and which form a visible manifestation or representation of it: "an error" will mean a particular magnitude.

5. If we alter the ordinate and abscissa of every point in the curve C′D′ in a given ratio, changing the limits a, $-b$ of the Error in the same ratio, we find the curve $c'd'$ represented by

$$\frac{y}{i}=\phi\left(\frac{x}{i}\right), \quad \dots\dots\dots\dots\dots \quad (3)$$



which may be called a *similar Error* to (1) or C′D′. The number of observations will be different in the two cases, being represented by the areas of the two figures. We may find it convenient to suppose the number of observations the same; if so

$$y=\frac{1}{i}\phi\left(\frac{x}{i}\right) \quad \dots\dots\dots\dots\dots \quad (4)$$

will be a *similar function of Error* to $y=\phi(x)$, the number of observations being the same for both, the limits of the error in (3) and (4) being ia, $-ib$.

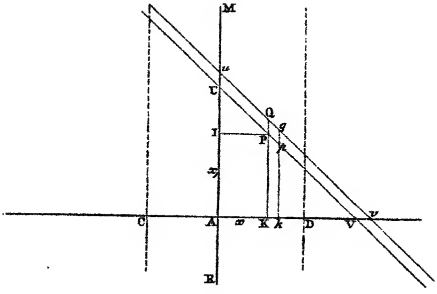
6. *To find the function of Error resulting from the combination of a given Error whose equation is*

$$y_1=f(x_1) \quad \dots\dots\dots\dots\dots \quad (5)$$

(the limits being $\pm\infty$*) with another independent Error*

$$y=\phi(x), \quad \dots\dots\dots\dots\dots \quad (6)$$

whose limits are a, $-b$.



We shall do this most clearly by help of a geometrical construction. Let the (N) values of the first Error be measured from A according to their signs along the indefinite line MR; likewise measure the (n) values of the second Error along CD, where AD$=a$, AC$=b$. Take any two values, AI$=x_1$ of the first, and AK$=x$ of the second; they give a value $x+x_1$ of the compound Error, to which will correspond a point P of the plane, whose *coordinates* are x, x_1. The number of such points contained within the element

$dx\,dx_1$, each point corresponding to a compound error, will be

$$yy_1\,dx\,dx_1 \text{ or } f(x_1)\phi(x)dS,$$

dS being the element of the area. Draw through P a line UV equally inclined to the axes, then $x+x_1$ is constant along this line; put $\xi=x+x_1=AV$, take $Vv=d\xi$, and draw uv parallel to UV; take $Kk=dx$, then the number of points within the elementary parallelogram $PQpq$ will be

$$f(\xi-x)\phi(x)d\xi\,dx.$$

Hence the whole number of points between the parallels UV and uv (that is, *the number of compound errors whose magnitudes lie between* ξ and $\xi+d\xi$) will be

$$d\xi\int_{-b}^{a} f(\xi-x)\phi(x)dx.$$

The total number of compound errors thus obtained will be Nn; however, for uniformity, we will suppose the number of observations taken, affected with the compound Error, to be N, the same as for (5). This will oblige us to divide by

$$n=\int_{-b}^{a}\phi(x)dx.$$

Thus if we represent the compound Error by a curve whose coordinates are (ξ, η), it will be

$$\eta=\frac{\int_{-b}^{a} f(\xi-x)\phi(x)dx}{\int_{-b}^{a}\phi(x)dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus if we wish to find the Error resulting from the combination of the two Errors whose equations are

$$y=\frac{N}{\theta\sqrt{\pi}}e^{-\frac{(x-\alpha)^2}{\theta^2}},\quad y=\frac{N'}{\phi\sqrt{\pi}}e^{-\frac{(x-\beta)^2}{\phi^2}},$$

we have from formula (7) (N, N′ denoting the numbers of observations),

$$\eta=\frac{N}{\pi\theta\phi}\int_{-\infty}^{\infty} e^{-\frac{(\xi-x-\alpha)^2}{\theta^2}}\,e^{-\frac{(x-\beta)^2}{\phi^2}}\,dx,$$

whence

$$\eta=\frac{N}{\sqrt{(\theta^2+\phi^2)\pi}}e^{-\frac{(\xi-\alpha-\beta)^2}{\theta^2+\phi^2}}.$$

Hence it is easy to see that if any number of Errors of the forms

$$y=\frac{N}{\theta\sqrt{\pi}}e^{-\frac{(x-\alpha)^2}{\theta^2}},\quad y=\frac{N'}{\phi\sqrt{\pi}}e^{-\frac{(x-\beta)^2}{\phi^2}},\quad y=\frac{N''}{\psi\sqrt{\pi}}e^{-\frac{(x-\gamma)^2}{\psi^2}}, \text{ \&c.}$$

be combined, the resultant Error will be

$$y=\frac{N}{\sqrt{\pi(\theta^2+\phi^2+\psi^2+\dots)}}e^{-\frac{(x-\alpha-\beta-\gamma-\dots)^2}{\theta^2+\phi^2+\psi^2+\dots}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Expanding $f(\xi-x)$ in formula (7), it becomes

$$\eta=f(\xi)-\alpha f'(\xi)+\frac{\lambda}{2}f''(\xi)-\frac{\sigma}{2.3}f'''(\xi)+\text{\&c.},$$

where

$$\alpha=\frac{\int_{-b}^{a}x\phi(x)dx}{\int_{-b}^{a}\phi(x)dx},\quad \lambda=\frac{\int_{-b}^{a}x^2\phi(x)dx}{\int_{-b}^{a}\phi(x)dx},\quad \sigma=\frac{\int_{-b}^{a}x^3\phi(x)dx}{\int_{-b}^{a}\phi(x)dx},\ \&c.,$$

α being the mean value of the Error $y=\phi(x)$, λ its mean square, σ its mean cube, &c.

7. In the problem of finding the law of error resulting from the superposition of a great number of Errors, each of very small importance by itself, we will consider each component Error as the *diminutive* of some Error of finite importance* (see art. 5). Thus if $y=\mathrm{F}(x)$ be some possible finite Error, and we reduce its dimensions in the ratio i, where i is infinitesimal, the diminished Error will be $\frac{y}{i}=\mathrm{F}\left(\frac{x}{i}\right)$; and if the mean value, mean square, mean cube, &c. of the former be called

$$\mathrm{E}_1,\ \mathrm{E}_2,\ \mathrm{E}_3,\ \ldots\ldots,$$

it is easy to see that the same means, for the reduced Error, will be

$$i\mathrm{E}_1,\ i^2\mathrm{E}_2,\ i^3\mathrm{E}_3,\ \ldots\ldots$$

Now adopting the usual axiom that *no function can represent a finite Error unless* $\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \ldots$ *are finite,* it follows that the mean cube, mean 4th power, &c. of the diminutive Error may be rejected in comparison with its mean square*. We infer, therefore,

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* Thus all conceivable cases of Errors whose extreme limits, or amplitude, are very small, are contained in the above method of proof; also those small Errors which, though their extreme amplitude be not very small, are merely possible finite Errors (of great or infinite amplitude) on a reduced scale. It is necessary, however, to observe, in examining the nature of all the minute simple Errors which our hypothesis in its generality comprises, that there are cases quite conceivable, and involving no absurdity, of simple Errors of trivial or infinitesimal importance which come under neither of these categories, and to which the method in the text will not apply. To give a simple instance, imagine an *occasional* source of Error, which rarely operates, but which, when it does, gives a fixed finite error k (thus we may conceive an observer to mistake, once in a thousand times, the succeeding division of his instrument for the true one). Let this happen on an average once for n times that the cause is not in operation (n being supposed very great); then the mean value of the Error is $\frac{k}{n+1}$, its mean square is $\frac{k^2}{n+1}$, &c. It is therefore of infinitesimal importance whether, with LAPLACE, we estimate the *importance* of an Error by its mean value (irrespective of sign), or, with GAUSS, by its mean square; but as its mean cube &c. cannot be rejected in comparison with the mean square, the above analysis cannot be applied to it. Minute *simple* Errors of such a description must then be excepted from those which are supposed to enter into the composition of the actual errors of observations. If an appreciable number of them did enter, the received exponential law could not hold for the compound Error. Thus were we to combine a large number of small Errors of the nature of the simple instance just cited, the resultant Error would be of a discontinuous nature, represented by groups of coincident points, with finite intervals between them.

Though it is necessary clearly to understand that the full generality of the hypothesis is restricted by the exceptions explained in this note, yet there seems every reason to suppose that such cases are too rare in practice to cause any sensible deviation from the exponential law of error, the great majority of the minute component Errors which jointly affect any observation *in rerum naturâ* having each, it is natural to suppose, a very minute range or amplitude.

If $y=f(x)$ represent any Error of indefinite amplitude, and if a new Error, $y=\phi(x)$, of indefinitely small importance as compared with it, be superposed, the resulting compound Error will be represented by the equation

$$y=f(x)-\alpha\frac{d}{dx}f(x)+\frac{\lambda}{2}\frac{d^2}{dx^2}f(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where α, λ are infinitesimal constants, viz. the mean value of the new Error and the mean value of its square†, the number of observations being supposed the same for the Error (9) as for the former, $y=f(x)$.

If we now conceive $y=\phi(x)$ in the above to be one of a large number of independent infinitesimal Errors, and $y=f(x)$ to be the compound finite Error resulting from the combination of all the others, we infer from (9) that each elementary Error $y=\phi(x)$ affects the law of the combined Errors in a manner which only involves (α) the mean value of the elementary Error, and (λ) its mean square. But if this be so, we may, for our present purpose, substitute for $y=\phi(x)$ any other Error whatever which has the same mean value and mean square (provided of course its mean cube &c. may be neglected in comparison with its mean square). We may therefore *for our purpose* replace $y=\phi(x)$ by‡

$$y=\frac{n}{\sqrt{2\pi(\lambda-\alpha^2)}}e^{-\frac{(x-\alpha)^2}{2(\lambda-\alpha^2)}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which fulfils these conditions.

Likewise, if there be another elementary Error whose mean value is β and mean square μ, we may replace it by

$$y=\frac{n'}{\sqrt{2\pi(\mu-\beta^2)}}e^{-\frac{(x-\beta)^2}{2(\mu-\beta^2)}},$$

and so on, for all the elementary Errors. Hence (see equation 8) the Error compounded of any number of them will be

$$y=\frac{\mathrm{N}}{\sqrt{2\pi(\lambda+\mu+\nu+\ldots.-\alpha^2-\beta^2-\gamma^2-\ldots)}}e^{-\frac{(x-\alpha-\beta-\gamma-\ldots.)^2}{2(\lambda+\mu+\ldots.-\alpha^2-\beta^2-\ldots)}}.$$

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* We cannot neglect the mean square as compared with the mean 1st power, as the latter is the algebraical sum of a number of positive and negative elements, which sum may be of any amount, however small, and may sometimes vanish altogether; whereas the former is the sum of a number of *positive* elements, and therefore cannot vanish.

† If the new Error be such as to give any discontinuous distribution of points (see art. 4), it is easy to satisfy ourselves, by the method of art. 6, that the above proposition still holds good. In fact, if the n values of the new Error be x_1, x_2, x_3, ..., we shall have, instead of the formula (7),

$$\eta=\frac{1}{n}\{f(\xi-x_1)+f(\xi-x_2)+f(\xi-x_3)+\&c.\},$$

which is true in all cases, whether the distribution be continuous or discontinuous, or a mixture of both; and hence the formula (9) will follow.

‡ This suggestion is due to Professor J. C. ADAMS, one of the Referees charged by the Royal Society with the duty of reporting upon the present Paper. The remainder of the proof, which was of a different nature in the Paper as originally presented, is much simplified thereby.

We conclude therefore that *if a great number of minute independent Errors be combined, and if we write*

$$\left.\begin{aligned} m&=\alpha+\beta+\gamma+\dots=\text{sum of mean Errors,}\\ h&=\lambda+\mu+\nu+\dots=\text{sum of mean squares of Errors,}\\ i&=\alpha^2+\beta^2+\gamma^2+\dots=\text{sum of squares of mean Errors,}\end{aligned}\right\}^{*} \quad . \quad . \quad . \quad . \quad (11)$$

the resulting function of Error will be

$$y=\frac{\mathrm{N}}{\sqrt{2\pi(h-i)}}e^{-\frac{(x-m)^2}{2(h-i)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The Probability of an error being found to lie between x and $x+dx$ is of course

$$\frac{1}{\sqrt{2\pi(h-i)}}e^{-\frac{(x-m)^2}{2(h-i)}}\,dx\dagger.$$

If positive and negative errors in the observation are equally probable, as generally can be secured in practice, at least approximately, then $m=0$; that is, the sum of the mean values of the elementary component Errors vanishes, and the Probability is expressed by the usual value

$$\frac{1}{c\sqrt{\pi}}e^{-\frac{x^2}{c^2}}\,dx.$$

If we calculate by integration from equation (12) the mean value of the composite Error (or, as GAUSS calls it, *the constant part* of the Error) and the mean value of its square, we shall find

Mean Error $=m=$ sum of mean values of component Errors,
Mean Square of Error $=h+m^2-i$.

We have thus a verification of the correctness of our analysis, as the same results may be found from independent algebraical computation[‡].

8. Considering the celebrity of the question, it may not be superfluous to show how the result might have been obtained without any antecedent knowledge of the peculiar property of combination of the Errors in equation (8).

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[*] We may observe that $h-i$ *is always positive*; for if we take any set of numbers, positive or negative, the mean of their squares is always greater than the square of the mean (see TODHUNTER's 'Algebra,' p. 407). Therefore

$$\lambda>\alpha^2,\ \text{also}\ \mu>\beta^2,\ \nu>\gamma^2,\ \&\text{c}.$$

Consequently $h>i$.

[†] This expression will be found to agree with POISSON's final result in the memoir already cited.

[‡] If

$$\mathrm{U}=a+b+c+d+\&\text{c.},$$

where each of the quantities a, b, c, d, &c. may take any number (different for each quantity) of different independent values, adopting for shortness the symbol M(K) for "the mean value of K," it is not difficult to prove, by elementary algebra, that

$$\mathrm{M}(\mathrm{U})=\mathrm{M}(a)+\mathrm{M}(b)+\mathrm{M}(c)+\&\text{c.}=\Sigma\mathrm{M}(a),$$

$$\mathrm{M}(\mathrm{U}^2)=\mathrm{M}(a^2)+\mathrm{M}(b^2)+\mathrm{M}(c^2)+\dots+2\Sigma\{\mathrm{M}(a)\mathrm{M}(b)\},$$

or

$$\mathrm{M}(\mathrm{U}^2)=\Sigma\mathrm{M}(a^2)+\{\Sigma\mathrm{M}(a)\}^2-\Sigma\{\mathrm{M}(a)\}^2.$$

Let us suppose all the infinitesimal simple Errors which it is proposed to combine to be successively superposed upon some assumed function of Error $y=f(x)$; then by equation (9) the new function arising from the first of them will be, putting $D=\frac{d}{dx}$,

$$y=\left(1-\alpha D+\frac{\lambda}{2}D^2\right)f(x).$$

If another be now superposed upon this, we shall have

$$y=\left(1-\beta D+\frac{\mu}{2}D^2\right)\left(1-\alpha D+\frac{\lambda}{2}D^2\right)f(x),$$

and finally the function arising from the superposition of all the given Errors upon the assumed Error $y=f(x)$ will be

$$y=\left(1-\alpha D+\frac{\lambda}{2}D^2\right)\left(1-\beta D+\frac{\mu}{2}D^2\right)\left(1-\gamma D+\frac{\nu}{2}D^2\right)\ldots\ldots f(x). \quad . \quad . \quad (13)$$

But as α, λ are infinitesimals, we have, retaining the square of α,

$$1-\alpha D+\frac{\lambda}{2}D^2=e^{-\alpha D+\frac{1}{2}(\lambda-\alpha^2)D^2}.$$

Thus (13) will become

$$y=e^{-(\alpha+\beta+\gamma+\ldots\ldots)D+\frac{1}{2}(\lambda-\alpha^2+\mu-\beta^2+\ldots\ldots)D^2}f(x),$$

or, adopting the notation (11),

$$y=e^{\frac{1}{2}(h-i)D^2}e^{-mD}f(x)=e^{\frac{1}{2}(h-i)D^2}f(x-m). \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

9. Let us now take as the assumed function of Error

$$y=f(x)=\frac{N}{\theta\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

(where N is the number of observations), and imagine the whole given system of small Errors superposed upon it; the resulting function is

$$y=\frac{N}{\theta\sqrt{\pi}}e^{\frac{1}{2}(h-i)D^2}e^{-\frac{(x-m)^2}{\theta^2}}.$$

Now by a theorem in the Differential Calculus*,

$$e^{aD^2}e^{-kx^2}=\frac{1}{\sqrt{1+4ak}}e^{-\frac{kx^2}{1+4ak}};$$

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* This theorem, which is new to the present writer, may be proved in various ways. Thus if we put

$$u=e^{aD^2}e^{-kx^2},$$

and differentiate with regard to a, we have

$$\frac{du}{da}=e^{aD^2}D^2e^{-kx^2}=e^{aD^2}(4k^2x^2-2k)e^{-kx^2}:$$

again,

$$\frac{du}{dk}=e^{aD^2}(-x^2e^{-kx^2});$$

we thus obtain the partial differential equation

$$\frac{du}{da}+4k^2\frac{du}{dk}+2ku=0,$$

hence

$$y=\frac{\mathrm{N}}{\sqrt{\pi}\sqrt{2(h-i)+\theta^2}}e^{-\frac{(x-m)^2}{2(h-i)+\theta^2}}.$$

Now we may here assume θ as small as we please*,—that is, we may assume the Error (15) upon which the given system was superposed, to be of as small importance as we please. We conclude, then, rejecting this Error altogether, that a system of very small Errors, when combined, give for the resulting function of Error

$$y=\frac{\mathrm{N}}{\sqrt{2\pi(h-i)}}e^{-\frac{(x-m)^2}{2(h-i)}}$$

as before.

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the integral of which is

$$u=k^{-\frac{1}{2}}\phi\left(4a+\frac{1}{k}\right).$$

To determine the arbitrary function ϕ, we remark that if $a=0$, $u=e^{-kx^2}$,

$$\therefore \phi\left(\frac{1}{k}\right)=k^{\frac{1}{2}}e^{-kx^2},$$

hence

$$u=k^{-\frac{1}{2}}\left(4a+\frac{1}{k}\right)^{-\frac{1}{2}}e^{-x^2\left(4a+\frac{1}{k}\right)^{-1}}=(1+4ak)^{-\frac{1}{2}}e^{-\frac{kx^2}{1+4ak}}.$$

Another proof may be obtained by employing Poisson's ingenious transformation (Traité de Mécanique, tom. ii. p. 356), which gives

$$e^{a\mathrm{D}^2}\phi(x)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-\omega^2}\phi(x+2\omega\sqrt{a})d\omega.$$

* In order that we may retain the three first terms only in the expansion

$$y=f(x)-\alpha f'(x)+\frac{\lambda}{2}f''x-\&c.,$$

it is necessary to show that $f'''(x)$ and the succeeding differential coefficients are not infinite. Now they generally will be infinite in the case where $y=f(x)$ is an infinitesimal Error, as $f(x)$ will be of the form $\mathrm{K}\phi\left(\frac{x}{\epsilon}\right)$, where ϵ is infinitesimal; but in the case where

$$y=f(x)=\frac{\mathrm{N}}{\theta\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}},$$

we may take θ as small as we please, and yet retain only the three first terms above, because the differential coefficients of y do not here become infinite; in fact it is easy to see that any differential coefficient $\frac{d^ny}{dx^n}$ will consist of a series of terms of the form

$$\mathrm{C}\frac{x^p}{\theta^r}e^{-\frac{x^2}{\theta^2}};$$

now by the rules in the Differential Calculus for evaluating indeterminate forms, this quantity tends to zero as θ diminishes.

X. *On the Mineral Constituents of Meteorites.* *By* Nevil Story-Maskelyne, *M.A., Professor of Mineralogy, Oxford, and Keeper of the Mineral Department, British Museum. Communicated by Professor* H. J. Stephen Smith, *F.R.S.*

Received October 9, 1869,—Read January 13, 1870.

I. *The applications of the Microscope in the investigation of Meteorites.*

The mineralogical investigation of a meteoric stone presents difficulties very similar to those which have hitherto rendered the analyses and descriptions of many of the finer-grained terrestrial rocks unsatisfactory; for a meteoric stone is in fact a fragment of a rock, though formed under conditions in some respects widely differing from those which have produced the rocks of our globe.

The difficulties alluded to arise from the minute size and imperfectly developed crystallisation of the mineral constituents alike of the rock and the aërolite; and they have in general baffled the efforts of the chemist on the one hand to effect their separate analyses, and of the crystallographer on the other hand to determine the forms of these constituents. The chemist indeed has endeavoured to overcome the difficulty by attempting a chemical separation of the constituent minerals of these fine-grained mixtures into one group of such as are soluble and another group of those which are insoluble in acids, and then treating the numbers obtained from the analyses of these groups by the light of theoretical considerations founded on the formulæ and properties of known minerals. This method is necessarily only an approximative one. Even granting that by its means we could divide a rock into two classes of ingredients, which we cannot in fact accurately do, there remains the question of how to separate from each other the mingled minerals in, for instance, its insoluble portion.

But the great interest that attaches to whatever may throw light on the history of aërolitic rocks seemed to render it very desirable that some more reliable method should be sought for their investigation. With this end in view, and also with the purpose of basing on such an investigation a scientific classification of the now very extensive collection of aërolites in the British Museum, I some six years ago commenced a systematic examination of these bodies by the microscope. While the meteorites were being cut in order to show their polished surfaces, a small fragment of the portion detached was fastened by its flat side to a strip of glass and carefully worked down to the utmost tenuity. The transparent section thus formed was then examined in the microscope. The results to be obtained by the study of such sections may be divided into such as are structural, throwing light on the physical conditions under which the meteorite was formed, and such as are mineralogical and concern purely the particular minerals that are the ingredients of the stone. From the former class, we learn that a meteorite has had a history; that it has undergone change subsequently to its first consolidation in its present form. The crystalline character of all the constituent minerals; the fissures at one time formed, then filled and then, in many cases, broken across and 'heaved' and filled again, like some mineral lode; the 'chondritic' structure that G. ROSE has illustrated*; the fracture, in at least one meteorite, of the spherular 'chondra,' which have been split and severed and recemented into a compact mass,—these are among the many facts imprinted on a meteorite which are so many records belonging to its history, and which by the aid of the microscope we may read and interpret. But to found a classification on the structural characters of meteorites is not the same thing as to arrange them according to their mineralogical composition. The two must be combined for a philosophical arrangement.

I propose dealing with the mineralogical side of the problem in the present memoir, and to recur hereafter to the structural composition of meteorites, when the nature of their ingredient minerals shall have been rendered clear.

The general features of the microscopic sections of certain meteorites were described by me in the years 1863–64; and the examination in this way has been extended to above 140 distinct aërolites. The crystallography, however, of the numerous crystals seen in such a microscopic section is almost hopelessly difficult. In cases where crystallographic directions are indicated by cleavage-planes or by the 'traces' on the section of determinable crystal faces, some conclusion as to the symmetry and system of the crystal can be drawn from the directions of its optical principal sections as indicated by light polarised in a known plane. And occasionally a section is met with so nearly parallel to one of the important faces of the crystal as to allow some reliance to be placed on the angles of its bounding planes as measured by a delicate eyepiece goniometer. A long series of measurements and determinations of the directions parallel to the principal sections in the crystals met with in these microscopic slides has convinced me that, however useful the microscope may be in revealing the structure of a meteorite and helping to determine its place in a collection of such bodies classified according to their physical constitution, it is only partially of use in determining the mineralogical character of the constituents. But if the applications of polarised light and the eyepiece goniometer are thus limited, the microscope has another function to perform in such an investigation; for, from the carefully bruised débris of particular meteorites selected for the frequent recurrence in them of recognisable minerals, and for the magnitude of the grains of these, one is able to pick out under the microscope the distinct particles of each such mineral.

Such particles occasionally offer cleavage-planes, or even a crystal face or two, to the goniometer. In a very few cases crystals have been found sufficiently complete to lead to a reliable crystallographic result†. The chief advantage of this method is, however, that it affords the means of analysing the separate minerals selected by it; but even thus the difficulties are considerable. The quantity of material at one's disposal amounts generally to a very few grains, and of these but a small proportion consists of the desired mineral; in fact half a gramme is as much as one is generally able to obtain in a state of purity.

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* Abhandl. der Königl. Akademie der Wissensch. Berlin, 1864, p. 84.

† The crystals of Anorthite in the Juvinas Meteorite were thus measured in the British Museum by my late colleague, Professor V. von LANG, Sitzungsber. Akad. der Wissensch. Wien, 1867.

The analysis of so small an amount of a silicate is a difficult problem. To break up the silicate by fusion with alkalies implies the introduction of foreign and non-volatile ingredients; to estimate the silica by its loss on treating the mineral by ammonium or hydrogen fluoride were to lose what, in the analysis of such small quantities, is the necessary check afforded by the summing of the percentages of its constituents.

To distil the silica in the form of silicium fluoride, and then to determine it either as silica or as potassium fluosilicate, suggested itself as a means by which this check might be secured. A long series of experiments, undertaken in order to ascertain the best process for thus determining the silica, has resulted in a completely successful application of the method; and by it so small an amount as two-tenths of a gramme of enstatite or of augite has been analysed with a satisfactory result in the Laboratory at the British Museum.

II. *On a new Method of analysing Silicates that do not gelatinise with Acids.*

The method adopted for the analysis of silicates in small quantity, to which reference has already been made, was the following.

Hydrogen fluoride formed from picked fluor-spar was conducted into water. When the saturation had reached the point at which the liquid gave off fumes, the acid solution was treated in a platinum dish with potassium fluoride so long as any precipitate was formed. After decantation the acid was distilled, the first and last portions being omitted, and the distilled acid preserved in a platinum bottle. A leaden bottle, even when lined with pure gutta percha deposited from its solution in benzole, appears to be attacked by the acid.

A small platinum retort of a capacity of 30 cub. centims. (fig. 1) has fitted into it a tubulated stopper (fig. 2) reaching nearly to the bottom of the retort; a small tube (*b*) enters the straight tube (*k*) of the stopper (*a*) at an angle above the neck of the retort, for the delivery of hydrogen. The straight tube can be stopped either by a small platinum

Fig. 1.

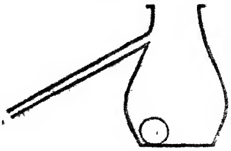


Fig. 2.

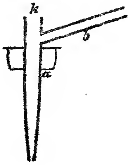


Fig. 3.

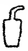
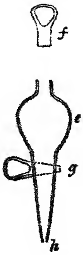


Fig. 4.



stopper (fig. 3), or by a funnel of that metal (fig. 4) with a stopper (*f*) at the top and a fine orifice at its lower extremity (*h*).

In the side of the retort just below the neck a straight delivery-tube is fixed, which again fits into another platinum tube (fig. 5) that, after taking a curve into a vertical position, is enlarged into a long cylinder capable of passing nearly to the bottom of a test-tube. The test-tube, into which it is fitted by a cork, holds when properly charged 7·5 cub. centims.=6·6 grammes of a strong solution of ammonia (of specific gravity =0·88), corresponding to 2·03 of H_3N, and a glass delivery-tube passes to the bottom of another test-tube, also containing a little of that alkali.

The mineral to be analysed is first powdered extremely carefully in an agate mortar; of this a quantity, that may be from 0·2 to 0·5 gramme, is introduced into the retort together with a small platinum ball. The tubulated stopper (fig. 2) is now introduced into its place and cemented by the aid of a little gutta-percha varnish, and by winding over its edge and that of the retort a strip of thin sheet india-rubber. The funnel, with its top closed, is now put into the tubulure of the stopper and filled with the hydrogen fluoride, which has a strength of about 32 per cent. of acid: it contains 1·12 grm. of pure hydrogen fluoride, capable of rendering gaseous 0·84 grm. of silicic acid (SiO_2), and of neutralising 0·95 of ammonia (H_3N). The acid is next admitted to the silicate, and the funnel removed to give place to the small stopper, which, and the joint of the platinum delivery-tubes, are now sealed with gutta-percha varnish. The apparatus, as prepared for use, is represented in fig. 6*. Pure dry hydrogen is next allowed slowly to traverse the whole apparatus, and the retort is placed in a water-bath at 100° C. for two hours, and occasionally shaken to set the ball in motion. During this operation only a minute trace of silicium difluoride comes over.

Fig. 5.

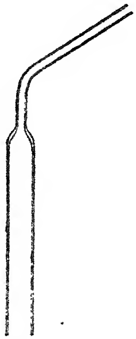
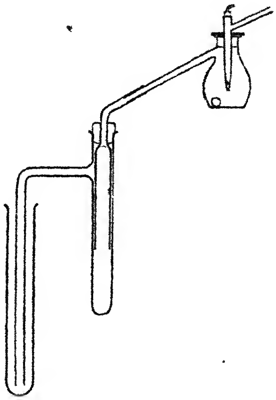


Fig. 6.



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[*] The scale of figs. 1 to 5 is one-half the actual size of the apparatus; that of fig. 6 is one-third the actual

The retort is next transferred to a bath of paraffin and carefully heated in it. At first hydrogen fluoride comes over; and at this point of the process the flow of the hydrogen requires a little attention. At about 132° C., with the silicates described in this memoir, the silica first becomes visible in fine flocks in the ammonia solution, and in another minute the whole is cloudy. In from five to ten minutes the temperature has risen to 142°–145°, and so much of the fluoride has come over that the contents of the tube are of a semi-solid consistency, and nearly the whole of it has in fact passed over. The temperature is allowed to increase to 150°, and the retort then permitted to cool. The process is repeated by introducing a fresh charge of hydrogen fluoride into the retort and of ammonia into the test-tube, and again heating in the paraffin bath. If the quantity of the silicate taken be not more than 0·2 grm., twice charging the retort is sufficient; if it amount to 0·5, three or four repetitions of the process are required. The process must in fact be repeated so long as any fresh flocks of silica can be seen to form in the ammonia tube. Finally, 0·75 cub. centim. of sulphuric acid are introduced into the retort, and the temperature raised to 160°, the stream of hydrogen being continued as before*.

The several ammoniacal charges of the ammonia tube are now brought together into a platinum dish with all the washings from the test-tubes and the connecting tubes; and these are now slowly evaporated in a water-bath with continual stirring.

At a certain point of the evaporation, just before the solution becomes neutral, and the ammonium fluoride begins to become acid, all the silica in the dish is dissolved by the fluoride. The process is gradual, but the moment is easily determined when it is complete. Then the dish being removed from the water-bath, potassium chloride is added in slight excess; and absolute alcohol equal in bulk to the liquid in the platinum vessel is poured on it. Potassium fluosilicate precipitates, and after standing twenty-four hours it is filtered and washed with a mixture of equal volumes of absolute alcohol and water, and then dried and weighed. The results are accurate.

In the platinum retort are the bases, in the form of sulphates, the treatment of which calls for no further remark.

A specimen of diopside, pulverised and analysed by the method here described, yielded 53·46 per cent.; on treatment by fusion with potassium and sodium carbonate, it gave in two analyses 53·51 and 53·54 per cent. of silicic acid.

III. *The Busti Aërolite of* 1852.

Among the meteorites with ingredients sufficiently large in the grain to offer an opportunity for isolating and determining their constituent minerals, is a stone that fell in India on the 2nd of December, 1852, near the station named Busti, situate about halfway between Goruckpur on the east and Fyzabad on the west, and consequently some 45 miles from Goruckpur, and nearly in 26° 45′ north latitude and 82° 42′ east longitude.

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* In a series of analyses made with a view to determine the degree of energy with which the acid attacks various silicates and the forms of silica itself, it was found that the first turbidity of the ammonia will, if sufficient time be allowed, commence at 120° C. At this temperature to 121° C. twenty-three minutes were requisite for the bulk of the silicium compound to come over from the retort. It was not found, however, that the action of the acid was more energetic on one silicate than on any other, or on quartz.

For the account of the circumstances attending the fall of this meteorite I am indebted to Mr. GEORGE OSBORNE, at that time Resident at the Busti Station, to whose care science owes the preservation of the stone that fell there. He presented it to the East-India Company, and for several years it stood in the Library of the India House. It was presented to the British Museum when Lord HALIFAX was Secretary of State for India, by the Secretary of State in Council. Mr. OSBORNE describes the fall as having taken place at 10^h 10^m A.M., announcing itself by a sudden explosion much louder and of a more detonating character than an ordinary thunderclap, increasing in intensity towards its termination. There was no trace of cloud in the sky, and the report lasted for a time that Mr. OSBORNE estimated at from three to five minutes. At Busti the report was not accompanied by the effects of concussion, while at Goruckpur it shook the glass and doors in the houses, the sound appearing at the latter station to approach in a direction from W.N.W. At Busti it seemed to one facing the north to come from the zenith; and though heard so loudly at Busti, and apparently still more loudly at Goruckpur, it was not noticed at a station thirty miles west of Busti. The course of the stone was probably a north-easterly to a south-westerly one, the explosion that shattered it having occurred soon after it had passed the longitude of Goruckpur. The stones fell at a place six miles south of Busti, and Mr. OSBORNE obtained a small one weighing about three pounds. How many fell was not ascertained, but all the others have been lost sight of, Mr. OSBORNE having in vain endeavoured to obtain a second.

The aspect of the specimen of this aërolite which Mr. OSBORNE preserved is in many respects very similar to the stone that fell at Bishopville, in South Carolina, U.S.A., on the 25th of March, 1843. The form and actual size of the stone are represented in Plate XXIII., in which two views from opposite points are given, the orientation being shown by the position of the letters A, B, C, D in the two views. The crust which coated the larger part of the stone was exceptional in character. At the flat end this crust was of a dark yellowish brown, with a few yellowish-white porphyritic-looking patches where the brown crust was thinner. In the large hollow portion on one side, near to C in the lower view, a yellowish enamel mingled with a very dark grey enamel is also relieved by white markings like the white felspar of a porphyry. In other places these white or yellowish-white markings, with their angular but unsymmetrical outlines, are seen sharply contrasted with the black-grey enamel only. It is difficult to connect the outlines of these porphyry-like markings with those of crystals of any mineral underlying them. Over augite and enstatite alike, where they occur in this stone, the crust seems to be similar in its features. I can only suppose the natural hue of the crust, due to the fusion of the silicates, to be a pale yellow; but that the metallic nickeliferous iron, found here and there in grains of considerable size, has, during the fusion and dispersion of the outer parts of the stone in its career through the air, fused down with portions of the silicates into a dark and perhaps more fusible enamel that has mixed unevenly as it flowed over the less fluid glass of the silicate. The enamel, it should be added, has generally vesicular appearances when seen under a high power, as if gases had escaped from it during its fusion.

One end of the meteorite presented a remarkable feature; small round chestnut-brown spherules, coated with yellowish-white fused silicate, stood out from a well-defined nodule that was imbedded in this portion of the stone near N in the upper view. In these small spherules there might in one or two places be seen, with a lens, minute octahedral crystals with the lustre and colour of gold. These two minerals seemed scarcely to have been affected by the heat that fused the silicates in which they are imbedded, and which protected their surfaces from the action of the atmospheric oxygen; but in one or two cases the crust at these spots was rendered darker in colour by the influence of the nodule it covered.

It was indeed fortunate that this meteorite came in its entirety into the British Museum. A blow from a hammer (the too usual fate of Indian meteorites) would have scattered the contents of this nodule as dust, for its peculiarities were only visible from the outside on very careful inspection. A section was made so as to pass through this nodule; it was then seen to be definitely bounded by a black line, within which two distinct silicates could be detected, and the polished surface was dotted with the round spherules of the chestnut-brown mineral that has been alluded to. A representation of this section parallel to the line N is given in Plate XXIII.

The powder produced in the cutting of the meteorite and a few fragments of the separated portion, too small for distribution to other museums, were retained for chemical examination, and slides of the different minerals were worked from some of these fragments for the microscope. A small nodule of the metallic iron was also preserved for analysis. From the fragments the different minerals were picked out under the microscope, and among these a few specimens were found sufficiently complete to throw light on their crystallography.

IV. *Oldhamite—Sulphide of Calcium.*

I gave the name of Oldhamite to this mineral in 1862, when it first attracted my attention, though I had not then the opportunity of properly investigating it*.

Oldhamite is a pale chestnut-brown and, where pure, transparent mineral occurring in the Busti aërolite, and apparently also sparsely in that of Bishopville, in small nearly round spherules imbedded in enstatite or augite, or in a mixture of both. The outer surface of the spherules is generally partly coated by calcium sulphate, the result of the oxidation of the sulphide. When the adhering silicate and this crust have been removed, the mineral is readily cleaved in three directions. The mean of nearly 200 measurements of the normal angles of these cleavage-planes gave 89° 57′. That this angle is really 90° and the mineral cubic in its system, rendered probable by the equal facility of the three cleavages, is placed beyond doubt by the fact that transparent sections made along either plane; when examined by polarised light, afford no indications whatever of double refraction. The hardness of the mineral is nearly 4; its density is 2·58. Boiled with water it breaks up, yielding a bright-yellow solution of calcium polysulphides and an insoluble residue. With acids it readily dissolves with evolution of hydrogen sulphide and deposition of sulphur.

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* I named it in compliment to Dr. Oldham, Director of the Indian Geological Survey, who in that year acted on behalf of the Asiatic Society of Calcutta on the occasion of that Society giving, in the most liberal spirit, to the British Museum large portions of several important aërolites that had fallen on Indian territory, and were preserved in its Museum.

The small amount of the mineral at command for analysis, the desirability of excluding as far as possible during the process all non-volatile materials from admixture with the constituents of the meteorite, and, as it afterwards proved, the unnecessary precaution of not using any reagent that might destroy or prevent the subsequent selection of the microscopic octahedra that have been alluded to, seemed to render a special method of analysis necessary.

Experimental analyses of calcium sulphide, formed by passing first hydrogen and subsequently hydrogen sulphide over caustic lime ignited in a glass tube, led to the employment of the following method[*].

0·4696 grm. of the mineral, dried over sulphuric acid for thirty-six hours, were placed in a small flask with a stoppered funnel filled with previously boiled but cold water. Hydrogen, purified by traversing a solution of lead acetate and a U-tube containing glass moistened by that liquid, was then passed by a tube through the cork into the flask, whence it was conducted by a delivery-tube into a solution of 3 grms. of pure silver nitrate, and thence, finally, by a second tube through a test-tube similarly charged. First a little of the boiled water and strong hydrogen chloride were introduced by the funnel tube, and the hydrogen sulphide formed was swept out by the hydrogen as fast as it was generated. The action having ceased, the flask was carefully heated for six or seven hours, when the hydrogen no longer discoloured lead test-paper. The silver sulphide was then thrown on a weighed filter, washed with ammonia and water, dried at 100°, and weighed. Some sulphur separated in the flask; this, together with the undissolved silicates and golden-yellow crystals, was collected on a dry filter and weighed, and the sulphur was then removed by carbon disulphide, and its amount determined by the difference of weight. The yellow crystals were picked out from the undissolved residue, and there remained the silicates that had surrounded or been entangled with the spherules of Oldhamite.

The results of the first analysis are given below as No. I.; those of a second, in which 0·5061 grm. of the mineral in rather large spherules were taken, are given as No. II.

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[*] This calcium sulphide, in colour and in all but its want of cohesion and crystalline characters, resembles Oldhamite. Thus formed it contained 44·3 per cent. (it should contain 44·44) of sulphur. Formed without the previous treatment with hydrogen it contained only 39·5 per cent.

TABLE I.

	I.			II.		
Undissolved silicate	—	—	7·644		—	8·456
Octahedra (Osbornite) . . .	—	—	0·277		—	0·297
Dissolved silicate (enstatite). .	SiO_3 . Mg . .	1·159 0·366	1·525		0·875 0·276	1·151
Residuary magnesium (as magnesium monosulphide). . .	Mg . . S . .	1·257 1·676	2·933		1·229 1·638	2·867
Calcium monosulphide . . .	S . . Ca . .	35·888 44·860	80·748		35·227 44·034	79·261
Calcium sulphate (gypsum) . .	Ca . . SO_4 . (H_2O .	0·830 1·993 0·747)	3·570		0·856 2·054 0·770	3·680
Calcium carbonate	Ca . . (CO_3 .	1·241 1·861)	3·102			
Iron sulphide (Troilite) . . .	. . . . .	. . . . .		Fe . S. .	1·288 0·736	2·024
Iron (probably metallic) . . .	Fe		0·507	. . . . . .		0·261
			100·306			97·997

The dissolved silicate, as will be seen in the sequel, is most likely to be enstatite, that mineral being the more soluble of the two silicates in which the Oldhamite is imbedded. That a portion of the magnesium is present as sulphide is a necessary conclusion from the proportions of sulphur and of metal in the spherules. I have assumed the sulphuric acid to correspond to the ordinary hydrated calcium sulphate (gypsum), and the residuary calcium in analysis I. to be present as calcium carbonate. The result of this interpretation of the above analyses is that, if we deduct the silicates encrusting the Oldhamite together with the Osbornite and the iron that the spherules contain, we have the following composition for these spherules and their oxidised coating.

TABLE II.

		I.		II.	
			per cent.		per cent.
Oldhamite . .	Calcium monosulphide . . .	80·748	89·369	79·261	90·244
	Magnesium monosulphide . .	2·933	3·246	2·867	3·264
Incrustation . .	Gypsum	3·570	3·951	3·680	4·189
	Calcium carbonate	3·102	3·434		
	Troilite	—	—	2·024	2·303
		90·353	100·000	87·832	100·000

The magnesium sulphide may be looked on either as a mechanically mixed ingredient or as a constituent of the Oldhamite*.

The existence of either of these substances in a meteorite serves to prove that the conditions under which the ingredients of that rock came into their present form were very unlike those met with on the surface of our globe. Water and free oxygen must alike have been absent, or only present in inappreciable quantities; indeed the existence of the metallic iron in the state of minute division in which it so frequently occurs in meteorites would lead to a similar conclusion. But the evidence afforded by the aërolite of Busti seems, further, to point to a reducing agent having been present during the formation of its constituent minerals; while the crystalline structure of Oldhamite, and of the Osbornite next to be described, must certainly have been the result of fusion at an enormous temperature.

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* In the second analysis the deficiency in the percentage may have been due to some small error in the determination of the lime; and any such small error, where the quantity of the mineral disposable for the analysis is so minute, becomes magnified considerably on calculating the percentage results.

The detection of hydrogen by Professor GRAHAM in meteoric iron tends to confirm the probability of the presence of a reducing agent among the conditions under which these meteoric minerals were formed.

V. *Osbornite.*

The golden-yellow microscopic octahedra that have been mentioned in the description of Oldhamite were furnished by the analysis of that mineral to the amount of only 0·0028 grm., the first analysis having yielded 0·0013, and the second 0·0015 grm. Yet even this minute weight, forming less than 0·3 per cent. of that of the Oldhamite, was divided between upwards of 150 crystals. These crystals were nevertheless capable of being measured by the goniometer.

This microscopic mineral I wish to name Osbornite in honour of Mr. OSBORNE, and in order to commemorate the important service that gentleman rendered to science in preserving and transmitting to London in its entirety the stone which his zeal saved at the time of its fall, and in recording all he could collect about the circumstances associated with that fall.

That the octahedra of Osbornite are "regular" octahedra will be apparent from the following results of their measurement. A supplementary lens applied to the object-glass of the telescope of the goniometer enables the observer to determine that position of the face of a crystal in which the illumination from a narrow slit in a distant window is at its maximum. In this way the angles between faces can be measured when the faces themselves are too small, or are too dull, or too much striated for use as reflectors of the image of the slit.

This approximate method of measuring the angles of so minute a crystal, when used with some crystals of Osbornite, gave for:

					Regular octahedron.
111	$11\bar{1}$	a mean of fifteen	measurements on two crystals	70° 27′	70° 31′
111	$1\bar{1}\bar{1}$	nine	measurements on ditto ditto	109° 31′	109° 28′
111	$1\bar{1}1$	six	measurements on ditto ditto	69° 58′	
111	$11\bar{1}$	six	measurements on a third crystal	70° 37′	

The analysis of this mineral presented a very difficult problem, the total amount available being too minute for any quantitative results to be expected from it. Moreover it was found that these little crystals resisted the action of the acids when in their integrity, and when crushed their minute size rendered the manipulation most difficult and the results uncertain. Boiled for a long while in nitric acid they were unchanged; and even hydrogen fluoride had no apparent action on them. They passed unscathed through a fusion of a small amount of Oldhamite with potassium-sodium carbonate; though when fused with potassium chlorate, a crystal of Osbornite entirely disappeared—perhaps from its escaping notice.

The crystals are very brittle, and after being crushed the powder retains the beautiful golden colour of the surface, which is therefore intrinsic and not due to a tarnish; to which cause, however, a ruddy hue in two of the crystals may have been due.

A few of the crystals placed in a glass tube and ignited in a current of dry oxygen, underwent only an external oxidation. Similarly treated in a slow current of chlorine they were decomposed; but they sank into the glass before the decomposition was complete. An experiment was therefore made in which the crystals were supported in a cavity scooped into a small fragment of the thinnest Japanese porcelain. The chlorine was passed through the apparatus and the tube ignited previously to the introduction of the octahedra, in order to determine that the chlorine was without action on any part of the apparatus. This apparatus consisted of a small tube in which the porcelain splinter was placed; one end of it was drawn out so as to form a U tube of considerable length, surrounded at the bend by a freezing-mixture, and dipping ultimately into a tube of pure water.

The whole of the available material, except a few crystals reserved for the goniometer, was placed in the cavities of the porcelain splinter, and dry chlorine allowed slowly to traverse the tube. On the application of heat shortly below redness, a glow was seen to commence among the minute crystals, which, extending itself through the whole, lasted for a few seconds.

The crystals appeared to have somewhat increased in bulk; they still retained their forms, but their metallic lustre had left them, and their colour became of a pale honey-yellow.

The tube had become slightly iridescent in front of the assayed mineral; the drawn-out portion of it contained a small amount of a white sublimate, and a slight fuming came with each chlorine bubble through the water.

The altered crystals on being exposed to the air soon began to deliquesce and assumed a pasty consistence; treated with water, they only partially dissolved and gave the solution an alkaline reaction. This solution gave no precipitate with ammonia, but yielded one to ammonium oxalate. The residue undissolved by this water dissolved, though neither with ease nor quite completely, in hydrogen chloride.

This acid solution gave with ammonia a slight precipitate, which was redissolved in acid and reprecipitated: it seemed to contain a little iron. Filtered from this, the solution further gave a very distinct precipitate on being treated with ammonium oxalate and kept for some time warm. Hydrogen disodium phosphate gave a very slight flocculent turbidity to the filtrate from that precipitate, but it had not the appearance of the magnesium salt.

The interior of the portion of the tube drawn out and kept cool at the bend by a cooling-mixture was lined by a slightly yellowish-white sublimate.

This white body was treated with hydrogen chloride, in which it was at first somewhat difficult of solution, and was added to the water into which the chlorine had been passed. That liquid, which was acid in its reaction, was evaporated down as a clear solution till it had become but a few drops, and barium chloride was added to it. A precipitate of barium sulphate was formed, which after some time was filtered off and weighed. It weighed 0·0008 grm. There can be no doubt, therefore, that Osbornite contains sulphur as an important constituent. The excess of barium was removed and ammonia added, which threw down a very decided flocculent nearly white precipitate. The filtrate from this precipitate left no visible residue on evaporation. The precipitate itself readily redissolved in hydrogen chloride, and was again reprecipitated by ammonia; it resembled alumina in appearance, but it proved to be entirely insoluble in potash. The experiment, more than once repeated, of redissolving and reprecipitating it, as before, invariably gave a body insoluble in potash.

Dissolved again in acid and all excess of the acid having been removed, the addition of sodium hyposulphite produced, on warming it, a white precipitate. Restored to its former condition of as nearly neutral a solution as possible, on being treated by an excess of potassium sulphate, it gave a white precipitate.

The insolubility of the ammonia-precipitate in caustic potash having pointed to the probability that either titanium or zirconium oxide was present, some preliminary comparative experiments were made which led to the following method.

Three small glass tubes, similar in all respects, were taken, and into one a portion of the precipitate formed by ammonia from the Osbornite was transferred; into a second a quantity to appearance rather less of titanium oxide, formed in a similar manner by solution in acid and reprecipitation by ammonia, was put; and the third was similarly charged with zirconium oxide, formed in as similar a manner as possible by passing chlorine over a heated mixture of zirconia and charcoal, and treating the sublimate as the mineral sublimate had been treated*.

Hydrogen chloride and water were next added in equal amounts to each tube. A minute bit of magnesium wire was dropped into each, and the changes were watched under the microscope with an inch objective. After a certain period black patches began to appear on the magnesium wire in the titanium oxide tube, and a bluish coloration could be faintly discerned: in the tube with the precipitate from the Osbornite the magnesium retained its silvery brightness entirely unstained, as did the wire in the zirconia tube. Repetition of the experiment confirmed the delicacy of this test for the presence of titanium oxide, a test tried by which the precipitate from the Osbornite failed invariably to show any evidence of the presence of that oxide. Phosphorus was carefully looked for, but ammonium molybdate failed to give any trace of an indication of its presence.

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* It is remarkable that in this experiment the same internal iridescence of the glass tube was observed as in the experiment with the mineral, and the chloride sublimed in the same manner and with the same appear-

A negative test of the above kind would not afford sufficient ground for asserting the presence of zirconium; on the other hand, it would seem a fair presumption that titanium at least is not present in Osbornite. That the metal to which these reactions are attributable is not zirconium, however, may be affirmed with some certainty. Mr. Sorby has made the pyrognostic characters of this element a special study; and I gave that gentleman rather more than half of the minute amount I possessed of the precipitated oxide.

He examined it in a microscopic borax bead, and asserts that he failed to obtain the crystals characteristic of zirconium, and that the chief and probably the only constituent of this substance is titanic acid, as the crystalline deposit in the bead exactly accords with the very peculiar forms assumed under the same conditions by that oxide.

With so infinitesimal an amount of substance at one's disposal it seems impossible to investigate further the nature of this element. Even the methods of the spectrum-analysis are not yet reduced to a form available for determining the nature of an element of this group in so minute an amount. That it is not zirconium the evidence of so accurate an experimentalist as Mr. Sorby may be taken to prove; that it is titanium seems scarcely compatible with the comparative experiments I made with it and with titanium chloride. It is certain, however, that Osbornite consists of calcium, and what may be provisionally termed a titanoid element, possibly titanium itself, with a trace of iron and combined with sulphur in some peculiarly stable form. This form can hardly be that of a combination of the sulphides of the metals merely; one cannot well conceive such a compound resisting the action of acids. The sulphides of the metals of this class, however, are little known; while those of calcium associated with oxygen in the form of what are termed oxysulphides need also further investigation.

The fact of the Osbornite crystals being met with occasionally in the variety of augite, which will be presently described, as an ingredient of this meteorite, and which is for the most part confined to that nodule of the meteorite in which the Oldhamite occurs, suggested a search in that silicate for an oxide corresponding to what had been found in the Osbornite. The precipitates thrown down by ammonia from the acid solution of the bases in the different analyses of that augite were therefore brought together and examined. They contained some ferric oxide; but this was associated with a small amount of a colourless oxide entirely insoluble in potash, which, when tried by the tests that had been employed with the Osbornite precipitate, gave exactly the same results as these had given. The dichroism of this augite is very marked; and on looking through one of its faces (the face 0 1 0) a tint (like that of the bluish anatase from Brazil) is seen that appears to be due to certain minute interlaminated layers permeating the augite, but which require the microscope for their exhibition.

In whatever manner, whether as a constituent base in the augite itself, or as a foreign body interlaminated with it in the direction of the planes 0 0 1, 0 1 0, the evidence of the augite goes to confirm that of the analysis of the Osbornite, namely, that a metal nearly related to, if it be not titanium is present in both minerals. Possibly the minute interlaminated mineral alluded to may consist of Osbornite of sufficient thinness to be transparent, and to give the colour alluded to. It is remarkable that the metallic sheen on the plane 1 0 0 of the augite is of a golden yellow by reflected light, and exhibits the bluish tint by transmitted light.

It may not be out of place here to call attention to a singular golden-yellow incrustation, cubic in the form of its particles, obtained by Professor Mallet, of Alabama University, U. S. A., by heating metallic zirconium to an intense heat in a furnace with lime and aluminium. These crystals were not analysed, but it is not impossible that sulphur from the fuel might have supplied that ingredient, and that these crystals were in their nature analogous to those revealed to us in this meteorite, for like the Osbornite crystals they were not attacked by the strongest acids (see American Journal of Science, Series 2, vol. xxviii. 1859, p. 346).

VI. *The Augitic Constituent of the Busti Aërolite.*

Associated with the spherules of calcium sulphide that have been described as occurring in a nodule in this aërolite, and also less plentifully distributed through the rest of its mass, is the silicate, to which allusions have already been made as a variety of augite, and as containing traces of an element with some of the chemical characteristics of titanium. This silicate is of a pale violet-grey colour, intimately mixed in the form of crystalline grains with another silicate presently to be described. These crystalline lilac-grey grains, when isolated as much as possible from the other minerals, present a few crystal-faces, among which one as a cleavage-plane is prominent. The rest are very imperfect; and it is extremely difficult to get any measurements that are at all reliable from them. The goniometrical observations, however, were sufficient, together with the optical characters of the mineral, to determine that it belonged to the oblique system. These measurements gave the following approximate values:—

			Those of diopside being
001	100 =	about 75° 30′	73° 59′
001	110 =	„ 81	79 29
110	100 =	45° 54′ to 47 26	46 27
110	$\bar{1}10$ =	85 8 „ 86 20	87 5
100	111 ?=	53 25 „ 54 15	53 50
001	$\bar{1}10$ =	100 8	100 57

A slide cut for the microscope from a fragment of the nodule was found to exhibit a section of one of the crystals of this mineral cut very nearly parallel to the plane of symmetry. Two of the edges bounding this section were parallel, the one to a series of lines running through the crystal corresponding to its cleavage-planes, the other to certain bands that are constantly present in this augite, generally parallel to the plane 0 0 1, and formed of a white doubly refracting silicate, no doubt of the enstatite next to be described, intercalated in microscopic layers through the augite. These two edges represent the planes 1 0 0 and 0 0 1 as seen in a section nearly parallel to the plane of symmetry. They gave a normal angle of 0 0 1, 1 0 0=75° 15′. In diopside this angle is 73°·59.

Light traversing this section of the crystal between crossed Nicols is at its maximum of extinction when polarised in a plane parallel or perpendicular to a line, making with 0 0 1 an angle very near to 22° 45′, and with 1 0 0 an approximate angle of 52° 30′. In diopside the second mean line makes corresponding angles of 22° 5′ and 51° 6′ with these normals. These measurements were made by an eyepiece goniometer fitted to the microscope, and having a fixed spider-line to indicate the plane of polarisation, while a rotating line is employed to measure on a graduated circle the inclinations of the edges and other directions in the section.

A section made parallel to the plane 1 0 0 exhibited one of the optic axes on the limit of the field of view in a Nörremberg's polariscope. The plane containing the optic axes is perpendicular to the edge [1 0 0, 0 0 1], and the optical character in the centre of the field is negative.

In all the above respects the mineral accords with diopside.

When looked through in a direction nearly normal to 0 0 1 or 0 1 0, or indeed in any direction parallel to the zone circle [0 0 1, 0 1 0], the crystals show a remarkable dichroism, which is, however, especially conspicuous when the direction is nearly normal to the plane 0 0 1.

If the section is parallel to the plane of symmetry, light polarised in a plane perpendicular to the principal section containing the acute mean line and the axis of symmetry is transmitted, of a pale pink lilac; when the crystal is turned 90°, so as to bring the same principal section into parallelism with the plane of polarisation, the transmitted tint is bluer, exhibiting a pale slate-blue or lavender.

The plane 1 0 0 presents a somewhat facile cleavage, much more readily obtained than cleavages which are also met with on the planes of the form 1 1 0, the latter being interrupted and uneven. The plane 1 0 0 is also conspicuous for a remarkable metallic lustre, recalling that seen on some kinds of diallage, but of a fine golden hue.

Two analyses of this mineral by the method already described, the silica being distilled as silicium difluoride and determined as potassium fluosilicate, gave the following results:—

	I.	II.	Mean.	Oxygen ratios.
Silicic acid . .	55·389	55·594	55·491	29·28
Magnesia . . .	23·621	23·036	23·328	9·331
Lime	20·02	19·942	19·981	5·709
Iron oxide . .	0·78	0·309		
Soda	0·554	[0·554]		
Lithia	a trace	[a trace]		
	100·364	99·435		

Viewed as a magnesium and calcium silicate the percentage composition becomes—

		The formula ($\frac{5}{8}$Mg $\frac{3}{8}$Ca) Si O_3 requires
Silicic acid . .	56·165	56·604
Magnesia . . .	23·612	23·585
Lime	20·223	19·811
	100·000	100·000

This does not accord with the analyses of the ordinary varieties of augite, in which the calcium is usually in excess of the magnesium.

It is, however, to be observed that a small deduction of the corresponding magnesium silicate (enstatite) has to be made by reason of the presence of the white mineral intercalated in layers along the direction parallel to the plane 0 0 1, and sometimes also to a second plane of the crystal. This mineral is doubtless the enstatite next to be described, and its presence would only modify the true formula of the augite by adding to the proportion of the magnesian constituent. The amount of one equivalent of enstatite to three of augite that this explanation would require, is more than microscopic observations would warrant; and it is probable that the augite itself is richer in magnesium than is usual in terrestrial augites.

The small amount of the oxide that in this augite corresponds to the ingredient of Osbornite that I identify with a titanoid metal, is met with in the precipitate by ammonia from the solution of the bases, and is included with the iron oxide in the above analyses.

VII. *Enstatite as a Constituent of the Busti Meteorite.*

Besides the augitic mineral that has just been described, there is present in this meteorite another silicate which is in fact its most important ingredient. The augite is present in greatest quantity in the nodule that contains the calcium sulphide, though it is met with in smaller amount in the other parts of the meteorite. But associated with it everywhere, and otherwise forming the mass of the stone, is the mineral I have next to describe. As seen in a microscopic section, it presents the appearance of a number of more or less fissured crystals with different degrees of transparency, sometimes quite clear, sometimes nearly opaque, and with a more or less symmetrical polygonal outline. These crystals are imbedded in a magma of fine-grained silicate, through which a sort of irregular meshwork of an opaque white mineral is seen to ramify. When the ingredients are mechanically separated and examined, it is not difficult to distinguish what seem to be three different minerals. One is rare; it is colourless and transparent, and may be obtained in small splinters that have the appearance of being the result of a definite cleavage. The little planes thus obtained are too often merely divisional surfaces without crystallographic significance; and where they possess a more definite character, they present such rude faces that the values obtained for the angles can rarely be relied on. Another form of the mineral mass is that of a grey semitransparent splintery mineral, the fragments being generally very composite. From these two varieties I failed in obtaining the measurements of an entire zone, the planes in which belonged to the same individual, and the attempt to cleave these minute individuals apart only serves to destroy them. The third form is that of a dark grey glistening crystalline substance tabular in form and very opaque. It presented cleavages indistinctly marking the faces of a prism, for which the mean of several measurements gave an angle of $\left\{\begin{matrix}88^\circ\ 35'\\ 91^\circ\ 25'\end{matrix}\right\}$; and to the planes (1 1 0) of this prism a dull face (0 0 1) is perpendicular, which seems in this case to be a second and less facile cleavage.

The results subjoined were obtained from seven selected fragments of the other forms of this mineral. They lack the important check which the polariscope affords; for the substance was usually too opaque for the use of this instrument, or else too composite to give any value to the results obtained with it. The fragments experimented upon were extremely minute and fragile, often breaking into powder while being mounted for the goniometer, and the angles are necessarily only approximate.

		Found.	In Breitenbach enstatite.
1 0 0	1 1 0	about 46°	45° 52′
1 1 0	$\bar{1}$ 1 0	87 10 to 88	88 15
1 0 0	1 0 1	41 34	41 12
0 1 0	0 1 1*	about 40	40 21

The planes 1 0 0 and 1 1 0 are cleavages. In some cases, generally where the crystals are very composite, a cleavage seems to run parallel to a plane inclined at 73° to 74° to the face (1 0 0) and 90° to the face (0 1 0). As the forms of the mineral presenting this plane contain calcium, I have been uncertain whether to attribute the importance of this plane in certain specimens to an intermixture of augite with the enstatite. The plane 1 0 4 is also a conspicuous one on the crystals of enstatite in the Breitenbach meteorite, and the angle (74° 4′) which it makes with the plane 1 0 0 in that mineral is very near that of the inclination (73° 59′) of the planes 1 0 0 and 0 0 1 of diopside.

The chemical analysis of these three minerals shows that they are really enstatite under different aspects. Where the substance contains no lime it presents itself as a simply prismatic mineral, the dark grey tabular variety; where lime is present, though to the amount of less than 2 per cent., the crystalline structure becomes more complex, and it is far more difficult to obtain pieces in which it is sufficiently definite in character to allow of any measurements at all. It seems probable that the augite is in these cases blended in minute quantity, by a sort of tesselation, with the enstatite, somewhat as the enstatite is seen to be intercalated in narrow bands between layers of the augite already described, although the enstatite in the latter case is in much larger relative amount. But I have failed to obtain satisfactory proof of the actual presence of the augite from the optical characters of the sections of the mineral as seen in the microscope; though these frequently exhibit a structure in a high degree composite in its crystalline characters, the principal sections of the different parts of the mineral being in these cases disposed at all sorts of angles of mutual inclination. The analysis of these minerals yielded the following numbers:—

---

* A dubious plane.

	Dark grey tabular variety.		Transparent white variety.		
	Percentages.	Oxygen ratios.	Percentages.	Oxygen ratios.	
Silicic acid . .	57·597	30·718	58·437	31·166	
Magnesia . .	40·64	16·238	38·942	15·564	=16·043
Lime	——	——	1·677	0·479	
Iron oxide . .	1·438		1·177		
Potash . . .	0·394		0·332		
Soda	0·906		0·357		
	100·975		100·922		

Semitransparent grey variety.

	I.			II.			III.		
	Percentages.	Oxygen ratios.		Percentages.	Oxygen ratios.		Percentages.	Oxygen ratios.	
Silicic acid .	57·037	30·419		57·961	30·912		57·754	30·802	
Magnesia .	40·574	16·217	16·8	39·026	15·598	16·0	38·397	15·347	16·01
Lime . . .	2·294	0·655		1·524	0·435		2·376	0·678	
Iron oxide .	0·867			0·154			0·423		
Soda . . .	——			0·68			0·657		
Potash . .	——			0·569			0·569		
Lithia . .	——			——			0·016		
	100·772			99·914			100·192		

The greater part of the soda and part of the potash in these analyses, as in those of the augite, is certainly due to an impurity traceable to a minute amount of these bases contaminating the hydrochloric acid employed. The iron is present partly as metallic iron in a state of minutest subdivision, in small part also without doubt in combination in the magnesian silicate. In every case the bases are slightly in excess of the amount requisite for the formula of enstatite. It would seem highly probable, from the comparison of these with the known analyses and with such as I shall have to offer of other meteorites, that where in these bodies the conditions under which the rock was formed were such that the silicic acid was in excess of that required by the formula for enstatite, it has remained uncombined in the form of a crystallised silica with the specific gravity of *fused* quartz; but that where the magnesium and other bases were in excess, a basic silicate with the formula of olivine absorbed the supplementary portion of these bases. Where calcium is present, it probably converts into an augite a portion of the materials that otherwise would go to constitute enstatite.

In none of the particular meteorites hitherto examined in the Museum Laboratory has a trace of alumina been found, though it has been carefully looked for, and consequently no felspathic ingredient has been detected in them.

VIII. *General Analysis of the Busti Meteorite.*

In order to determine approximately the proportions in which the different ingredient minerals were present in the meteorite, and to ascertain whether any other mineral had escaped detection, an analysis of the fragments and dust of the stone from the neighbourhood of the nodule containing the sulphides and the augite was made. The material employed was that obtained on cutting the meteorite by a dry wheel-saw, used to prevent the introduction of foreign substances. 1·874 grm. were taken for analysis. The sulphur was determined, as in the case of the Oldhamite, as sulphide of silver, and as separated by means of carbon disulphide. Heated with hydrogen chloride, and afterwards with potash, there was dissolved by those reagents and by the carbon disulphide 16·873 per cent.,

disulphide	16·873 per cent.,
the residue being	83·127
	100·000

The soluble part gave the results in column I., the insoluble part those in column II.; the sulphur and sulphuric acid being supposed to be present as calcium sulphide and sulphate respectively.

	I.	Oxygen ratios.		II.	Oxygen ratios.	
Calcium sulphate . . .	0·442					
Calcium monosulphide . .	4·133					
Iron oxide	0·194			0·891		
Silicic acid	6·514	3·474		46·357	24·727	
Lime	0·022	0·006	2·073	12·375	3·535	12·974
Magnesia	5·055	2·02		23·266	9·299	
Potash	0·099	0·017		0·14	0·023	
Soda	0·118	0·03		0·455	0·117	
Lithia	——	——		0·019	0·01	
	16·577			83·503		

The insoluble part in this analysis would correspond to a composition $Si\,O_3\,(Mg_{\frac{3}{4}}\,Ca_{\frac{1}{4}})$, which, if we consider the calcium as being present as a constituent of the augite and the formula of this mineral to be $Si\,O_3\,(Mg_{\frac{5}{8}}\,Ca_{\frac{3}{8}})$, will give for the insoluble silicate of the rock in the neighbourhood of the nodule a composition of two equivalents of augite to one of enstatite. As the analysis of the soluble portion showed that some of the above minerals had been dissolved, it was thought advisable to determine what and how much of them were rendered soluble by the action of hydrogen chloride in the cold. For this purpose some fresh material, selected partly from the neighbourhood of the nodule, partly from a portion of the meteorite consisting entirely of silicates, was submitted to the action of a mixture of one of hydrogen chloride and two of water for sixty-six hours at ordinary temperatures. Some sulphuretted hydrogen was given off and 4·419 per cent. dissolved. By a further treatment of the insoluble portion with soda 1·204 per cent. was removed from it. The acid had dissolved 0·501 per cent. silica, an amount of calcium corresponding to 1·285 per cent. of Oldhamite, 1·896 per cent. of magnesia, and 0·564 per cent. of iron. The oxygen ratios of the silica and magnesia dissolved are as 0·91 to 0·758, and show that of the 3·601 per cent. magnesian silicates extracted about 2·65 per cent. was olivine, the residue being enstatite*.

A formula for the augite rather richer in lime would no doubt give a truer statement of the composition; but it is as impossible to separate the small amount of enstatite intercalated in the layers of the augite, as it is to distinguish and remove the latter mineral from the enstatite with which it appears in general to be so intimately blended.

IX. *The Action of Acids on the Enstatite and Augite.*

As it appeared of some importance to determine the degree to which these meteoric minerals were soluble in the acids used for separating the silicates of a meteorite, and whether an olivinous constituent could be found in the Busti aërolite associated with the enstatite, or with some other silicate, the augite and the enstatite described in the previous sections were submitted to this solvent action. Alternately digested for many hours at 100° C. in strong hydrogen chloride, diluted with its volume of water and in caustic potash for ten or twelve hours to remove the separated silica, each of the three forms of enstatite proved to be acted upon; and the results in each case showed that the acid exercises simply a solvent action upon the mineral without separating it into two or more distinct silicates.

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* In 1863 Mr. William Dancer analysed some of this powder from the cutting of this meteorite in the Laboratory of Professor Bunsen at Heidelberg.

The results, which he was so good as to place in my hands, were as follow:—

SiO_2 = 52·73
MgO = 37·22
FeO = 4·28
MnO = 0·01
CaO = 1·18
NiO = 0·78
$CaSO_4$ = 1·58
Ca_3PO_4 = trace
$CaCl$ = 0·01
NaS = 0·76
KO = trace
LiO = trace
HO = 0·92
99·47

"The mass of the stone," he says, "is evidently a monosilicate of magnesia, lime, potash, and lithia; the iron and nickel existing in small particles as nickel-iron together with a small portion of manganese. The percentage of substance soluble in water is 1·03; this consists of sulphide of sodium, chloride of calcium, sulphate of lime, and traces of lithia and potash." Of course from such material as was in Mr. Dancer's hands it was impossible for him to separate the different minerals.

The subjoined Table records these experiments. The different degrees in which the acid dissolved the minerals in either case was due to the more or less complete character of the trituration to which the minerals had been subjected.

I deemed it desirable in one case (and I selected the transparent variety for this purpose) to repeat the process three times so as to remove any doubt as to the nature of the action exercised by the acid.

Of the greyish-white variety, 1·0686 grm. was submitted to the action of acid for twenty hours at 100°, and subsequently to that of potash, to remove separated silica, for twelve hours. It yielded

$$\begin{array}{llr} & \text{per cent.} & \text{per cent.}\\ \text{Mineral dissolved}=0{\cdot}1006\text{ grm., viz.}\left\{\begin{array}{ll}\text{in acid,} & 0{\cdot}0475=4{\cdot}445\\ \text{in potash,} & 0{\cdot}0531=4{\cdot}969\end{array}\right. & \left.\right\}= & 9{\cdot}414\\ \text{Mineral unacted on}=0{\cdot}968\ \ldots\ldots & \ldots\ldots\ldots = & \underline{90{\cdot}586}\\ & & 100{\cdot}000\end{array}$$

The analysis of the 9·414 per cent. dissolved is given in column I. of the Table.

Of the grey tabular variety of enstatite 0·1478 grm. were treated by hydrogen chloride for sixteen hours at 100°, and subsequently by potash for a similar time.

$$\begin{array}{llr} & \text{per cent.} & \text{per cent.}\\ \text{The dissolved portion was }0{\cdot}0115\text{, viz.}\left\{\begin{array}{ll}\text{by acid,} & 0{\cdot}0047=3{\cdot}179\\ \text{by potash,} & 0{\cdot}0068=4{\cdot}6\end{array}\right. & \left.\right\}= & 7{\cdot}779\\ \text{The residue unacted on was }0{\cdot}1363\ \ldots & \ldots\ldots\ldots = & 92{\cdot}19\end{array}$$

The 7·779 per cent. dissolved gave on analysis numbers the approximate composition of which is given in column II.

Of the white variety of the enstatite 0·2082 grm. yielded on a *first treatment* for twenty hours with acid, and subsequently with potash,

$$\begin{array}{llr} & \text{per cent.} & \text{per cent.}\\ \text{Dissolved mineral}=0{\cdot}0264\text{, viz.}\left\{\begin{array}{ll}\text{by acid,} & 0{\cdot}0164=7{\cdot}877\\ \text{by potash,} & 0{\cdot}01\ \ \ =4{\cdot}803\end{array}\right. & \left.\right\}= & 12{\cdot}68\\ \text{and undissolved}=0{\cdot}1818\ \ldots & \ldots\ldots\ldots = & 87{\cdot}319\end{array}$$

The approximate composition of the 12·68 of dissolved mineral is given in column III.

On a *second* treatment of the undissolved portion, whereof after two hours further trituration 0·1674 grm. were operated on as before with acid for thirty, and with potash for twelve hours,

$$\begin{array}{llr} & \text{per cent.} & \\ \text{The mineral dissolved}=0{\cdot}1137\text{, viz.}\left\{\begin{array}{ll}\text{by acid,} & 0{\cdot}0437=41{\cdot}766\\ \text{by potash,} & 0{\cdot}07\ \ \ =26{\cdot}074\end{array}\right. & \left.\right\}= & 67{\cdot}84\\ \text{Mineral unacted on}=0{\cdot}0539\ \ldots & \ldots\ldots\ldots = & 32{\cdot}16\end{array}$$

On a *third* treatment in a similar way 0·0424 grm. yielded:

$$\text{Of mineral dissolved}=0\cdot0217,\ \text{viz.}\left\{\begin{array}{ll} & \qquad\quad \text{per cent.}\\ \text{by acid,} & 0\cdot0091 = 21\cdot47\\ \text{by potash,} & 0\cdot0126 = 29\cdot71\end{array}\right\}=51\cdot18$$

$$\text{Of mineral unacted on}=0\cdot0207 \ldots\ldots\ldots\ldots = 48\cdot82$$

	I.	II.	III.
Silicic acid	5·408	5·141	6·724
Magnesia	2·367	1·353	4·61
Lime	1·048	0·270	0·432
Iron oxide	0·187	0·676	0·576
Potash	0·121	0·528	0·504
Lithia	——	trace	trace
	9·131	7·968	12·846
Soda found	0·126	1·217	1·042

In the last two treatments of the white silicate the quantities, 0·0437 and 0·0091 grm., of ingredients dissolved by the acid and 0·07 and 0·0126 of silicic acid dissolved by the potash were too small for even approximate analysis. The ratio of silicic acid to the bases, neglecting the small amount of the former dissolved by the acid, is in the last case in the ratio of 58·4 SiO_2: 42 bases, that of the analysis of this white variety giving a ratio of 58·4 : 41·6.

The degree to which the augite is soluble was determined by subjecting this mineral to a treatment similar to that by which the enstatite was dissolved; 0·2714 grm. so treated for eighteen hours by acid, and a similar time by potash at 100° C., gave 0·2614 of unchanged, and 0·0193 of dissolved mineral. This corresponds to 7·384 per cent. of the latter and 92·616 of the former.

There can be little doubt from these results that the action of acid on the minerals with the formula of enstatite or of augite is that simply of a solvent.

X. *The Iron of the Busti Meteorite.*

A small pepita of the iron, weighing 0·1997 grm., contained in the meteorite was analysed. A small quantity of silicates and of glistening Schreibersite was left at first undissolved by hydrogen chloride. A second treatment with acid dissolved the latter.

The first solution contained a trace of phosphoric acid, and a small quantity of hydrogen sulphide came off from the iron during its solution. The metallic constituents of the dissolved portion were separately determined, and an analysis was also made of the Schreibersite. The results were:

		per cent.
Silicates		16·725
Schreibersite:		
Iron	=0·736	
Nickel	=0·195	
Phosphorus	=0·07	
		1·001
Iron		79·069
Nickel		3·205
		100·000

or, omitting the silicate,

Iron-nickel alloy	Iron	94·949	98·798
	Nickel	3·849	
Schreibersite	Iron	0·884	1·202
	Nickel	0·234	
	Phosphorus	0·084	
		100·000	

The quantity was far too small to encourage a search for cobalt and other metals.

Besides the nickeliferous iron, which is disseminated very sparsely, and in particles singularly unequal as regards their size and distribution, and with which troilite is associated in very small quantity, chromite is present as a constituent of small but appreciable amount.

The crystals of this mineral are distinct, and sometimes present minute brilliant faces and good angles for measurement. One of these gave the solid angle of a regular octahedron.

XI. *On the Manegaum Meteorite of* 1843.

Of the circumstances attending the fall of the Manegaum meteorite, and of its appearance as seen in section in the microscope, I gave some account in the Philosophical Magazine of 1863.

I was precluded at that time from making chemical analyses at the British Museum, and was unable to investigate with any precision the nature of this stone, which is one of those that, in the well-defined character of a chief ingredient, offers considerable advantages for the inquiry on which I am engaged.

The meteorite fell on the 29th of June*, 1843, at Manegaum in Khandeish, at half past three o'clock P.M. A very small part of it was preserved; and of this a little fragment was sent to the British Museum, as one among many valuable contributions of the kind from the Asiatic Society of Bengal.

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* The date usually assigned to this fall, viz. the 16th of July, is erroneous. The true day of the fall is given, in the Mahratta account of it, as the third day of the month Asarh sudi, on Thursday. I am indebted to General CUNNINGHAM for the identification of this date with the 29th of June, 1843. [For the account see Journal As. Soc. Bengal, vol. xiii. p. 880.]

The examination and the analysis of this meteorite had therefore to be performed on very minute quantities; in fact on a few grains of *dêbris* that had become detached from the brittle little stone.

The conspicuous ingredient in this meteorite is a pale yellow-green, or primrose-coloured mineral, with a tint similar to that of a very pale peridot or chrysolite, occurring in crystalline grains cemented together, in a state of very slight aggregation, by a white opaque silicate, which in a microscopic section has a flocculent appearance.

The granules of green minerals present in the microscope the appearance of tolerably symmetrical crystals, and are seen of every size, from that of a small pin's head to that of a microscopic dust.

In separating this green mineral from the fragile mass, I have never succeeded in obtaining a crystal of it entire.

The mineral is enstatite; if, at least, we are to include under this name every isomorphous mixture of iron and magnesium silicates with the formula $M\,Si\,O_3$ and crystallised in the prismatic system, but without the distinguishing features either of hypersthene or of diaclasite.

Two of the grains selected from the picked green mineral for measurement gave the following results,—

	Manegaum enstatite.	Breitenbach enstatite.
100, 110	= about 46°	45° 52′
100, 101	= 49° 4′	48 49
110, $\bar{1}$10	= about 88°	88 16
110, 101	= 58° 39′	58 24

The comparison of these measurements with those obtained from the enstatite of the Breitenbach siderolite given in the second column will suffice for the identification of the two minerals as enstatite.

The analysis of the Manegaum mineral was performed by the method of distillation already described. 0·2658 grm. was taken.

		per cent.	Oxygen ratios.	
Silicic acid	= 0·14805	55·699 =		29·706
Magnesia	= 0·0606	22·799 =	9·119	14·059
Iron monoxide . . .	= 0·0546	20·541 =	4·564	
Lime	= 0·0035	1·316 =	0·376	
	0·26675	100·355		

If we allow for the probable presence of a little augite, corresponding proportionately to the lime found in the analysis, this Manegaum mineral will have the formula of an enstatite richer in iron than even that of the Breitenbach siderolite, the formula for which is $(\frac{4}{5}\,Mg\,\frac{1}{5}\,Fe)\,Si\,O_3$. The formula $(\frac{2}{3}\,Mg\,\frac{1}{3}\,Fe)\,Si\,O_3$ requires a percentage composition of $Si\,O_2=54{\cdot}2$; $Mg\,O=24$; $Fe\,O=21{\cdot}7$, which would accord very closely with that of the Manegaum enstatite if we deduct the 1·5 per cent. of silica that the analysis gives in excess.

The specific gravity of the Manegaum enstatite is 3·198, its hardness is 5–6.

A small portion of the meteorite was taken for analysis in its entirety. A black mineral disseminated in a band running through it in minute crystalline particles is chromite; its formula is assumed as $Fe\,Cr_2\,O_4$. 0·4078 grm. was analysed by the hydrogen fluoride method, and gave the following results:—

			per cent.			Oxygen ratios.
Silicic acid	=	0·2187	53·629	=		28·602
Magnesia	=	0·0951	23·32	=	9·328	14·305
Iron monoxide . . .	=	0·0835	20·476	=	4·55	
Iron monoxide . . .	=	0·0013	1·029			
Chromium sesquioxide	=	0·0029				
Lime	=	0·0061	1·495	=	0·427	
		0·4076	99·949			

The silicic acid in this sample of the entire meteorite is in the exact proportion requisite for the enstatite formula; it is therefore not improbable that the excess found in the green enstatite may have been due to an error in the analysis rather than to the presence of either free silica or of a silicate with a higher proportion of this ingredient.

The Manegaum meteorite contains a very minute amount of meteoric iron, far too small for isolation and analysis; indeed the portion taken for analysis could hardly have contained a trace of it.

This meteorite is interesting as presenting us with an instance of a meteoric rock constituted of a single silicate, and that enstatite. It differs from the mass of the Busti meteorite in that the latter is a nearly pure magnesian enstatite, while that of Manegaum is a highly ferriferous one. The two meteorites concur also in the light they throw on the nature of the flocculent opaque white mineral seen in the microscopic sections of many meteorites. In these two cases, at least, that mineral is enstatite.

In concluding this memoir, in which I have endeavoured to deal as exhaustively as possible with the constitution and characters of two remarkable meteorites, I wish to record the great services rendered me in its investigation by Dr. Flight, Assistant in my Department at the British Museum, to whose manipulatory skill and care I am greatly indebted in the chemical part of the inquiry.

EXPLANATION OF THE PLATES.

PLATE XXII.

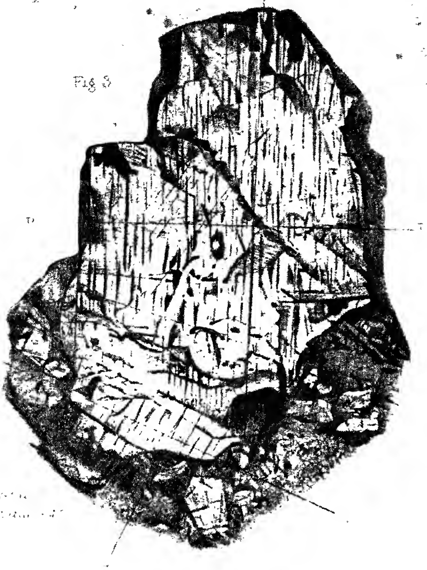
Figs. 1 & 2 represent sections of crystals of the augite as seen in the microscope with a power of 45 linear; that in fig. 1 is nearly in the zone [1 0 0, 0 1 0] and a little oblique to the plane 1 0 0; that in fig. 2 is slightly oblique to the plane 0 0 1, and a little so also to the zone [0 0 1, 0 1 0]. The lines P, P indicate the planes of vibration.

Figs. 3 & 4 represent sections, as seen in a microscopic slide cut from the meteorite, of crystals of enstatite, that in fig. 3 being nearly in the zone of the prism planes, and that in fig. 4 being nearly perpendicular to these.

PLATE XXIII.

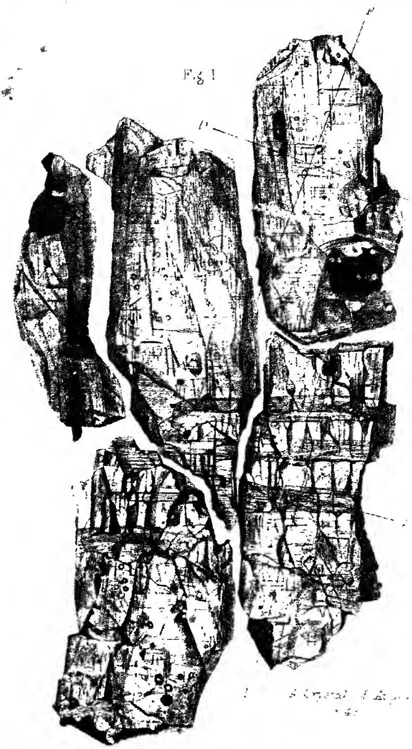
Representation of the Busti meteoric stone, in two views taken from opposite points. The letter N indicates the position of the nodule containing the Oldhamite and Osbornite, of which a section is also represented in the Plate. All the drawings are of the natural size.

Fig. 3

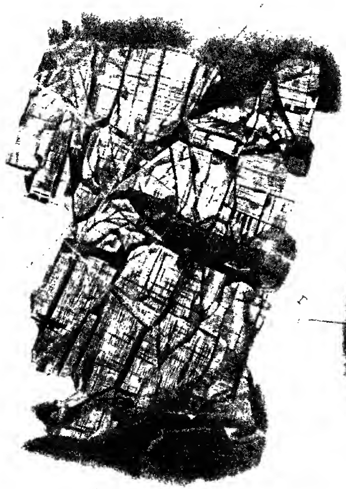


[illegible]

Fig. 1

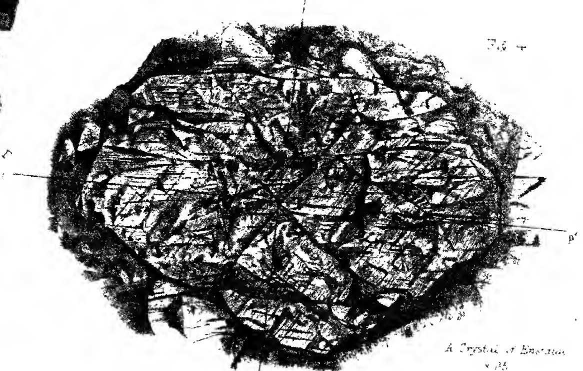


A Crystal of Augite × 45



A Crystal of Augite × 45

Fig. 4

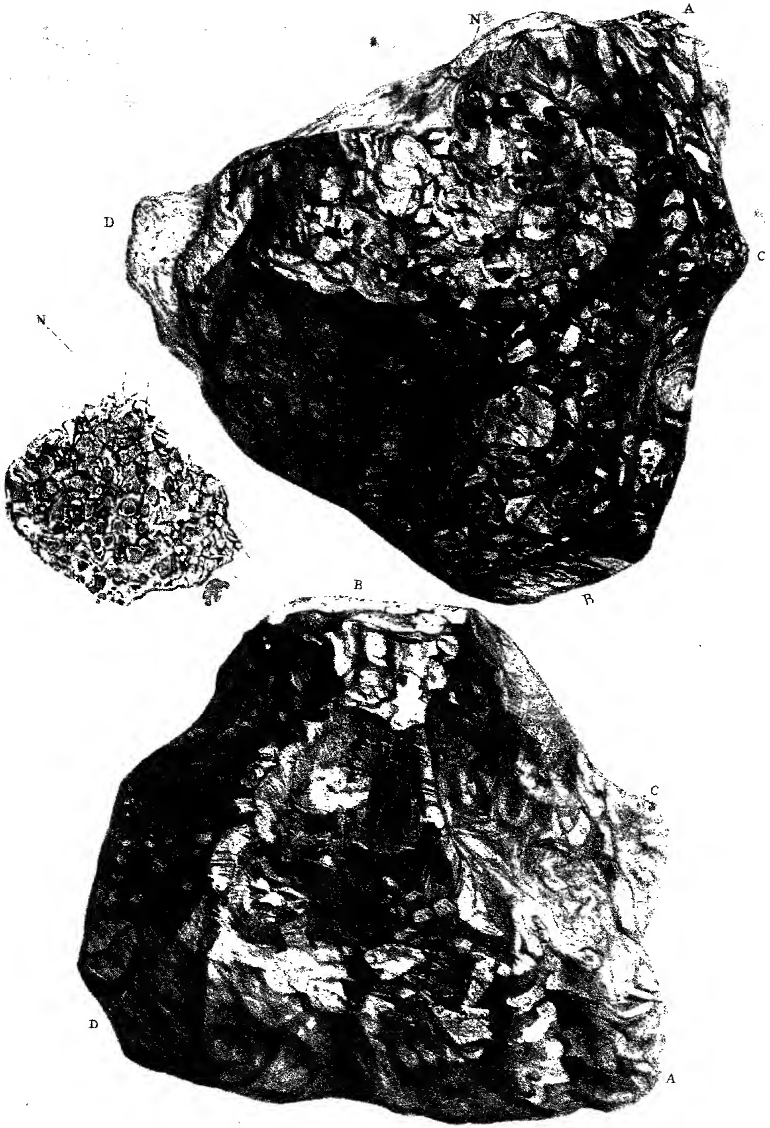


A Crystal of Enstatite × 35

W. H. Bathe del et lith

W. West imp.

Crystals seen in Section in Microscopic slides cut from the Meteorite that fell at Busti in India on Decr 2 1852

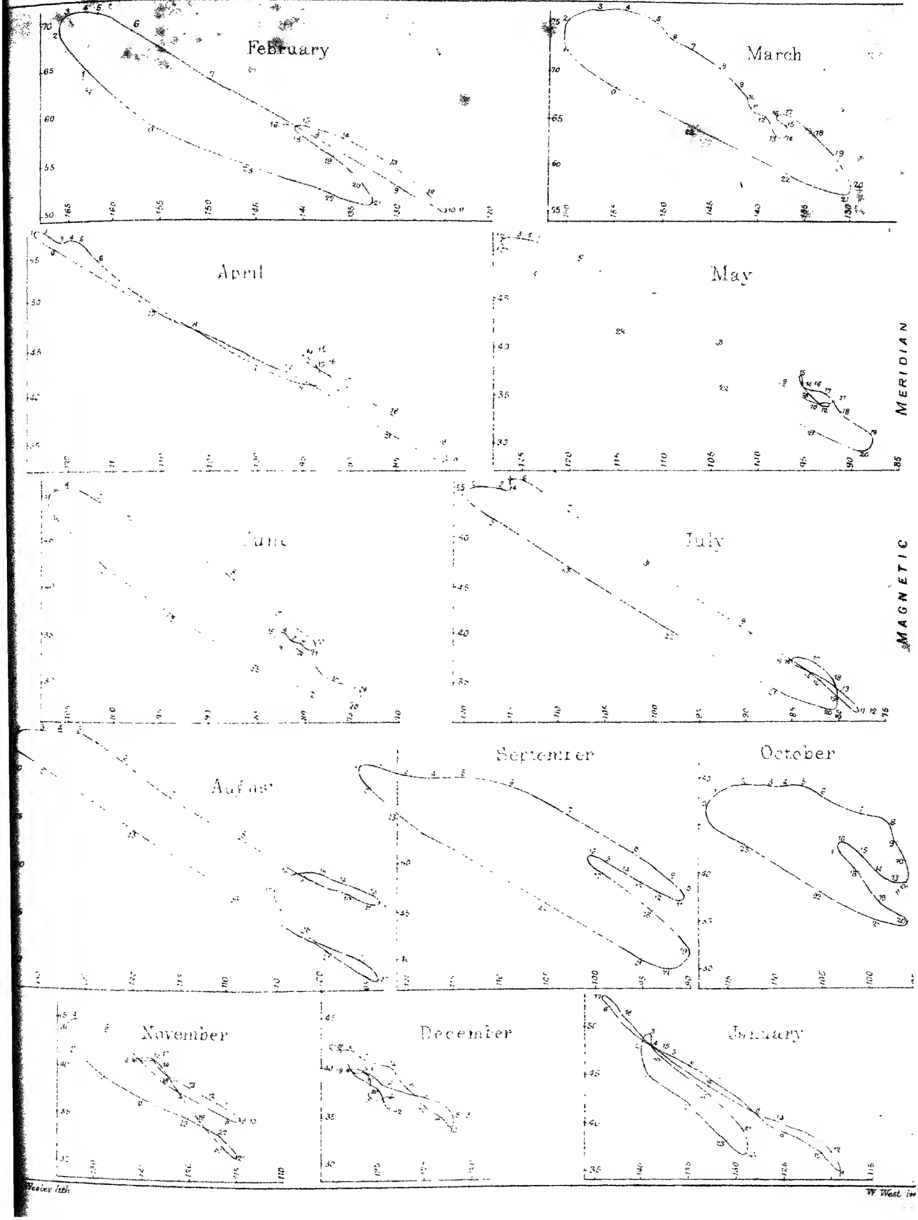


om nat on stone by J. Erxleben

W West imp.

Meteorite that fell at Busti near Goruckpur, India, on Dec 2^{nd} 1852.

Curves [illegible] Inequalities of Magnetism inferred from the Galvanic Currents registered at the Roya Observatory, Greenwich. The ordinates are measured from the Zero found by interrupting galvanic circuit, their u is 0·00001 of Total Horizontal Force. The numerals upon the curves denote Hours of Greenwich Mean Solar Time.



Wesley lith

W West imp

XI. *Note on an Extension of the Comparison of Magnetic Disturbances with Magnetic Effects inferred from observed Terrestrial Galvanic Currents; and Discussion of the Magnetic Effects inferred from Galvanic Currents on days of Tranquil Magnetism.*

By GEORGE BIDDELL AIRY, *Astronomer Royal, F.R.S.*

Received December 22, 1869,—Read February 3, 1870

IN a communication to the Royal Society, which was honoured by publication in the Philosophical Transactions for 1868, I described the methods and gave the results of comparing the Magnetic Disturbances which might be expected as consequent on the Terrestrial Galvanic Currents recorded by the self-registering galvanometers of the Royal Observatory of Greenwich, with the Magnetic Disturbances actually registered by the self-registering magnetometers. The comparison was limited to seventeen days (1865, October 5 and 31; 1866, October 4; 1867, April 4, 5, 7, 8, 9, 11, May 4, 14, 28, 31, June 1, 2, 7, 24), various days having been omitted in consequence of a doubt on the uniformity of the clock-movement of the registering-barrel, which afterwards proved to be unfounded. The results of the comparison were exhibited in curves, engraved copies of which are given in the volume of publication. I expressed my opinion that it was impossible to doubt the general causal connexion of the Galvanic Currents with the Magnetic Disturbances, but that some points yet remained to be cleared up.

As soon as circumstances permitted, I undertook the examination of the whole of the Earth-currents recorded during the establishment of the Croydon and Dartford Wires (namely from 1865 April 1 to 1867 December 31), as far as they should appear to bear upon this and similar questions. For this purpose the days of observation were divided by Mr. GLAISHER into three groups. Group No. 1 contained days of considerable magnetic disturbance (or days of considerable galvanic disturbance, which are always the same), including, besides the seventeen days above-mentioned, the thirty-six days of the following list:—1865, April 15, 16, 17, 18, 19, May 14, 17, July 7, 15, August 14, 19, 26, September 8, 16, 28, October 4, 6, 10, 12, 14, November 1; 1866, August 11, 23, September 8, 9, 12, 13, 17, 18, 25, October 6, 7, 10, 30, November 26; 1867, February 8; making in all fifty-three days of considerable magnetic disturbance. Group No. 2 consisted of days of moderate magnetic disturbance, and of these no further notice was taken. Group No. 3 contained the days of tranquil magnetism, and the discussion of these will form the principal part of the present Memoir.

The comparisons of the additional thirty-six disturbed days were made in every respect by the same process as those of the seventeen days in the former paper; the operations of every kind were directed, as before, by Mr. GLAISHER; and the results are exhibited in the same way, in curves, drawn with great care by Mr. **WILLIAM CARPENTER NASH**, Assistant in the Magnetical and Meteorological Department of the Royal Observatory. While submitting these curves to the examination of the Royal Society, as presenting to the Society the evidence on which conclusions as to the relation between the galvanic currents and the magnetic disturbances must rest, I remark that the class and completeness of the evidence which they afford appear to be precisely similar to those offered by the curves appended to the First Memoir, and that the necessity for multiplying copies of them is not, perhaps, very pressing.

The conclusions arrived at in the former investigation were these:—

1. The general agreement of the curves, especially in the bold inequalities, is very striking; particularly in the curves relating to Northerly Force.

2. The small irregularities in the curves of galvanic origin are more numerous than those in the curves of magnetic origin.

3. The irregularities in the curves of galvanic origin usually precede, in time, those of magnetic origin, especially as regards Westerly Force.

4. The proportions of the magnitudes of rise and fall in the curves often differ sensibly, especially as regards Westerly Force.

5. The Northerly Force appears, on these days of magnetic storms, to be increased; whereas general experience leads us to expect that it would be diminished.

These conclusions are all supported by examination of the curves formed from the new investigations; I am still unable to suggest any explanation of the 2nd, 3rd, and 4th, and I still offer them as subjects worthy of the most careful inquiry. In considering the possibility of explaining any of them by instrumental causes, it appeared to me that the only one, for the effects of which there could be any opening, is, fault in the Declination-Magnetometer. By the courtesy of the Committee of the Kew Observatory, I was permitted to compare the Greenwich Declination-Photograms with the Kew Declination-Photograms, and I found them absolutely identical. I therefore abandon the expectation of explaining the conclusions as the effect of instrumental error.

On the 5th conclusion, much light will be thrown by the examination of the phenomena of days of tranquil magnetism.

I now proceed with the discussion of the curves exhibited by the Earth-current Photograms on days of tranquil magnetism. No comparison was made here between the results of the Earth-current Curves and the Magnetometer Curves; my object being merely to examine the laws, as regards diurnal inequality, of the Terrestrial Galvanic Currents, or rather of the Northerly and Westerly Magnetic Forces which those currents might be expected to produce.

It was necessary that the process to be employed should be precisely equivalent to that used on the days of magnetic disturbance; but there was advantage in changing the form. For, where every individual disturbance was to be depicted, it was necessary to measure every individual ordinate by two different scales; here, where the mean of results for the same nominal hour on numerous days was to be used, it was better to measure the ordinate (whether of the curve or of the zero formed by breaking connexions) by one scale (a scale of inches was in fact used); to take the means of all corresponding to the same hour; and then to multiply the means by the two factors obtained from the theory explained in the former paper. This being done for the two Galvanic Curves, the results were combined in the way explained in the former paper to exhibit the Inferred Northerly Force and the Inferred Westerly Force.

The general multiplier of the geometrical factors used in the former investigation was determined tentatively, to satisfy this condition, that on the whole the magnitudes of the sudden changes of the large ordinates of the curves representing Inferred Northerly Force and Inferred Westerly Force should be sensibly equal to the similar magnitudes in the curves given by the Magnetometers. Considering it as proved that the great disturbances are really produced by the galvanic currents, it is evident that we have thus a fairly accurate scale for converting galvanic indications into magnetic forces (referred, as is done all through, to the total horizontal magnetic force as unit), which will also apply accurately to the days of tranquil magnetism. Also, the zero-indications being formed in the same way for the disturbed and the tranquil days, any error which we may discover in the zeros for tranquil days, or in the references of the ordinates to those zeros, will apply to the zeros or references of disturbed days.

I now proceed with the numerical treatment of the observations of the tranquil days.

The readings in inch-measures of the galvanic ordinates for each nominal hour being grouped by months, and, where there were observations in the same months in different years, the different years being combined, the means were taken, and were converted into Magnetic Forces by the following formulæ:—

In the scale of Horizontal-Force Photograms, 0·01 of Horizontal Force is represented by 2·3565 inches (Introduction to Greenwich Magnetical and Meteorological Observations, 1866 and 1867); and for the graduation of "Scale E for Dartford," the value $\frac{1}{e}\times$graduation of Horizontal-Force Photogram (Phil. Trans. 1868, p. 470), or $\frac{2\cdot3565 \text{ inches}}{0\cdot68259\times0\cdot5437}$ (p. 469) must be used. This number reduced gives for Scale E for Dartford,

0·01 of Horizontal Force=6·350 inches.

Similarly, for Scale F for Croydon,

0·01 of Horizontal Force=7·768 inches.

For Scale G for Dartford,

0·01 of Horizontal Force=5·471 inches.

For Scale H for Croydon,

0·01 of Horizontal Force=4·901 inches.

With these elements, Tables were prepared for converting inch-measures into measures of Horizontal Force. In the original adaptation to base-lines below the photographic curves, the measures with E and F, both used negatively, were to be added, to form

Northerly Magnetic Tendency; and the measures with G taken positively and H taken negatively were to be added, to form Westerly Magnetic Tendency. In the present operations, in which all measures were taken from the photographic base-lines, which reversed the direction of measure for Croydon, E was to be applied to the complement of actual measure of ordinate to 5 inches, and F, G, H to the actual measures of ordinates. I have entered into these minutiæ with the view of facilitating any future reference to the calculations* preserved in the Royal Observatory.

The reading for the zero of each curve (or the indication when the galvanic communications are broken) is found by taking the mean of a group of such zeros, as far as there appears to be no probability of instrumental change; and these readings are treated in the same way as those for the ordinates of the curves. Subtracting these mean zeros from the mean monthly ordinates at each nominal hour, the first Tables of Hourly indications of the Magnetic Effect of Galvanic Currents are formed. It is important to observe that all the numbers have the positive sign.

It will be remarked that, in a few instances, the register has been defective at one or two hours; in those cases it has been thought best still to use the registered hours, though imperfect in number, in the formation of means. The number of days employed to form the means for different hours is thus somewhat variable, as is indicated by the list at the bottom of the Tables. The "Means for the whole day" will appear again, in a following Table, and will explain an important difficulty.

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* In the original calculations, by a mistake, E was applied to the actual measures of ordinates, and F, G, H to the complements of measure to 5 inches. The error was corrected by changing the signs of all the ultimate results.

TABLE I.

Hour. Greenwich Mean Solar Time.	Magnetic Tendencies to the North, inferred from the Monthly Means of Galvanic Currents at every hour of the day, expressed in terms of the Total Horizontal Magnetic Force.											
	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
0	+0·00046	+0·00056	+0·00069	+0·00062	+0·00050	+0·00044	+0·00053	+0·00063	+0·00062	+0·00047	+0·00035	+0·00038
1	46	69	74	56	51	49	55	65	59	50	40	39
2	48	70	76	56	56	51	57	65	61	51	42	39
3	50	73	77	56	47	49	52	63	60	45	44	40
4	50	70	77	56	50	51	54	65	57	49	48	40
5	46	72	76	58	53	50	57	66	59	51	47	42
6	47	72	69	55	51	49	60	62	62	47	41	39
7	43	65	77	52	46	44	56	59	54	49	34	35
8	42	60	68	49	39	44	48	52	52	40	35	34
9	38	48	67	39	34	34	41	48	45	47	34	35
10	36	48	67	41	34	30	30	45	46	38	36	34
11	35	48	68	38	32	34	30	47	46	41	36	34
12	30	52	64	42	33	35	30	42	46	37	34	34
13	43	55	61	43	37	33	35	48	48	38	39	38
14	44	59	62	45	36	35	37	49	48	39	40	41
15	45	61	65	44	37	35	39	49	50	42	42	42
16	57	59	64	43	37	35	38	52	54	45	42	42
17	55	60	66	43	35	33	37	49	51	43	42	44
18	50	58	66	39	34	31	36	43	46	41	40	42
19	50	57	61	34	29	26	32	38	39	36	36	41
20	42	53	57	30	27	27	29	35	35	34	29	38
21	39	48	53	31	29	28	29	37	35	30	29	34
22	35	51	59	41	33	30	38	45	44	38	33	35
23	32	53	63	50	45	34	52	57	58	45	35	36
Means for the whole day...	+0·00044	+0·00059	+0·00067	+0·00046	+0·00040	+0·00038	+0·00043	+0·00052	+0·00051	+0·00043	+0·00038	+0·00038
Number of Measures employed in forming the Means for each Hour.												
Dartford Line...	36 to 40	21 to 23	24 to 28	38 to 45	34 to 41	55 to 58	54 to 58	46 to 51	47 to 50	40 to 49	32 to 38	56 to 58
Croydon Line...	35 to 40	20 to 23	26 to 28	39 to 47	49 to 54	53 to 57	53 to 61	44 to 60	45 to 52	43 to 57	53 to 67	58 to 63

TABLE II.

Magnetic Tendencies to the West, inferred from the Monthly Means of Galvanic Currents at every hour of the day, expressed in terms of the Total Horizontal Magnetic Force.

Hour. Greenwich Mean Solar Time.	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
0	+0·00141	+0·00158	+0·00163	+0·00131	+0·00127	+0·00104	+0·00122	+0·00135	+0·00131	+0·00119	+0·00122	+0·00131
1	142	172	164	129	130	111	123	138	128	118	134	133
2	138	166	161	121	132	109	122	132	122	116	133	130
3	141	166	156	118	119	104	112	127	120	103	133	131
4	139	160	154	120	125	106	113	129	114	110	132	134
5	136	164	155	120	126	103	116	128	117	111	132	136
6	139	165	141	120	122	100	118	123	115	103	127	129
7	132	152	153	112	111	93	113	116	99	102	115	122
8	128	139	143	112	104	92	102	110	94	94	117	120
9	123	123	139	94	93	78	90	99	89	101	111	122
10	123	123	139	95	93	75	75	95	90	91	117	123
11	116	124	145	91	90	79	77	95	90	97	115	122
12	110	125	140	94	92	81	75	88	90	97	115	123
13	131	130	135	93	95	80	80	96	98	95	123	127
14	133	138	136	97	94	82	84	99	96	97	123	130
15	138	144	140	93	96	84	86	99	97	102	123	131
16	147	140	136	98	96	83	86	104	105	104	126	134
17	147	142	138	94	94	81	81	102	102	105	125	136
18	143	141	139	85	91	77	81	98	95	104	124	136
19	141	140	132	79	86	71	79	93	85	97	120	133
20	129	132	126	72	84	71	77	89	85	95	112	130
21	130	126	125	80	90	78	82	93	88	90	113	126
22	126	135	136	97	99	84	96	107	103	105	116	130
23	123	142	144	111	119	93	115	123	122	119	120	128
Means for the whole day...	+0·00133	+0·00144	+0·00143	+0·00102	+0·00105	+0·00088	+0·00096	+0·00109	+0·00103	+0·00103	+0·00122	+0·00129

Number of Measures employed in forming the Means for each Hour.

Dartford Line...	36 to 40	21 to 23	24 to 28	38 to 45	34 to 41	55 to 58	54 to 58	46 to 51	47 to 50	40 to 49	32 to 38	56 to 58
Croydon Line...	35 to 40	20 to 23	26 to 28	39 to 47	49 to 54	53 to 57	53 to 61	44 to 60	45 to 52	43 to 57	53 to 67	58 to 63

On constructing curves of each month, with the northerly and westerly tendencies for coordinates, it appeared desirable to eliminate some of the irregularities by a systematic process. The numbers in each ordinate, for any month, being arranged in order, and extended as necessary by repetition of the first terms of the series, the means of adjacent numbers were taken to form a second series; the means of adjacent numbers of the second series to form a third series, and so on. This operation was repeated six times. It will be proper to examine the effect of this process upon periodical terms of different orders.

The inequalities which we seek being essentially periodical with respect to one day, the expression for the inequality at the pth hour may be expressed in the form

$$\left\{\begin{matrix}+a_1\text{ cosine}\\+b_1\text{ sine}\end{matrix}\right\}p\times15^\circ+\left\{\begin{matrix}+a_2\text{ cosine}\\+b_2\text{ sine}\end{matrix}\right\}2p\times15^\circ+\left\{\begin{matrix}+a_3\text{ cosine}\\+b_3\text{ sine}\end{matrix}\right\}3p\times15^\circ+\text{\&c.}$$

$$+\left\{\begin{matrix}+a_n\text{ cosine}\\+b_n\text{ sine}\end{matrix}\right\}np\times15^\circ.$$

Confining our attention to the cosine of the general term; three successive terms in the series for hours, for the $p-1$ hour, the p hour, and the $p+1$ hour, will be

$$a_n\text{ cosine }\{np\times15^\circ-n\times15^\circ\}$$
$$a_n\text{ cosine }\{np\times15^\circ\}$$
$$a_n\text{ cosine }\{np\times15^\circ+n\times15^\circ\};$$

then, taking the means of the adjacent terms, and again taking their means, we arrive by this double operation at the expression

$$a_n.(\text{cosine }\{n\times7^\circ\ 30'\})^2\times\text{ cosine }\{np\times15^\circ\}.$$

And, repeating the operation six times, we obtain

$$a_n.(\text{cosine }\{n\times7^\circ\ 30'\})^6\times\text{ cosine }\{np\times15^\circ\}.$$

The argument $np\times15^\circ$ remains unaltered; but the coefficient is diminished, not much for $n=1$, but very much when n is large, as for instance, $=8$; which makes

$$(\text{cosine }\{n\times7^\circ\ 30'\})^6=\tfrac{1}{64}.$$

It appears therefore that the effect of this process is, practically to annihilate the advanced terms of the series, and to diminish the earlier terms in different degrees. And, when we have formed a smoothed series by the process described, and resolve it into terms depending on the arguments $p\times15^\circ$, $2p\times15^\circ$, $3p\times15^\circ$, &c., we must, in order to find the terms of the same kind in the original or unsmoothed series, multiply the terms found by the following factors:

The first, by $(\text{secant }7^\circ\ 30')^6$;
The second, by $(\text{secant }15^\circ)^6$;
The third, by $(\text{secant }22^\circ\ 30')^6$;

and so on. And the effects of these ought, if possible, to be introduced into the curves which we may form, using the smoothed terms for ordinates; an introduction, however, which will not in practice be easy.

The two following Tables are formed by smoothing the numbers in Tables I. and II., by the process described above.

TABLE III.

Hour. Greenwich Mean Solar Time.	Magnetic Tendencies to the North, formed by smoothing the numbers of Table I. The unit is 0·00001 of Total Horizontal Magnetic Force, and the sign is everywhere positive.											
	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
0	42	59	68	55	48	42	52	60	58	47	37	37
1	45	65	72	57	51	47	54	63	60	48	39	39
2	48	69	75	57	51	49	55	64	60	49	42	39
3	49	71	76	56	51	50	55	64	59	49	44	40
4	48	71	76	56	51	50	55	64	59	49	45	40
5	47	71	75	56	51	49	56	64	59	49	44	40
6	46	69	73	54	49	47	56	61	58	48	41	38
7	44	64	72	51	45	45	53	57	55	46	37	37
8	41	58	70	47	40	41	47	53	51	45	35	35
9	39	53	68	43	36	37	41	49	48	43	35	35
10	37	50	67	41	34	34	35	47	47	41	35	34
11	35	50	66	41	34	33	32	46	46	39	36	35
12	37	52	65	41	34	34	32	46	47	39	37	36
13	40	55	63	43	35	34	34	47	48	39	38	38
14	44	58	63	44	36	35	36	48	49	40	40	40
15	48	59	64	44	37	35	37	49	50	42	41	41
16	52	59	65	43	36	34	37	49	51	43	41	42
17	53	59	65	41	35	33	37	47	49	42	41	42
18	51	58	63	38	33	30	35	43	45	40	39	42
19	47	56	61	35	31	29	33	40	41	37	35	40
20	43	53	58	33	29	28	32	38	39	35	32	38
21	39	51	57	36	31	29	34	41	40	35	31	36
22	37	52	59	42	36	32	40	47	46	38	33	36
23	38	55	64	49	42	37	47	54	53	43	35	37

TABLE IV.

Hour. Greenwich Mean Solar Time.	Magnetic Tendencies to the West, formed by smoothing the numbers of Table II. The unit is 0·00001 of Total Horizontal Magnetic Force, and the sign is everywhere positive.											
	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
0	135	156	155	121	123	101	117	129	124	117	125	131
1	139	163	160	124	127	106	120	133	125	116	129	131
2	140	166	160	123	127	107	119	132	123	113	132	131
3	139	165	157	121	125	106	116	130	120	110	133	132
4	139	163	154	120	124	105	115	128	117	109	132	133
5	137	162	151	119	123	102	115	126	114	107	129	132
6	135	158	149	117	119	99	114	121	109	105	125	129
7	132	150	147	113	111	94	109	116	103	101	120	125
8	128	139	144	107	104	88	101	109	96	98	116	122
9	125	130	142	100	97	83	91	102	92	97	115	122
10	121	125	141	95	93	79	82	97	91	96	115	122
11	119	125	141	93	92	79	78	95	91	96	116	123
12	120	127	139	93	93	80	78	94	93	96	118	125
13	126	131	138	94	94	81	80	95	95	97	120	127
14	133	136	137	95	95	82	83	98	97	99	123	129
15	139	140	137	95	95	83	85	100	99	101	124	132
16	143	141	138	94	95	82	85	103	100	103	125	133
17	144	141	137	91	93	80	83	105	99	103	124	135
18	142	140	134	86	91	77	81	102	94	101	122	134
19	138	137	132	81	88	74	80	96	90	99	119	133
20	133	134	130	80	89	75	81	94	92	96	116	130
21	129	133	131	86	94	79	87	99	95	99	115	129

The next step of treatment was, to resolve the series of terms for each month into a diurnal, a semidiurnal, and a tertio-diurnal series. With such numbers as we have here before us, I have found the following process very convenient. I take the numbers for the Westerly Inequality in January as an example.

Arrange the numbers in two columns, 0^h to 11^h and 12^h to 23^h, side by side, take the difference of corresponding numbers for double diurnal, and the sum (when corrected by subtracting its mean) for double semidiurnal. For convenience, the numbers may be left in the double form. Half the mean above-mentioned will be the true mean of the twenty-four hours.

Hours.		Westerly Tendencies.		Differences for double diurnal.		Sums for double semidiurnal.	
						Uncorrected.	Corrected.
0	12	35	20	+15	−15	55	−11
1	13	39	26	+13	−13	65	− 1
2	14	40	33	+ 7	− 7	73	+ 7
3	15	39	39	0	0	78	+12
4	16	39	43	− 4	+ 4	82	+16
5	17	37	44	− 7	+ 7	81	+15
6	18	35	42	− 7	+ 7	77	+11
7	19	32	38	− 6	+ 6	70	+ 4
8	20	28	33	− 5	+ 5	61	− 5
9	21	25	29	− 4	+ 4	54	−12
10	22	21	29	− 8	+ 8	50	−16
11	23	19	31	−12	+12	50	−16
					Mean	66	
				Half, or mean of the 24 hours	......	33	

The tertio-diurnal term will contribute no part to the semidiurnal, but it will contribute a part to the diurnal. To find its value, arrange the diurnal terms (which I shall now call 'uncorrected') in three columns of eight hours each; take the means of the corresponding numbers, which will be the tertio-diurnal terms, and apply them negatively to correct the diurnal terms.

Hours.			Uncorrected double diurnal.			Sums.	Means or double tertio-diurnal.	Corrected double diurnal.		
0	8	16	+15	− 5	+ 4	+14	+5	+10	−10	− 1
1	9	17	+13	− 4	+ 7	+16	+5	+ 8	− 9	+ 2
2	10	18	+ 7	− 8	+ 7	+ 6	+2	+ 5	−10	+ 5
3	11	19	0	−12	+ 6	− 6	−2	+ 2	−10	+ 8
4	12	20	− 4	−15	+ 5	−14	−5	+ 1	−10	+10
5	13	21	− 7	−13	+ 4	−16	−5	− 2	− 8	+ 9
6	14	22	− 7	− 7	+ 8	− 6	−2	− 5	− 5	+10
7	15	23	− 6	0	+12	+ 6	+2	− 8	− 2	+10

A quarto-diurnal series would readily be formed from the semidiurnal series by the same process by which we have formed the semidiurnal from the original numbers; and the semidiurnal would be corrected as in the operation for correcting the diurnal. But the numbers would be small, and would scarcely repay the trifling trouble.

The numbers thus found ought to be multiplied by 0·5; and then the diurnal numbers ought to be multiplied by (secant 7° 30′)^6, the semidiurnal numbers by (secant 15°)^6, the tertio-diurnal numbers by (secant 22° 30′)^6.

The following Tables exhibit all the results.

TABLE III.

Magnetic Tendencies to the North, formed by smoothing the numbers of Table I.
The unit is 0·00001 of Total Horizontal Magnetic Force, and the sign is everywhere positive.

Hour. Greenwich Mean Solar Time.	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
0	42	59	68	55	48	42	52	60	58	47	37	37
1	45	65	72	57	51	47	54	63	60	48	39	39
2	48	69	75	57	51	49	55	64	60	49	42	39
3	49	71	76	56	51	50	55	64	59	49	44	40
4	48	71	76	56	51	50	55	64	59	49	45	40
5	47	71	75	56	51	49	56	64	59	49	44	40
6	46	69	73	54	49	47	56	61	58	48	41	38
7	44	64	72	51	45	45	53	57	55	46	37	37
8	41	58	70	47	40	41	47	53	51	45	35	35
9	39	53	68	43	36	37	41	49	48	43	35	35
10	37	50	67	41	34	34	35	47	47	41	35	34
11	35	50	66	41	34	33	32	46	46	39	36	35
12	37	52	65	41	34	34	32	46	47	39	37	36
13	40	55	63	43	35	34	34	47	48	39	38	38
14	44	58	63	44	36	35	36	48	49	40	40	40
15	48	59	64	44	37	35	37	49	50	42	41	41
16	52	59	65	43	36	34	37	49	51	43	41	42
17	53	59	65	41	35	33	37	47	49	42	41	42
18	51	58	63	38	33	30	35	43	45	40	39	42
19	47	56	61	35	31	29	33	40	41	37	35	40
20	43	53	58	33	29	28	32	38	39	35	32	38
21	39	51	57	36	31	29	34	41	40	35	31	36
22	37	52	59	42	36	32	40	47	46	38	33	36
23	38	55	64	49	42	37	47	54	53	43	35	37

TABLE IV.

Magnetic Tendencies to the West, formed by smoothing the numbers of Table II.
The unit is 0·00001 of Total Horizontal Magnetic Force, and the sign is everywhere positive.

Greenwich Mean Solar Time.	1866 and 1867.	1867.		1865 and 1867.	1866 and 1867.		1865, 1866, and 1867.					
	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
0	135	156	155	121	123	101	117	129	124	117	125	131
1	139	163	160	124	127	106	120	133	125	116	129	131
2	140	166	160	123	127	107	119	132	123	113	132	131
3	139	165	157	121	125	106	116	130	120	110	133	132
4	139	163	154	120	124	105	115	128	117	109	132	133
5	137	162	151	119	123	102	115	126	114	107	129	132
6	135	158	149	117	119	99	114	121	109	105	125	129
7	132	150	147	113	111	94	109	116	103	101	120	125
8	128	139	144	107	104	88	101	109	96	98	116	122
9	125	130	142	100	97	83	91	102	92	97	115	122
10	121	125	141	95	93	79	82	97	91	96	115	122
11	119	125	141	93	92	79	78	95	91	96	116	123
12	120	127	139	93	93	80	78	94	93	96	118	125
13	126	131	138	94	94	81	80	95	95	97	120	127
14	133	136	137	95	95	82	83	98	97	99	123	129
15	139	140	137	95	95	83	85	100	99	101	124	132
16	143	141	138	94	95	82	85	103	100	103	125	133
17	144	141	137	91	93	80	83	105	99	103	124	135
18	142	140	134	86	91	77	81	102	94	101	122	134
19	138	137	132	81	88	74	80	96	90	99	119	133
20	133	134	130	80	89	75	81	94	92	96	116	130
21	129	133	131	86	94	79	87	99	95	99	115	129
22	129	137	137	97	103	85	98	109	105	105	117	129
23	131	146	147	111	114	94	109	120	117	113	120	129

The next step of treatment was, to resolve the series of terms for each month into a diurnal, a semidiurnal, and a tertio-diurnal series. With such numbers as we have here before us, I have found the following process very convenient. I take the numbers for the Westerly Inequality in January as an example.

Arrange the numbers in two columns, 0^h to 11^h and 12^h to 23^h, side by side, take the difference of corresponding numbers for double diurnal, and the sum (when corrected by subtracting its mean) for double semidiurnal. For convenience, the numbers may be left in the double form. Half the mean above-mentioned will be the true mean of the twenty-four hours.

Hours.		Westerly Tendencies.		Differences for double diurnal.		Sums for double semidiurnal.	
						Uncorrected.	Corrected.
0	12	35	20	+15	−15	55	−11
1	13	39	26	+13	−13	65	− 1
2	14	40	33	+ 7	− 7	73	+ 7
3	15	39	39	0	0	78	+12
4	16	39	43	− 4	+ 4	82	+16
5	17	37	44	− 7	+ 7	81	+15
6	18	35	42	− 7	+ 7	77	+11
7	19	32	38	− 6	+ 6	70	+ 4
8	20	28	33	− 5	+ 5	61	− 5
9	21	25	29	− 4	+ 4	54	−12
10	22	21	29	− 8	+ 8	50	−16
11	23	19	31	−12	+12	50	−16
				Mean		66	
			Half, or mean of the 24 hours			33	

The tertio-diurnal term will contribute no part to the semidiurnal, but it will contribute a part to the diurnal. To find its value, arrange the diurnal terms (which I shall now call 'uncorrected') in three columns of eight hours each; take the means of the corresponding numbers, which will be the tertio-diurnal terms, and apply them negatively to correct the diurnal terms.

Hours.			Uncorrected double diurnal.			Sums.	Means or double tertio-diurnal.	Corrected double diurnal.		
0	8	16	+15	− 5	+ 4	+14	+5	+10	−10	− 1
1	9	17	+13	− 4	+ 7	+16	+5	+ 8	− 9	+ 2
2	10	18	+ 7	− 8	+ 7	+ 6	+2	+ 5	−10	+ 5
3	11	19	0	−12	+ 6	− 6	−2	+ 2	−10	+ 8
4	12	20	− 4	−15	+ 5	−14	−5	+ 1	−10	+10
5	13	21	− 7	−13	+ 4	−16	−5	− 2	− 8	+ 9
6	14	22	− 7	− 7	+ 8	− 6	−2	− 5	− 5	+10
7	15	23	− 6	0	+12	+ 6	+2	− 8	− 2	+10

A quarto-diurnal series would readily be formed from the semidiurnal series by the same process by which we have formed the semidiurnal from the original numbers; and the semidiurnal would be corrected as in the operation for correcting the diurnal. But the numbers would be small, and would scarcely repay the trifling trouble.

The numbers thus found ought to be multiplied by 0·5; and then the diurnal numbers ought to be multiplied by $(\text{secant } 7^\circ\ 30')^6$, the semidiurnal numbers by $(\text{secant } 15^\circ)^6$, the tertio-diurnal numbers by $(\text{secant } 22^\circ\ 30')^6$.

The following Tables exhibit all the results.

Table V.—Diurnal Magnetic Inequality in North Direction, inferred from Galvanic Earth-Currents. The unit is 0·00001 of Total Horizontal Magnetic Force.

Hours.	1866 and 1867. January.	1867. February.	1867. March.	1865 and 1867. April.	1866 and 1867. May.	1866 and 1867. June.	1865 to 1867. July.	1865 to 1867. August.	1865 to 1867. September.	1865 to 1867. October.	1865 to 1867. November.	1865 to 1867. December.
Mean of the Inequality for the twenty-four hours.												
	+44	+59	+67	+46	+40	+38	+43	+52	+51	+43	+38	+38
Simple Diurnal Inequality (to be multiplied by 0·526).												
0	+ 3	+ 7	+ 2	+ 9	+11	+ 6	+14	+ 9	+ 6	+ 4	0	+ 1
1	+ 1	+10	+ 6	+12	+14	+11	+17	+14	+ 9	+ 6	0	0
2	+ 1	+12	+ 9	+14	+16	+14	+21	+17	+11	+ 8	+ 1	− 1
3	+ 1	+12	+11	+16	+17	+17	+24	+18	+13	+ 9	+ 2	− 1
4	− 2	+12	+12	+18	+18	+18	+24	+20	+13	+10	+ 4	− 2
5	− 2	+12	+13	+17	+18	+18	+22	+19	+13	+10	+ 4	− 1
6	− 2	+10	+13	+15	+15	+17	+19	+17	+13	+ 9	+ 3	− 4
7	− 3	+ 8	+12	+12	+11	+14	+14	+14	+10	+ 7	+ 3	− 3
8	− 4	+ 5	+11	+ 9	+ 8	+11	+ 9	+10	+ 7	+ 6	+ 3	− 3
9	− 4	+ 2	+ 8	+ 5	+ 3	+ 6	+ 4	+ 6	+ 5	+ 5	+ 3	− 2
10	− 3	− 1	+ 5	0	− 1	+ 2	− 3	+ 1	+ 1	+ 2	+ 1	− 2
11	− 3	− 5	+ 1	− 4	− 5	− 2	− 9	− 5	− 3	− 2	0	− 2
12	− 3	− 7	− 2	− 9	−11	− 6	−14	− 9	− 6	− 4	0	− 1
13	− 1	−10	− 6	−12	−14	−11	−17	−14	− 9	− 6	0	0
14	− 1	−12	− 9	−14	−16	−14	−21	−17	−11	− 8	− 1	+ 1
15	− 1	−12	−11	−16	−17	−17	−24	−18	−13	− 9	− 2	+ 1
16	+ 2	−12	−12	−18	−18	−18	−24	−20	−13	−10	− 4	+ 2
17	+ 2	−12	−13	−17	−18	−18	−22	−19	−13	−10	− 4	+ 1
18	+ 2	−10	−13	−15	−15	−17	−19	−17	−13	− 9	− 3	+ 4
19	+ 3	− 8	−12	−12	−11	−14	−14	−14	−10	− 7	− 3	+ 3
20	+ 4	− 5	−11	− 9	− 8	−11	− 9	−10	− 7	− 6	− 3	+ 3
21	+ 4	− 2	− 8	− 5	− 3	− 6	− 4	− 6	− 5	− 5	− 3	+ 2
22	+ 3	+ 1	− 5	0	+ 1	− 2	+ 3	− 1	− 1	− 2	− 1	+ 2
23	+ 3	+ 5	− 1	+ 4	+ 5	+ 2	+ 9	+ 5	+ 3	+ 2	0	+ 2
Semidiurnal Inequality (to be multiplied by 0·616).												
0 12	− 9	− 7	− 1	+ 4	+ 2	0	− 1	+ 3	+ 3	0	+ 2	− 3
1 13	− 3	+ 2	+ 1	+ 8	+ 6	+ 5	+ 3	+ 7	+ 6	+ 1	+ 1	+ 1
2 14	+ 4	+ 9	+ 4	+ 9	+ 7	+ 8	+ 6	+ 9	+ 7	+ 3	+ 6	+ 3
3 15	+ 9	+12	+ 6	+ 8	+ 8	+ 9	+ 7	+10	+ 7	+ 5	+ 9	+ 5
4 16	+12	+12	+ 7	+ 7	+ 7	+ 8	+ 7	+10	+ 8	+ 6	+10	+ 6
5 17	+12	+12	+ 6	+ 5	+ 6	+ 6	+ 8	+ 8	+ 6	+ 5	+ 9	+ 6
6 18	+ 9	+ 9	+ 2	0	+ 2	+ 1	+ 6	+ 1	+ 1	+ 2	+ 4	+ 4
7 19	+ 3	+ 2	− 1	− 6	− 4	− 2	+ 1	− 6	− 6	− 3	− 4	+ 1
8 20	− 4	− 7	− 6	−12	−11	− 7	− 6	−12	−12	− 6	− 9	− 3
9 21	−10	−14	− 9	−13	−13	−10	−10	−13	−14	− 8	−10	− 5
10 22	−14	−16	− 8	− 9	−10	−10	−10	− 9	− 9	− 7	− 8	− 6
11 23	−15	−13	− 4	− 2	− 4	− 6	− 6	− 3	− 3	− 4	− 5	− 4
Tertio-diurnal Inequality (to be multiplied by 0·804).												
0 8 16	+ 2	0	+ 1	+ 5	+ 3	+ 2	+ 6	+ 5	+ 5	+ 4	0	0
1 9 17	+ 4	0	+ 3	+ 2	+ 2	+ 2	+ 3	+ 2	+ 3	+ 3	+ 1	+ 1
2 10 18	+ 3	− 1	+ 3	− 1	− 1	0	− 2	− 1	0	+ 1	+ 1	0
3 11 19	0	0	+ 1	− 4	− 3	− 2	− 6	− 3	− 4	− 2	+ 1	0
4 12 20	− 2	0	− 1	− 5	− 3	− 2	− 6	− 5	− 5	− 4	0	0
5 13 21	− 4	0	− 3	− 2	− 2	− 2	− 3	− 2	− 3	− 3	− 1	− 1
6 14 22	− 3	+ 1	− 3	+ 1	+ 1	0	+ 2	+ 1	0	− 1	− 1	0
7 15 23	0	0	− 1	+ 4	+ 3	+ 2	+ 6	+ 3	+ 4	+ 2	− 1	0

TABLE VI.—Diurnal Magnetic Inequality in West Direction, inferred from Galvanic Earth-Currents. The unit is 0·00001 of Total Horizontal Magnetic Force.

Hours.	1866 and 1867. January.	1867. February.	1867. March.	1865 and 1867. April.	1866 and 1867. May.	1866 and 1867. June.	1865 to 1867. July.	1865 to 1867. August.	1865 to 1867. September.	1865 to 1867. October.	1865 to 1867. November.	1865 to 1867. December.
Mean of the Inequality for the twenty-four hours.												
	+133	+144	+143	+103	+105	+88	+96	+110	+103	+103	+122	+129
Simple Diurnal Inequality (to be multiplied by 0·526).												
0	+10	+25	+11	+18	+25	+17	+29	+27	+25	+15	+ 7	+7
1	+ 8	+29	+16	+25	+31	+23	+36	+31	+26	+15	+ 8	+4
2	+ 5	+30	+19	+30	+34	+26	+40	+33	+27	+14	+ 8	+2
3	+ 2	+28	+20	+34	+35	+27	+41	+35	+27	+12	+ 8	−1
4	+ 1	+26	+21	+36	+34	+27	+40	+33	+23	+12	+ 7	−1
5	− 2	+24	+20	+33	+32	+24	+36	+28	+19	+ 8	+ 6	−3
6	− 5	+18	+19	+29	+26	+21	+29	+20	+14	+ 4	+ 4	−5
7	− 8	+10	+15	+24	+18	+16	+19	+15	+ 7	− 1	+ 2	−7
8	−10	+ 1	+ 9	+17	+10	+ 9	+10	+ 7	− 2	− 4	0	−7
9	− 9	− 6	+ 5	+ 9	+ 1	+ 2	0	− 4	− 7	− 6	− 1	−7
10	−10	−12	0	0	− 8	− 5	−12	−13	−13	− 9	− 3	−7
11	−10	−18	− 6	−10	−17	−11	−21	−20	−20	−14	− 5	−7
12	−10	−25	−11	−18	−25	−17	−29	−27	−25	−15	− 7	−7
13	− 8	−29	−16	−25	−31	−23	−36	−31	−26	−15	− 8	−4
14	− 5	−30	−19	−30	−34	−26	−40	−33	−27	−14	− 8	−2
15	− 2	−28	−20	−34	−35	−27	−41	−35	−27	−12	− 8	+1
16	− 1	−26	−21	−36	−34	−27	−40	−33	−23	−12	− 7	+1
17	+ 2	−24	−20	−33	−32	−24	−36	−28	−19	− 8	− 6	+3
18	+ 5	−18	−19	−29	−26	−21	−21	−20	−14	− 4	− 4	+5
19	+ 8	−10	−15	−24	−18	−16	−19	−15	− 7	+ 1	− 2	+7
20	+10	− 1	− 9	−17	−10	− 9	−10	− 7	+ 2	+ 4	0	+7
21	+ 9	+ 6	− 5	− 9	− 1	− 2	0	+ 4	+ 7	+ 6	+ 1	+7
22	+10	+12	0	0	+ 8	+ 5	+12	+13	+13	+ 9	+ 3	+7
23	+10	+18	+ 6	+10	+17	+11	+21	+20	+20	+14	+ 5	+7
Semidiurnal Inequality (to be multiplied by 0·616).												
0 12	−11	− 4	+ 7	+ 9	+ 7	+ 4	+ 3	+ 4	+10	+ 7	− 1	−2
1 13	− 1	+ 7	+11	+13	+12	+10	+ 8	+ 9	+13	+ 7	+ 5	0
2 14	+ 7	+15	+10	+13	+13	+12	+10	+11	+13	+ 6	+11	+2
3 15	+12	+18	+ 7	+11	+11	+12	+ 9	+11	+12	+ 5	+13	+6
4 16	+16	+17	+ 5	+ 9	+10	+10	+ 8	+12	+10	+ 6	+13	+8
5 17	+15	+16	+ 1	+ 5	+ 7	+ 5	+ 6	+12	+ 6	+ 4	+ 9	+9
6 18	+11	+11	− 4	− 2	+ 1	− 1	+ 3	+ 4	− 4	0	+ 3	+5
7 19	+ 4	0	− 8	−11	−10	− 9	− 3	− 7	−14	− 6	− 5	0
8 20	− 5	−14	−13	−18	−16	−14	−10	−16	−19	−18	−12	−6
9 21	−12	−24	−14	−19	−18	−15	−14	−18	−20	−10	−14	−7
10 22	−16	−25	− 9	−13	−13	−13	−12	−13	−11	− 5	−12	−7
11 23	−16	−16	+ 1	− 1	− 3	− 4	− 5	− 4	+ 1	+ 3	− 8	−6
Tertio-diurnal Inequality (to be multiplied by 0·804).												
0 8 16	+ 5	+ 4	+ 5	+10	+ 5	+ 4	+10	+ 8	+ 6	+ 6	0	−1
1 9 17	+ 5	+ 3	+ 6	+ 5	+ 2	+ 2	+ 4	+ 7	+ 4	+ 4	+ 1	0
2 10 18	+ 2	0	+ 4	− 2	− 2	− 1	− 4	+ 1	− 1	0	+ 1	0
3 11 19	− 2	− 3	0	− 8	− 5	− 4	−10	− 5	− 6	− 3	+ 1	+1
4 12 20	− 5	− 4	− 5	−10	− 5	− 4	−10	− 8	− 6	− 6	0	+1
5 13 21	− 5	− 3	− 6	− 5	− 2	− 2	− 4	− 7	− 4	− 4	− 1	0
6 14 22	− 2	0	− 4	+ 2	+ 2	+ 1	+ 4	− 1	+ 1	0	− 1	0
7 15 23	+ 2	+ 3	0	+ 8	+ 5	+ 4	+10	+ 5	+ 6	+ 3	− 1	−1

It must be observed that the investigations of any one month are totally unconnected with the investigations of every other month. Bearing this in mind, and remarking the strong similarity in the laws of the numbers (under each division of the Table) in proceeding from month to month, with change in the magnitude of the numbers and small change in the epochs of the argument evidently depending on the season, it is impossible to doubt that these numbers are real, the true representation of a galvanic and consequent magnetic action, with remarkable diurnal variation, in the surface-materials of the earth.

In every month there is a constant term of considerable magnitude (in reference to the scale of forces before us) towards the North. Of the origin of this term we can give no certain account; but it may not improbably arise from the different oxidabilities of the terminal plates. The variations of magnitude probably depend on the changes which were made from time to time in the earth-connexions. In any case, there is no reason to doubt that the same term exists in the exhibition of forces on days of great disturbance. And, referring to the tabular values of these constant terms, and to the apparent increase of northern force in the disturbed days as measured by the scales at the sides of their diagrams, it will be seen that the magnitude of these constant terms fully explains the apparent increase in northerly force which was remarked in the discussion of the magnetic effects of earth-currents on the days of great disturbance. The last of the apparent anomalies, which exhibited itself in that discussion, is therefore entirely removed.

In every month there is a constant term of still greater magnitude towards the west. And, on referring to the diagrams applying to the days of great disturbance, it will be seen that there is in them a greater increase of force to the west, well corresponding in magnitude to that larger constant term.

The peculiarities of the law of diurnal inequality will be well seen in the diagrams attached to this paper. The general type of the curve is a double lobe, somewhat modified in one or two months, but always preserving the duplicity. It must be remembered that these curves, which are formed by use of the smoothed numbers, are slightly inaccurate in regard to the more rapid inequalities.

Neither in magnitude nor in law are these inequalities, consequent on the galvanic currents, competent to explain the ordinary diurnal inequalities of magnetism.

The discussion of the galvanic currents on the Croydon and Dartford Lines may now, perhaps, be considered as exhausted.

XII. *On Fluoride of Silver.* *By* George Gore, *F.R.S.*

Received October 5, 1869,—Read January 13, 1870.

*Formation**.

Anhydrous or gaseous hydrofluoric acid had no visible action on metallic silver; by electrolysis of the aqueous acid with a silver anode, the metal was dissolved and argentic fluoride formed; by electrolysis of anhydrous hydrofluoric acid also, a silver anode was rapidly corroded. Metallic silver in contact with platinum, in a mixture of dilute hydrofluoric acid and aqueous nitrate of potassium, was not corroded even with the aid of heat. A solution of nitrate of silver, mixed with dilute hydrofluoric acid, did not yield fluoride of silver on evaporation to dryness; nor did it show any signs of decomposition on the addition of solutions of any of the soluble fluorides, except stannous fluoride, or a saturated solution of fluoride of potassium. The effects of anhydrous hydrofluoric acid upon oxide, peroxide, nitrate, chloride, iodide, and carbonate of silver have been already briefly described (Phil. Trans. Roy. Soc. 1869, pages 191 and 192).

Preparation.

I prepared the salt as follows:—A solution of pure nitrate of silver was precipitated by pure carbonate of sodium, and the well-washed carbonate dissolved in dilute hydrofluoric acid; a little heat was absorbed. The clear liquid was heated to boiling, filtered, evaporated, and heated to incipient fusion; transferred whilst hot to a bottle of platinum, and when partly cool retransferred to a gutta-percha bottle.

The earthy-brown salt thus prepared contains a small amount of free silver (especially if it has been stirred when nearly dry), some water, and traces of hydrofluoric acid. To remove the water and acid, I have sometimes heated it in a platinum cup covered with layers of filtering-paper kept cold by a vessel of water placed upon them. When nearly dry it had a greenish metallic lustre, due to the free silver; and when made perfectly anhydrous, by heating nearly to its point of softening, it was black, with a grey satin metallic lustre.

Analysis.

To determine the amount of silver, I passed hydrogen, steam, or ammonia over the gently fused salt, or I precipitated the aqueous solution by hydrochloric acid. To find the amount of fluorine, I precipitated the solution by nitrate of lime; I also employed the following process, and found it much more accurate and satisfactory. About 50 or 100 grains of the salt was weighed in a narrow closed cup of platinum, the cup then placed in a platinum tube-retort about 150 millims. long, and the salt heated to fusion to expel all moisture; the retort was then closed by a tube containing fragments of chloride of calcium and cooled. The cup was removed, instantly closed, and reweighed. The salt was then dissolved in hot water, the solution filtered in the dark into a two-ounce platinum bottle, the washed filter burned, and the residuary silver weighed. The well-washed cup was also heated to faint redness, and its gain of weight of silver ascertained. By deducting these weights from that of the fused fluoride, nearly the true weight of soluble fluoride was found. For every single grain of soluble fluoride, there was now added to the contents of the bottle ·22047 grain of perfectly pure caustic lime in powder (prepared as described, Phil. Trans. Roy. Soc. 1869, p. 179), which had just been heated nearly to whiteness until it ceased to lose weight. The bottle was now placed in a large platinum dish, with a small inverted platinum funnel supported above its mouth, and the mixture continuously evaporated to perfect dryness, the products of combustion from the gas-flame being deflected on one side (see page 232). The bottle was then heated to incipient redness until it ceased to lose weight. This method is capable of yielding very accurate results when sufficient care is exercised.

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* All the operations upon fluorine compounds described in this paper were conducted in platinum vessels, unless otherwise stated.

The reaction upon which this method is based is represented by the following equation:—

$$2\mathrm{Ag\,F} + \mathrm{Ca\,O} = \mathrm{Ca\,F_2} + 2\mathrm{Ag} + \mathrm{O}.$$

Two molecules, or 254 parts, of the silver salt, and one molecule, or 56 parts, of caustic lime lose 16 parts of oxygen, and leave a mixture of 78 parts of fluoride of calcium and 216 parts of metallic silver. The 16 parts of oxygen lost by the above weights of materials equal 38 parts of fluorine present; a *greater* proportion of loss would occur if the silver salt contained *less* fluorine and an equal weight of oxygen or water was present to supply the deficiency, because the lime prevents the expulsion of the fluorine by heat, but not that of any oxygen or water.

The following are the particulars of an analysis of this kind:—100·92 grains of the brown fluoride was fused in the cup and retort; it effervesced, and a little silver was set free upon the surface of the cup; the loss of weight was 1·08 grain =1·070 per cent., consisting of acid watery vapour, and some oxygen produced by the action of that vapour upon the heated salt. On dissolving the salt and filtering, 2·80 grains of silver was found upon the filter, and ·13 grain upon the cup. The quantity of salt dissolved therefore was 96·91 grains =21·366 grains of CaO; 21·39 grains of the lime was then added, and the mixture evaporated and ignited; the process occupied about nine hours. The residue was perfectly neutral to test-paper, and weighed 112·20 grains = a loss of 6·10 grains, theory requiring 6·104 grains.

The contents of the bottle was now dissolved by warm dilute nitric acid, the solution filtered and precipitated; 108·545 grains of argentic chloride, =81·692 grains of silver, was obtained, theory requiring 82·411 grains*.

To determine the amount of silver more accurately I made the following experiment:—

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* Platinum vessels heated to low redness in contact with finely divided silver from fused argentic fluoride show an increase of weight even after having been cleaned by nitric acid (see page 229).—Added April 1870, G. G.

55·44 grains of the brown fluoride was put into a platinum boat and heated to fusion in the retort-tube; it lost ·91 grain =1·641 per cent. The boat and contents was then put into a tube of platinum 114 millims. long, in the middle part of another platinum tube 508 millims. long, and heated to low redness during 55 minutes in a slow current of gaseous ammonia previously dried by caustic lime. A very bulky white and deliquescent sublimate of fluoride of ammonium, having the appearance of down, was produced, and the silver salt crept over the edge of the boat at the exit end. The heat was continued some time after the sublimation ceased, and the boat was cooled in the current of gas. The whole of the salt was reduced to metal, and the amount of silver found was 46·09 grains. A small amount, ·305 grain, had been carried forward into the long tube during the process, thus making a total of 46·395 grains, theory requiring 46·610 grains, viz. 1·599 grains* of free silver, and 45·011 grains of combined silver, in the salt after fusion. After thoroughly digesting the boat in warm dilute nitric acid and washing, it was jet-black (whilst wet) in those parts which had been in contact with the fused salt, and when dry those parts were grey and rough with finely divided platinum (see page 234); the boat had also increased 1·14 grain in weight, probably by absorption of metallic silver: igniting the boat did not alter this weight. The short tube enclosing the boat had, after similar cleaning, lost ·05 grain in weight. The reaction which occurred in this case was probably in accordance with the following equation:—

$$4H_3N + 3AgF = 3(H_4NF) + 3Ag + N;$$

the twelve atoms of hydrogen lose one atom of trivalent nitrogen and gain three atoms of monovalent fluorine by substitution.

To ascertain the amount of free silver in a specimen of the brown fluoride, 43·04 grains of the salt was dissolved in cold water, and the solution filtered in the dark; a little grey powder of silver separated. The residue was well washed, ignited, and weighed; ·575 grain of silver was found after deducting the weight of filter ash.

The foregoing results show that 100 grains by weight of the brown fluoride contained 1·336 grain of silver, ·795 grain of aqueous hydrofluoric acid, and 97·869 grains of pure fluoride. By fusing this quantity (*i. e.* 100 grains) of the brown salt in a nearly closed platinum vessel it loses 1·070 grain†, consisting of water and hydrofluoric acid. The fused residue contained 2·903 grains of free silver, of which 1·336 grain was in the salt when first taken, and 1·567 grain was set free during the fusion; it also contained 96·027 grains of pure fluoride of silver.

	Dried salt.		Fused salt.				
AgF	97·869	AgF	96·027	or	Silver . . .	82·544	=97·066 AgF.
Free silver about	1·336	Free silver about	2·903	,,	Fluorine . . .	14·522	
Aqueous H F. .	·795		98·930		Free silver about	2·934	
	100·000	Loss by fusion . .	1·070			100·000	
			100·000				

---

* This number is necessarily variable owing to the salt suffering different amounts of decomposition in the process of drying.

† This number is also variable.

All these numbers, especially those representing the amounts of free silver and of water, are slightly variable, owing to the extreme deliquescence of the salt and its easy decomposability by heated aqueous vapour*.

According to M. PRAT (Comptes Rendus, No. 9, August 26th, 1867), ordinary fluoride of silver in the anhydrous state is composed of oxygen, fluorine, and silver, its composition being represented by the formula AgF, AgO; and true fluoride of silver, prepared by him in a different way, and possessing very different properties from the ordinary fluoride, has the following composition,—

Silver, 1 equiv. or . . .	108·0	0·785
Fluorine, 1 equiv. or . .	29·6	0·215
AgF	137·6	1·000

He considers the equivalent or atomic weight of fluorine to be 29·6; and he represents the composition of fluor-spar as follows:—

Calcium, 2 equivs. or	40·0
Fluorine, 1 equiv. or	29·6
Oxygen, 1 equiv. or	8·0
	77·6

The results obtained by me in the foregoing analytical experiments, as well as those obtained in determining the molecular volume of anhydrous hydrofluoric acid in the gaseous state (Phil. Trans. Roy. Soc. 1869, pages 179–183), do not agree with the view that ordinary fluoride of silver in the anhydrous state, prepared in the way I have described, contains oxygen.

Physical Properties.

The salt is a troublesome one to prepare, vessels of silver or platinum are essential for the evaporation of its solution. The solution must be heated and filtered before evaporating. When its solution was nearly concentrated by evaporation skin-like films formed upon its surface, and the liquid beneath was filled with very minute silky fibres, probably crystals of the salt. When heated nearly to dryness it became a tenacious sticky mass, requiring a rigid and sharp-edged spatula to remove it. The dried salt was in brown earthy fragments, which rapidly attracted moisture and deliquesced. In a fused state it formed a highly lustrous, mobile, and jet-black liquid, which on cooling became hard and tough†. On dissolving the cooled salt in hot water, filtering, and evaporating the clear solution by heat, some jet-black crystalline powder separated. On several occasions a clear solution of the pure salt, containing free hydrofluoric acid, became blackish with separation of a little black powder on heating.

The salt is very soluble, and evolves a small amount of heat whilst dissolving in the minimum amount of water. 103·58 grains of its aqueous solution (saturated at 15°·5 C.), placed in a platinum cup 51 millims. deep and 16 millims wide, and evaporated to dryness and heated to gentle fusion, left 66·16 grains of solid residue. After allowing ·34 grain for the fluorine set free during the fusion, the result shows that 66·5 parts of it dissolved in 37·08 parts of water, or 1 part of water dissolves about 1·79 part of the salt at 15°·5 C., or 1 part of the salt is soluble in ·55 part of water at that temperature. The aqueous solution is strongly alkaline to litmus-paper. On adding distilled water to a hot saturated solution, a white cloud appeared which afterwards dissolved; it probably consisted of a basic salt. On several occasions I have had reason to suspect a slight degree of solubility of finely divided metallic silver in a strong solution of the salt, similar to that of lead and silver in strong aqueous solutions of their respective nitrates. The salt is nearly insoluble in anhydrous alcohol.

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* (Added April 1870.)—I have since made numerous analyses and quantitative experiments relating to fluoride of silver, and they have all confirmed the above composition of the salt.—G. G.

† In a state of fusion it corrodes silver. See page 233.

By weighing a known amount of the brown salt in a specific-gravity bottle filled up with tetrachloride of carbon, the specific gravity of it was found to be =5·852 at 15°·5 C.

The saturated solution of it at 15°·5 C. was very heavy, its specific gravity being =2·61. I have not obtained crystals from it at that temperature*, but by placing it in a covered platinum cup in a freezing-mixture at −23°·5 C. it solidified to a mass of needle-shaped crystals, radiating towards the centre of the cup, and finally to a solid mass of colourless salt with deep fissures in its surface produced by contraction. In another similar experiment the solution became a mass of crystalline plates at −2°·25 C., very similar to those of the acid fluoride of potassium. During the formation of these crystals a yellowish-brown sediment formed at the bottom of the vessel, and on dissolving the crystals by means of gentle heat the sediment remained. The crust was now broken to pieces, and a little cold water added; this caused it to cohere to a rather hard mass, which soon dissolved to a colourless liquid, leaving only a few blackish particles. There was no impurity present, and the probable explanation of the formation of the crust is that there was not sufficient water in the original solution to form the crystalline hydrate, and when the hydrate crystallized it withdrew the water from the remaining portion of salt, and left it in the anhydrous state as a heavy powder which sank to the bottom. This result is remarkable if we consider the great degree of attraction for water which the anhydrous salt possesses, and shows that its tendency to form a crystalline hydrate is even more powerful. I have not experimentally determined the composition of the crystals, but from several circumstances I consider them likely to prove *penta*-hydrated. A saturated solution of the salt chilled to −18° C. exhibited the phenomenon of supersaturation on immersing a platinum wire in it. Melted fluoride of silver which had been gradually cooled was covered with crystalline markings.

A fragment of the brown salt in a well-stoppered colourless glass bottle, exposed to daylight and sunlight during ten weeks, showed no signs of decomposition. A saturated solution of the salt in a platinum cup in a similar bottle similarly exposed during fourteen days evolved traces of hydrofluoric acid, but liberated no silver.

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* According to FREMY, 'Chemist,' New Series, vol. i. pp. 556 & 557; Comptes Rendus, February 27, 1854, a concentrated solution of the salt yields very regular crystals.

Fluoride of silver in a platinum crucible within a heated cast-iron muffle with a bar of zinc near the crucible, melted at a temperature much below redness, and at exactly the same time as the zinc; its fusion-point therefore is about 434° C.

50 grains of the salt was melted in a platinum crucible; 20 grains of scraps of platinum was now added, and the lower end of the retort (enclosed within a glass test-tube) heated in a bath of melted zinc during half an hour to a temperature varying both above and below its fusion-point, the exit-tube of the retort being connected with a glass receiver over mercury. Only a trace of gas was evolved, and the retort and its contents lost only ·05 grain in weight. The salt therefore is not decomposed by heat alone at the temperature of melting zinc. According to Fremy also (Chemist, New Series, vol. i. pp. 556 & 557) the anhydrous salt is undecomposable by heat.

In some cases of evaporation of fluorides I have used the following arrangement to prevent the products of combustion from the gas-flame coming into contact with the substances. It consists of a triangular sheet-iron cap, fig. 1, without a bottom, fitting over the top of a common iron tripod. In the upper surface of the cap is a large hole A, to receive the evaporating dish. The cap has a short chimney, B, and two flaps of sheet iron, C, C, with hinges to admit of a gas-burner being placed beneath.

In other experiments in which tubes were heated to redness, I have employed a row of Bunsen's burners (fig. 2), with a moveable table A A of sheet iron fixed to them at any desired height by the screws B, B. The table was for the purpose of supporting the tube, which lay in two notches in its edges in the direction C C; it was also used for supporting notched plates of fireclay to increase the heat. To fix the tube securely to the table, two peculiar hooked-shaped pieces of iron, provided with screws, were employed, as shown in fig. 3.

Fig. 1.

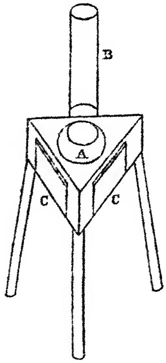


Fig. 2.

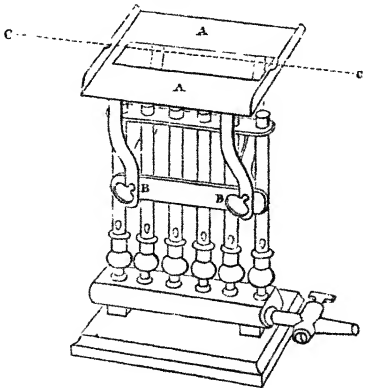
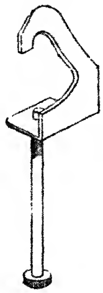


Fig. 3.



In numerous experiments requiring the fusion of fluorides &c., I have employed a thin platinum cup or muffle (fig. 4), made without solder, heated by means of a furnace (fig. 5), in which the substances could be raised to a high temperature without exposing them to contact with the products of combustion. A (fig. 5) is an open clay cylinder, B B is a perforated clay plate to receive the cup D. With the aid of this arrangement and a corrugated gas-burner, I have been able to electrolyze substances at the temperature of melted cast iron, and observe the phenomena taking place at the electrodes, and by means of additional arrangements have also collected the gases evolved at those temperatures without admission of air.

Fig. 4.

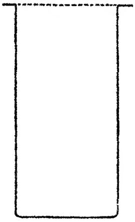
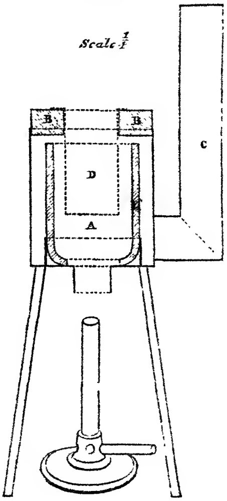


Fig. 5.



On cooling melted fluoride of silver in an open platinum vessel, it effervesced on every occasion whilst solidifying; and on remelting it out of contact with the gases of the burner, more gas was evolved as soon as it acquired a dull red heat. 100 grains of the brown salt, alternately heated to redness and cooled nearly to solidifying, thus, repeatedly during thirty-five minutes, lost 3·86 grains in weight; and the residue, on solution in water, left 17·04 grains of silver, chiefly as a fine powder. A second 100 grains, kept at a dull red heat in an open crucible within the covered muffle during forty-five minutes, lost 2·47 grains, and yielded 12·22 grains of free silver. In a third experiment, 50 grains kept at just visible redness in a closely covered crucible in the closely covered muffle during $1\frac{1}{2}$ hour, lost only ·62 grain, and yielded 3·18 grains of free silver. In a fourth similar experiment, in which the silver-salt was partially exposed to the air during one hour, the quantity of silver set free was equivalent to about five-sixths of the total amount of fluorine present. The losses in all these experiments were due, 1st, to the watery vapour expelled from the brown salt itself, and 2nd, to that of the outer air; the more the air was excluded the less was the percentage of loss. In each experiment some of the silver was set free in a coherent deposited state upon the crucible, chiefly at the junction with the surface of the liquid, and the crucible was slightly corroded. On dissolving this silver by nitric acid, jet-black powder of platinum appeared beneath; this always occurs in such cases.

A current of dried air was passed over 20 grains of argentic fluoride at a gentle red heat in a platinum boat within a glass tube during half an hour; the fluoride was not decomposed. A current of undried air was also similarly passed over the salt in a state of fusion; a little silver was set free at the ingoing end of the boat, and where the salt was not kept perfectly fused effervescence occurred, which caused the salt to creep up the sides of the boat.

Electrolysis.

A number of experiments were made of electrolyzing argentic fluoride in a fused state, with platinum electrodes and six SMEE's elements, in a covered platinum cup. In each case conduction commenced before the salt had fused. When the salt was liquid the conduction was *perfect*, so that on closing the circuit by a copper wire no additional action could be detected in the battery. No gas was evolved at the cathode, and only a little from the anode, most on each occasion at the first moment of conduction. The anode exhibited no more signs of corrosion than the cathode, nor than what always occurs when argentic fluoride is fused in contact with platinum. Metallic silver was set free in each case, but apparently only by the usual influence of moisture of the air. No film of coherent silver was deposited upon the cathode. I have met with similar results of perfect conduction without manifest electrolysis whilst passing electric currents through certain other metallic fluorides in a state of fusion.

In one of these experiments, supposing that fine fibres of silver might have stretched across and united the electrodes, as actually does occur in some cases, I frequently stirred the liquid around the anode by means of a platinum wire, but no alteration of conduction was thereby produced, and no fibres of silver could be seen. Had such fibres been formed they would probably have been highly heated by the electric current and rendered visible.

On electrolyzing the fused salt with an anode composed of a rod of highly ignited charcoal of lignum-vitæ and ten SMEE's elements, only a small amount of conduction took place in consequence of the resistance offered by the charcoal. Gas was evolved from the anode, and the anode was corroded, but no special odour besides that of hydrofluoric acid was perceived. According to FREMY (Chemist, New Series, vol. ii. p. 548; Comptes Rendus, April 25, 1855), who electrolyzed the fused salt in a platinum vessel, it decomposed easily, but the liberated silver perforated the platinum vessel in a few minutes.

A special investigation is necessary to determine the amount of conduction which occurs without electrolysis in this and certain other fluorides in a state of fusion.

Various aqueous solutions of the salt were also electrolyzed. In two of these experiments a saturated solution, not containing free acid, in a platinum dish as the cathode, and a slightly immersed thick platinum wire as the anode, was electrolyzed by means of six large GROVE's cells. Free conduction occurred. No gas or odour was evolved. A thick, hard, and strongly adherent crust quickly formed upon the anode, and a rapid deposit of loose brilliant yellow scales of silver upon the cathode. The crystals soon extended upwards and united the electrodes. By still less immersion of the anode sufficient heat was evolved to boil the liquid and set free metallic silver. In another experiment with a large platinum anode suspended inside an inverted funnel of gutta percha to collect evolved gas, no gas was evolved and a similar crust was formed. The black crust, after washing with water, effervesced with ozone-like odour in strong nitric acid, and formed a deep brown opake solution. With strong hydrochloric acid it effervesced and evolved an odour like that of an oxide of chlorine. In concentrated sulphuric acid it evolved gas and an odour of ozone. It also evolved gas in strong aqueous ammonia. Probably, therefore, it was a peroxide of silver, such as occurs in the electrolysis of a solution of argentic nitrate. 13·2 grains of it (which had been kept a long time in a glass bottle with but little corrosive effect upon the glass, and contained scales of free silver) was heated gradually to redness in a narrow platinum tube-retort; much acid fume was evolved at first, and then a gas which repeatedly reinflamed a red-hot charcoal splint explosively. The loss of weight was 1·9 grain. If the substance was peroxide of silver, $Ag_2 O_2$, the loss should have been 1·7 grain. Metallic silver was left. By the electrolysis of a more dilute solution a similar crust was formed, but gas was also evolved at the anode.

Anhydrous hydrofluoric acid, artificially chilled, was electrolyzed by means of ten SMEE's elements with a silver anode. The acid conducted more freely than with an anode of palladium, and still more so than with one of gold (see Phil. Trans. Roy. Soc. 1869, p. 189). The anode corroded rapidly, and became covered, first with a little black powder upon its edges, and then with a grey powder (probably metallic silver) which contained only a trace of soluble silver-salt.

The following is the order of electrical relation of several metals in the fused salt at a barely visible red heat, the most positive being named first:—silver, platinum, charcoal of lignum-vitæ, palladium, gold. The silver was rapidly corroded and apparently dissolved. The charcoal emitted much gas when first immersed. Silicon could not be tried because it decomposes the salt rapidly, setting free silver; it is not, however, necessarily electro-positive to silver on that account, for I have met with substances which are electro-negative to silver in argentic solutions from which they liberate silver rapidly. For a similar reason the base metals could not be tried.

Magnesium was rather strongly electro-positive, and silver, palladium, and a rod of charcoal of lignum-vitæ weakly positive to platinum in a saturated solution of the salt at 16° C.

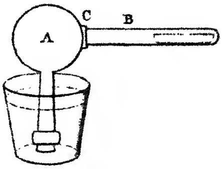
The following is the order of electrical relation found with a moderately dilute solution of the salt at ordinary temperature:—aluminium, magnesium, silicon, *iridium*, *rhodium*, and *carbon of lignum-vitæ* nearly equal, platinum, palladium, tellurium[*], gold. The iridium was much more strongly positive if it was covered with a film of oxide.

Chemical Behaviour.

In many of the experiments great corrosion and injury of the vessels was caused by reduced silver alloying with the platinum; and the products of the reactions were very difficult and tedious to remove. The solvents employed for cleaning the vessels were usually dilute nitric or hydrochloric acids, strong aqueous ammonia, or saturated solutions of iodide of potassium or nitrate of mercury[†]. In some cases the residue of silver-salt had to be reduced to metal at a red heat by means of hydrogen or alkaline carbonates.

With Hydrogen.—The reduction of the salt by hydrogen has been already described (Phil. Trans. Roy. Soc. 1869, p. 183). Fragments of the brown salt in hydrogen in a colourless glass vessel at 60° Fahr., exposed to sunlight during two days, exhibited no chemical change. A solution of the salt in a platinum vessel at 60° Fahr., containing a little free hydrofluoric acid, was not decomposed by passing a stream of hydrogen through it.

Fig. 6.



With Nitrogen.—A globe of glass of the form A, fig. 6, was filled with pure nitrogen; the platinum retort B enclosing a platinum boat, containing a weighed amount of the previously fused salt, was now fixed on air-tight by means of a washer of vulcanized india-rubber at C, and its outer end heated to redness during half an hour, the mouth of the vessel being closed air-tight by a bung of vulcanized india-rubber, and immersing it in a basin of mercury. On opening the mouth of the cooled vessel under mercury, no alteration of volume of the gas was found, and the loss of weight of the boat and its contents was only ·26 grain. The residual gas was found by proper tests to be nitrogen. Nitrogen, therefore, is not absorbed by, and does not decompose, argentic fluoride at a low red heat.

With Ammonia.—A fragment of the brown salt, weighing 10½ grains, was fixed in an iron clip and introduced into a colourless 4-ounce bottle filled with dry ammonia gas over mercury, a stick of caustic potash having been previously placed in the gas; the bottle was then exposed to sunlight. In a few minutes the salt became darker in colour, and a slow absorption of the gas continued during about twenty-six days; a slight odour of ammonia then remained, and the salt had absorbed about 98·3 cub. centims. or 1·0914 grain, or about 844 times its volume of the gas. Light was not necessary, as the absorption was observed to occur during the night. The ammoniated salt was odourless, and was probably analogous to the corresponding compound of argentic chloride and ammonia. The behaviour of argentic fluoride in a state of fusion in a current of dry gaseous ammonia has been already described (see p. 229). A saturated aqueous solution of argentic fluoride was instantly decomposed with powerful action by the strongest aqueous ammonia, but no precipitate was observed.

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[*] The tellurium contained traces of copper.

[†] The most convenient plan I have employed for recovering noble metals from the acid liquids used in cleaning the pieces of apparatus, has been to immerse a plate of zinc in them in a vessel of ebonite.

With Oxygen.—24·6 grains of oxide of silver previously heated to 600° Fahr. was introduced into a platinum tube-retort containing 49·86 grains of the previously fused salt, and 20 grains of platinum scraps, and the retort (enclosed in a test-tube) heated in a bath of melted zinc to an incipient red heat during fifteen minutes until evolution of gas ceased. The loss of weight was 1·8 grain, and about 5½ cubic inches of gas was collected over mercury. The gas was proved to be oxygen, and the residue contained soluble undecomposed fluoride. Fluoride of silver therefore is not decomposed by heating with oxide of silver. A globe (see fig. 6) was filled with pure and dry oxygen. The platinum retort B, containing the residue of the last experiment, was fixed air-tight on the tubulure C, and the outer end of the retort heated to low redness during half an hour, the open neck of the receiver being immersed in mercury. No alteration of volume or properties of the gas took place, and the loss of weight was only ·16 grain. This result, therefore, confirms the previous one. The effect of dry air upon the salt in a state of fusion has been already described (see p. 234).

With Water.—The behaviour of the salt with liquid water has been already described (see p. 231). 20 grains of the brown salt was heated to fusion in a platinum boat within a glass tube, and a current of steam passed over it; rapid decomposition took place; the salt lost its fluidity and became of a brown colour for a short time, as if converted into oxide of silver; abundance of hydrofluoric acid was set free and the glass corroded powerfully; soon, however, by more heat the whole of the salt was changed into white silver. The loss of weight was 4·45 grains, consisting no doubt of fluorine (united to hydrogen), substance carried away mechanically by the violence of the action, and a small amount of acid moisture originally contained in the brown salt. This confirms the results obtained by heating the salt to fusion in undried atmospheric air (see p. 233).

With Nitrous Oxide.—71·64 grains of the recently fused salt in a platinum boat inside a platinum tube within a platinum tube-retort at a low red heat was subjected to a current of previously dried nitrous oxide during one hour. No chemical change took place, and the loss of weight was only ·27 grain. Argentic fluoride, therefore, is not decomposed by nitrous oxide at a low red heat.

With Nitric Oxide.—A current of dried nitric oxide from a mixture of nitre, sulphuric acid, water, and ferrous sulphate, was passed during forty-five minutes over 75·2 grains of the previously fused salt in a platinum boat within a short platinum tube placed in the middle of another platinum tube 508 millims. long, the salt being kept at an incipient red heat. Traces only of hydrofluoric acid were set free, and the salt lost only ·5 grain in weight; the loss might have been due to traces of moisture still remaining in the gas. Argentic fluoride, therefore, is not decomposed by nitric oxide at a low red heat.

With Nitrous Anhydride.—Nitrous anhydride from a mixture of starch and partly diluted nitric acid, perfectly dried by passing through a LIEBIG'S condenser and then over burnt chloride of calcium, was passed during two hours over a weighed amount of the previously fused salt at a low read heat in platinum vessels. The salt was wholly converted into chloride, evidently in consequence of my having employed chloride of calcium as the drying agent.

With Peroxide of Nitrogen.—Highly dried and hot nitrate of lead in powder in a glass retort was heated until the retort was full of a red-brown vapour; the vapour then passed over 69·68 grains of *gently warmed* fluoride of silver (which had been previously fused) in a platinum boat within the long platinum tube (used in the nitric oxide experiment) during seventy minutes. The salt gained 4·70 grains in weight, but exhibited no signs of decomposition; by heating the residue to near redness it freely evolved brown fumes of peroxide of nitrogen. Gently warmed argentic fluoride, therefore, is not decomposed by peroxide of nitrogen, but only absorbs it. The residuary salt of this experiment was now heated just to redness in the same apparatus, and the vapour similarly prepared passed over it during $1\frac{1}{2}$ hour. During the whole of the first hour fumes were evolved which corroded dry glass freely; the corrosive action then ceased. After the process the boat was empty of silver-salt, and much reduced silver (more than 24 grains) was found in the boat and tube. The undecomposed portion of the salt had passed out of the boat by capillary action. The results were probably due to moisture still remaining in the nitrate of lead.

Fluoride of silver dissolved in hot concentrated nitric acid.

With Hydrofluoric Acid.—The behaviour of the brown salt with liquid anhydrous hydrofluoric acid has been already described (Phil. Trans. Roy. Soc. 1869, p. 191). 41·14 grains of the recently fused salt in a platinum boat within a short tube of platinum was placed in the middle part of the long platinum tube, and near the exit end of the tube was placed a second boat containing 37·88 grains of similar fused fluoride, that end of the tube having a very small exit-tube of platinum. Near the opposite end of the tube was placed a third platinum boat containing pure and anhydrous acid fluoride of potassium, and that end closed by a stopper of platinum. A red heat being now applied to the middle boat, and also gentle heat to the third boat very gradually, a current of anhydrous hydrofluoric acid vapour passed over the two boats during $1\frac{3}{4}$ hour, the second boat being kept below 16° C. When the current of vapour had ceased, the cold boat contained a large quantity of liquid anhydrous hydrofluoric acid, which was then expelled by application of a gentle heat. The boats were transferred to a closed platinum tube and separately reweighed. The residue in the cold boat was of a grey-white colour, and in micaceous scales, totally unlike ordinary fluoride of silver; a large portion of the fluoride, however, remained unchanged. The weight of this residue was 41·48 grains; had it all been converted into a salt of the formula AgF, HF the weight would have been 43·54 grains. The heated boat and its contents had lost only ·08 grain; the residue consisted of unaltered fluoride, and showed no signs of free silver as it would have done if any moisture had been present. No special corrosion of any of the vessels occurred. From these results, supported by other reasons, I consider that a double fluoride of hydrogen and silver exists, similar to the corresponding salt of hydrogen and potassium, but more easily decomposable by heat. The non-reduction of any of the heated salt to metal affords further confirmation of the fact that the liquid expelled by heat from the acid fluoride of potassium is quite free from water (see Phil. Trans. Roy. Soc. 1869, pp. 184 & 185).

With Chlorine.—Several investigators have subjected argentic fluoride to the action of chlorine. AIME[*] passed chlorine over the salt at 60° Fahr. Sir H. DAVY[†] heated perfectly dry fluoride of silver in a glass vessel filled with chlorine; the salt was converted into chloride, the retort was violently corroded, much chlorine was absorbed, fluoride of silicon and free oxygen was produced, but no new gas was discovered.

The brown fluoride, in the proportion of 6 grains to each ounce of chlorine, was exposed to the sun's rays in a bottle full of that gas in the dry state; the yellow colour of the gas disappeared, and strong rarefaction was found on opening the vessel.

A glass receiver A (see p. 236), of 262 cub. centims. capacity, was filled with dry chlorine. A stout tube B of refractory glass (lined within its outer half with a sheet of platinum), containing a platinum boat with an equivalent quantity of recently fused argentic fluoride, was fixed to the globe air-tight by a washer of vulcanized india-rubber coated with paraffin[‡]. The neck of the globe was closed by a cork coated with paraffin and immersed in mercury. A gentle red heat was applied to the boat, and the tubulure was kept cold by a stream of water. The yellow colour of the gas disappeared in less than one hour, and the heat was continued $3\frac{3}{4}$ hours. The receiver was then opened under mercury, on doing which the mercury entered with force until it filled about $\frac{5}{6}$ of the vessel. The mercury acquired a slight film as if a little free chlorine still remained. The level of the mercury was not lowered by now applying a low red heat to the boat. No leakage of gas took place, and the bottle was comparatively but little corroded.

In a second similar experiment the retort was of platinum constructed without solder. 59 grains of the fused fluoride was employed, and a red heat applied during one hour and twenty-two minutes. The colour disappeared in three quarters of an hour, and no leakage of gas took place. On admitting the mercury it immediately filled the apparatus within 41 cub. centims. of its capacity at standard pressure. The boat was much corroded, and the contents of the tube had gained 9·5 grains in weight, the missing chlorine being 221·2 cub. centims.=10·3 grains. By heating the retort to redness its contents lost 1·06 grain; and an acid fuming vapour (probably fluoride of silicon) was evolved, which extinguished a red-hot splint.

To exclude the interferences caused by corrosion of the glass, I have employed an apparatus composed entirely of platinum, its parts being constructed without solder; it is represented by the annexed sketch, fig. 7. A is a tube-retort 6½ inches (=165·1 millims.) long and 1 inch (=25·4 millims.) wide. B and C are the two halves of a cylindrical receiver 2 inches (=50·8 millims.) diameter and 6 inches (=152·4 millims.) high, the upper half, B, being 3 inches (=76·2 millims.) high, and fitting very accurately and tightly to about ½ an inch (=12·7 millims.) of its height within the part C; it has a tubulure D, into which the end of the retort A has been ground nearly air-tight to a distance of about ½ an inch (=12·7 millims.); and also a smaller tubulure E, ¼ of an inch (=6·35 millims.) diameter, with a plug F, also ground nearly air-tight; the lower half, C, has a tubulure G, ¼ of an inch (=6·35 millims) diameter, with a small hole I, 0·1 inch (=2·54 millims.) diameter in its lower surface; it has also a plug H ground in nearly air-tight, which is perforated with a hole about ⅛ of an inch (=3·17 millims.) diameter in the direction of the dotted lines, so as to serve the purpose of a tap.

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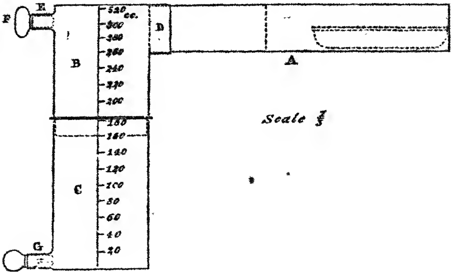
[*] GMELIN's Handbook of Chemistry, vol. ii. p. 359; Annal. de Chimie, vol. lv. (1833) p. 443; LIEBIG's Annal. vol. xvi. (1835) p. 174; POGGENDORFF's Annalen, vol. xxxii. (1834) p. 576.

[†] Sir H. DAVY's Works, v. p. 416.

[‡] Paraffin protects vulcanized india-rubber bungs very effectually from injury by chlorine.

Fig. 7.



The receiver was graduated as shown in the figure, its contents being 19·77 cubic inches (=324 cub. centims.), and the contents of the retort 4·943 cubic inches (=81 cub. centims.), or total contents of the apparatus 24·7 cubic inches (=405 cub. centims.) =18·895 grains of chlorine, or 67·578 grains of argentic fluoride=10·11 grains of fluorine. The total weight of the apparatus was about 5650 grains. The junctions of its different parts were made gas-tight by the mixture of melted paraffin and lampblack.

One experiment made with this apparatus was as follows:—Closed the receiver portion with sticks of potash in it during sixteen hours. Reheated to near redness the retort, containing a boat with 69·71 grains of recently fused argentic fluoride, and (after having removed the potash) at once fixed the retort on the receiver, and cemented the junction. Inserted a horizontal glass exit-tube, about 8 inches (=203 millims.) long, in the tubulure E, with its inner end close to the boat. Passed a stream of pure chlorine into the apparatus by the opening G, and out by the exit-tube during about thirty or forty minutes, until I knew for certain (by means of a separate special experiment I made) that not more than $\frac{1}{50}$ of the total contents of the apparatus consisted of atmospheric air. Removed the exit-tube and inserted the plugs F and H, and cemented the junctions very carefully. Supported the receiver vertically with its lower end in mercury; placed a wet rag upon the retort and tubulure at D, and kept it cold by a constant stream of water. Heated about 3 inches of the outer end of the retort to gentle redness during seventy-six minutes. Allowed the apparatus to cool, and slightly opened the lower tap of the receiver (which was in an inclined position to prevent the mercury rising into contact with the boat) under the surface of a large and measured bulk (600 cub. centims.) of mercury in a graduated glass vessel; great rarefaction was found, and about 315 cub. centims. of mercury rushed in during four or five minutes; the bulk of gas which had disappeared was 326·37 cub. centims. (=15·236 grains of chlorine) when corrected for difference of pressure.

The tap H was now closed, and the retort separated from the receiver; the residuary gas had a strong odour of chlorine, filmed mercury powerfully, and fumed in the air; the reaction was evidently not complete. The stout platinum boat had a large hole corroded in it, and with the stout sheet platinum surrounding it, and saline residue, showed an increase of 16·46 grains in weight. The weight of the salt alone could not be definitely ascertained. The saline matter was red-brown, and contained much dissolved platinum; traces of it (weighing ·15 grain) were found upon the retort. It was a little deliquescent and contained a small quantity of undecomposed fluoride, but consisted nearly wholly of a double salt in which chloride of silver was united to tetrafluoride of platinum. After cleaning the apparatus the following losses of weight were found:—retort, 1·67 grain; sheet platinum, ·64 grain; boat, 20·58 grains; total, 22·89 grains (=16·754 grains of chlorine). This includes some particles of metallic platinum imbedded in the salt. These results show that the chlorine was absorbed without liberating much (if any) fluorine, and that 1·67 grain of platinum had been transferred from the retort to the saline mass.

Thinking that the platinum boat may have conduced to the formation of the double salt, I made another similar experiment, using, however, a boat of pure gold and 70·16 grains of recently fused fluoride. In this experiment the outer end of the retort became concave (indicating rarefaction) long before the end of the heating process. On opening the apparatus under mercury powerful rarefaction was found, and 300 cub. centims. of mercury rapidly ran in, and more would have entered but was prevented. Much free chlorine was again found, the gas fumed in the air, and the retort and its contents had gained 16·20 grains in weight. The gold boat and sheet platinum were powerfully corroded where the saline matter touched them. The saline residue contained very little soluble matter. After cleaning the apparatus the losses of weight were as follows:—retort, 1·13 grain; sheet platinum, 14·59 grains; boat, 11·36 grains; total, 27·08 grains. After dissolving the chloride of silver by aqueous ammonia, and the salts of gold and platinum by hydrochloric acid, 6·32 grains of fragments and dust of those metals was found, thereby reducing the total loss by corrosion to 20·76 grains. The general and numerical results of this experiment closely agree with those of the preceding one*.

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* A number of other experiments were also made with this apparatus, but with its lower half composed of glass; in each instance the glass was more or less corroded, and there was less rarefaction than with the receiver formed wholly of platinum; I have not described the results obtained because of the interference produced by this corrosion.

I conclude, therefore, that chlorine does not set free fluorine from fluoride of silver at a low red heat in vessels of gold or platinum, but unites chemically with that salt and with the metal of the vessel to form a compound from which fluorine is not set free at a low red heat.

I now tried a method of getting boats of pure carbon in order to repeat the foregoing experiment. The method consisted in making boats of lignum-vitæ, baking them very gradually until they were quite black and evolved no visible liquid, then heating them with extreme slowness in a nearly closed copper tube, with frequent turning until the tube was quite red-hot, and cooling them very slowly. On heating fluoride of silver in one of them inside a platinum tube-retort, it became nearly wholly reduced to metal, probably in consequence of the presence of hydrogen in the boat. To purify the boats from that substance, a number of them were heated to redness in a porcelain tube with a current of chlorine passing over them during three hours; nearly the whole of them fell to pieces during the process.

I also obtained a large variety of specimens of native graphite, and made many boats from the purest and hardest varieties*, and also from selected pieces of gas-carbon†. To purify them, they were digested about six days in hot concentrated hydrochloric acid until soluble matter ceased to come out, then in boiling water, then in cold and pure aqueous hydrofluoric acid during six days, and again in boiling water until all acid was removed. They were now dried and ignited to expel sulphur; and to remove hydrogen they were heated to redness in a platinum tube-retort in a current of bromine‡ vapour during more than one hour; and to remove any remaining silica they were similarly heated in vapour of pure aqueous hydrofluoric acid during one hour; then finally heated to redness in an open platinum tube. Boats thus prepared had no chemical effect upon melted fluoride of silver at a low red heat, provided they were heated to redness immediately before use.

The following kinds of native graphite were tried. Ceylon graphite; the hardest specimens were too fragile. Graphite from Iceland; much too impure. Compressed Cumberland graphite; fell to powder when heated in hydrochloric acid. Graphite from Greenland; split into layers in the process of carving. SIDOROFF'S graphite from the Lower Tongooska River, Siberia§; boats made of it cracked when heated with fluoride of silver in chlorine. Graphite from "Marbella, Andulasia," also a specimen of "Siberian graphite"‖; formed good boats, but were rather soft. Excellent boats were made from a specimen of "Graphite from Spain;" but the best were made from ALIBERT'S graphite from Irkoutsk, and that from Borrodale. Many specimens contained veins and nodules of quartz.

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* The best I have found have been the purest varieties of ALIBERT'S Siberian plumbago, obtained from Messrs. FABER, of Stein, near Nuremberg; and also selected pieces of Borrodale graphite.

† Boats of this substance have been made for me in a satisfactory manner by Mr. J. THORP, 3 Charles Street, Wandsworth Road, London.

‡ If chlorine was used the boats became covered with minute spangles of platinum.

§ Given to me by Mr. BRANDT, Palmerston Buildings, Bishopsgate Street, London.

‖ Given to me by the Plumbago Crucible Company, Battersea, London.

Pure chlorine was passed in a slow stream during 1½ hour over 30·66 grains of previously fused argentic fluoride in a boat of purified gas-carbon in a short platinum tube within a long one at a barely visible red heat; a fuming vapour, which corroded glass freely, was evolved during the earlier part of the process, and a small amount of platinum salt sublimed. The saline residue weighed 35·14 grains, theory requiring 34·82 grains of argentic chloride, if the correction be made for the amount of free silver in the original fluoride. The residue was non-deliquescent, and had the physical properties of argentic chloride slightly coloured by traces of platinum salt. After cleaning the two tubes by means of aqueous ammonia and by hydrochloric acid, the short one had lost 3·58 grains, and the long one 1·31 grain by corrosive action of the gas. In this experiment I consider the fluorine passed away in chemical union with the carbon of the boat, and that the corrosive action upon the platinum was due to the chlorine.

69·59 grains of recently fused fluoride in a partly purified and recently ignited boat of Siberian graphite, was heated to incipient redness in the graduated platinum receiver apparatus during 3¼ hours, and then stood all night. No leakage occurred. On opening the receiver under mercury less powerful rarefaction occurred than when a boat of platinum or gold was employed, and about 260 cub. centims. (=12·1 grains of chlorine) of mercury ran in. A slow absorption of gas then took place, after which the residuary gas was 130 cub. centims. at ordinary pressure and temperature. On opening the receiver free chlorine was found, and the gas fumed in the air. A film of brown sublimed salt of platinum was spread over the inside of the vessel. The saline residue weighed 76·18 grains (=14·11 grains or 302·27 cub. centims. of effective chlorine); it consisted of argentic chloride containing some undecomposed fluoride. After cleaning the platinum articles by suitable solvents, they were found to have lost 4·81 grains=3·467 grains, or 74·3 cub. centims. of chlorine taken up by them and rendered non-effective. Some chlorine was also absorbed by the mercury on its admission. The carbon boat was slightly corroded around the silver-salt, and its weight was increased 2·07 grains; by heating it to low redness in a nearly closed platinum retort, it evolved a strong acid odour and a vapour which corroded glass rapidly, and lost 1·57 grain in weight. I consider that the residuary gas was a compound of fluorine and carbon containing free chlorine, and that there were interferences in the results caused by several circumstances.

To obtain a more accurate result with the same apparatus, I took *an excess*, or 117·11 grains, of recently fused fluoride in a very pure boat of Borrodale graphite (weighing 119·4 grains) which had just been heated nearly to redness. A slight leakage took place, and the receiver had to be refilled with gas, thereby causing the silver-salt to take up ·4 grain of chlorine. The end of the retort containing the boat was heated to low redness during four hours, and the apparatus then set aside thirty-six hours at 60° Fahr. On opening the receiver under mercury, 282 cub. centims. ran in under ordinary pressure=123 cub. centims. of residual gas. Warming the receiver did not much expand the enclosed gas, showing thereby the absence of any highly volatile liquid. On separating the retort from the receiver no chlorine remained, and the mercury was not filmed; the excess of silver-salt employed had made the reaction complete. The enclosed gas was colourless, heavy, fumed in the air, had no odour of chlorine, but a peculiar characteristic dusty odour very distinctly, similar to that evolved when bisulphide of carbon is digested with iodine and a large excess of dry argentic fluoride. The gas did not attack mercury or *dry* glass. It was freely absorbed by purified plumbago; the boat of graphite absorbed 2·88 grains of it at 60° Fahr. The retort and contents had gained 12·90 grains (including the 2·88 grains of gas absorbed by the boat). The saline residue weighed 125·62 grains=8·11 grains gain of weight=17·448 grains, or 379·27 cub. centims. of *effective* chlorine=9·338 grains of expelled fluorine; it contained undecomposed fluoride of silver freely. The boat was corroded all over its inner surface only, and smelt strongly of the residuary gas. By heating it nearly to redness in a nearly closed platinum tube during half an hour, the gas was expelled, and it lost 2·88 grains in weight. Its final weight was 117·72 grains=1·68 grain lost by the heating in chlorine. The platinum apparatus was less corroded than in previous experiments, owing to the more rapid absorption of the chlorine by the excess of argentic fluoride, and had lost only 1·33 grain =·958 grain, or 20·53 cub. centims. of chlorine rendered non-effective.

The general results of this experiment may be approximately explained as follows:—The apparatus, when ready for heating, contained 394·5 cub. centims. or 18·406 grains of chlorine, and 10·5 cub. centims. (or a little less) of air, and about 117·5 grains of silver-salt. Of this quantity of chlorine, about ·958 grain was rendered non-effective; the remaining 17·448 grains united with the silver of the fluoride and expelled 9·338 grains of fluorine, which united with 1·474 grain of carbon of the boat and produced 10·812 grains of tetrafluoride of carbon, of which 2·88 grains was in an absorbed state in the boat when the gas was measured, and the remaining 7·932 grains occupied about 123 cub. centims. in the apparatus.

To obtain a still more accurate result I employed a still larger excess of the fluoride, and diminished the heated platinum surface by discarding a boat and cup I had previously employed to contain the carbon boat, and also heated a less length of the retort. The same boat was used; it contained 153·1 grains of the fused salt. The boat and contents was heated during three hours, and then cooled during 1½ hour. On opening the receiver under mercury about 162 cub. centims. of residuary gas (=a loss of about 1·48 grain of carbon from the boat) was found; it contained no free chlorine. On applying a red heat to the outer end of the retort the gas was considerably increased in volume, and returned only partly to its original bulk on cooling, showing that the boat or its contents absorbed some of the gas. Some of the gas was transferred to a glass vessel over mercury; it was colourless, clear, and did not corrode mercury or glass in twenty days. On taking the retort off the receiver the gas exhibited the same properties as in the last experiment, and contained no free chlorine. The amount of sublimed platinum salt was now very small, only ·49 grain=·20 grain of chlorine rendered non-effective. The total gain of the retort and its contents was only 7·35 grains, chiefly in consequence of the boat and outer end of the retort having been heated to redness before disconnecting them from the receiver. The saline residue weighed 161·17 grains=a gain of 7·87 grains=16·93 grains, or 362·8 cub. centims. of effective chlorine. After heating the boat to just below redness for half an hour in a nearly closed platinum retort, it was found to weigh 1·63 grain less than after heating to redness immediately before the experiment. The results are substantially the same as those of the last experiment, and may be explained in a similar manner.

In another experiment made for the purpose of collecting some of the gas, the same boat, containing 177·2 grains of the fused fluoride, was employed, and the retort was heated 2½ hours. About 156·4 cub. centims. of gas was found*, and the properties of the gas were the same as those previously found. Three separate small portions of it were mixed with an equal volume of hydrogen in one instance, and with twice its volume in the other instances, and a light applied; combustion only, without explosion, took place, hydrofluoric acid was formed, and the glass became corroded on the entrance of atmospheric air. The gas therefore was not free fluorine. The odour of the gas was the same as that observed in other experiments in which fluoride of carbon was formed by different reactions.

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* The true volume of gas could not be accurately determined in these experiments on account of its absorption by the carbon boat, and also on account of the opacity of the receiver.

XIII. *On a distinct form of Transient Hemiopsia**. *By* Hubert Airy, *M.A.*, *M.D. Communicated by the Astronomer Royal.*

Received January 6,—Read February 17, 1870.

It is certainly matter of surprise that a morbid affection of the eyesight, so striking as to engage the attention of Wollaston, Arago, Brewster, Herschel, and the present Astronomer Royal, should have received but little notice from that profession to whose province it exclusively belongs. But it must be borne in mind that the votaries of Natural Philosophy are especially qualified by their habits of accurate observation to contemplate attentively any strange apparition, without or within, and, I had almost said, are especially exposed to the risk of impairment (temporary or permanent) of the eyesight, by the severity of the eye-work and brain-work they undergo, and therefore possess especial advantages for the study of visual derangements; whereas the physician, unless personally subject to the malady, must depend, for his acquaintance with its phenomena, on the imperfect or exaggerated accounts of patients untrained to observe closely or record faithfully. The complaint cannot be a rare one; each writer on the subject, in addition to his own personal experience, has mentioned instances of the same affection among his friends. In the whole body of the medical profession there must be many who are at once liable to the disease and able to describe it. And it is not unimportant. I have seen a person, terribly subject to these attacks, shudder at the very name, and turn away in horror from a drawing of the ugly sight, quite content to bear serious illness "if only the 'half-blindness' would keep away."

I think it will appear from the various accounts to which I shall refer, and from the different instances which I shall bring forward, that there are more forms than one, of transient hemiopsia.

The characters of that form to which I wish chiefly to direct attention, as described in the latter part of this paper, are so remarkable, that it is difficult to believe that such observers as Wollaston, Arago, and Brewster could have failed to notice them if present, or could have refrained from recording them if noticed.

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* Most writers have used the word 'Hemiopia' (ἡμιωπία), apparently relying on the analogy of 'Amblyopia' (*ἀμβλυωπία*, Hippocrates, Aphorism. III. 31). But I conceive that *ἀμβλυωπία*, from *ἀμβλυωπός*, ought in grammatical strictness to be used of the eye, not of the eyesight (compare φαιδρωπός, bright-eyed; πυρωπός, fire-eyed; γοργωπός, fierce-eyed), though it seems to admit of the same laxity of meaning as our word 'dull-eyed.' 'Hemiopia' would mean 'half-eyedness.' The form 'Hemiopsia' rests on the analogy of words like αὐτοψία and ὑποψία, referring purely to the eyesight.

It is hardly necessary to indicate, in the outset, the broad distinction between Transient and Permanent Hemiopsia. The latter, as the more important, has naturally received the larger share of attention from the ophthalmists, and a reference to their works will show its characters to be markedly different from those of the transient forms. In Dr. BADER's recent work on 'The Natural and Morbid Changes of the Human Eye,' p. 447, 'Hemiopia' is clearly described as presenting three varieties, of which the first is the only one that comes into comparison with the transient forms, and the chief characteristic of that first variety is concisely stated in these words: "The line of demarcation between the sensitive and blind part of the retina is vertical and sharply defined." This fact, together with the permanency of the graver form, and its association with intra-cranial tumours, apoplexy with hemiplegia, &c., will be found to offer strong points of distinction between the permanent and the transient forms of Hemiopsia.

Most writers on the subject refer, as to a fountain-head, to Dr. WOLLASTON's well-known paper "On Semi-decussation of the Optic Nerves" (Phil. Trans. 1824, I. p. 222), in which he gives a graphic account of two attacks of 'half-blindness,' with an interval of twenty years between them.

On comparing his account with later descriptions, I think it will be recognized that WOLLASTON, ARAGO, and BREWSTER are describing one form, while Sir JOHN HERSCHEL, Sir CHARLES WHEATSTONE, the Astronomer Royal, and Professor DUFOUR, with myself, are describing another (which had also been noticed by Dr. FOTHERGILL and Dr. PARRY). The striking facts of gradual increase, motion, form, and colour which characterize the records of the later group, could not have escaped the notice of the three earlier observers, if present; and we must conclude from their silence that these features were wanting or at least were inconspicuous.

In Dr. WOLLASTON's paper, the passage of chief importance for comparison with other accounts is the following:—"This blindness was not so complete as to amount to absolute blackness, but was a shaded darkness without definite outline. The complaint was of short duration, and in about a quarter of an hour might be said to be wholly gone, having receded with a gradual motion from the centre of vision obliquely upwards towards the left." The author lays stress on the equal affection of both eyes, and concludes that corresponding tracts of the two retinæ receive fibres from the seat of disease.

WOLLASTON's paper attracted much attention abroad as well as at home. A full translation of it appears in the 'Annales de Chimie et de Physique' (tom. xxvii. p. 102), and the editor, M. ARAGO, appends a note in which he illustrates the subject from his own experience. There can be no doubt that ARAGO and WOLLASTON describe the same affection, but in both accounts we miss the remarkable features that characterize the particular form of hemiopsia on which I shall chiefly have to dwell.

A great part of Dr. WOLLASTON'S paper is quoted and corroborated by Dr. PRAVAZ, in the 'Archives Générales de Médecine' (tom. viii. p. 76, 1ère série).

From Sir DAVID BREWSTER'S interesting paper "On Hemiopsy, or Half-vision" (published in the Edinburgh Transactions, vol. xxiv. part i., and also in the Philosophical Magazine for 1865, vol. xxix. p. 503), I must quote one or two passages for the sake of illustration.

"The blindness," he says, "or insensibility to distinct impressions, exists chiefly in a small portion of the retina to the right or left hand of the *foramen centrale**, and extends itself irregularly to other parts of the retina on the same side, in the neighbourhood of which the vision is uninjured."

"In the case of ordinary hemiopsy, as observed by myself, there is neither darkness nor obscurity, the portion of the paper from which the letters disappear being as bright as those upon which they are seen. Now this is a remarkable condition of the retina. While it is sensible to luminous impressions, it is insensible to the lines and shades of the pictures which it receives of external objects; or, in other words, the retina is in certain parts of it in such a state that the light which falls upon it is irradiated, or passes into the dark lines or shades of the pictures upon it, and obliterates them."

"The parts which are in these cases affected extend irregularly from the *foramen centrale* to the margin of the retina, as if they were related to the distribution of its blood-vessels, and hence it was probable that the paralysis of the corresponding parts of the retina was produced by their pressure. This opinion might have long remained merely a reasonable explanation of hemiopsy, had not a phenomenon presented itself to me which places it beyond a doubt. When I had a rather severe attack, which never took place unless I had been reading for a long time the small print of the 'Times' newspaper, and which was never accompanied either with headache or gastric irritation, I went accidentally into a dark room, when I was surprised to observe that all the parts of the retina which were affected were slightly luminous, an effect invariably produced by pressure upon that membrane. If these views be correct, hemiopsy cannot be regarded as a case of amaurosis, or in any way connected, as has been supposed, with cerebral disturbance."

"The two great facts of hemiopsy in both eyes, and of what is called single vision with two eyes, do not require the hypothesis of semidecussation to explain them. If hemiopsy is produced by the distended blood-vessels of the retina, these vessels must be similarly distributed in each eye, and similarly affected by any change in the system; and consequently must produce the same effect upon each retina, and upon the same part of it."

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* Under this term BREWSTER appears to have confounded what are now called the "optic disk" and the "yellow spot." For when he localizes the first beginning of the affection "to the right or left hand of the *foramen centrale*," he certainly means the centre of vision, as also below when he speaks of vision being perfect at the *foramen centrale*: yet when he afterwards speaks of the parts affected extending from the *foramen centrale* to the margin of the retina, "as if they were related to the distribution of its blood-vessels," he is certainly thinking of the optic disk, the punctum cæcum, whence the vessels radiate over the retina.

It is quite inconceivable that the blood-vessels of one retina should be disposed in perfect unsymmetrical correspondence with those of the other (the left side of one agreeing with the left side of the other, and the right with the right); and in fact they are not so disposed. And even if they were, and if the phenomenon were due to their disturbance, its spread would be ruled by geometrical radiation from the punctum cæcum, where nerve and vessels enter the eye, not from the 'yellow spot' at the centre of vision; and we should have two unconformable spectra, one on each side of the centre, perhaps overlapping, but certainly not coinciding, since the puncta cæca are not corresponding points in vision.

Sir DAVID BREWSTER'S experience seems to differ a little from Dr. WOLLASTON'S, the "shaded darkness" of the latter contrasting with the words "neither darkness nor obscurity" of the former; yet I think they are describing the same affection. The difference was probably due to the different circumstances of light &c. under which the observations were made. WOLLASTON was in the full light of the open air; BREWSTER speaks of his severe attacks as brought on by reading the 'Times,' probably therefore in his study, or at least indoors.

I believe BREWSTER'S is the earliest mention (except Dr. PARRY'S) of the self-luminous state of the parts affected, when observed in a dark room, and of the sensibility to general impressions of light which they retain.

Sir DAVID BREWSTER'S paper in the Philosophical Magazine gave occasion to another paper in the same Journal (Phil. Mag. July 1865, vol. xxx. p. 19) by the Astronomer Royal, who adds many particulars of great interest, and gives us a vivid picture in place of the imperfect sketches of previous writers. Indeed it is difficult to avoid the belief that we are dealing here with a new form of the disease. The outward spread of the cloud, its arched shape, its serrated outline, with smaller teeth at one end than at the other, its remarkable tremor, greater where the teeth are greater, its "boiling," its tinge of scarlet, and its sequel of partial aphasia and loss of memory, are all new features, not mentioned by any previous observer, but most important for the identification of the complaint*. This form of hemiopsia is the one to which I desire chiefly to direct attention, the one which I am able to illustrate from my own experience.

A translation of Professor AIRY'S paper appeared in 'Les Mondes' (April 16, 1868), and not long afterwards he received a letter (April 24, 1868) from Professor DUFOUR, of Lausanne, from which I extract the following:—

"C'est avec un intérêt particulièrement vif, que j'ai lu cet article: car j'ai été plusieurs fois atteint de l'affection optique que vous décrivez. Jusqu'ici, j'en avais parlé à deux médecins qui n'ont pas paru connaître ce cas curieux, et je me figurais qu'il s'agissait d'un accident auquel seul j'étais sujet. Votre description de l'hémiopsie décrit si *exacte-*

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* I am able to add, from later information, that in my father's case the phenomenon exhibits conspicuous luminosity in a dark room.

ment et si *parfaitement* ce que je ressens, qu'il ne peut y avoir aucun doute. Permettez-moi de vous signaler les observations suivantes que j'ai faites sur moi-même à l'endroit de cette maladie.

"1. L'attaque commence très brusquement et toujours vers le *centre* du champ de la vision. Je cesse de voir les points que je fixe, conservant très bien, à ce premier moment, la vision latérale.

"2. Les lignes avec zig-zags se produisent alors bientôt; mais, dans les premières minutes, les zig-zags me semblent plus petits. Ils deviennent plus grands ensuite.

"3. Je crois que, chez moi, les deux yeux sont affectés en même temps; mais dans l'un l'affection optique est généralement plus intense.

"4. L'attaque dure toujours une demi-heure environ.

"5. Je n'ai pas remarqué que durant l'attaque j'éprouvais plus de peine à parler. Il m'est arrivé de terminer une leçon après avoir été atteint; je continuais à parler, mais je n'ai pu tracer des lignes sur le tableau noir.

"6. Ces attaques se sont produites à intervalles irrégulières et sans que je puisse en soupçonner la cause immédiate, depuis une dizaine d'années. J'en ai eu, en somme, une douzaine.

"7. Autrefois, l'attaque était *invariablement* suivie d'un très violent mal de tête, qui me rendait très souffrant durant plusieurs heures. J'ai eu le bonheur de trouver un moyen qui neutralise, presque tout-à-fait, cette suite pénible, et c'est surtout pour vous indiquer ce moyen que j'ai pris la liberté de vous écrire à ce sujet. Dès que l'attaque a commencé, je me mouille abondamment le front, les tempes, la nuque, les yeux avec *de l'eau froide*, plus froide et mieux. Je répète cette operation plusieurs fois pendant que le phénomène optique dure, et je tâche d'ailleurs de demeurer tranquille avec les yeux fermés. J'ai pratiqué cet abondant lavage à l'eau froide, pour la première fois, il y a quatre ans: l'accident s'est passé sans mal de tête. La seconde fois, même operation et même succés. La troisième fois, j'ai été atteint loin de chez moi et dans des circonstances où je n'ai pu avoir de l'eau: le mal de tête est arrivé comme jadis avec toute sa violence. Enfin, à ma dernière attaque, il y a quelques mois, j'ai pu de nouveau employer l'eau, et le mal de tête n'a pas paru d'une façon pénible."

Thus far we notice that the several records to which I have referred hang together by an interesting chain of historical succession. WOLLASTON'S memoir seems to have led to those of ARAGO and BREWSTER, BREWSTER'S gave occasion to Professor AIRY'S, and Professor AIRY'S in turn evoked the orderly evidence of Professor DUFOUR.

The next account I have to quote is wholly independent of all earlier descriptions. It is Sir JOHN HERSCHEL'S, and is to be found in his 'Familiar Lectures on Scientific Subjects,' p. 406, in Lecture IX. "On Sensorial Vision" (delivered before the Philosophical and Literary Society of Leeds, Sept. 30th, 1858).

"I was sitting one morning very quietly at my breakfast-table, doing nothing and

thinking of nothing, when I was startled by a singular shadowy appearance at the outside corner of the field of vision of the left eye. It gradually advanced into the field of view, and then appeared to be a pattern in straight-lined-angular forms, very much in general aspect like the drawing of a fortification, with salient and re-entering angles, bastions, and ravelins, with some suspicion of faint lines of colour between the dark lines. The impression was very strong, equally so with the eyes open or closed, and it appeared to advance slowly from out of the corner till it spread all over the visual area, and passed across to the right side, where it disappeared. I cannot say how long it lasted, but it must have been a minute or two. I was a little alarmed, looking on it as the precursor of some disorder of the eyes, but no ill consequence followed. Several years afterwards the same thing occurred again, and I recognized, not indeed the same precise form, but the same general character—the fortification outline, the dark and bright lines, and the steady progressive advance from left to right. I have mentioned this to several persons, but have only met with one to whom it has occurred. This was a lady of my acquaintance, who assured me that she had often experienced a similar affection, and that it was always followed by a violent headache, which was not the case with me. In this case the regularity of the pattern was not great, but the lines were quite straight and the angles sharp and well defined. Had it remained stationary, it might be assumed that the retina had a structure corresponding to the figure, and that some undue pressure might render that structure visible. But such an hypothesis is precluded by the gradual transit of the lines over every part of the visual area."

The following extract from a letter from Sir JOHN HERSCHEL to myself (May 4, 1868) will show that in his later experience the affection has begun near the centre of vision.

"It is very strange, and I am sure more than a coincidence, that two or three hours after I had read your letter, and while in the act of reading a printed book, I caught the impression of the commencement of an attack in the obliteration of one or two letters a little to the left and below the point of vision. Soon after, further out to the left in a wavy course, the printed letters ran into large angular black zigzags, and then I knew what was coming, and shutting my eyes I watched the development of the luminous bastions, &c. It was, however, by no means so well developed or striking an instance as I *have* had, but it is its recurrence evidently as a *consequence* of the mind dwelling on its description that I look on as worth notice."

Sir JOHN HERSCHEL has very kindly communicated to me his latest experience in a letter dated Nov. 17, 1869.

"Since I wrote to you I have been very frequently visited with the phenomenon—in a greater or less degree,—never, however, with the extreme vividness of colour and distinctness of form as heretofore; and it has assumed some new features, viz. patches of a kind of coloured chequer work in some of the corners of the fortification forms.

"It always now begins with a small glimmer *near* the middle of the field of view, and spreads out. I am now most distinctly able to say that it sometimes opens out from left to right, and sometimes from right to left.

"Here is what I find recorded in a memorandum of June 22 ult.—'The fortification pattern twice in my eyes today. The first was turned *leftwards*. Colours red and black, or red, yellow and black with little blue, and at moments only black and white. Also a sort of chequer worked filling in, in (?) rectangular patches, and a carpet-work pattern over the rest of the [internal] visual area.

"'The second and far the brightest and most beautiful in colouring was turned to the right. Colours very vivid. Red, blue, yellow, black. Not sure of any green.'

"I have sometimes had an impression that *one eye only* was affected—the right eye being affected with the right-handed and the left with the left-handed spectrum; but I never could devise any means of coming to a conclusion as to this point, and on the whole I lean to the opinion that both eyes are concerned in either case."

Very recently (1870, Jan 15) I have become acquainted, through the kindness of Professor STOKES, with the following description by Sir C. WHEATSTONE of a form of hemiopsia differing from my own in nothing but the total absence of colour. With the writer's permission I insert the whole.

"I will here subjoin the note I made at the time I was first attacked with this affection.

"'Sept. 30th, 1849.—This evening I had a curious affection of vision. Whilst I was writing, characters near the centre of vision became invisible. Thus fixing my eyes on the figure 6 in the group $4\frac{6}{7}$, 4 and 7 were completely obliterated. On closing each eye alternately, I found precisely the same result. This did not arise from an ocular spectrum, for neither a black nor a coloured spot was projected on the paper, the disappearance was exactly that of an object when placed in the projection of the entrance of the optic nerve. After a short time the spot became larger, spreading towards the left in both eyes until it occupied a large oval space; objects at and near the centre of vision reappeared, but nearly the left half of each retina was blinded. The phenomenon in its later stages was accompanied by an effect like the motion of a luminous liquid. At the time the luminous mist entirely passed away, about half an hour after its commencement, a slight fainting sensation came over me.'

"I have frequently, though generally at very distant intervals, been subject to this affection. It has usually occurred whilst reading. It has always commenced near the centre of the retina, and ordinarily expanded towards the left. The zigzag luminous lines which border the spectrum externally do not commence until it has received some expansion, and they become brighter as it enlarges; before it disappears vision is restored to the central part of the retina, and when the zigzag lines arrive at the limit of the field of view, the entire vision becomes clear. On one occasion I drew with a pen the outline of the spot, a short time after its first development, as it appeared to each eye separately projected on the paper; both outlines exactly corresponded. I have never suffered any inconvenience after these attacks, and my vision, although I have at times tried my eyes severely with optical experiments, is I believe as good as ever, though I have been subject occasionally to this affection for more than twenty years. The only difference between the phenomena as they appear to me and as they are described by Dr. AIRY is, that in my case they are always unaccompanied with colour"*.

This case, though clearly belonging to the later group (p. 248), yet in absence of colour shows a step towards the earlier, and suggests that all the forms of transient hemiopsia are but varieties of one and the same affection, differing only in degree of prominence of their different features. We must look for more connecting links before we can be satisfied that it is so.

Hitherto I have quoted only from non-medical authorities, and I think no one can fail to be struck by the amount of attention that this obscure malady has received from so many writers of such high scientific attainments.

Most medical works that I have consulted give but little information concerning Transient Hemiopsia. The fullest notice of the subject that I have met with is in Tyrrell's 'Diseases of the Eye' (vol. ii. p. 231), under the head of "Functional Amaurosis from Cerebral Disturbance."

But very lately (April 7, 1870) my attention has been drawn to two passages in the writings of earlier authors in which the disease in question is plainly to be recognized. The first is to be found in the works of Dr. FOTHERGILL, "Remarks on the Sick Headach" (a paper read before the Select Society of Licentiates, Dec. 14, 1778). Speaking of butter as an article of diet, he says, "Nothing more speedily and effectually gives the sick-headach, and sometimes within a very few hours. After breakfast, if much toast and butter has been used, it begins with a singular kind of glimmering in the sight; objects swiftly changing their apparent position, *surrounded with luminous angles, like those of a fortification*. Giddiness comes on, headach, and sickness. An emetic, and warm water soon wash off the offending matter, and remove these disorders."

The other passage to which I refer occurs at pages 557, 558 of the first volume of 'Collections from the unpublished medical writings of the late CALEB HILLIER PARRY, of Bath, 1825.'

"After violent fatigue, more especially when accompanied with fasting eight or ten hours, which has often happened to me, and now, Sept. 26, 1808, I have frequently experienced a sudden failure of sight. The general sight did not appear affected; but when I looked at any particular object, it seemed as if something brown, and more or less opake, was interposed between my eyes and it, so that I saw it indistinctly or sometimes not at all. Most generally it seemed to be exactly in the middle of the object, while what my sight comprehended all round it, was as distinct and clear as usual, in consequence of which, if I wished to see anything, I was obliged to look on one side. At other times, though much more rarely, the cloud was on one side of the direct line of vision. After it had continued a few minutes, the upper or lower edge, I think always the upper, appeared *bounded by an edging of light of a zigzag shape*, and coruscating nearly at right angles to its length. The coruscation always seemed to be in one eye; but both it and the cloud existed equally, whether I looked at an object with one or both eyes open. When I shut both eyes, covering them with my hand so as to exclude all rays of light, the coruscation was still perceptible in the same place, and what had been a semi-opake cloud appeared lighter than the rest. When I raised or lowered the axes of my eyes, or squinted, the cloud and coruscation, though it moved its place, still bore the same relation to the object at which I looked. In this way they would remain from twenty minutes sometimes to half an hour, the cloud lessening as the coruscation continued, and the latter sometimes rather suddenly going off. They were in me never followed by headach, but seemed evidently connected with the state of the stomach; for though they sometimes occurred without any feeling of indisposition at the time, either there or elsewhere, they generally went off with a movement in the stomach, producing eructation; and anything which produced a glow in the stomach, with eructation, and perhaps without it, such as brandy, hot water, &c., always hasten their departure."

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* Sir CHARLES WHEATSTONE had seen my description before the paper was finally presented.

My own experience of Hemiopsia dates from 1854. I was so much struck by the first attack that I made a record of it at the time, which I allow myself to transcribe here as an authentic and independent, though very incomplete, account.

"Friday, Oct. 6th.—This morning (the last before the Michaelmas Holidays) I had an attack of that half-blindness to which —— is subject; she had one yesterday. It came on while I was with Mr. Drew*, and I noticed it first by being unable to see the 't' in "tan A" when I looked at the top. At first it looked just like the spot which you see after having looked at the sun or some bright object; I thought it might be an eyelash in the way, or something of that sort, but I was soon undeceived when it began to increase. I then bethought me that it must be the same thing that —— suffered from, so I let it alone, knowing that it would go off in time, which it did, leaving a most terrible headache behind it, which is the worst part of it, the blindness itself giving no pain whatever. When it was in its height it seemed like a fortified town with bastions all round it, these bastions being coloured most gorgeously. If I put my pen into the space where there was this dimness, I could not see it at all, I could not even distinguish the colour of the ink at the end. All the interior of the fortification, so to speak, was boiling and rolling about in a most wonderful manner as if it was some thick liquid all alive. It did not belong only to one eye, but to both, the right eye having the most."

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* Now Professor of Mathematics at King's College, London, formerly Vice-Principal of the Blackheath Proprietary School.

This account of my first attack may very well stand for a description of my last, about a month ago. The type has remained unaltered from that time to this. It is not stated, and I cannot remember, whether that first attack was on the right side or on the left: from the last words I should think it was the former.

Since then I have very frequently been revisited by this affection, perhaps as often as a hundred times, possibly much oftener, sometimes at intervals of a month or two, or a week or two, or a day or two, sometimes on successive days, sometimes twice in the same day, sometimes twice in the same hour, the second attack beginning before the first had quite passed away. The circumstances and features of the complaint have varied somewhat in different attacks. I will first describe its usual course, and then refer to varieties.

Usually after two or three hours' close reading, especially if I have had insufficient exercise, I become aware that part of the letter I am looking at, or a word at some little distance from the sight-point* (in most cases, below, to the left), is eclipsed by a dim cloud-spot that would not be noticed except for this obliteration. Even at this very earliest stage, the tremor, that is so characteristic of the developed disease, can be detected, and as the cloud enlarges, it begins to assume its proper zigzag outline, enriched with tinges of colour.

At this early stage the spot is but faintly luminous in a dark room, or with the eyes shut and shaded, and scarcely shows at all against a bright sky. Its shape and colours are best seen by looking at a shady part of the ceiling or a neutral-tinted wall.

When this blind spot makes its appearance close to the centre of vision, as soon as it begins to spread, and shows a serrated margin, it at once presents the irregular horseshoe shape, with one arm adherent to the sight-point, and the other receding from it outward. The teeth of the adherent arm are small and fine, those of the receding arm grow larger and larger (Plate XXV. figs. 1–4).

But when the blind spot takes origin at some distance from the centre of vision, as it spreads it preserves its contour unbroken, stellate, nearly circular, until its margin nears the centre of vision; then the serration at the point nearest the centre shows irregularity, and a breach appears in the outline: one branch of the incomplete circle takes a smaller pattern of zigzag, and attaches itself to the centre of vision, the other branch takes a larger pattern of zigzag and recedes (Plate XXV. figs. 5–8).

The enlargement is slow at first, and gradually quickens.

Almost from the very first it may be noticed that parts of the faint cloud have a slow rolling heaving swaying motion to and fro, by which the outline is altered from time to time and again restored in the gradual outward spread; and superadded to this slow rolling there is rapid flickering tremor (about five vibrations per second) of the marginal rays, affecting especially such parts as are rolling at the same time.

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* I have employed indifferently the expressions "sight-point," "point of sight," "centre of vision," "centre of the field of view," to signify the point on which the eyes are fixed.

These three orders of motion, (1) gradual outward spread of the whole, (2) slow rolling of parts, (3) rapid tremor of the margin, are especially characteristic of this affection.

As the cloud extends its borders, its central region begins to fade, and clear vision begins to return in the concavity of the seething crescent. As fast as the trembling rolling jagged margin encroaches on the clear field outside, so fast the power of the enemy is waning and the faculty of sight is reasserting itself in the rear of his advance (Plate XXV. figs. 4 & 8).

The sight of both eyes is affected exactly in the same manner and in the same degree (though naturally that eye *seems* most affected which corresponds to the obliterated side of the field of view, because the nasal half of the field of view of either eye is more limited, and vision there is less distinct than on the temporal side). Whether the one eye, or the other, or both, be open, the same strange sight is seen—no mere general resemblance, but absolute identity, indubitable, unmistakable. Every angle of the outline, every gleam of colour, is still there, in its place,—survives the ordeal of alternate closure of the eyes, unaltered except by its own gradual outward spread.

When the eyes rest on a printed page, the cloud is seen as a faint shade of horseshoe shape, with serrated margin, bright-lined in some places, and varied with changing gleams of red and blue and yellow and green and orange, in order of frequency, now one colour, now another, slowly waxing and waning in harmony with the unrest around. All words and letters covered by this strange intruder are completely blotted out; those immediately adjoining the margin seem smeared into it, not cut off sharp; while inside the horseshoe there is gradual transition from the unseen to the seen, again. The tremor and rolling are plainly recognized.

Looking at any surface of uniform colour, as a green wall-paper, or a red table-cover, or a mahogany table, the cloud is scarcely to be seen at all: it partakes of the general hue of the field on which it lies, and only reveals itself by the bright lines and gleams of colour at the margin, by the tremor and rolling that belong to them, and by the obliteration of the grain of the wood and minor markings.

Looking at the bright sky, the affected portion of the field of view appears as a faint shadowy curved cloud. The bright lines at its margin are not conspicuous, except when they show gleams of gay colours. The boiling and tremor are well seen.

Looking at a white ceiling in shade, the display is seen perhaps most favourably. The bright lines of the margin contrast with dark lines behind them; the space immediately within the margin is seen to be partly broken up into wedges and angular figures of faint light and shade, especially towards the larger end of the curved cloud, which is receding from the centre of sight and shows a larger pattern in every respect,—larger zigzags, larger wedges, larger boiling, and stronger tremor. Any gleams of colour are well seen.

When the eyes are so directed that part of the cloud is seen against the sky, and part against the dark wall, it may be noticed that the jagged arch appears faintly dark against the sky, and faintly light against the wall; but the part seen against sky is decidedly brighter than the part seen against wall. At the junction between the two, I find great difficulty in recognizing the transition from lighter to darker.

The eyesight thus affected, though wholly blind to boundaries of light and shade, is not disqualified for the reception of light, in quantity. The light is seen, almost unimpaired, though all definition is blurred and blotted out.

But, besides being open to general impressions of light and shade from without, the clouded patch on the field of view has an inherent luminosity of its own. If the observer shuts his eyes and carefully shades them, or (better) retires into a dark room with his eyes open, he sees a faintly luminous curved figure in the dark, brilliantly edged along its serrated outline,—the bright margin supported by a trench of black, and in different places gleaming with red and blue and other colours. The tremor and boiling are beautifully seen (Plate XXV. figs. 1–9).

Meanwhile the disease extends with gradually increasing rapidity, and spreading outwards invades the more distant parts of the field of vision. Still the small end of the curve points ever to an imaginary cynosure at the centre of sight, the rest of the arch sweeping away into outer regions, where the mind has great difficulty in watching its form. Towards the outer extremity, where the zigzag pattern is much larger and less defined, and where the cloud dies away confusedly, as the disease attains its height, the turbulence of motion becomes greatly exalted; the outskirts of the visual area seem to be boiling over with tumultuous light, that may be seen at times to collect itself in a rallying-point here and there, and presently to stream away again along the shore of the seething sea, splendid with large gleams of blue and red and green.

The climax is generally reached in twenty or twenty-five minutes from the first beginning. Then the large arm, having overspread the margin of the field, begins to fade and leave the lower part to recover slowly from the storm. The small arm is the last to perish; it remains in strength while the large arm is dying away; but soon the outward spread carries it in turn to the upper margin of the field, and it there exhibits the same fervour that characterized the career of the larger end. The whole duration of the phenomenon is just half an hour, often with curious exactness.

When the disease is about halfway advanced, I generally observe the rudiments of a fresh attack, beginning nearly where the first began, and sometimes advancing so far as to exhibit its bastioned margin, with rolling and tremor, as though the performance were to be rehearsed from beginning to end (Plate XXV. fig. 9, *b*). But there it stops and fades away, unless it arise on the opposite side, when I have known a second attack develope itself immediately after the first.

The sight feels somewhat dazed for ten or fifteen minutes after the final disappearance of the phenomenon.

Throughout the earlier part of this visual derangement I feel no discomfort at all; my faculties are free to observe the phenomena closely and carefully. It is not till near the end, when the boiling is at its height, that the eyes feel oppressed, and the head has a presage that it is going to ache. The headache comes on gradually: it is not localized in any particular part: it lasts for five or six hours or more, accompanied with slight nausea.

Such is the normal course of an attack of transient Hemiopsia in my case. I will presently mention such abnormal forms as seem to have interest.

Reviewing the nomenclature of the disease, I think we must recognize the inaccuracy and insufficiency of the name 'Hemiopsia' or 'Hemiopia,' used by Brewster and others, especially as the same name has been given, appropriately, to the graver and more permanent half-blindness, in which "the line of demarcation between the sensitive and blind part of the retina is vertical and sharply defined" (Bader, p. 447). I am tempted to look for a single word which shall express the most striking feature of the morbid vision. In Sir John Herschel's account (above, p. 251), it will be seen that he was much struck by the resemblance to a fortified wall "with salient and re-entering angles, bastions, and ravelins;" and writing last year, he speaks of the visual phenomenon as "the 'Fortification Pattern.'" Dr. Fothergill's words (above, p. 254) are, "surrounded with luminous angles, *like those of a fortification.*" Other persons also have habitually used this expression in describing their own experience. I think this similitude may furnish me with the word I seek, and I venture to propose the name 'Teichopsia' (τεῖχος, town-wall, ὄψις, vision) to represent the bastioned form of transient Hemiopsia which I have been describing, not without a reminiscence of some words of Tennyson's:

> ". . . . . . . as *yonder walls*
> Rose slowly to a music slowly breathed,
> *A cloud that gathered shape.*"

Various particulars, which would have burdened the description, I have reserved for consideration here.

Circumstances of the attack.—Want of exercise, sedentary employment, close reading and writing, are the usual antecedents of Teichopsia. It generally comes on while the eyes are engaged with toilsome reading. Several times I have believed the attack to be favoured by bad windy weather, for the reason that different members of my family have been affected on the same day, in such weather, though unaware of the synchronism till afterwards. I am careful to add this last caution, because I have thought (and by reference to the quotation above, page 252, it will be seen that Sir John Herschel entertained the same idea) that an attack might be induced by the mind dwelling on the description or imagination of the thing.

Sudden change of air and living have sometimes seemed to be the exciting cause. On one occasion, some years ago, on going into the country for a winter holiday, I had three or four attacks in the first two days.

Over exercise may bring it on, I believe. It has come on after a long walk before breakfast. It will be remembered that Dr. Wollaston attributes his first attack to a similar cause.

I have been attacked when I have been called early after insufficient sleep. Sometimes the attack has been nocturnal, mingling with a dream, from which I wake and find the spectacle in full fervour. Sometimes, I believe, I have passed through it without waking, for I have been half aware of it in my sleep, and have found the dull headache on me in the morning.

Not unfrequently I have found it impossible to assign any cause for the attack.

Position in the field.—The first spot of blindness never springs up (with me) *exactly* in the centre of vision. Even when most central, it is recognized as lying a very little to one side or the other; and this slight excentricity determines the side of the field which the disease will occupy in its development. With me, the left side is affected more frequently than the right. The most usual position of origin is 3° or 4° to the left of, and 3° or 4° below, the centre. I remember one case in which the attack began at a much greater distance in the same direction; but I have never had any experience of such a course of the cloud as Sir J. Herschel describes in his paper quoted above (p. 251), coming into view at the extreme left, and gradually extending to the right over the whole field.

Anomalies in the course of the Disease.—In one or two cases, after reaching a certain point of development, the phenomenon has died away without ripening; it has suffered breach of continuity in its walls opposite the natural gap, and then each part has faded separately (Plate XXVI. fig. 3). In one of these cases the disease began at the beginning of a walk before breakfast in summer, and died prematurely as I walked briskly on. In the case above alluded to as having taken origin at an unusual distance below and to the left of the sight-point, the cloud preserved its contour unbroken for an unusual length of time, and took a marked oval shape, until the gradual approach of its wall towards the centre led to an opening on that side, with adhesion of one arm (small-toothed) to the centre, and retreat of the other (large-toothed) towards the periphery of the field. In only one instance have I noticed the small end of the curve refusing allegiance to the centre of sight, and then the course of the phenomenon corresponded to that given in the diagram that accompanies Professor Airy's paper on Hemiopsy.

I believe the small end of the curve is always the upper of the two. It is so in all the cases of which I have kept any record, and I cannot remember any instance of the reverse*.

The shape of the curve has varied considerably in different instances. Sometimes it has been as flat as is represented in Professor Airy's diagram, when the phenomenon has not been well developed in other respects; but in far the greater number of cases it assumes a full horseshoe shape.

When a second attack has followed close upon the first, I have noticed that, besides the flattening of the curve, the salient angles of the margin have been less defined, and the marginal lines of light less clear.

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* Since writing these words (1870, Jan. 23) I have had an attack of dextral teichopsia, in which the *lower* arm had the smaller pattern, and offered rather distant allegiance to the sight-point.

I cannot remember any instance in which the cloud, springing up and spreading on one side of the field, has ever transgressed the vertical median line. I believe it has never done so, but I cannot speak positively on the point. (In Sir JOHN HERSCHEL's case it is distinctly stated that the shadowy intruder swept over the whole field from left to right; and TYRRELL speaks of instances in which the blindness "increases so as to extend over the whole field of vision.")

Colour.—When viewed in the dark, the general hue of the self-luminous cloud is yellowish-white. The casual gleams with which it is adorned are, in order of frequency, red and blue, yellow, green, orange. They seem to belong to the bright lines of the margin, but are less definite, and appear sometimes to spread over a wider space; but it is by no means easy to determine the exact relations of the various tracts of light and shade in the turbulence and trembling that prevail; especially when it is borne in mind that in order to do so the attention must leave the centre of sight and by effort of will transfer itself to a point 20° or 30° or 40° removed from the centre.

Sequelæ.—The headache has been very slight in some cases; but generally is very oppressive, with some degree of "eyeball pain." The nausea, usually slight, was sufficient to produce vomiting on one occasion. I have never experienced any affection of speech with or after these attacks. But lately they have been followed by a slight disturbance of hearing, in which external sounds gave rise to a momentary 'rumbling' in my ears.

I have once or twice tried to relieve the headache by the plan which M. DUFOUR recommends—abundant application of cold water to head and face,—but without success. The action of an emetic in no way prevented the cephalalgia. Indeed I have little reason to regard any gastric derangement as the cause of the affection in my case; and though the stomach is secondarily affected, yet the primary disease is not easily reached by simply acting on the stomach.

In one case (among my friends) with which I am well acquainted, these attacks have been very frequent from an early age till middle life. The bastioned outline is a striking feature in this case; but I am not able to say whether the blindness does or does not transgress the median vertical line of the field of view. It is always spoken of as 'half-blindness.' Formerly the attendant headache used to be very severe, accompanied with prolonged vomiting. Latterly the visual affection has been more oppressive than the headache, and its advent greatly dreaded. Sometimes the speech is affected, and the memory at the same time; on one occasion the mouth was seen to be drawn to one side. The cause has been easily recognized in previous anxiety and mental distress, troublesome letter-writing, and the like.

In another case the phenomena are much less definite. The first sign of the approach of an attack is a half-puzzled suspicion that the eye does not see all it ought, and it requires some gazing at surrounding objects to settle the doubt. Once it began with singular minuteness; the eyes and hands were engaged in scaling-off a certain distance from a fine linear mark, and in bringing the zero of the scale to coincide with the linear mark the zero became invisible just at the moment when the two lines were on the point of coinciding. Again and again the trial was made, ineffectually, till the nature of the failure was recognized in the gradual development of the blindness. The spot of origin must have been exactly central. In this case there is no definite serrated margin, no colour, no curve, nothing of which a picture can be made. The obscurity gathers like a cloudy film or gauze over the field, oppressive to the eyes, and accompanied with headache and nausea, and passes away after a doubtful period, leaving the impression that it is caused by disorder of the stomach. To this case the name Teichopsia is quite inapplicable; but the spread of the blindness from a small central spot, its extension over a great part of the field, and its final disappearance, establish it as a distinct variety of the same affection.

In yet a third case which I have recently met with, the blindness is sometimes brought on by looking at a striped wall-paper or a striped dress. The appearance before the eyes is described as zigzag, wavy, quivering, without colour. The first attack, in adult age, was followed by partial paralysis of one side; and later attacks have almost always had a sequel of defective speech, and tingling at the tip of the tongue, at the tip of the nose, and in the fingers and thumb.

I think I have accumulated evidence enough in the foregoing pages to establish the fact that there is a distinct form of transient hemiopsia, presenting the following main characteristics:—

1. Dependence on mental anxiety, bodily exhaustion, overwork to the eyes, gastric derangement, want of exercise.
2. Origin from a small spot near the centre of vision.
3. Orderly outward spread from the original spot.
4. Blindness to boundaries, but not to general impressions of light and colour.
5. Luminosity in the dark.
6. Bright bastioned margin, with gleams of various colours.
7. Tremor and 'boiling.'
8. Gradual occupation of one (lateral) half of the field of view.
9. Gradual recovery of clear vision in rear of the outward-spreading cloud.
10. Disappearance of the phenomenon after about half an hour.
11. Sequelæ: headache and nausea, and sometimes affection of speech and hearing, and even an approach to hemiplegia.

As to the actual seat of the visual derangement, all the facts of the case are in support of Dr. Wollaston's conclusion, that it is in the brain. The point that most distinctly bears upon this question is the exact agreement of the two eyes in the nature, extent, and degree of their affection.

Closing either eye, the scope of the other eye is blurred by the selfsame bastioned cloud*; and the effort of the will is powerless to disregard it and see through it.

The sight of both eyes being thus equally affected, we must conclude, assuming the semidecussation of the optic nerves at the chiasma, that the seat of the affection must lie at some point behind the chiasma of these nerves.

The main division (that seems to offer itself) into dextral and sinistral teichopsia will correspond then to the distinction between right and left permanent hemiopsia, and will depend upon temporary affection of the *left* or *right* optic tract, or its origin in the corresponding optic thalamus

The circumstances which determine the attack, whether those of bodily exhaustion, or mental fatigue and distress, or gastric derangement, all seem to me more likely to affect the inner integral parts of the brain than such outlying dependencies as the optic tracts, and lead me to suppose that the affection is in the former rather than in the latter. The partial paralysis, the loss of speech and of memory, and the derangement of hearing, that sometimes follow an attack of teichopsia, all point to the same conclusion. But, in truth, the radical connexions of the optic tracts seem to be so wide, that it is impossible at present to do more than guess at the locus morbi. Such cases as Sir John Herschel's, where the cloud passed over the whole field from left to right, can only be explained by supposing the disturbance to lie in some region of the brain where the opposite halves are in contact

Finally, as to the nature of the local mischief:—Is it a temporary suspension of function, propagated by contiguity, among the nerve-cells of the visual sensorium (wherever that may be), due to vascular congestion, and relieved by the relief of that congestion? Does the headache, following close upon the departure of the morbid vision, tell of the further propagation of the nervous disturbance into parts of the brain where disturbance is ache, as in the visual tract disturbance is abnormal sensation of light? And the detriment to speech and hearing that has occasionally been noticed, does it mean extension of the same disturbance still further into the regions of brain-substance appropriate to those functions?

The phenomena are so definite and so localized, and their course is so regular, that we can hardly avoid the conviction that their cause is equally definite and equally localized; and it is difficult to admit so vague an agent as nervous sympathy with gastric derangement, except as acting through the medium of some secondary local manifestation in the brain.

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* This is invariably the case with myself and with most of those who have noted these phenomena; but while preparing these pages for the press (May 12th, 1870) I have received a note from Sir John Herschel, with the following postscript.—"On the 16th ult., at waking, I found the 'Fortification pattern' *certainly* in my left eye *only*, and much more vivid with the eye open and looking at paper than when closed."

Regarded merely as a disease, Teichopsia, though by no means unimportant, may be thought hardly deserving of the attention of scientific men; but regarded as a veritable 'Photograph' of a morbid process going on in the brain, its interest and importance cannot be too strongly insisted upon.

In conclusion, I regret to be obliged to leave so much doubt upon so many points of the subject I have been dealing with. As more evidence arises, and more systematic observations are gathered, I hope these doubts may be removed. Meanwhile our duty is to collect and record *facts*, in confidence that they will arrange themselves into a theory sooner or later. No two cases of this disease present exactly the same features: every one is an illustration of the rest; and by the accumulation and comparison of accurate records we may hope that the transition from facts isolated to facts linked by the clue of theory will be soon attained.

Explanation of Plates.

PLATE XXV.

Figs. 1–4. Early stages of sinistral Teichopsia (see p. 256) beginning close to the sight-point, as seen in the dark. The letter O marks the sight-point in every figure.

Figs. 5–8. A similar series of the early stages of sinistral Teichopsia beginning a few degrees below and to the left of the sight-point.

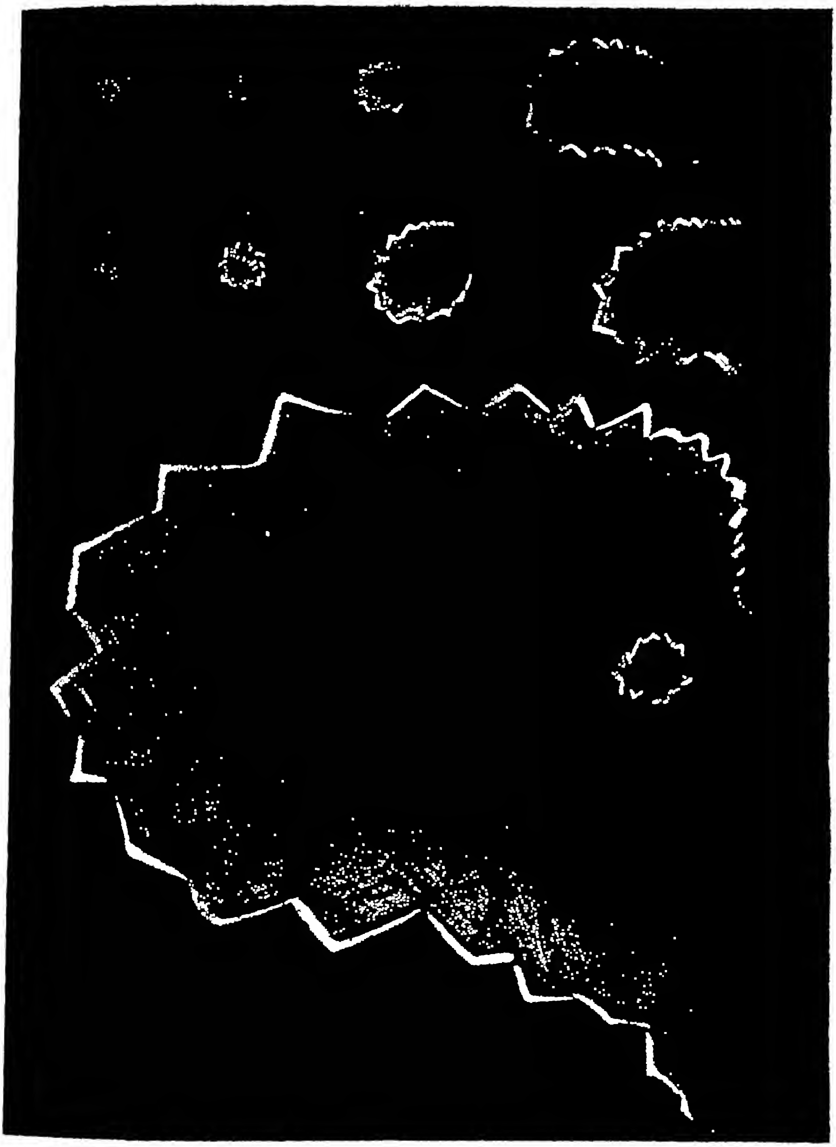
Fig. 9. Sinistral Teichopsia fully developed. *b*. Beginning of a secondary attack, which never attains full development unless it arise on the opposite side

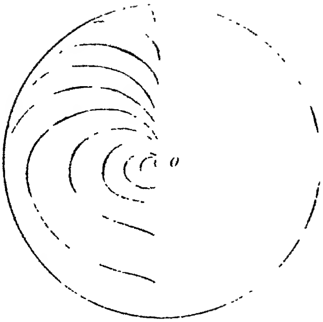
PLATE XXVI.

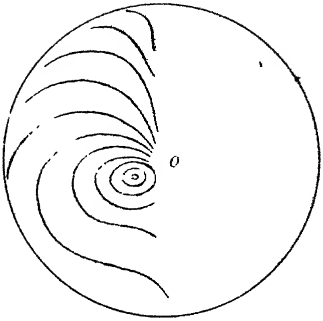
Fig. 1. Diagram to show the progress of the attack in any given case of sinistral Teichopsia beginning close to the sight-point but a little to the left. The successive curves denote the successive stages of the attack and positions of the arch, not transgressing the median vertical line.

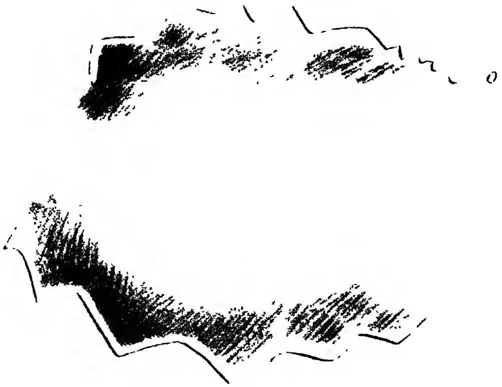
Fig. 2. Similar to fig. 1, for a case of sinistral Teichopsia beginning some distance below and to the left of the sight-point.

Fig. 3. Teichopsia, fading prematurely, losing continuity opposite the natural gap in its wall, and terminating by resolution.



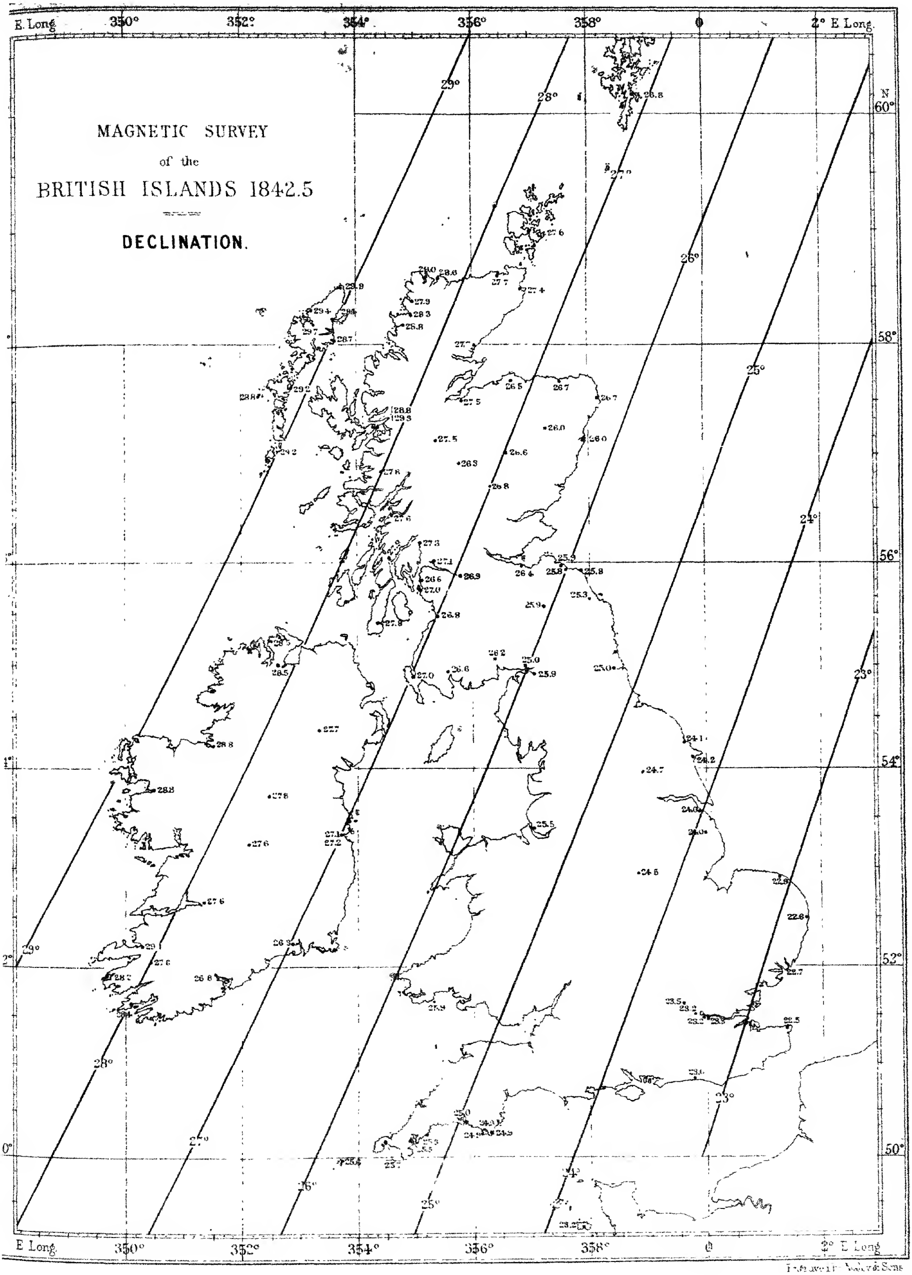






H Airy del. G West lith.

W West imp.

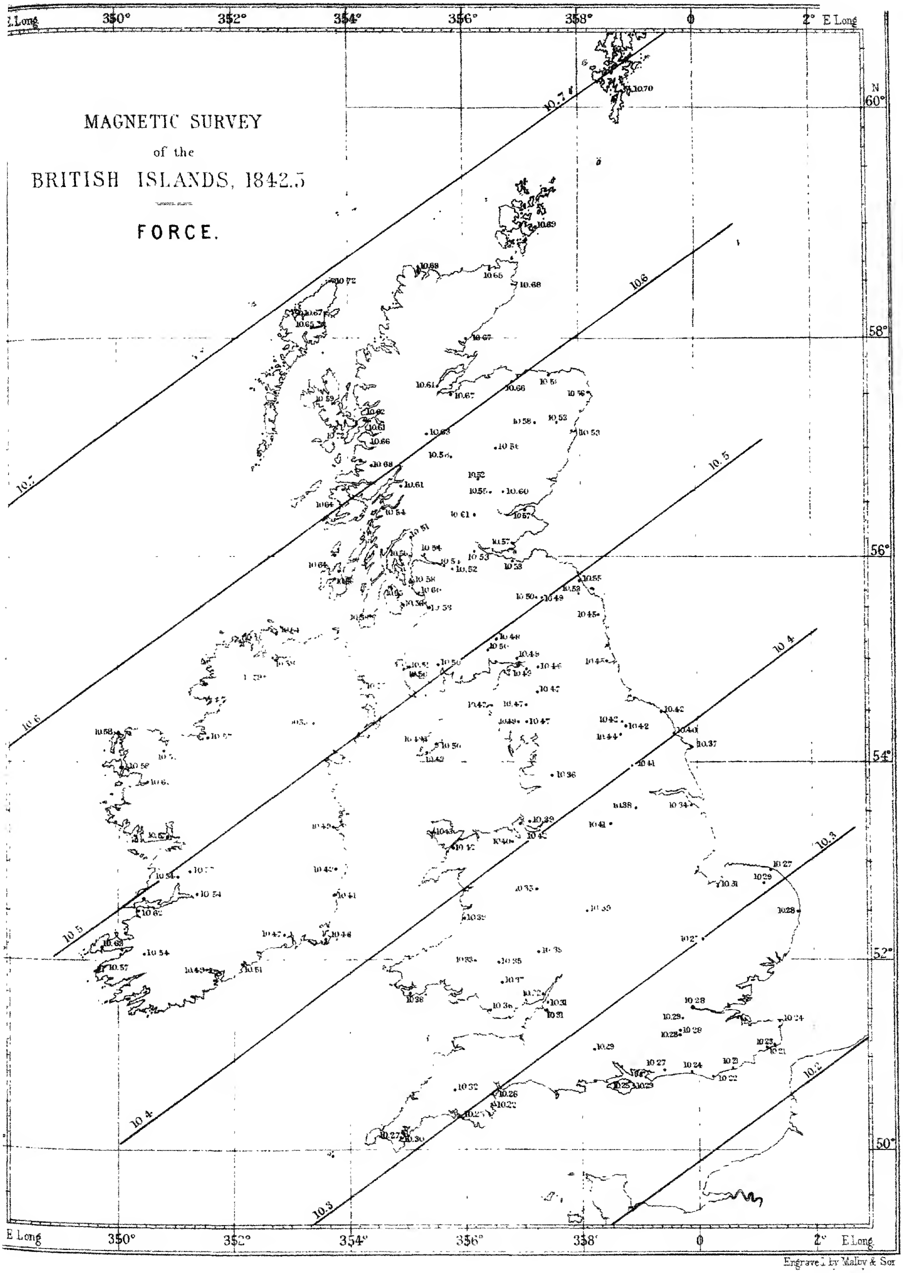


MAGNETIC SURVEY
of the
BRITISH ISLANDS 1842.5

DECLINATION.

E. Long.
350°
352°
354°
356°
358°
0
2° E Long.
N
60°
58°
56°
54°
52°
50°
29°
28°
27°
26°
25°
24°
23°
22°

Engraved by Malby & Sons



MAGNETIC SURVEY

of the

BRITISH ISLANDS, 1842.5

FORCE.

E Long
350°
352°
354°
356°
358°
0
2°
N
60°
58°
56°
54°
52°
50°
10.7
10.6
10.5
10.4
10.3
10.2

Engraved by Malby & Son

XIV. *Contributions to Terrestrial Magnetism.—No.* XII. *The Magnetic Survey of the British Islands, reduced to the Epoch* 1842·5. *By General Sir* Edward Sabine, *K.C.B., President of the Royal Society.*

Received June 15,—Read June 16, 1870.

The Magnetic Survey of the British Islands originated with a few persons interested in that branch of experimental science who attended the third Meeting of the British Association for the Advancement of Science, held at Cambridge in June 1833.

On his return to Dublin from attendance at that Meeting, Dr. Humphry Lloyd, the present Provost of Trinity College, Dublin, who was then its Professor of Natural Philosophy, proposed to myself, then serving on the Staff of the Army in Ireland, to unite with him in an endeavour to realize such an undertaking, by a commencement which should be at first limited to Ireland. Fortunately I had with me at the time the instruments which I had employed for similar purposes in several arctic and equatorial voyages; and being then quartered in the South-West District of Ireland, I found it not incompatible with my military duties to undertake the Southern portion of the island, whilst Professor Lloyd occupied himself in the Northern portion. Our observations were continued at intervals throughout 1834 and until the autumn of 1835, in the summer of which year we were joined by Captain James Clark Ross, R.N., who had been associated with me in similar undertakings in Arctic countries.

A provisional report of our operations, drawn up by Professor Lloyd, was presented to the British Association assembled in Dublin in 1835, and was printed in 1836 in the 4th volume of the Reports of the Association.

Mr. Robert Were Fox, who was present at the Dublin Meeting of the Association in 1835, brought with him an apparatus for magnetic observation on a new construction of his own invention, which, when the Meeting terminated, he employed in the course of a tour in the West and North of Ireland, the results of which were incorporated in Professor Lloyd's report adverted to in the last paragragh.

In 1836, having obtained two months' leave of absence from military duties in Ireland, I employed them in extending the Survey to twenty-seven stations in Scotland, well distributed over that country, and forming the basis of a memoir on the Scottish Isoclinal and Isodynamic Lines, which was printed in the fifth volume of the Reports of the Association, published in 1837.

In the summer of 1837 Professor Lloyd commenced the magnetic survey of England by observations at fourteen stations, principally in the midland and southern districts; and in the same summer Professor John Phillips, who as one of the Secretaries of the

British Association had participated in the early interest which had originated at the Meetings of the Association, visited and observed at twenty-four stations, chiefly in the north of England. In the same summer Mr. Fox observed at twenty stations in the north of England and south of Scotland; and in the summer of 1838 at eight additional stations in the south of England. In the same years (1837 and 1838) Captain James Ross employed himself almost unremittingly in magnetic observation, visiting for the purpose fifty-eight stations, extending over England, Ireland, and Scotland generally; whilst in the same years (1837 and 1838) my own observations comprehended twenty-two stations, distributed for the most part round the coast of England and Wales, and extending into Ireland and Scotland, so as to effect a more perfect connexion of the different Lines. A general provisional account of the results which had been thus obtained was drawn up by myself at the request of my colleagues, and was printed in 1839 in the 7th volume of the Reports of the British Association.

In the year 1856, twenty years having elapsed since the proceedings which have been thus referred to, the General Committee of the British Association deemed it expedient that the Survey should be resumed, partly with a view of adding other stations to those which had been included in the earlier operations, and partly for the purpose of repeating the observations at some of the earlier stations, with the view of examining the amounts of secular change which might appear to have taken place in the interval. The five persons who had taken part in the operations of 1835–38 were requested by the General Committee to continue their services, with the addition of Mr. John Welsh, Superintendent of the Magnetic Establishment at Kew. I was not myself present at the Cheltenham Meeting of the British Association in 1856 when the resolution was passed, requesting that a repetition of the survey of 1837 should be made by the same persons by whom the earlier survey had been accomplished; but on my return to England in 1856, finding my own name standing first in the list of the Committee by whom the work was to be accomplished, I lost no time in proposing to my colleagues such arrangements as seemed suitable for the accomplishment of the object which the Association had in view. The survey of the Scottish portion of the British Islands was entrusted to the very able hands of Mr. Welsh: who, in the summer and autumn of 1857, determined the Magnetic Elements at several stations in the Interior and on the East Coast of Scotland; and in the same season of the following year, extended the Survey to the West Coast, the Hebrides, and the Orkney and Shetland Islands. This work was performed with all Mr. Welsh's wonted accuracy and completeness; and with a devotion which was but too great, for the exposure to inclement weather acting on a previous delicacy of health proved the immediate occasion of the illness which at last terminated fatally. Science lost in him not only a zealous and accomplished worker, but one of rare gifts and qualities, affording yet higher promise of usefulness, if his had been a prolonged life. A provisional account of the results of Mr. Welsh's operations, drawn up by Mr. Balfour Stewart, his successor in the superintendence of the Magnetic Observatory at Kew, was printed in the Report of the Aberdeen Meeting of the British Association in 1859.

There was reason to hope that the Irish portion of the Survey would be repeated by Professor Lloyd, with the aid of other members of the Royal Irish Academy, who proposed that, when made, it should be printed in the Transactions of that Academy.

For some time I cherished the hope that the English Survey would be accomplished, as before, by the joint labours of the original observers; and accordingly I commenced my share of the work in the summer of 1858; but as time advanced, it became evident that circumstances of health and the pressure of other employments and duties stood in the way of the hoped for combined operation. My own avocations would not permit me to devote to this object so much time consecutively as would have been required for its accomplishment in a single year; but by employing in it portions of the summers of 1858, 1859, 1860, 1861, and 1862, I obtained observations for the determination of the isodynamic and isoclinal lines at twenty-four well-distributed stations. The Declination was supplied for several points on the coasts of the United Kingdom by Captain Frederic John Evans, R.N., F.R.S., from observations made by several naval officers between the years 1855 and 1861. A general notice of the results thus obtained was printed in the volume of the British Association Reports for 1861, together with the details of my own observations.

Having premised this general historical statement I proceed to a more circumstantial notice of some points of detail.

Corrections employed for secular change.

A. *Declination.*—The observations of the Declination comprised in the Survey supply six stations in Scotland at which that element was determined with suitable accuracy, at an interval in each case of nineteen or twenty years. The observers in all these instances were Captain James Ross at the earlier date, *i. e.* in 1838·5, and Mr. Welsh at the later date, *i. e.* 1857·5 or 1858·5. The details are as follows:—

Stations.	Lat. N.	Long. E.	Observer.	Date.	Delination observed.	Interval.	Average annual decrease.	Mean secular change.
						years.		
Lerwick	60° 09′	358° 53′	Ross.	1838·5	27° 09′ w.	20	5′·5	5′·3
			Welsh.	1858·5	25 18 w.			
Kirkwall	58 59	357 02	Ross.	1838·5	27 47 w.	20	4·5	
			Welsh.	1858·5	26 17 w.			
Wick	58 25	356 55	Ross.	1838·5	27 41 w.	20	4·9	
			Welsh.	1858·5	26 04 w.			
Golspie	57 58	356 03	Ross.	1838·5	27 54 w.	20	5·0	
			Welsh.	1858·5	26 15 w.			
Inverness	57 28	355 49	Ross.	1838·5	27 39 w.	19	6·4	
			Welsh.	1857·5	25 57 w.			
Aberdeen	57 09	357 55	Ross.	1838·5	26 21 w.	19	5·5	
			Welsh.	1857·5	24 36 w.			

In the Philosophical Transactions for 1863, Art. XII., p. 291, it is shown that, at the Kew Magnetic Observatory (lat. 51° 29′ N., long. 359° 42′ E.), the average rate of diminution of the Westerly Declination in 1858–59 was 6′·8 yearly (the decrease having been a little less in the years immediately preceding 1858, and a little more in the years immediately following that epoch). We may infer, consequently, that the annual decrease of the Declination was somewhat greater in the south-east of England, than it was at the same period in Scotland: and we find it stated by Dr. Lloyd, in vol. i. page 80 of the Magnetical Observations at Trinity College in Dublin, that in the years 1840 to 1843 the rate of decrease in Dublin was about 6′·9.

In conformity with these premises, the secular change of the Declination, at the Epoch of 1842·5 to which the observations of the British Survey are referred, has been assumed to have been an annual decrease of West Declination of approximately 5′·6 in Scotland and the north of England, increasing to 6′·2 in the middle and southern part of England, and to 6′·9 in Ireland.

B. *Inclination.*—In Scotland and the adjacent islands the survey supplies seventeen stations at which observations of the Inclination were made at intervals of nineteen years and upwards. They are brought together in the following Table.

Station.	Lat. N.	Long. E.	Observer.	Date.	Inclination observed.	Interval.	Annual secular decrease.
						years.	
Lerwick	60° 09′	358° 53′	Sabine. Ross. Welsh.	1818·5 1838·5 1858·5	74° 22′ 73 45 73 13	20 20	1′·85 1′·60 } 1′·72
Kirkwall.........	59 00	357 02	Ross. Welsh.	1838·5 1858·5	73 20 72 42	20	1·90
Wick	58 24	356 55	Ross. Welsh.	1838·5 1858·5	73 20 72 41	20	1·95
Golspie	57 58	356 03	Sabine. Ross. Welsh.	1836·5 1838·5 1858·5	72 56 73 04 72 26	21	1·62
Inverness	57 28	355 49	Sabine. Ross. Welsh.	1836·5 1838·5 1857·5	72 46 72 46 72 08	20	1·90
Melrose	55 35	357 16	Sabine. Fox. Welsh.	1836·5 1837·5 1857·5	71 37 71 38 70 55	20·5	2·07
Berwick	55 45	358 00	Ross. Welsh.	1838·5 1857·5	71 42 70 55	19	2·47
Alford	57 13	357 15	Sabine. Welsh.	1836·5 1857·5	72 22 71 46	21	1·71
Fort Augustus .	57 08	355 20	Sabine. Welsh.	1836·5 1857·5	72 40 72 03	21	1·76
Edinburgh	55 57	356 49	Sabine. Fox. Welsh.	1836·5 1837·5 1857·5	71 50 71 50 71 11	20·5	1·92
Gretna	55 01	356 56	Fox. Welsh.	1837·5 1857·5	71 29 70 46	20	2·15
Braemar.........	57 01	356 35	Sabine. Welsh.	1836·5 1857·5	72 14 71 31	21	2·05
Jordanhill	55 54	355 39	Sabine. Sabine.	1838·5 1859·5	72 14 71 30	21	2·10
Campbelton ...	55 23	354 22	Sabine. Welsh.	1836·5 1857·5	71 56 71 14	21	2·00
Helensburgh ...	56 00	355 19	Sabine. Welsh.	1836·5 1857·5	72 17 71 30	21	2·24
Cumbray	55 48	355 08	Sabine. Welsh.	1836·5 1857·5	72 01 71 29	21	1·52
Glasgow.........	55 51	355 46	Sabine. Fox. Welsh.	1836·5 1837·5 1857·5	72 02 72 05 71 26	20·5	1·83

The mean secular change shown by this Table is an annual diminution of 1′·94, or approximately 2′·0; which has been accordingly employed in the reduction to the mean epoch (1842·5) of all the observations at the Scotch Stations.

In England the number of stations at which we have determinations of the Inclination at intervals of considerable duration (within the limits of the time comprehended by the survey) are much fewer, but they are sufficient to supply a satisfactory approximation to the amount of the secular change for the short intervals for which the corrections are required. On the eastern side of England we have five localities, at all of which the Inclination was observed at intervals comprehending from 21·5 to 24 years. These, with their respective geographical positions and the resulting annual decrease of the Inclination at each, are shown in the following Table, viz.:—

Stations.	Lat. N.	Long. E.	Observer.	Date.	Inclination observed.	Interval.	Annual secular change.
						years.	
Scarborough	54° 17′	359° 37′	Phillips. Ross. Sabine.	1837·5 1838·5 1859·5	70° 42′ 70 43 69 59	21·5	2′·23
Cromer	52 56	1 19	Ross. Sabine.	1838·5 1861·5	69 46 68 55	23·0	2·22
Lowestoft	52 28	1 50	Ross. Sabine.	1838·5 1861·5	69 29 68 39	23·0	2·17
Cambridge	52 13	0 07	Lloyd. Sabine.	1836·5 1860·5	69 42 68 42	24·0	2·50
Margate	51 23	1 23	Sabine. Ross. Sabine.	1837·5 1838·5 1860·5	69 03 68 57 68 06	22·5	2·40

On the western side of England we have four stations at which the Inclination was observed at nearly similar intervals; these are:—

Stations.	Lat. N.	Long. E.	Observer.	Date.	Inclination observed.	Interval.	Annual secular change.
						years.	
Stackpole Court...	51° 38′	355° 05′	Ross. Sabine.	1837·5 1860·5	69° 56′ 68 58	23·0	2′·52
Lew Trenchard...	50 40	355 50	Sabine. Sabine.	1838·5 1859·5	69 19 68 18	21·0	2·90
Falmouth	50 09	354 54	Ross. Sabine. Fox. Sabine.	1837·5 1838·5 1838·5 1859·5	69 16 69 12 69 14 68 08	21·5	3·07
Plymouth	50 22	355 51	Ross. Sabine.	1837·5 1859·5	69 06 68 06	22·0	2·73

We have, further, one English station, viz. Kew, where the secular change of the Inclination has been the subject of a very careful and persistent inquiry for a much longer period (Proceedings of the Royal Society, vol. xi. pp. 144–162). In accordance with that discussion, we may safely regard an annual decrease of 2′·7 as approximately applicable at Kew to the whole interval from 1834 to 1861. And from all these results we may infer the existence of a small but tolerably well-assured progressive augmentation in the amount of the secular change of the Inclination,—1°, with a diminishing latitude, and 2°, with an increasing westerly position.

In the corrections to the mean epoch of 1842·5 I have employed in Scotland an annual decrease of 2′·0. In England, I have increased this rate with progressively diminishing latitude, on the eastern side to 2′·65 in 51° 30′, and on the western side to 2′·85 in lat. 50° 30′. In Ireland I have taken 2′·3 in lat. 55°, increasing to 2′·8 in lat. 51°.

C. *Force.*—In the determinations of the Intensity of the Magnetic Force, the Kew Observatory has been regarded as supplying the fundamental station of the British Survey. From the Philosophical Transactions, 1863, Art. XII. p. 302, we may assume the total force at Kew in absolute measure (British units), at the definite epoch of July 1, 1860, to have been 10·302; subject to an annual increase from secular change of ·00125, as derived from the Kew Observations between April 1857 (when the regular series of absolute determinations at that observatory commenced) and March 1862. In accordance with these values the total force at Kew at the mean epoch (1842·5), for which the present maps are constructed, is taken as 10·280, and the corrections applied to the several determinations to reduce them to the mean epoch are proportional to an annual increase of ·00125.

The maps of the three magnetic elements which accompany this paper have been prepared at the Hydrographic Office, with the sanction of Admiral RICHARDS, under the superintendence of Captain FREDERICK JOHN EVANS, R.N.

The Stations are arranged in the Table in order of latitude; the initials in the column of Observers denote:—L, Professor HUMPHREY LLOYD; F, Mr. ROBERT WERE FOX; R, Sir JAMES CLARK ROSS; P, Professor JOHN PHILLIPS; W, Mr. JOHN WELSH; and S, Sir EDWARD SABINE. The Naval Officers whose Declinations have been employed are Captains EVANS, OTTER, BEDFORD (B^d), BEECHEY (B^y), COX (C^x), CHURCH (C^h), WILLIAMS (W^s), THOMAS and ALDRIDGE (who are distinguished in the Table by their initials).

Stations.	Lat. N.	Long. E.	Observer.	Date.	Declination.			Inclination.			Force.		
					Observed.	Secular change.	Corrected: degrees and decimals.	Observed.	Secular change.	Corrected: degrees and decimals.	Observed.	Secular change.	Corrected.
					° ′	° ′	° °						
Lerwick	60° 09′	358° 53′	S.	1818·5	......	......	......	74° 22′	−48′	73°·6 } 73·7°	10·71	+·03	10·74 } 10 70
			R.	1838·5	27 09	−0 22	26 8 } 26·8	73 45	− 8	73·6	10·65	......	10·65
			W.	1858·5	25 18	+1 30	26·8	73 13	+32	73·8	10 74	−·02	10·72
Kirkwall	59 00	357 02	R.	1838 5	27 47	−0 22	27·4 } 27·6	73 20	− 8	73·2 } 73·2	10·67	+·01	10·68 } 10·69
			W.	1858·5	26 17	+1 30	27·8	72 42	+32	73·2	10 72	−·02	10·70
Stromness	58 57	356 44	W.	1858·5	......	......	......	72 46	+32	73·3	......	......	......
Thurso	58 35	356 28	A.	1856·5	26 08	+1 18	27·4 } 27·7	......	......	......	......	......	......
			W.	1858 5	26 30	+1 30	28·0	72 33	+32	73 1	10 67	−·02	10·65
Loch Eribol	58 34	355 20	O.	1856 5	27 17	+1 18	28 6	......	......	......	......	......	......
Durness	58 34	355 16	W.	1858·5	27 30	+1 30	29·0	72 50	+32	73·4	10·70	−·02	10·68
Cross	58 29	353 43	W.	1858 5	28 25	+1 30	29·9	72 49	+32	73·4	10·74	−·02	10·72
Wick	58 24	356 55	R.	1838·5	27 41	−0 22	27·3 } 27·4	73 20	− 8	73 2 } 73·2	......	......	......
			W.	1858 5	26 04	+1 30	27·6	72 41	+32	73 2	10·70	−·02	10·68
Luxford	58 23	354 56	O.	1846·5	27 34	+0 22	27·9	......	......	......	......	......	......
Carloway	58 17	353 13	O.	1856 5	28 05	+1 18	29·4	......	......	......	......	......	......
Stornoway	58 15	353 37	W.	1858 5	27 56	+1 30	29·4	72 33	+32	73·1	10·69	−·02	10·67
Gowan	58 15	354 57	O.	1846·5	27 58	+0 22	28 3	......	......	......	......	......	......
Loch Inver	58 10	354 48	O.	1848·5	27 59	+0 34	28·6 } 28 8	......	......	......	......	......	......
			W.	1858·5	27 30	+1 30	29·0	72 36	+32	73 1	......	......	......
Callinish	58 10	353 16	W.	1858 5	28 09	+1 30	29 7	72 34	+32	73·1	10·67	−·02	10·65
Loch Shell	58 01	353 34	O.	1856·5	27 24	+1 18	28·7	......	......	......	......	......	......
Golspie	57 58	356 03	S.	1836·5	......	......	......	72 56	−12	72·7 } 72·9	......	......	......
			R.	1838·5	27 55	−0 22	27·6 } 27·7	73 04	− 8	72·9	10·65	+·01	10·66 } 10·67
			W.	1858 5	26 15	+1 30	27·8	72 26	+32	73·0	10·49	−·02	10·67
Elgin	57 39	356 41	W.	1857·5	25 07	+1 24	26 5	72 08	+30	72 6	......	......	......
Banff	57 39	357 29	W.	1857·5	25 17	+1 24	26 7	71 57	+30	72 5	10·60	−·02	10·58
Gordon Castle	57 37	356 51	S.	1836·5	......	......	......	72 41	−12	72·5	10·65	+·01	10·66
Loch Maddy	57 36	352 52	O.	1859 5	27 38	+1 35	29 2	......	......	......	......	......	......
Dingwall	57 34	355 35	W.	1858·5	......	......	......	72 25	+32	73 0	10·63	−·02	10·61
Peterhead	57 31	358 14	W.	1857 5	25 17	+1 24	26 7	71 55	+30	72·4	10·58	−·02	10·56
Shillay	57 31	352 18	O.	1859·5	27 15	+1 35	28 8	......	......	......	......	......	......
Inverness	57 28	355 49	S.	1836 5	......	......	......	72 46	−12	72·6 } 72 6	10·66	+·01	10·67
			R.	1838·5	27 59	−0 22	27·6 } 27·5	72 46	− 8	72·6	10·67	......	
			W.	1857·5	25 59	+1 24	27·4	72 08	+30	72·6	10·67	−·02	
Portree	57 26	353 48	W.	1858·5	......	......	......	72 01	+32	72 6	10·61	−·02	10·59
Rhynie	57 20	357 10	S.	1836 5	......	......	......	72 26	−12	72·2	......	......	......
Balmacarra	57 17	354 21	W.	1858·5	27 33	+1 30	29·1	72 13	+32	72 8	10·63	−·02	10·61
Kyleakin	57 16	354 16	O.	1856·5	27 29	+1 18	28 8	......	......	......	......	......	......
			W.	1858·5	......	......	......	72 11	+32	72·7	10·64	−·02	10·62
Broadford	57 15	354 09	W.	1858 5	......	......	......	72 16	+32	72·8	10 68	−·02	10·66
Kintore	57 15	357 37	W.	1857·5	......	......	......	71 37	+30	72·1	10 55	−·02	10·53
Loch Slapin	57 14	353 58	S.	1836·5	......	......	......	73 02	−12	72 8	10·71	+·01	10 72
Loch Scavig	57 14	353 53	S.	1836 5	......	......	......	73 05	−12	72·9	......	......	......
Alford	57 13	357 15	S.	1836·5	......	......	......	72 22	−12	72·2 } 72·2	10·57	+·01	10·58
			W.	1857·5	24 36	+1 24	26 0	71 46	+30	72 3	10 60	−·02	
Aberdeen	57 09	357 55	R.	1838 5	26 21	−0 22	26 0 } 26·0	......	......	......	10·55	......	10·53
			W.	1857·5	24 31	+1 24	25 9	71 49	+30	72·3	10·54	−·02	
Fort Augustus	57 09	355 20	S.	1836·5	......	......	......	72 40	−12	72·5 } 72·5	......	......	......
			W.	1857·5	26 04	+1 24	27·5	72 03	+30	72 6	10·65	−·02	10·63
Barra Sound	57 03	352 37	O.	1861·5	27 27	+1 46	29 2	......	......	......	......	......	......
Braemar	57 01	356 35	S.	1833 5	......	......	......	72 14	−12	72 0 } 72·0	10·55	+·01	10·56
			W.	1857·5	25 09	+1 24	26·6	71 31	+30	72 0	10·59	+·02	
Dalwhinnie	56 56	355 43	W.	1857·5	25 55	+1 24	26 3	71 40	+30	72·2	10 60	−·02	10·58
Corpach	56 51	354 52	W.	1857·5	26 22	+1 24	27 8	71 53	+30	72·4	10·70	−·02	10·68
Pitlochry	56 42	356 17	W.	1857·5	25 23	+1 24	26 8	71 35	+30	72 1	10·54	−·02	10·52
Tobermorie	56 39	353 58	W.	1858 5	......	......	......	......	......	......	10·66	−·02	10·64
Glencoe	56 39	354 53	S	1836 5	......	......	......	72 17	−12	72·1	10·60	+·01	10·61
Glenmorven	56 38	354 02	W.	1858·5	......	......	......	72 02	+32	72·6	......	......	......
Blairgowrie	56 36	356 42	S.	1836·5	......	......	......	71 55	−12	71·7	10·59	+·01	10·60
Dunkeld	56 35	356 27	R.	1838·5	......	......	......	72 23	− 8	72 2	10 55	......	10·55
Artornish	56 33	354 53	S.	1836·5	......	......	......	72 43	−12	72 5	......	......	......
Castle Duart	56 31	354 15	S.	1836·5	......	......	......	72 15	−12	72·1	......	......	......
Oban	56 26	354 33	W.	1857 5	26 12	+1 24	27·6	71 30	+30	72·0	10·56	−·02	10·54

TABLE (continued).

Stations.	Lat. N.	Long. E.	Observer.	Date.	Declination.			Inclination.				Force.			
					Observed.	Secular change.	Corrected: degrees and decimals.	Observed.	Secular change.	Corrected: degrees and decimals.		Observed.	Secular change.	Corrected.	
	° ′	° ′			° ′	° ′	°	° ′	′	°	°				
Newport	56 25	357 05	S.	1836·5	......	......	....	72 17	−12	72·1		10·56	+·01	10·57	
Fern Tower	56 22	356 10	S.	1859·5	......	......	....	71 24	+34	72·0		10·63	−·02	10·61	
Inverary	56 15	354 56	F.	1837·5	......	......	....	72 07	−10	72·0		......	......	......	
Loch Lomond	56 13	355 20	F.	1837·5	......	......	....	72 15	−10	72·1		......	......	......	
Loch Goil Head	56 10	355 06	W.	1857·5	25 52	+1 24	27·3	71 17	+30	71·8		10·53	−·02	10·51	
Kirkaldy	56 07	357 00	S.	1836·5	......	......	....	72 11	−12	72·0		10·56	+·01	10·57	
Loch Gilphead	56 04	354 32	S.	1836·5	......	......	....	72 08	−12	71·9		......	......	......	
Tarbert	56 02	356 11	W.	1857·5	......	......	....	71 34	+30	72·1		10·55	−·02	10·53	
Helensburgh	56 00	355 19	S.	1836·5	......	......	....	72 01	−12	71·8	71·9	10·54	+·01	10·55	10·54
			W.	1857·5	......	......	....	71 30	+30	72·0		10·54	−·02	10·52	
Ardrishaig	56 00	354 33	W.	1857·5	26 30	+1 24	27·9	71 26	+30	71·9		10·58	−·02	10·56	
Broxburn	56 00	357 31	T.	1856·5	24 36	+1 18	25·9	......	......	......		......	......	......	
Linlithgow	55 59	356 23	F.	1837·5	......	......	....	71 59	−10	71·8		......	......	......	
Edinburgh	55 58	356 49	S.	1836·5	......	......	....	71 50	−12	71·6	71·7	10·51	+·01	10·52	10·53
			F.	1837·5	......	......	....	71 50	−10	71·7		......	......	......	
			W.	1858·5	24 56	+1 30	26·4	71 11	+31	71·7		10·56	−·02	10·54	
Loch Ridan	55 57	354 50	S.	1836·5	......	......	....	72 17	−12	72·1		......	......	......	
Fast Castle	55 55	357 36	T.	1856·5	24 29	+1 18	25·8	......	......	......		......	......	......	
St. Abbs	55 55	357 52	T.	1856·5	24 28	+1 18	25·8	......	......	......		......	......	......	
Jordanhill	55 54	355 39	R.	1838·5	......	......	....	......	......	......		10·52	+·01	10·53	10·54
			S.	1838·5	......	......	....	......	......	......		10·54	+·01	10·55	
Glasgow	55 51	355 46	S.	1836·5	......	......	....	72 02	−12	71·8	71·9	10·51	+·01	10·52	
			F.	1837·5	......	......	....	72 05	−10	71·9		......	......	......	
			W.	1857·5	25 28	+1 24	26·9	71 26	+30	71·9		......	......	......	
Port Askeg	55 52	353 52	W.	1858·5	......	......	....	71 14	+32	71·8		10·66	−·02	10·64	
Brisbane	55 48	355 08	W.	1857·5	25 14	+1 24	26·6	71 33	+30	72·1		......	......	......	
Bridgend	55 48	353 44	W.	1858·5	......	......	....	71 15	+32	71·8		10·68	−·02	10·66	
Cumbray	55 48	355 08	S.	1836·5	......	......	....	72 01	−12	71·8	71·9	10·57	+·01	10·58	
			W.	1857·5	25 37	+1 24	27·0	71 29	+30	72·0		......	......	......	
Berwick	55 45	358 00	R.	1838·5	......	......	....	71 42	− 8	71·6	71·5	10·54	+·01	10·55	
			W.	1857·5	......	......	....	70 55	+30	71·4		......	......	......	
Carstairs	55 43	356 20	W.	1857·5	......	......	....	70 56	+30	71·4		......	......	......	
Loch Ranza	55 42	354 43	S.	1836·5	......	......	....	72 23	−12	72·2		10·54	+·01	10·55	
Port Ellen	55 40	353 50	W.	1858·5	......	......	....	71 04	+32	71·6		......	......	......	
Ardrossan	55 39	355 13	W.	1858·5	......	......	....	71 14	+22	71·8		10·62	−·02	10·60	
Makerstoun	55 35	357 59	*	1842·5	25 27	......	25·5	......	......	......		......	......	......	
			W.	1857·5	23 56	+1 24	25·3	70 50	+30	71·3		10·55	−·02	10·53	
Melrose	55 35	357 16	S.	1836·5	......	......	....	71 37	−12	71·4	71·4	10·49	+·01	10·50	
			F.	1837·5	......	......	....	71 38	−10	71·5		......	......	......	
			W.	1857·5	24 27	+1 24	25·9	70 55	+30	71·4		......	......	......	
Dryburgh	55 34	357 21	S.	1836·5	......	......	....	71 34	−12	71·4		10·48	+·01	10·49	
Lamlash	55 31	354 55	W.	1857·5	......	......	....	71 06	+30	71·6		10·60	−·02	10·58	
Ayr	55 28	355 22	W.	1857·5	25 26	+1 24	26·8	71 06	+30	71·6		10·55	−·02	10·53	
Alnwick Castle	55 25	358 18	S.	1838·5	......	......	....	71 23	− 8	71·3		10·44	+·01	10·45	
Campbelton	55 23	354 22	S.	1836·5	......	......	....	71 56	−12	71·7	71·7	10·57	+·01	10·58	
			W.	1857·5	26 24	+1 24	27·8	71 14	+30	71·7		......	......	......	
Moffat	55 20	356 33	F.	1837·5	......	......	....	71 40	−10	71·5		......	......	......	
Loch Swilly	55 17	352 33	B^{d}.	1855·5	27 01	+1 30	28·5	......	......	......		......	......	......	
Carn	55 15	352 45	L.	1834·5	......	......	....	72 01	−19	71·7		10·63	+·01	10·64	
Jardine Hall	55 10	356 36	S.	1859·5	......	......	....	70 44	+34	71·3		10·50	−·02	10·48	
Belsay	55 07	358 07	F.	1837·5	......	......	....	71 17	−10	71·1		......	......	......	
Dumfries	55 05	356 24	W.	1857·5	24 47	+1 24	26·2	70 44	+30	71·2		10·52	−·02	10·50	
Cushendal	55 04	353 55	F.	1835·5	......	......	....	72 00	−14	71·8		......	......	......	
Gretna	55 01	356 56	F.	1837·5	......	......	....	71 29	−10	71·3	71·3	......	......	......	
			W.	1857·5	23 33	+1 24	25·0	70 46	+30	71·3		10·51	−·02	10·49	
Londonderry	54 59	352 36	R.	1839·5	28 47	−0 20	28·5	72 02	− 7	71·9		10·58	......	10·58	
Culgruff	54 58	356 00	R.	1838·5	......	......	....	71 36	− 8	71·5		......	......	......	
Newcastle	54 58	358 24	P.	1837·5	......	......	....	71 18	−10	71·1	71·1	10·45	+·01	......	
			S.	1838·5	......	......	....	71 09	− 8	71·0		10·43	+·01	10·45	
			R.	1838·5	......	......	....	71 13	− 8	71·1		10·44	+·01	......	
Newton Stewart	54 56	355 32	W.	1857·5	25 10	+1 24	26·6	70 54	+30	71·4		10·52	−·02	10·50	
Loch Ryan	54 55	355 01	S.	1836·5	......	......	....	71 43	−12	71·5		10·50	+·01	10·51	
Stranraer	54 54	354 58	W.	1857·5	25 38	+1 24	27·0	70 55	+30	71·4		10·52	−·02	10·50	
Stonehouse	54 54	357 16	R.	1838·5	......	......	....	71 24	− 8	71·3	71·3	10·45	+·01	10·46	10·46
			S.	1838·5	......	......	....	71 20	− 8	71·2		10·45	+·01	10·46	
Carlisle	54 54	357 06	P.	1837·5	......	......	....	71 29	−10	71·3		10·48	+·01	10·49	
Strabane	54 49	352 32	L.	1834·5	......	......	....	72 00	−16	71·7		10·58	+·01	10·59	
Skull	54 43	358 00	F.	1837·5	......	......	....	71 14	−10	71·1		......	......	......	
Skiddaw	54 40	356 51	F.	1837·5	......	......	....	71 15	−10	71·1		......	......	......	
Penrith	54 40	357 15	P.	1837·5	......	......	....	71 23	−10	71·2		10·46	+·01	10·47	
Bangor	54 39	354 18	S.	1836·5	......	......	....	71 39	−12	71·5		10·54	+·01	10·55	
Keswick	54 37	356 51	F.	1837·5	......	......	....	71 14	−10	71·1		......	......	......	
Whitehaven	54 33	356 27	S.	1838·5	......	......	....	71 11	− 8	71·1		10·46	+·01	10·47	

* At the Makerstoun Observatory.

TABLE (continued).

Stations.	Lat. N.	Long. E.	Observer.	Date.	Declination.				Inclination.				Force.			
					Observed.	Secular change.	Corrected: degrees and decimals.		Observed.	Secular change.	Corrected: degrees and decimals.		Observed.	Secular change.	Corrected.	
Patterdale	54° 32′	357° 04′	P.	1837·5	......	......	......		71° 20′	−10′	71°·2		10·46	+·01	10·47	
Darlington	54 32	358 27	P.	1837·5	......	......	......		71 07	−10	71·0		......	......	......	
Whitby	54 29	359 23	P.	1837·5	......	......	......		70 58	−10	70·8		10·41	+·01	10·42	
Grassmere	54 27	356 59	F.	1837·5	......	......	......		71 13	−10	71·1		......	......	......	
Lissadel	54 23	351 27	*	1838·5	......	......	......		71 56	− 8	71·8		......	......	......	
Bowness	54 22	357 05	P.	1837·5	......	......	......		71 18	−10	71·1		10·46	+·01	10·47	
Osmotherly	54 22	358 42	P.	1837·5	......	......	......		71 03	−10	70·9		10·41	+·01	10·42	
Coniston	54 22	356 55	P.	1837·5	......	......	......		71 20	−10	71·2		10·48	+·01	10·49	
Enniskillen	54 21	352 22	L.	1834·5	......	......	......		72 00	−16	71·7		......	......	......	
Armagh	54 21	353 21	L.	1834·5	......	......	......		71 42	−16	71·4	71·5	10·52	+·01	10·53	
			R.	1839·5	......	......	......		71 41	− 6	71·6		......	......	......	
Hambleton	54 20	358 45	P.	1837·5	......	......	......		71 04	−10	70·9		10·41	+·01	10·42	
Scarborough	54 17	359 37	P.	1837·5	......	......	......		70 42	−10	70·5	70·6	10·38	+·01	10·39	10·40
			R.	1838·5	......	......	......		70 43	− 8	70·6		......	......	......	
			S.	1859·5	......	......	......		69 59	+34	70·6		10·42	−·02	10·40	
Thirsk	54 14	358 39	P.	1837·5	......	......	......		70 59	−12	70·8		10·43	+·01	10·44	
Belmullet	54 13	350 03	L.	1835·5	......	......	......		72 13	−16	72·0		10·57	+·01	10·58	
Peel Town	54 13	355 17	P.	1837·5	......	......	......		71 24	−11	71·2		10·48	+·01	10·49	
Markree	54 12	351 34	L.	1835·5	......	......	......		72 06	−16	71·8	71·8	10·56	+·01	10·57	
			R.	1839·5	......	......	......		72 02	−10	71·9		......	......	......	
Douglas	54 10	355 32	P.	1837·5	......	......	......		71 22	−11	71·2	71·2	10·49	+·01	10·50	
			R.	1837·5	......	......	......		71 20	−11	71·2		......	......	......	
Studley Park	54 08	358 26	F.	1837·5	......	......	......		70 56	−11	70·8		......	......	......	
Bridlington	54 08	359 46	R.	1838·5	24 39	−0 24	24·3	24·2	70 39	− 9	70·5		......	......	......	
				1854·5	22 44	+1 25	24·2		......	......	......		......	......	......	
Flamborough	54 08	359 52	P.	1837·5	......	......	......		70 37	−11	70·4		10·36	+·01	10·37	
Ballina	54 07	350 53	L.	1835·5	......	......	......		72 11	−18	71·9	71·9	10·55	+·01	10·56	
			F.	1835·5	......	......	......		72 07	−18	71·8		......	......	......	
Castleton	54 04	355 20	P.	1837·5	......	......	......		71 23	−12	71·2		10·48	+·01	10·49	
Carlingford	54 02	353 49	L.	1834·5	......	......	......		71 30	−19	71·2		......	......	......	
York	53 58	358 54	P.	1837·5	......	......	......		70 49	−11	70·6	70·6	10·40	+·01	10·41	
			R.	1838·5	......	......	......		70 45	− 9	70·6		......	......	......	
Loch Conn	53 58	350 50	F.	1835·5	......	......	......		72 08	−17	71·9		......	......	......	
Achil Ferry	53 56	350 08	L	1835·5	......	......	......		72 07	−17	71·8		10·57	+·01	10·58	
Garstang	53 54	357 13	F.	1837·5	......	......	......		70 59	−12	70·8		......	......	......	
Stonyhurst	53 51	357 32	S.	1858·5	......	......	......		70 00	+38	70·6		10·38	−·02	10·36	
Westport	53 48	350 31	F.	1835·5	......	......	......		72 03	−17	71·8	71·8	......	......	......	
			R.	1839·5	29 09	−0 21	28·8		71 59	− 8	71·8		10·61	+·01	10·62	
Edgeworthstown	53 42	352 27	R.	1839·5	28 08	−0 21	27·8		71 30	− 8	71·4		......	......	......	
Busko Bridge	53 39	357 10	F.	1837·5	......	......	......		70 45	−12	70·6		......	......	......	
Grimsby	53 34	359 55	R.	1856·5	22 30	+1 27	24·0		......	......	......		......	......	......	
Cleethorpe	53 32	0 00	S.	1861·5	......	......	......		69 30	+44	70·2		10·36	−·02	10·34	
Doncaster	53 31	358 53	P.	1837·5	......	......	......		70 30	−12	70·3		10·37	+·01	10·38	
Clifden	53 29	350 01	F.	1835·5	......	......	......		71 52	−18	71·6		......	......	......	
Wadworth	53 28	358 53	R.	1838·5	......	......	......		70 28	−10	70·3		......	......	......	
Manchester	53 28	357 46	L.	1836·5	......	......	......		70 48	−14	70·6		......	......	......	
Liverpool	53 25	356 59	F.	1837·5	......	......	......		70 39	−12	70·5		......	......	......	
			†	1856·5	24 00	+1 27	25·5		......	......	......		......	......	......	
Gallhorick	53 25	350 55	F.	1835·5	......	......	......		71 41	−18	71·4		......	......	......	
Birkenhead	53 24	357 00	L.	1836·5	......	......	......		70 49	−14	70·6	70·5	10·39	+·01	10·40	10·42
			P.	1837·5	......	......	......		70 39	−12	70·5		10·42	+·01	10·43	
			S.	1837·5	......	......	......		70 35	−12	70·4		......	......	......	
			R.	1837·5	......	......	......		70 36	−12	70·4		......	......	......	
Calderstone	53 23	357 07	P.	1837·5	......	......	......		70 44	−12	70·5		10·38	+·01	10·39	
Sheffield	53 22	358 29	P.	1837·5	......	......	......		70 30	−11	70·3		10·40	+·01	10·41	
Dublin	53 21	353 44	F.	1835·5	......	......	......		70 59	−17	70·7	70·75	......	......	......	
			S.	1836·5	......	......	......		71 03	−15	70·8		10·48	+·01	10·49	10·49
			R.	1838·5	27 35	−0 26	27·2	27·2	71 00	−10	70·8		10·48	+·01	10·49	
			L.	1842·5	27 13	......	27·2		70 42	......	70·7		10·48	+·01	10·49	
Lowth	53 19	0 00	R.	1838·5	24 26	−0 24	24·0		70 19	− 9	70·2		......	......	......	
Holyhead	53 19	355 23	F.	1835·5	......	......	......		71 04	−17	70·8	70·9	......	......	......	
			L.	1836·5	......	......	......		71 09	−14	70·9		10·42	+·01	10·43	
Galway	53 17	350 56	F.	1835·5	......	......	......		71 26	−18	71·1	71·2	......	......	......	
			L.	1835·5	......	......	......		71 33	−18	71·3		10·56	+·01	10·57	
Shannon Harbour	53 14	352 07	R.	1838·5	28 03	−0 28	27·6		71 23	−10	71·2		......	......	......	
Bangor	53 14	355 54	F.	1835·5	......	......	......		71 02	−17	70·8		......	......	......	
Coed	53 11	356 48	P.	1837·5	......	......	......		70 41	−12	70·5		10·39	+·01	10·40	
Carnarvon	53 09	355 46	F.	1835·5	......	......	......		70 58	−17	70·7		......	......	......	
Glangwnna	53 08	355 46	S.	1858·5	......	......	......		70 03	+40	70·7		10·44	−·02	10·42	
Matlock	53 06	358 28	L.	1836·5	......	......	......		70 29	−14	70·3	70·2	......	......	......	
			F.	1837·5	......	......	......		70 19	−12	70·1		......	......	......	

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* Observed by Mr. ARCHIBALD SMITH.

† Observed by Mr. RUNDELL of Liverpool.

TABLE (continued).

Stations.	Lat. N.	Long. E.	Ob-server.	Date.	Declination.			Inclination.				Force.			
					Ob-served.	Secular change.	Corrected: degrees and decimals.	Ob-served.	Secular change.	Corrected: degrees and decimals.		Ob-served.	Secular change.	Corrected.	
					° ′	° ′	°				°				
Llanberris	53° 07′	355° 57′	F.	1835·5	......	......	......	70° 57′	−17′	70·7°		......	......	......	
Capelcarrig	53 06	356 07	F.	1835·5	......	......	......	70 48	−17	70·5		......	......	......	
Nottingham	52 57	358 52	R.	1838·5	24 53	−0 24	24·5	70 16	−10	70·1		......	......	......	
Cromer	52 56	1 19	R.	1838·5	23 21	−0 24	22·9	69 46	−10	69·6	69·6	......	......	......	
			S.	1861·5	......	......	......	68 55	+44	69·7		10·29	−·02	10·27	
Pwllheli	52 55	355 37	R.	1837·5	......	......	......	70 33	−12	70·4		......	......	......	
Rathdrum	52 55	353 46	L.	1835·5	......	......	......	70 54	−18	70·6		10·41	+·01	10·42	
Kiltanon	52 52	351 17	S.	1834·5	......	......	......	71 21	−21	71·0	71·0	10·56	+·01	10·57	
			S.	1835·5	......	......	......	71 20	−19	71·0		......	......	......	
Ennis	52 51	351 02	L.	1835·5	......	......	......	71 13	−19	70·9		10·53	+·01	10·54	
Stafford	52 48	357 54	R.	1837·5	......	......	......	70 10	−12	70·0		......	......	......	
Cawston	52 47	1 12	S.	1860·5	......	......	......	68 54	+43	69·6	69·6	10·31	−·02	10·29	
			S.	1861·5	......	......	......	68 52	+46	69·6		......	......	......	
Lynn	52 47	0 25	L.	1836·5	......	......	......	69 53	−14	69·7		10·30	+·01	10·31	
Shrewsbury	52 43	357 15	L.	1836·5	......	......	......	70 28	−15	70·2	70·2	10·35	+·01	10·36	10·35
			S.	1837·5	......	......	......	70 25	−13	70·2		10·33	+·01	10·34	
Gorey	52 40	353 43	L.	1835·5	......	......	......	70 56	−18	70·6		10·40	+·01	10·41	
Limerick	52 40	351 24	S.	1834·5	......	......	......	71 04	−22	70·7	70·7	......	......	......	
			S.	1835·5	......	......	......	71 06	−19	70·7		......	......	......	
			L.	1835·5	......	......	......	71 03	−19	70·7		......	......	......	
			S.	1836·5	......	......	......	71 01	−16	70·8		10·53	+·01	10·54	10·54
			R.	1838·5	28 03	−0 28	27·6	71 00	−11	70·8		10·52	+·01	10·53	
Ballybunian	52 30	350 19	S.	1835·5	......	......	......	71 20	−20	71·0		10·61	+·01	10·62	
Lowestoft	52 28	1 50	R.	1838·5	23 00	−0 25	22·6	69 29	−10	69·3	69·3	......	......	......	
			S.	1851·5	......	......	......	68 39	+47	69·4		10·30	−·02	10·28	
Birmingham	52 28	358 07	P.	1837·5	......	......	......	70 07	−12	69·9	69·9	10·38	+·01	10·39	
			R.	1837·5	......	......	......	70 01	−12	69·8		......	......	......	
Aberysthwith	52 24	355 35	S.	1837·5	......	......	......	70 24	−13	70·2		10·38	+·01	10·39	
Daventry	52 16	358 52	R.	1837·5	......	......	......	69 41	−12	69·5		......	......	......	
Waterford	52 15	352 52	L.	1835·5	......	......	......	70 51	−19	70·5	70·5	10·44	+·01	10·45	10·47
			R.	1838·5	26 44	−0 28	26·3	70 46	−11	70·6		10·48	+·01	10·49	
Blennerville	52 15	350 16	Br.	1857·5	27 24	+1 44	29·1	......	......	......		......	......	......	
Broadway	52 13	353 36	L.	1835·5	......	......	......	70 36	−20	70·3		10·45	+·01	10·46	
Cambridge	52 13	0 07	L.	1836·5	......	......	......	69 42	−15	69·5	69·4	10·27	+·01	10·28	10·27
			S.	1860·5	......	......	......	68 42	+44	69·4		10·28	−·02	10·26	
Fermoy	52 07	351 44	S.	1834·5	......	......	......	70 48	−22	70·4		......	......	......	
Dingle	52 08	319 43	S.	1836·5	......	......	......	71 08	−23	70·7		10·62	+·01	10·63	
Malvern	52 07	357 31	F.	1835·5	......	......	......	70 11	−18	69·9		......	......	......	
Hereford	52 04	357 16	L.	1836·5	......	......	......	70 07	−15	69·9		10·32	+·01	10·33	
Killarney	52 03	350 29	S.	1834·5	......	......	......	71 05	−22	70·7	70·7	......	......	......	
			R.	1838·5	28 11	−0 28	27·6	70 51	−11	70·7		10·53	+·01	10·54	
Llandovery	52 01	356 15	S.	1860·5	......	......	......	69 10	+29	69·7		10·35	−·02	10·33	
Youghal	51 57	352 10	S.	1835·5	......	......	......	70 39	−19	70·3		10·50	+·01	10·51	
Brecon	51 57	356 39	S.	1837·5	......	......	......	70 03	−13	69·8		10·34	+·01	10·35	
Valencia	51 56	349 43	S.	1835·5	......	......	......	71 05	−20	70·8	70·7	10·57	+·01	10·58	10·57
			R.	1838·5	28 42	−0 28	28·2	70 52	−11	70·7		10·55	+·01	10·56	
Harwich	51 56	1 13	R.	1838·5	23 08	−0 25	22·7	69 15	−10	69·1		......	......	......	
Ross	51 55	357 25	F.	1835·5	......	......	......	70 00	−14	69·8		......	......	......	
Cork	51 54	351 34	L.	1835·5	......	......	......	70 43	−20	70·4	70·5	10·47	+·01	10·48	10·49
			R.	1838·5	27 14	−0 28	26·8	70 39	−11	70·5		10·49	+·01	10·50	
Glengariffe	51 45	350 29	S.	1834·5	......	......	......	71 02	−23	70·7		......	......	......	
Merthyr	51 43	356 39	S.	1837·5	......	......	......	70 04	−13	69·9		10·36	+·01	10·37	
St. Catherine's	51 40	355 17	A.	1856·5	24 30	+1 27	25·9	......	......	......		......	......	......	
Meath	51 40	356 14	F.	1835·5	......	......	......	69 57	−18	69·7		......	......	......	
Bearhaven	51 39	350 09	Ch.	1857·0	26 42	+1 40	28·4	......	......	......		......	......	......	
Bushey	51 38	359 38	R.	1837·5	23 59	−0 31	23·5	69 25	−13	69·2		......	......	......	
Stackpole Court	51 38	355 05	R.	1837·5	......	......	......	69 56	−13	69·7	69·7	......	......	......	
			S.	1860·5	......	......	......	68 58	+49	69·8		10·40	−·02	10·38	
Chepstow	51 38	357 20	F.	1835·5	......	......	......	69 48	−19	69·5		......	......	......	
			L.	1836·5	......	......	......	69 48	−18	69·5		10·31	+·01	10·32	
Swansea	51 36	356 05	R.	1837·5	......	......	......	69 47	−13	69·6		......	......	......	
London	51 32	359 50	S.	1837·5	......	......	......	69 21	−13	69·1	69·1	......	......	......	
			F.	1838·5	......	......	......	69 18	−11	69·1		......	......	......	
			L.	1836·5	......	......	......	69 23	−15	69·1		......	......	10·28	
			P.	1837·5	......	......	......	69 19	−13	69·1		......	......	......	
			R.	1848·5	......	......	......	69 16	−11	69·1		......	......	......	
Combe House	51 31	357 26	F.	1838·5	......	......	......	69 32	−11	69·4		10·30	+·01	10·31	
Kew	51 29	359 42	S.	1838·5	......	......	......	69 17	−11	69·1	69·1	......	......	......	
			Oby.	1842·5	23 13	......	23·2	......	......	69·1		10·28	......	10·28	
			S.	1858·5	......	......	......	68 24	+43	69·1		......	......	......	
Woolwich	51 29	0 02	E.	1857·5	21 46	+1 33	23·3	......	......	......		......	......	......	
Greenwich	51 28	0 00	Oby.	1842·5	23 11	......	23·2	......	......	......		......	......	......	
Dunraven Castle	51 28	356 23	S.	1837·5	......	......	......	69 46	−14	69·5		10·35	+·01	10·36	

TABLE (continued).

Stations.	Lat. N.	Long. E.	Observer.	Date.	Declination.			Inclination.				Force.			
					Observed.	Secular change.	Corrected: degrees and decimals.	Observed.	Secular change.	Corrected: degrees and decimals.		Observed.	Secular change.	Corrected.	
	° ′	° ′			° ′	° ′	°	° ′	′	°	°				
Clifton	51 27	357 25	L.	1836·5	......	......	....	69 43	−16	69 5	69·4	10·30	+·01	10·31	
			R.	1837·5	......	......	....	69 34	−14	69·3		......	......	......	
Tooting	51 26	359 50	F.	1838·5	......	......	....	69 15	−11	69·1		......	......	......	
Marlborough	51 25	358 17	R.	1837·5	......	......	....	69 25	−14	69 2		......	......	......	
Worcester Park	51 23	359 43	S.	1838·5	......	......	....	69 07	−11	68·9		10 28	+·01	10·29	
Margate	51 23	1 23	S.	1837·5	......	......	....	69 03	−14	68·8	68·8	10·24	+·01	10·25	10·24
			R.	1838·5	22 54	−0 25	22·5	68 57	−11	68·8		......		......	
			S.	1860·5	......	......	....	68 06	+49	68·9		10·25	−·02	10·23	
Eastwick Park	51 17	359 41	F.	1838·5	......	......	....	69 08	−11	69·0		10·27	+·01	10·28	
Broome Park	51 14	359 42	S.	1859·5	......	......	....	68 01	+16	68·8		10·25	−·02	10·23	
Guildford	51 14	359 26	R.	1837·5	......	......	....	69 05	−14	68·9		......	......	......	
Ilfracombe	51 12	355 54	R.	1837·5	......	......	....	69 37	−14	69·4		......	......	......	
Dover	51 08	1 19	S.	1837·5	......	......	....	68 52	−14	68·6		10 22	+·01	10 23	
Folkestone	51 05	1 10	S.	1860·5	......	......	....	67 51	+49	68·7		10 23	−·02	10·21	
Salisbury	51 04	358 12	L.	1836·5	......	......	....	69 23	−15	69 1	69·0	10·28	+·01	10·29	
			R.	1837·5	......	......	....	69 15	−13	69 0		......	......	......	
Shoreham	50 51	359 45	E.	1857·5	22 02	+1 33	23·6	......	......	......		......	......	......	
St. Leonards	50 51	0 33	S.	1858·5	......	......	....	67 49	+45	68·6		10·23	−·02	10·21	
Tortington	50 50	359 26	R.	1837·5	......	......	....	68 56	−14	68·7	68·7	......	......	......	
			S.	1837·5	......	......	....	68 57	−14	68·7		10 26	+·01	10·27	
Brighton	50 50	359 52	L.	1836·5	......	......	....	68 50	−17	68·6		10·23	+·01	10·24	
Southsea	50 48	359 02	R.	1837·5	......	......	....	69 00	−14	68·8		......	......	......	
Eastbourne	50 47	0 16	F	1838·5	......	......	....	68 45	−11	68·6		10·21	+·01	10·22	
St. Clairs	50 44	358 52	P.	1837·5	......	......	....	69 01	−14	68·8		10·28	+·01	10·29	
Ryde	50 41	358 50	L.	1836·5	......	......	....	69 01	−17	68·7		10·24	+·01	10·25	
Exeter	50 43	356 29	R.	1837·5	......	......	....	69 06	−14	68·9		......	......	......	
Lew Trenchard	50 40	355 30	S.	1838·5	......	......	....	69 19	−11	69·1	69·1	10.32	+·01	10·33	10·32
			S.	1859·5	......	......	....	68 18	+48	69 1		10·33	−·02	10·31	
Weymouth	50 37	357 33	R	1837·5	......	......	....	69 07	−14	68·9		......	......	......	
Padstow	50 33	355 04	R.	1837·5	......	......	....	69 25	−14	69·2		......	......	......	
Teignmouth	50 33	356 30	S.	1859·5	......	......	....	68 05	+48	68 9		10·28	−·02	10·26	
Torquay	50 28	356 28	S.	1859·5	......	......	....	68 04	+48	68 9		10·24	−·02	10·22	
Plymouth	50 22	355 51	R.	1837·5	......	......	....	69 06	−14	68 9	68·9	......	......	......	
			S.	1859·5	......	......	....	68 06	+48	68·9		10 27	−·02	10·25	
			C^{x}.	1857·5	23 28	+1 33	25·0	......	......	......		......	......	......	
Bigbury	50 17	356 07	C^{x}.	1857·5	23 20	+1 36	24 9	......	......	......		......	......	......	
Bolt Tail	50 14	356 08	C^{x}.	1857·5	23 19	+1 36	24·9	......	......	......		......	......	......	
Deadman	50 13	355 12	W^{s}.	1857·0	23 46	+1 34	25·3	......	......	......		......	......	......	
Salcombe	50 13	356 13	C^{x}.	1857·5	23 19	+1 36	24 9	......	......	......		......	......	......	
Falmouth	50 10	354 54	R.	1837·5	......	......	....	69 16	−17	69·0	69·0	......	......	......	
			S.	1838·5	......	......	....	69 12	−12	69·0		10·29	+·01	10·30	10·30
			F.	1838·5	......	......	....	69 14	−12	69·0		10·29	+·01	10·30	
			S	1859·0	......	......	....	69 08	+52	69·0		......	......	......	
			W^{s}.	1856·0	24 06	+1 25	25·5	......	......	......		......	......	......	
Mounts Bay	50 10	354 29	W^{s}.	1861·0	23 41	+1 58	25·7	......	......	......		......	......	......	
Lands End	50 05	354 20	R.	1837·5	......	......	....	69 19	−17	69·0		......	......	......	
St. Marys	49 55	353 43	F.	1838·5	......	......	....	69 26	−14	69·2		......	......	......	
St. Heliers	49 12	357 55	E.	1857·5	21 35	+1 34	23·2	......	......	......		......	......	......	

XV. *On the Thermodynamic Theory of Waves of Finite Longitudinal Disturbance.*

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§ 1. The object of the present investigation is to determine the relations which must exist between the laws of the elasticity of any substance, whether gaseous, liquid, or solid, and those of the wave-like propagation of a finite longitudinal disturbance in that substance; in other words, of a disturbance consisting in displacements of particles along the direction of propagation, the velocity of displacement of the particles being so great that it is not to be neglected in comparison with the velocity of propagation. In particular, the investigation aims at ascertaining what conditions as to the transfer of heat from particle to particle must be fulfilled in order that a finite longitudinal disturbance may be propagated along a prismatic or cylindrical mass without loss of energy or change of type: the word *type* being used to denote the relation between the extent of disturbance at a given instant of a set of particles, and their respective undisturbed positions. The disturbed matter in these inquiries may be conceived to be contained in a straight tube of uniform cross-section and indefinite length.

§ 2. *Mass-Velocity.*—A convenient quantity in the present investigation is what may be termed the *mass-velocity* or *somatic velocity*—that is to say, the mass of matter through which a disturbance is propagated in a unit of time while advancing along a prism of the sectional area unity. That mass-velocity will be denoted by m.

Let S denote the *bulkiness*, or the space filled by unity of mass, of the substance in the undisturbed state, and a the linear velocity of advance of the wave; then we have evidently

$$a=m\mathrm{S}. \qquad (1)$$

§ 3. *Cinematical Condition of Permanency of Type.*—If it be possible for a wave of disturbance to be propagated in a uniform tube without change of type, that possibility is expressed by the uniformity of the mass-velocity m for all parts of the wave.

Conceive a space in the supposed tube, of an invariable length Δx, to be contained between a pair of transverse planes, and let those planes advance with the linear velocity a in the direction of propagation. Let the values of the bulkiness of the matter at the foremost and aftermost planes respectively be denoted by s_1 and s_2, and those of the velocity of longitudinal disturbance by u_1 and u_2. Then the linear velocities with which the particles traverse the two planes respectively are as follows: for the foremost plane u_1-a, for the aftermost plane u_2-a. The uniformity of type of the disturbance involves as a condition, that equal masses of matter traverse the two planes respectively in a given time, being each, in unity of time, expressed by the *mass-velocity*; hence we have, as the *cinematical condition* of uniformity of type, the following equation:

$$\frac{a-u_1}{s_1}=\frac{a-u_2}{s_2}=\frac{a}{\mathrm{S}}=m. \qquad (2)$$

Another way of expressing the same condition is as follows:

$$\Delta u=-m\Delta s. \qquad (3)$$

§ 4. *Dynamical Condition of Permanency of Type.*—Let p_1 and p_2 be the intensities of the longitudinal pressure at the foremost and aftermost advancing planes respectively. Then in each unit of time the difference of pressure, p_2-p_1, impresses on the mass m the acceleration u_2-u_1, and consequently, by the second law of motion, we have the following value for the difference of pressure:

$$p_2-p_1=m(u_2-u_1). \qquad (4)$$

Then substituting for the acceleration u_2-u_1 its value in terms of the change of bulkiness as given by equation (3), we obtain, for the *dynamical condition* of permanency of type, the following equation,

$$p_2-p_1=m^2(s_1-s_2), \qquad (5)$$

which may also be put in the form of an expression giving the value of the square of the mass-velocity, viz.

$$m^2=-\frac{\Delta p}{\Delta s}=-\frac{dp}{ds}. \qquad (6)$$

The square of the linear velocity of advance is given by the following equation:

$$a^2=m^2\mathrm{S}^2=-\mathrm{S}^2\frac{dp}{ds}. \qquad (7)$$

The integral form of the preceding equations may be expressed as follows. Let S, as before, be the bulkiness in the undisturbed state, and P the longitudinal pressure; then in a wave of disturbance of permanent type, we must have the following condition fulfilled:

$$p+m^2s=\mathrm{P}+m^2\mathrm{S}. \qquad (8)$$

§ 5. *Waves of Sudden Disturbance.*—The condition expressed by the equations of the preceding section holds for any type of disturbance, continuous or discontinuous, gradual or abrupt. To represent, in particular, the case of a single abrupt disturbance, we must conceive the foremost and aftermost advancing planes already mentioned to coalesce into one. Then P is the longitudinal pressure, and S the bulkiness, in front of the advancing plane; p is the lonigtudinal pressure, and s the bulkiness, behind the advancing plane; and the advancing plane is a wave-front of *sudden compression* or of *sudden rarefaction**

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* (*Note, added* 1st *August*, 1870.) Sir William Thomson has pointed out to the author, that a wave of sudden rarefaction, though mathematically possible, is an unstable condition of motion; any deviation from absolute suddenness tending to make the disturbance become more and more gradual. Hence the only wave of sudden disturbance whose permanency of type is physically possible, is one of sudden compression; and this is to be taken into account in connexion with all that is stated in the paper respecting such waves.

according as p is greater or less than P. The squares of the *mass-velocity* and of the linear velocity of advance are respectively as follows:

$$m^2=\frac{p-\mathrm{P}}{\mathrm{S}-s}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$a^2=m^2\mathrm{S}^2=\frac{p-\mathrm{P}}{\mathrm{S}-s}.\mathrm{S}^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The velocity of the disturbed particles is as follows:

$$u=m(\mathrm{S}-s)=\frac{p-\mathrm{P}}{m}=\sqrt{(p-\mathrm{P}).(\mathrm{S}-s)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and it is forward or backward according as the wave is one of compression or of rarefaction.

The energy expended in unity of time, in producing any such wave, is expressed by pu; for the wave may be conceived to be produced in a tube closed at one end by a moveable piston of inappreciable mass, to which there is applied a pressure p different from the undisturbed pressure P, and which consequently moves with the velocity u. The way in which that energy is disposed of is as follows: actual energy of the disturbance, $\frac{mu^2}{2}$; work done in altering bulkiness, $\frac{m(p+\mathrm{P})(\mathrm{S}-s)}{2}$; and the equation of the conservation of energy is

$$pu=\frac{m}{2}\{u^2+(p+\mathrm{P})(\mathrm{S}-s)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11\text{A})$$

§ 6. *Thermodynamic Conditions.*—While the equations of the two preceding sections impose the constancy of the rate of variation of pressure with bulkiness during the disturbance $\left(\frac{dp}{ds}=-m^2\right)$ as an indispensable condition of permanency of type of the wave, they leave the limits of pressure and of bulkiness, being four quantities, connected by one equation only $\left(\frac{p_2-p_1}{s_1-s_2}=-\frac{dp}{ds}=m^2\right)$. Two only of those quantities can be arbitrary; therefore one more equation is required, and that is to be determined by the aid of the laws of thermodynamics.

It is to be observed, in the first place, that no substance yet known fulfils the condition expressed by the equation $\frac{dp}{ds}=-m^2=$ constant, between finite limits of disturbance, at a constant temperature, nor in a state of non-conduction of heat (called the *adiabatic* state). In order, then, that permanency of type may be possible in a wave of longitudinal disturbance, there must be both change of temperature and conduction of heat during the disturbance.

The cylindrical or prismatic tube in which the disturbance is supposed to take place being ideal, is to be considered as non-conducting. Also, the foremost and aftermost transverse advancing planes, or front and back of the wave, which contain between them the particles whose pressure and bulkiness are in the act of varying, are to be considered as non-conducting, because of there being an indefinite length of matter before the foremost and behind the aftermost plane, to resist conduction.

The transfer of heat, therefore, takes place wholly amongst the particles undergoing variation of pressure and bulkiness; and therefore for any given particle, during its passage from the front to the back of the wave, the *integral amount of heat received must be nothing*; and this is the thermodynamic condition which gives the required equation. That equation is expressed as follows:

$$\int_{\phi_1}^{\phi_2} \tau d\phi = 0; \quad \ldots \ldots \ldots \ldots \quad (12)$$

in which τ denotes absolute temperature, and ϕ the "thermodynamic function." The value of that function, as explained in various papers and treatises on thermodynamics, is given by the following formula:

$$\phi = Jc \text{ hyp log } \tau + \chi(\tau) + \frac{dU}{d\tau}, \quad \ldots \ldots \ldots \quad (12\text{ A})$$

in which J is the dynamical value of a unit of heat; c, the real specific heat of the substance; $\chi(\tau)$, a function of the temperature alone, which is $=0$ for all temperatures at which the substance is capable of approximating indefinitely to the perfectly gaseous state, and is introduced into the formula solely to provide for the possible existence of substances which at some temperatures are incapable of approximating to the perfectly gaseous state; and U, the work which the elastic forces in unity of mass are capable of doing at the constant temperature τ. The substitution for the integral in equation (12) of its value in terms of p and s for any particular substance, gives a relation between the limits of pressure p_1 and p_2, and the limits of bulkiness s_1 and s_2, which being combined with equation (5), or with any one of the equivalent equations (6), (8), or (9), completes the expression of the laws of the propagation of waves of finite longitudinal disturbance and permanent type in that particular substance.

§ 7. *Assumption as to Transfer of Heat.*—In applying the principles of the preceding section to the propagation of waves of longitudinal disturbance, it is obviously assumed that the transfer of heat takes place between the various particles which are undergoing disturbance at a given time, in such a manner as to ensure the fulfilment of the dynamical condition of permanency of type. It appears highly probable, that how great soever the resistance of the substance to the conduction of heat may be, that assumption as to the transfer is realized when the disturbance is *sudden*, as described in § 5; for then particles in all the successive stages of the change of pressure and bulkiness within the limits of the disturbance are at inappreciable distances from each other; so that the resistance to the transfer of heat between them is inappreciable.

But when the disturbance is not sudden, it is probable that the assumption as to the transfer of heat is fulfilled in an approximate manner only; and if such is the case, it follows that *the only longitudinal disturbance which can be propagated with absolute permanence of type is a sudden disturbance.*

§ 8. *Combination of the Dynamic and Thermodynamic Equations.*—In every fluid, and probably in many solids, the quantity of heat received during an indefinitely small change of pressure dp and of bulkiness ds is capable of being expressed in either of the following forms:

$$\frac{\tau d\phi}{\mathrm{J}} = c_s \frac{d\tau}{dp} dp + c_p \frac{d\tau}{ds} ds;$$

in which c_s and c_p denote the specific heat at constant bulkiness and at constant pressure respectively; and the differential coefficients $\frac{d\tau}{dp}$ and $\frac{d\tau}{ds}$ of the absolute temperature are taken, the former on the supposition that the bulkiness is constant, and the latter on the supposition that the pressure is constant. Let it now be supposed that the bulkiness varies with the pressure according to some definite law; and let the actual rate of variation of the bulkiness with the pressure be denoted by $\frac{ds}{dp}$. Then equation (12) may be expressed in the following form:

$$\int_{p_1}^{p_2} dp \cdot \left\{ c_s \frac{d\tau}{dp} + c_p \frac{d\tau}{ds} \cdot \frac{ds}{dp} \right\} = 0.$$

Now, according to the dynamic condition of permanence of type, we have by equation (6),

$$\frac{ds}{dp} = -\frac{1}{m^2};$$

which, being substituted in the preceding integrals, gives the following equations from which to deduce *the square of the mass-velocity*:

$$\int_{p_1}^{p_2} dp \cdot \left\{ m^2 c_s \frac{d\tau}{dp} - c_p \frac{d\tau}{ds} \right\} = 0. \qquad (13)$$

It is sometimes convenient to substitute for $c_p \frac{d\tau}{ds}$ the following value, which is a known consequence of the laws of thermodynamics:

$$c_p \frac{d\tau}{ds} = c_s \frac{d\tau}{ds} + \frac{\tau dp}{\mathrm{J} d\tau}, \qquad (13\,\mathrm{A})$$

the differential coefficient $\frac{dp}{d\tau}$ being taken on the supposition that s is constant. The equations (13) and (13 A) are applicable to all fluids, and probably to many solids also, especially those which are isotropic.

The determination of the squared mass-velocity, m^2, enables the bulkiness s for any given pressure p, and the corresponding velocity of disturbance u, to be found by means of the following formulæ, which are substantially identical with equations (8) and (3) respectively:

$$s = \mathrm{S} + \frac{\mathrm{P} - p}{m^2}; \qquad (14)$$

$$u = m(\mathrm{S} - s) = \frac{p - \mathrm{P}}{m}. \qquad (15)$$

Equation (15) also serves to calculate the pressure p corresponding to a given velocity of disturbance u. It may here be repeated that the linear velocity of advance is $a=m$S (equation 1).

§ 9. *Application to a Perfect Gas.*—In a perfect gas, the specific heat at constant volume, c_s, and the specific heat at constant pressure, c_p, are both constant; and consequently bear to each other a constant ratio, $\frac{c_p}{c_s}$, whose value for air, oxygen, nitrogen, and hydrogen is nearly 1·41, and for steam-gas nearly 1·3. Let this ratio be denoted by γ. Also, the differential coefficients which appear in equations (13) and (13A) have the following values:—

$$\left.\begin{aligned} \frac{d\tau}{dp}&=\frac{\tau}{p}=\frac{s}{\mathrm{J}(c_p-c_s)}=\frac{s}{\mathrm{J}(\gamma-1)c_s}\,; \\ \frac{d\tau}{ds}&=\frac{\tau}{s}=\frac{p}{\mathrm{J}(c_p-c_s)}=\frac{p}{\mathrm{J}(\gamma-1)c_s}\,; \\ \frac{dp}{d\tau}&=\frac{p}{\tau}=\frac{\mathrm{J}(c_p-c_s)}{s}=\frac{\mathrm{J}(\gamma-1)c_s}{s}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (16)$$

When these substitutions are made in equation (13), and constant common factors cancelled, it is reduced to the following:

$$\int_{p_1}^{p_2} dp\,.\,\{m^2s-\gamma p\}=0. \quad \ldots\ldots\ldots\ldots \quad (17)$$

But according to the dynamical condition of permanence of type, as expressed in equation (8), we have $m^2s=m^2\mathrm{S}+\mathrm{P}-p$; whence it follows that the value of the integral in equation (17) is

$$\int_{p_1}^{p_2} dp\,.\,\{m^2\mathrm{S}+\mathrm{P}-(\gamma+1)p\}=(m^2\mathrm{S}+\mathrm{P})(p_2-p_1)-\frac{\gamma+1}{2}(p_2^2-p_1^2)=0\,;$$

which, being divided by p_2-p_1, gives for the square of the mass-velocity of advance the following value:

$$m^2=\frac{1}{\mathrm{S}}\left\{(\gamma+1)\,.\,\frac{p_2+p_1}{2}-\mathrm{P}\right\}. \quad \ldots\ldots\ldots\ldots \quad (18)$$

The square of the linear velocity of advance is

$$a^2=m^2\mathrm{S}^2=\mathrm{S}\left\{(\gamma+1)\,.\,\frac{p_2+p_1}{2}-\mathrm{P}\right\}. \quad \ldots\ldots\ldots \quad (19)$$

The velocity of disturbance u corresponding to a given pressure p, or, conversely, the pressure p corresponding to a given velocity of disturbance, may be found by means of equation (15).

Such are the general equations of the propagation of waves of longitudinal disturbance of permanent type along a cylindrical mass of a perfect gas whose undisturbed pressure and bulkiness are respectively P and S. In the next two sections particular cases will be treated of.

§ 10. *Wave of Oscillation in a Perfect Gas.*—Let the mean between the two extreme pressures be equal to the undisturbed pressure; that is, let

$$\frac{p_2+p_1}{2}=\mathrm{P}; \qquad (20)$$

then equations (18) and (19) become simply

$$m^2=\frac{\gamma \mathrm{P}}{\mathrm{S}}, \qquad (21)$$

and

$$a^2=\gamma \mathrm{PS}; \qquad (22)$$

the last of which is LAPLACE's well-known law of the propagation of sound. The three equations of this section are applicable to an indefinitely long series of waves in which equal disturbances of pressure take place alternately in opposite directions.

§ 11. *Wave of Permanent Compression or Dilatation in a Tube of Perfect Gas.*—To adapt equation (18) to the case of a wave of permanent compression or dilatation in a tube of perfect gas, the pressure at the front of the wave is to be made equal to the undisturbed pressure, and the pressure at the back of the wave to the final or permanently altered pressure. Let the final pressure be denoted simply by p; then $p_1=\mathrm{P}$, and $p_2=p$; giving for the square of the mass-velocity

$$m^2=\frac{1}{\mathrm{S}}\left\{(\gamma+1)\frac{p}{2}+(\gamma-1)\frac{\mathrm{P}}{2}\right\}, \qquad (23)$$

for the square of the linear velocity of advance

$$a^2=m^2\mathrm{S}^2=\mathrm{S}\left\{(\gamma+1)\frac{p}{2}+(\gamma-1)\frac{\mathrm{P}}{2}\right\}, \qquad (24)$$

and for the final velocity of disturbance

$$u=\frac{p-\mathrm{P}}{m}=(p-\mathrm{P})\sqrt{\left\{\frac{\mathrm{S}}{(\gamma+1)\frac{p}{2}+(\gamma-1)\frac{\mathrm{P}}{2}}\right\}}. \qquad (25)$$

Equations (23) and (24) show that a wave of condensation is propagated faster, and a wave of rarefaction slower, than a series of waves of oscillation. They further show that there is no upper limit to the velocity of propagation of a wave of condensation; and also that to the velocity of propagation of a wave of rarefaction there is a lower limit, found by making $p=0$ in equations (23) and (24). The values of that lower limit, for the squares of the mass-velocity and linear velocity respectively, are as follows:—

$$m^2(p=0)=\frac{(\gamma-1)\mathrm{P}}{2\mathrm{S}}; \qquad (26)$$

$$a^2(p=0)=\frac{(\gamma-1)\mathrm{PS}}{2}; \qquad (27)$$

and the corresponding value of the velocity of disturbance, being its negative limit, is

$$u(p=0)=-\sqrt{\left\{\frac{2\mathrm{PS}}{\gamma-1}\right\}}. \qquad (28)$$

It is to be borne in mind that the last three equations represent a state of matters which may be approximated to, but not absolutely realized.

Equation (25) gives the velocity with which a piston in a tube is to be moved inwards or outwards as the case may be, in order to produce a change of pressure from P to p, travelling along the tube from the piston towards the further end. Equation (25) may be converted into a quadratic equation, for finding p in terms of u; in other words, for finding what pressure must be applied to a piston in order to make it move at a given speed along a tube filled with a perfect gas whose undisturbed pressure and bulkiness are P and S. The quadratic equation is as follows:

$$p^2-\left(2\mathrm{P}+\frac{\gamma+1}{2\mathrm{S}}u^2\right)p-\frac{\gamma-1}{2}\cdot\frac{\mathrm{P}u^2}{\mathrm{S}}+\mathrm{P}^2=0;$$

and its alternative roots are given by the following formula:

$$p=\mathrm{P}+\frac{\gamma+1}{4\mathrm{S}}u^2\pm\sqrt{\left\{\frac{\gamma\mathrm{P}u^2}{\mathrm{S}}+\frac{(\gamma+1)^2u^4}{16\mathrm{S}^2}\right\}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The sign + or — is to be used, according as the piston moves inwards so as to produce condensation, or outwards so as to produce rarefaction. Suppose, now, that in a tube of unit area, filled with a perfect gas whose undisturbed pressure and volume are P and S, there is a piston dividing the space within that tube into two parts, and moving at the uniform velocity u: condensation will be propagated from one side of the piston, and rarefaction from the other; the pressures on the two sides of the piston will be expressed by the two values of p in equation (29); and the force required in order to keep the piston in motion will be the difference of these values; that is to say,

$$\Delta p=2u\,.\sqrt{\left\{\frac{\gamma\mathrm{P}}{\mathrm{S}}+\frac{(\gamma+1)^2u^2}{16\mathrm{S}^2}\right\}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Two limiting cases of the last equation may be noted: first, if the velocity of the piston is very small compared with the velocity of sound, that is if $\frac{\mathrm{S}u^2}{\gamma\mathrm{P}}$ is very small, we have

$$\Delta p \text{ nearly } =2u\,.\sqrt{\left(\frac{\gamma\mathrm{P}}{\mathrm{S}}\right)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30\text{ A})$$

secondly, if the velocity of the piston is very great compared with the velocity of sound, that is if $\frac{\gamma\mathrm{P}}{\mathrm{S}u^2}$ is very small, we have

$$\Delta p \text{ nearly } =\frac{(\gamma+1)u^2}{2\mathrm{S}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30\text{ B})$$

§ 12. *Absolute Temperature.*—The absolute temperature of a given particle of a given substance, being a function of the pressure p and bulkiness s, can be calculated for a point in a wave of disturbance for which p and s are given. In particular, the absolute temperature in a perfect gas is given by the following well-known thermodynamic formula:

$$\tau=\frac{ps}{(\mathrm{J}c_p-c_s)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

and if, in that formula, there be substituted the value of s in terms of p, given by equations (8) and (18) combined, we find, for the absolute temperature of a particle at which the pressure is p, in a wave of permanent type, the following value:

$$\tau=\frac{\mathrm{PS}}{\mathrm{J}(c\ \ -c_s)}\cdot\frac{(\gamma+1)(p_1+p_2)p-2p^2}{(\gamma+1)(p_1+p_2)\mathrm{P}-2\mathrm{P}^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

in which the first factor $\frac{\mathrm{PS}}{\mathrm{J}(c_p-c_s)}$ is obviously the *undisturbed* value of the absolute temperature. For brevity's sake, let this be denoted by T.

The following particular cases may be noted. In a wave of oscillation, as defined in § 10, we have $p_1+p_2=2\mathrm{P}$; and consequently

$$\tau=\mathrm{T}\,.\,\frac{(\gamma+1)\mathrm{P}p-p^2}{\gamma\mathrm{P}^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32\,\text{A})$$

In a wave of permanent condensation or rarefaction, as described in § 11, let $p_1=\mathrm{P}$, $p_2=\mathrm{P}$; then the final temperature is

$$\tau=\mathrm{T}\,.\,\frac{(\gamma+1)\mathrm{P}p+(\gamma-1)p^2}{(\gamma+1)\mathrm{P}p+(\gamma-1)\mathrm{P}^2}.$$

§ 13. *Types of Disturbance capable of Permanence.*—In order that a particular type of disturbance may be capable of permanence during its propagation, a relation must exist between the temperatures of the particles and their relative positions, such that the conduction of heat between the particles may effect the transfers of heat required by the thermodynamic conditions of permanence of type stated in § 6.

During the time occupied by a given phase of the disturbance in traversing a unit of mass of the cylindrical body of area unity in which the wave is travelling, the quantity of heat received by that mass, as determined by the thermodynamic conditions, is expressed in dynamical units by

$$\tau d\phi.$$

The time during which that transfer of heat takes place is the reciprocal $\frac{1}{m}$ of the mass-velocity of the wave. Let $\frac{d\tau}{dx}$ be the rate at which temperature varies with longitudinal distance, and k the conductivity of the substance, in dynamical units; then the same quantity of heat, as determined by the laws of conduction, is expressed by

$$\frac{1}{m}\,.\,d\left(k\frac{d\tau}{dx}\right).$$

The equality of these two expressions gives the following general differential equation for the determination of the types of disturbance that are capable of permanence:

$$m\tau d\,.\,\phi=d\,.\left(k\frac{d\tau}{dx}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

The following are the results of two successive integrations of that differential equation:—

$$\frac{dx}{d\tau}=\frac{k}{\mathrm{A}+m\int\tau d\phi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33\,\text{A})$$

$$x=\mathrm{B}+\int\frac{kd\tau}{\mathrm{A}+m\int\tau d\phi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33\,\text{B})$$

in which A and B are arbitrary constants. The value of A depends on the magnitude of the disturbance, and that of B upon the position of the point from which x is reckoned. In applying these general equations to particular substances, the values of τ and ϕ are to be expressed in terms of the pressure p, by the aid of the formulæ of the preceding section; when equation (33 B) will give the value of x in terms of p, and thus will show the type of disturbance required.

Our knowledge of the laws of the conduction of heat is not yet sufficient to enable us to solve such problems as these for actual substances with certainty. As a hypothetical example, however, of a simple kind, we may suppose the substance to be perfectly gaseous and of constant conductivity. The assumption of the perfectly gaseous condition gives, according to the formulæ of the preceding sections,

$$\tau=\frac{\mathrm{PS}}{(\gamma-1)\mathrm{J}c_s}\cdot\frac{(\gamma+1)(p_1+p_2)p-2p^2}{(\gamma+1)(p_1+p_2)\mathrm{P}-2\mathrm{P}^2},$$

and

$$\tau d\phi=\frac{\gamma+1}{m^2(\gamma-1)}\left\{\frac{p_2+p_1}{2}-p\right\}dp.$$

It is unnecessary to occupy space by giving the whole details of the calculation; and it may be sufficient to state that the following are the results. Let

$$p-\frac{p_1+p_2}{2}=q,$$

$$\frac{p_2-p_1}{2}=q_1;$$

then

$$\frac{dx}{dp}=\frac{dx}{dq}=\frac{k}{(\gamma+1)mJc_s}\cdot\frac{(\gamma-1)(p_1+p_2)-4q}{q_1^2-q^2} \quad \ldots\ldots\ldots\ldots \quad (34)$$

$$x=\frac{k}{(\gamma+1)mJc_s}\left\{\frac{(\gamma-1)(p_1+p_2)}{2q_1}\,.\,\text{hyp log}\,\frac{q_1+q}{q_1-q}+2\,\text{hyp log}\left(1-\frac{q^2}{q_1^2}\right)\right\}. \quad \ldots \quad (34\,\text{A})$$

In equation (34 A) it is obvious that x is reckoned from the point where $q=0$; that is, where the pressure $p=\frac{p_2+p_1}{2}$; a mean between the greatest and least pressures. The direction in which x is positive may be either the same with or contrary to that of the advance of the wave; the former case represents the type of a wave of rarefaction, the latter that of a wave of compression. For the two limiting pressures when $q=\pm q_1$, $\frac{dx}{dq}$ becomes infinite, and x becomes positively or negatively infinite; so that the wave is infinitely long. The only exception to this is the limiting case, when the conductivity k is indefinitely small; and then we have the following results: when $p=p_1$, or $p=p_2$, $\frac{dx}{dp}$ is infinite, and x is indefinite; and for all values of p between p_1 and p_2, $\frac{dx}{dp}$ and x are each indefinitely small. These conditions evidently represent the case of a wave of abrupt rarefaction or compression, already referred to in §§ 6 and 7.

Supplement to a Paper "On the Thermodynamic Theory of Waves of Finite Longitudinal Disturbance;" by W. J. Macquorn Rankine, *C.E., LL.D., F.R.SS. Lond. & Edin.*

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Note as to previous investigations.—Four previous investigations on the subject of the transmission of waves of finite longitudinal disturbance may be referred to, in order to show in what respects the present investigation was anticipated by them, and in what respects its results are new.

The first is that of Poisson, in the Journal de l'Ecole Polytechnique, vol. vii. Cahier 14, p. 319. The author arrives at the following general equations for a gas fulfilling Mariotte's law:—

$$\frac{d\varphi}{dx}=f\left\{x-at-\frac{d\varphi}{dx}t\right\},$$

$$\frac{d\varphi}{dt}+a\frac{d\varphi}{dx}+\frac{1}{2}\cdot\frac{d\varphi^2}{dx^2}=0;$$

in which φ is the velocity-function; $\frac{d\varphi}{dx}$ the velocity of disturbance, at the time t, of a particle whose distance from the origin is x; a is the limit to which the velocity of propagation of the wave approximates when $\frac{d\varphi}{dx}$ becomes indefinitely small, viz. $\sqrt{\frac{dp_0}{d\varrho_0}}$, p_0 being the undisturbed pressure and ϱ_0 the undisturbed density; and f denotes an arbitrary function. This equation obviously indicates the quicker propagation of the parts of the wave where the disturbance is forward (that is, the compressed parts) and the slower propagation of the parts where the disturbance is backward (that is, the dilated parts).

The second is that of Mr. Stokes, in the Philosophical Magazine for November 1848, 3rd series, vol. xxxiii. p. 349, in which that author shows how the type of a series of waves of finite longitudinal disturbance in a perfect gas alters as it advances, and tends ultimately to become a series of sudden compressions followed by gradual dilatations.

The third is that of Mr. Airy, Astronomer Royal, in the Philosophical Magazine for June 1849, 3rd series, vol. xxxiv. p. 401, in which is pointed out the analogy between the above-mentioned change of type in waves of sound, and that which takes place in sea-waves when they roll into shallow water.

The fourth, and most complete, is that of the Rev. Samuel Earnshaw, received by the Royal Society in November 1858, read in January 1859, and published in the Philosophical Transactions for 1860, page 133. That author obtains exact equations for the propagation of waves of finite longitudinal disturbance in a medium in which the pressure is any function of the density; he shows what changes of type, of the kind already mentioned, must go on in such waves; and he points out, finally, that in order that the type may be permanent $\varrho^2\frac{dp}{d\varrho}\left(=-\frac{dp}{ds}\right.$ in the notation of the present paper) must be a constant quantity; being the proposition which is demonstrated in an elementary way near the beginning of the present paper. Mr. **Earnshaw** regards that condition as one which cannot be realized.

The *new results*, then, obtained in the present paper may be considered to be the following:—the conditions as to transformation and transfer of heat which must be fulfilled, in order that permanence of type may be realized, exactly or approximately; the types of wave which enable such conditions to be fulfilled, with a given law of the conduction of heat; and the velocity of advance of such waves.

The *method of investigation* in the present paper, by the aid of *mass-velocity* to express the speed of advance of a wave, is new, so far as I know; and it seems to me to have great advantages in point of simplicity, enabling results to be demonstrated in a very elementary manner, which otherwise would have required comparatively long and elaborate processes of investigation.

XVI. *On the Contact of Conics with Surfaces.*

By WILLIAM SPOTTISWOODE, *M.A.*, *F.R.S.*

Received February 16,—Read March 10, 1870.

IT is well known that at every point of a surface two tangents, called principal tangents, may be drawn having three-pointic contact with the surface, *i.e.* having an intimacy exceeding by one degree that generally enjoyed by a straight line and a surface. The object of the present paper is to establish the corresponding theorem respecting tangent conics, viz. that "at every point of a surface ten conics may be drawn having six-pointic contact with the surface;" these may be called Principal Tangent Conics. In this investigation I have adopted a method analogous to that employed in my paper "On the Sextactic Points of a Plane Curve" (Philosophical Transactions, vol. clv. p. 653); and as I there, in the case of three variables, introduced a set of three arbitrary constants in order to comprise a group of expressions in a single formula, so here, in the case of four variables, I introduce with the same view two sets of four arbitrary constants. If these constants be represented by α, β, γ, δ; α', β', γ', δ', I consider the conic of five-pointic contact of a section of the surface made by the plane $\varpi-k\varpi'=0$, where $\varpi=\alpha x+\beta y+\gamma z+\delta t$, and $\varpi'=\alpha' x+\beta' y+\gamma' z+\delta' t$, and k is indeterminate; and then I proceed to determine k, and thereby the azimuth of the plane about the line $\varpi=0$, $\varpi'=0$, so that the contact may be six-pointic. The formulæ thence arising turn out to be strictly analogous to those belonging to the case of three variables, except that the arbitrary quantities cannot in general be divided out from the final expression. In fact, it is the presence of these quantities which enables us to determine the position of the plane of section, and the equation whereby this is effected proves to be of the degree 10 in $\varpi:\varpi'=k$, and besides this of the degree $12n-27$ in the coordinates x, y, z, t, giving rise to the theorem above stated.

Beyond the question of the principal tangents, it has been shown by CLEBSCH and SALMON that on every surface U a curve may be drawn, at every point of which one of the principal tangents will have a four-pointic contact. And if n be the degree of U, that of the surface S intersecting U in the curve in question will be $11n-24$. Further, it has been shown that at a finite number of points the contact will be five-pointic. The number of these points has not yet been completely determined; but CLEBSCH has shown (Crelle, vol. lviii. p. 93) that it does not exceed $n(11n-24)(14n-30)$. Similarly it appears that on every surface a curve may be drawn, at every point of which one of the principal tangent conics has a seven-pointic contact, and that at a finite number of points the contact will become eight-pointic. But into the discussion of these latter problems I do not propose to enter in the present communication.

§ 1. *Conditions for a Sextactic Point.*

Let $U=0$ be the equation to the given surface, and $V=0$ that to the surface whose section by the plane $\varpi-k\varpi'=0$ is to have a six-pointic contact with the corresponding section of U at the point (x, y, z, t). Also, following the method of Professor CAYLEY (Philosophical Transactions, vol. cxlix. p. 371, and vol. clv. p. 545), let the coordinates of a point of U be considered as functions of a single parameter; then for the present purpose the coordinates of a point consecutive to (x, y, z, t) may be taken to be

$$x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x+\tfrac{1}{120}d^5x,\ y+dy+\ldots,\ z+dz+\ldots,\ t+dt+\ldots; \quad . \quad . \quad (1)$$

and these values when substituted in U must satisfy the equation $U=0$. Then writing for shortness

$$\left.\begin{aligned}\partial_1&=dx\,\partial_x+dy\,\partial_y+dz\,\partial_z+dt\,\partial_t,\\ \partial_2&=d^2x\partial_x+d^2y\partial_y+d^2z\partial_z+d^2t\partial_t,\\ &\vdots\qquad\vdots\qquad\vdots\\ \partial_5&=d^5x\partial_x+d^5y\partial_y+d^5z\partial_z+d^5t\partial_t\,;\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

substituting the values (1) in U, expanding as far as terms of the fifth degree, and arranging the result in lines of the degrees 0, 1, .. 5, respectively, we shall have

$$\left.\begin{aligned}0=&\,U\\ &+\partial_1U\\ &+\tfrac{1}{2}(\partial_1^2+\partial_2)U\\ &+\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2+\partial_3)U\\ &+\tfrac{1}{24}(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+\partial_4)U\\ &+\tfrac{1}{120}(\partial_1^5+10\partial_1^3\partial_2+10\partial_1^2\partial_3+15\partial_1\partial_2^2+5\partial_1\partial_4+10\partial_2\partial_3+\partial_5)U,\end{aligned}\right\}\quad . \quad . \quad . \quad (3)$$

each line of which, being of an order different from the rest, must separately vanish.

Let us write, as usual,

$$\left.\begin{aligned}&\partial_xU=u\ , &&\partial_yU=v\,, &&\partial_zU=w\,, &&\partial_tU=k,\\ &\partial_x^2U=u_1, &&\partial_y^2U=v_1, &&\partial_z^2U=w_1, &&\partial_t^2U=k_1,\\ &\partial_y\partial_zU=u', &&\partial_z\partial_xU=v'\,, &&\partial_x\partial_yU=w', &&\\ &\partial_x\partial_tU=l'\,, &&\partial_y\partial_tU=m', &&\partial_z\partial_tU=n'. &&\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Then combining the equation $\partial_1U=0$ with the corresponding expression in V, viz. $\partial_1V=0$, we obtain the usual expressions for two-pointic contact, viz.

$$\frac{\partial_xV}{u}=\frac{\partial_yV}{v}=\frac{\partial_zV}{w}=\frac{\partial_tV}{k}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which, since U and V are both homogeneous in x, y, z, t, are equivalent to only two independent conditions. These conditions may be comprised in the single formula

$$\begin{vmatrix} \alpha, & \alpha', & u, & \partial_x V \\ \beta, & \beta', & v, & \partial_y V \\ \gamma, & \gamma', & w, & \partial_z V \\ \delta, & \delta', & k, & \partial_t V \end{vmatrix} = \square V = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\alpha, \beta, \gamma, \delta$; $\alpha', \beta' \gamma', \delta'$ are arbitrary quantities to which any values may be given, provided only that all the determinants of the matrix

$$\begin{matrix} \alpha, & \beta, & \gamma, & \delta \\ \alpha', & \beta', & \gamma', & \delta' \end{matrix}$$

do not simultaneously vanish, since in that case the equation (6) would become nugatory. Comparing (6) with the equation $\partial_1 U=0$, and observing that the differentials dx, dy, dz, dt may be replaced by the determinants

$$\left\| \begin{matrix} \alpha, & \beta, & \gamma, & \delta \\ \alpha', & \beta', & \gamma', & \delta' \\ u, & v, & w, & k \end{matrix} \right\| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(the usual rule of signs being adopted, viz. the columns being taken in the cyclic order, the determinants are to have the signs $+, -, +, -$, respectively), it is clear that the equation $(\partial_1^2+\partial_2)U=0$, when combined with the corresponding equation in V, is equivalent to $\square^2 V=0$. Similarly, the equation $(\partial_1^3+3\partial_1\partial_2+\partial_3)U=0$, combined with the corresponding equation in V, is equivalent to $\square^3 V=0$; and so on. Hence the series of conditions comprised in (3) may be expressed as follows:

$$V=0, \quad \square V=0, \quad \square^2 V=0, \quad \square^3 V=0, \quad \square^4 V=0, \quad \square^5 V=0, \quad . \quad . \quad . \quad (8)$$

which correspond to equations (3) of my paper on Sextactic Points above quoted.

§ 2. *Preliminary Transformation.*

The next step is to effect a transformation of the first three equations of the system (8), corresponding to that given in § 1 of the same paper. As was stated in the introduction, the transformation does not throw out as factors any lineo-linear functions of the arbitrary quantities and the variables; but it reduces the expressions transformed to functions of such lineo-linear functions, viz. ϖ, and ϖ'.

Taking the columns of the matrix (7) two and two in the usual cyclic order, viz. β, γ; γ, α; α, β; α, δ; β, δ; γ, δ, and calling the determinants so formed a, b, c, f, g, h; *i. e.* writing

$$\left\| \begin{matrix} \alpha, & \beta, & \gamma, & \delta \\ \alpha', & \beta', & \gamma', & \delta' \end{matrix} \right\| = a, b, c, f, g, h; \quad . \quad . \quad . \quad . \quad (9)$$

and writing, further,

$$\left.\begin{aligned} \alpha x+\beta y+\gamma z+\delta t&=\varpi \\ \alpha' x+\beta' y+\gamma' z+\delta t'&=\varpi' \\ \alpha\varpi'-\alpha\varpi'&=\mathrm{A} \\ \beta'\varpi-\beta\varpi'&=\mathrm{B} \\ \gamma'\varpi-\gamma\varpi'&=\mathrm{C} \\ \delta'\varpi-\delta\varpi'&=\mathrm{D}, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (10)$$

the quantities a, b, .. A, B, .. will be found to satisfy the following relations, useful in subsequent transformations, viz. :—

$$\left.\begin{array}{llll} & bz-cy-ft=\mathrm{A} & & \\ & cx-az-gt=\mathrm{B} & & \\ & ay-bx-ht=\mathrm{C} & & \\ & fx+gy+hz=\mathrm{D} & & \\ & \mathrm{A}x+\mathrm{B}y+cz+\mathrm{D}t=0 & & \\ & \mathrm{B}h-\mathrm{C}g+a\mathrm{D}=0 & & \\ & \mathrm{C}f-\mathrm{A}h+b\mathrm{D}=0 & & \\ & \mathrm{A}g-\mathrm{B}f+c\mathrm{D}=0 & & \\ & \mathrm{A}a+\mathrm{B}b+c\mathrm{C}=0 & & \\ & af+bg+ch=0 & & \\ \varpi\partial_x\mathrm{A}=\alpha\mathrm{A}-\alpha\mathrm{A}, & \varpi\partial_y\mathrm{A}=\alpha\mathrm{B}-\beta\mathrm{A}, & \varpi\partial_z\mathrm{A}=\alpha\mathrm{C}-\gamma\mathrm{A}, & \varpi\partial_t\mathrm{A}=\alpha\mathrm{D}-\delta\mathrm{A}, \\ \varpi\partial_x\mathrm{B}=\beta\mathrm{A}-\alpha\mathrm{B}, & \varpi\partial_y\mathrm{B}=\beta\mathrm{B}-\beta\mathrm{B}, & \varpi\partial_z\mathrm{B}=\beta\mathrm{C}-\gamma\mathrm{B}, & \varpi\partial_t\mathrm{B}=\beta\mathrm{D}-\delta\mathrm{B}, \\ \varpi\partial_x\mathrm{C}=\gamma\mathrm{A}-\alpha\mathrm{C}, & \varpi\partial_y\mathrm{C}=\gamma\mathrm{B}-\beta\mathrm{C}, & \varpi\partial_z\mathrm{C}=\gamma\mathrm{C}-\gamma\mathrm{C}, & \varpi\partial_t\mathrm{C}=\gamma\mathrm{D}-\delta\mathrm{C}, \\ \varpi\partial_x\mathrm{D}=\delta\mathrm{A}-\alpha\mathrm{D}, & \varpi\partial_y\mathrm{D}=\delta\mathrm{B}-\beta\mathrm{D}, & \varpi\partial_z\mathrm{D}=\delta\mathrm{C}-\gamma\mathrm{D}, & \varpi\partial_t\mathrm{D}=\delta\mathrm{D}-\delta\mathrm{D}. \end{array}\right\} \quad (11)$$

And in terms of this notation the developed form of $\square$ will be given by

$$-\square=(vh-wg+ka)\partial_x+(wf-uh+kb)\partial_y+(ug-vf+kc)\partial_z+(-ua-vb-wc)\partial_t. \quad (12)$$

This being premised, our first object is to investigate, as was done in the case of plane curves, an expression for $\square^2\mathrm{V}$, which in virtue of (12) will consist of two parts: first, terms of the form $(h\square v-g\square w+a\square k)\partial_x$, ..; and secondly, terms of the form $(vh-wg+ka)^2\partial_x^2$, .. Referring to (12), we have

$$h\square v-g\square w+a\square k=\begin{array}{llll} \alpha, & \alpha', & u, & hw'-gv'+al' \\ \beta, & \beta', & v, & hv_1-gu'+am' \\ \gamma, & \gamma', & w, & hu'-gw_1+an' \\ \delta, & \delta', & k, & hm'-gn'+ak_1 \end{array}$$

$$=\begin{vmatrix} . & . & . & \beta, & \gamma, & \delta \\ . & . & . & \beta', & \gamma', & \delta' \\ \alpha, & \alpha', & u, & w', & v', & l' \\ \beta, & \beta', & v, & v_1, & u', & m' \\ \gamma, & \gamma', & w, & u', & w_1, & n' \\ \delta, & \delta', & p, & m', & n', & k_1 \end{vmatrix} =(n-1)^{-1}. \begin{vmatrix} . & . & \alpha x-\varpi, & \beta, & \gamma, & \delta \\ . & . & \alpha' x-\varpi', & \beta', & \gamma', & \delta' \\ \alpha, & \alpha', & u_1 x, & w', & v', & l' \\ \beta, & \beta', & w' x, & v_1, & u', & m' \\ \gamma, & \gamma', & v' x, & u', & w_1, & n' \\ \delta, & \delta', & l' x, & m', & n', & k_1, \end{vmatrix},$$

and writing

$$\Phi=\begin{vmatrix} u_1, & w', & v', & l', & \alpha, & \alpha' \\ w', & v_1, & u', & m', & \beta, & \beta' \\ v', & u', & w_1, & n', & \gamma, & \gamma' \\ l', & m', & n', & k_1, & \delta, & \delta' \\ \alpha, & \beta, & \gamma, & \delta, & . & . \\ \alpha', & \beta', & \gamma', & \delta', & . & . \end{vmatrix} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

we may deduce

$$\left.\begin{aligned} h\Box v-g\Box w+a\Box p &= -\{(vh-wg+pa)\partial_x+\,.\,.\}(vh-wg+pa) \\ &=(n-1)^{-1}\left\{x\Phi-\begin{vmatrix} . & . & B, & C, & D \\ \alpha, & \alpha', & w', & v', & l' \\ \beta, & \beta', & v_1, & u', & m' \\ \gamma, & \gamma', & u', & w_1, & n' \\ \delta, & \delta', & l', & m', & k_1. \end{vmatrix}\right\} \end{aligned}\right\} \quad . \; . \; . \quad (14)$$

In the same way

$$(-vh-wg+pa)^2=\begin{vmatrix} . & . & . & \beta, & \gamma, & \delta \\ . & . & . & \beta', & \gamma', & \delta' \\ . & . & . & v, & w, & p \\ \beta, & \beta', & v, & v_1, & u', & m' \\ \gamma, & \gamma', & w, & u', & w_1, & n' \\ \delta, & \delta', & p, & m', & n', & k_1 \end{vmatrix} =(n-1)^{-2}. \begin{vmatrix} . & . & \alpha x-\varpi, & \beta, & \gamma, & \delta \\ . & . & \alpha' x-\varpi', & \beta', & \gamma', & \delta' \\ \alpha x-\varpi, & \alpha' x-\varpi, & u_1 x^2, & w' x, & v' x, & l' x \\ \beta, & \beta', & w' x, & v_1, & u', & m' \\ \gamma, & \gamma', & v' x, & u', & w_1, & n' \\ \delta, & \delta', & l' x, & m', & n', & k_1 \end{vmatrix}$$

$$=(n-1)^{-2}\left\{x^2\Phi-2x\begin{vmatrix} . & \alpha, & \beta, & \gamma, & \delta \\ . & \alpha', & \beta', & \gamma', & \delta' \\ B, & w', & v_1, & u', & m' \\ C, & v', & u', & w_1, & n' \\ D, & l', & m', & n', & k_1, \end{vmatrix}+\begin{vmatrix} . & B, & C, & D \\ B, & v_1, & u', & m' \\ C, & u', & w_1, & n' \\ D, & m', & n', & k \end{vmatrix}\right\}$$

or

$$(n-1)^2(vh-wg+ka)^2=x^2\Phi-2(n-1)x\Box(vh-wg+ka)-\left.\begin{array}{cccc} . & B & C & D\\ B & v_1 & u' & m'\\ C & u' & w_1 & n'\\ D & m' & n' & k_1.\end{array}\right\} \quad . \qquad (15)$$

Similarly, it will be found that

$$\begin{aligned}&(n-1)^2(wf-uh+pb)(ug-vf+kc)\\ &\quad =yz\Phi-2(n-1)\{y\Box(ug-vf+kc)+z\Box(wf-uh+kb)\}-\left.\begin{array}{cccc} . & A & C & D\\ A & u_1 & v' & l'\\ B & w' & u_1 & m'\\ D & l' & n' & k_1\,;\end{array}\right\} \quad . \qquad (16)\end{aligned}$$

so that, m being the degree of V, we shall have, on collecting all the terms of the forms (15) and (16), the following expression:—

$$\begin{aligned}&(n-1)^2\{(vh-wg+ka)^2\partial_x^2+\,.\,.+2(wf-uh-kb)(ug-vf+kc)\partial_y\partial_z+\,.\,.\}\mathrm{V}\\ &\quad =-2(n-1)(m-1)\{\Box(vh-wg+ka)\partial_x\mathrm{V}+\,.\,.\}-\begin{array}{cccc} . & B & C & D\\ B & v_1 & u' & m'\\ C & u' & w_1 & n'\\ D & m' & n' & k_1.\end{array}\partial_x^2\mathrm{V}-\,.\,.\end{aligned}$$

And if to each side of this equation we add

$(n-1)^2\{(vh-wg+ka)^2\partial'_x+\,.\,.\}\{(vh-wg+ka)\partial_x\mathrm{V}+\,.\,.\}=-(n-1)^2\{\partial_x\mathrm{V}\Box(vh-wg+ka)+$

in which the operations ∂'_x, .. affect the quantities u, .. only and not $\partial_x\mathrm{V}$, ..; then we shall have the full expression for $\Box^2\mathrm{V}$, viz.

$$-(n-1^2\left(1+\frac{2(m-1)}{n-1}\right)\{\partial_x\mathrm{V}\Box(vh-wg+ka)+\,.\,.\}-\begin{array}{cccc} . & B & C & D\\ B & v_1 & u' & m'\\ C & u' & w_1 & n'\\ D & m' & n' & k_1.\end{array}\partial_x^2\mathrm{V}-\,.\,.$$

Now referring to (5), and calling the ratios therein contained θ, and substituting θu for $\partial_x\mathrm{V}$, θv for $\partial_y\mathrm{V}$, . . , we have

$$u\Box(vh-wg+ka)+v\Box(wf-uh+kb)+w\Box(ug-vf+kc)+p\Box(-ua-vb-wc)$$

$$=\begin{array}{ccccccc} . & . & . & \alpha & \beta & \gamma & \delta\\ . & . & . & \alpha' & \beta' & \gamma' & \delta'\\ . & . & . & u & v & w & k\\ \alpha & \alpha' & u & u_1 & w' & v' & l'\\ \beta & \beta' & v & w' & v_1 & u' & m'\\ \gamma & \gamma' & w & v' & u' & w_1 & n'\\ \delta & \delta' & p & l' & m' & u' & k_1\end{array} =n(-1)^{-2}\begin{array}{ccccccc} . & . & -\varpi & \alpha & \beta & \gamma & \delta\\ . & . & -\varpi' & \alpha' & \beta' & \gamma' & \delta'\\ -\varpi & -\varpi' & . & . & . & . & .\\ \alpha & \alpha' & . & u_1 & w' & v' & l'\\ \beta & \beta' & . & w' & v_1 & u' & m'\\ \gamma & \gamma' & . & v' & u' & w_1 & n'\\ \delta & \delta' & . & l' & m' & n' & k_1.\end{array} =(n-1)^{-2}.\begin{array}{ccccc} & A & B & C & D\\ A & u_1 & w' & v' & l'\\ B & w' & v_1 & u' & m'\\ C & v' & u' & w_1 & n'\\ D & l' & m' & n' & k_1\end{array}$$

Further, if we agree upon the proper mode of development we may write

$$\begin{array}{cccc} . & B & C & D \\ B & v_1 & u' & m' \\ C & u' & w_1 & u' \\ D & m' & n' & k_1 \end{array} \partial_x^2 V + \,..\, + 2 \begin{array}{cccc} . & A & C & D \\ A & u_1 & v' & l' \\ B & w' & u' & m' \\ D & l' & n' & k_1 \end{array} \partial_y \partial_z V + \,..$$

$$= \begin{array}{cccccc} u_1 & w' & v' & l' & A & \partial_x \\ w' & v_1 & u' & m' & B & \partial_y \\ v' & u' & w_1 & n' & C & \partial_z \\ l' & m' & n' & k_1 & D & \partial_t \\ A & B & C & D & . & . \\ \partial_x & \partial_y & \partial_z & \partial_t & . & . \end{array} V = \Delta V \text{ suppose} \quad . \quad . \quad . \quad . \quad (17)$$

which expression, by giving obvious values to $\overline{\mathfrak{A}}, \,.\,.\, \overline{\mathfrak{F}}, \,.\,.$, may be written thus:

$$\overline{\mathfrak{A}} \partial_x^2 V + \,..\, + 2\overline{\mathfrak{F}} \partial_y \partial_z V + \,..\, = (\overline{\mathfrak{A}}, \overline{\mathfrak{B}}, \overline{\mathfrak{C}}, \overline{\mathfrak{D}}, \overline{\mathfrak{F}}, \overline{\mathfrak{G}}, \overline{\mathfrak{H}}, \overline{\mathfrak{L}}, \overline{\mathfrak{M}}, \overline{\mathfrak{N}})(\partial_x, \partial_y, \partial_z, \partial_t)^2 V. \quad . \quad (18)$$

Lastly, putting

$$H = \begin{array}{ccccc} u_1 & w' & v' & l' & A \\ w' & v_1 & u' & m' & B \\ v' & u' & w_1 & n' & C \\ l' & m' & n' & k_1 & D \\ A & B & C & D & . \end{array}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

the expression for $\square^2 V$ finally becomes

$$\Delta V - \left(1 + \frac{2(m-1)}{n-1}\right) \theta H = 0,$$

and consequently the system $V=0$, $\square V=0$, $\square^2 V=0$ may be replaced by

$$\frac{\partial_x V}{u} = \frac{\partial_y V}{v} = \frac{\partial_z V}{w} = \frac{\partial_t V}{k} = \left(1 + \frac{2(m-1)}{n-1}\right)^{-1} \frac{\Delta V}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If in the foregoing expressions we put $\alpha'=0$, $\beta'=0$, $\gamma'=0$, $\delta'=1$, $\delta=0$, $t=0$, we shall have the case of plane curves, and as the last suppositions give $\varpi'=0$, $A=0$, $B=0$, $C=0$, $D=\alpha x+\beta y+\gamma z$, the expression (20) then reduces itself, as it should, to that given in equation (16) of the memoir above quoted.

§ 3. *Elimination of the Constants of the Quadric* V.

Before proceeding to the application of the formulæ (20) to the present problem, it will be convenient to premise that if ϕ, ψ be any two rational integral and homogeneous functions of x, y, z, t, the nature of the operation Δ is such that

$$\Delta\phi\psi = \psi\Delta\phi + \phi\Delta\psi + 2(\overline{\mathfrak{A}}, \overline{\mathfrak{B}}, \overline{\mathfrak{C}}, \overline{\mathfrak{D}}, \overline{\mathfrak{F}}, \overline{\mathfrak{G}}, \overline{\mathfrak{H}}, \overline{\mathfrak{L}}, \overline{\mathfrak{M}}, \overline{\mathfrak{N}})(\partial_x\phi, \partial_y\phi, \partial_z\phi, \partial_t\phi)(\partial_x\psi, \partial_y\psi, \partial_z\psi, \partial_t\psi); \quad (21)$$

also that

$$\Delta \mathrm{U} = 4\mathrm{H}.$$

And further, if we write

$$\partial_x \mathrm{H} = p,\ \partial_y \mathrm{H} = q,\ \partial_z \mathrm{H} = r,\ \partial_t \mathrm{H} = s, \quad \ldots\ldots\ldots \quad (22)$$

and if we suppose that the first four columns in the four following determinants are the same as the first four in the expression for H given in (19), then

$$\Delta u = p - 2\begin{matrix} \cdot\cdot & 0, \\ \cdot\cdot & c \\ \cdot\cdot & -b \\ \cdot\cdot & f \\ \cdot\cdot & 0 \end{matrix}\quad \Delta v = q - 2\begin{matrix} \cdot\cdot & -c, \\ \cdot\cdot & 0 \\ \cdot\cdot & a \\ \cdot\cdot & g \\ \cdot\cdot & 0 \end{matrix}\quad \Delta w = r - 2\begin{matrix} \cdot\cdot & b, \\ \cdot\cdot & -a \\ \cdot\cdot & 0 \\ \cdot\cdot & h \\ \cdot\cdot & 0 \end{matrix}\quad \Delta k = s - 2\begin{matrix} \cdot\cdot & -f \\ \cdot\cdot & -g \\ \cdot\cdot & -h \\ \cdot\cdot & 0 \\ \cdot\cdot & 0, \end{matrix} \quad \ldots \quad (23)$$

from which it follows that

$$\left.\begin{aligned} \Delta(\ vh - wg + ka) &= \ gh - rg + sa \\ \Delta(\ wf - uh + kb) &= \ rf - ph + sb \\ \Delta(\ ug - vf + kc) &= \ pg - qf + sc \\ \Delta(-ua - vb - wc) &= -pa - qb - rc, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (24)$$

the outstanding terms which occur in (23) having cancelled one another.

Furthermore, applying the formula (21) to the following products, and bearing in mind that $\partial_x \mathrm{V}$, $\partial_y \mathrm{V}$, . . are all linear in x, y, z, t, we have, by means of (24),

$$\left.\begin{aligned} \Delta\partial_x \mathrm{V}(vh - wg + ka) &= \partial_x \mathrm{V}(gh - rg + sa) + 2\mathrm{H}(h\partial_x\partial_y \mathrm{V} - g\partial_x\partial_z \mathrm{V} + a\partial_z\partial_t \mathrm{V}) \\ \Delta\partial_y \mathrm{V}(wf - uh + kb) &= \partial_y \mathrm{V}(rf - ph + sb) + 2\mathrm{H}(f\partial_y\partial_z \mathrm{V} - h\partial_x\partial_y \mathrm{V} + b\partial_y\partial_t \mathrm{V}). \\ \vdots \qquad\qquad & \qquad\qquad \vdots \qquad\qquad\qquad \vdots \end{aligned}\right\} \quad . \quad (25)$$

so that

$$\Delta\{\partial_x \mathrm{V}(vh - wg + ka) + \partial_y \mathrm{V}(wf - uh + kb) + \ldots\} = \begin{matrix} \alpha & \alpha' & p & \partial_x \mathrm{V} \\ \beta & \beta' & q & \partial_y \mathrm{V} \\ \gamma & \gamma' & r & \partial_z \mathrm{V} \\ \delta & \delta' & s & \partial_t \mathrm{V}, \end{matrix} \quad \ldots \quad (26)$$

the coefficients of 2H in (25) having cancelled one another.

This being premised, let us return to the equations (20). These equations being perfectly general, we may replace V by any other function we please. Hence if we replace V by $\square \mathrm{V}$, the system resulting will be an equivalent for the system $\square \mathrm{V} = 0$, $\square^2 \mathrm{V} = 0$, $\square^3 \mathrm{V} = 0$, viz. the second, third, and fourth of the system (8). Making this substitution in (20), and remembering that $\square \mathrm{V}$ is of the degree n, so that the numerical factor $1 + \frac{2(m-1)}{n-1} = 3$, we have

$$\frac{1}{u}\partial_x \begin{matrix} \alpha, & \alpha', & u, & \partial_x V \\ \beta & \beta' & v & \partial_y V \\ \gamma & \gamma' & w & \partial_z V \\ \delta & \delta' & k & \partial_t V \end{matrix} = \frac{1}{v}\partial_y \begin{matrix} \alpha, & \alpha', & u, & \partial_x V \\ \beta & \beta' & v & \partial_y V \\ \gamma & \gamma' & w & \partial_z V \\ \delta & \delta' & k & \partial_t V \end{matrix} = .. = \frac{1}{3\mathrm{H}}\Delta \begin{matrix} \alpha, & \alpha', & u, & \partial_x V \\ \beta & \beta' & v & \partial_y V \\ \gamma & \gamma' & w & \partial_z V \\ \delta & \delta' & k & \partial_t V \, ; \end{matrix}$$

or taking the first and last of these expressions, and substituting from (26),

$$\frac{3\mathrm{H}}{u}\left\{ \begin{matrix} \alpha & \alpha' & u_1 & \partial_x V \\ \beta & \beta' & w' & \partial_y V \\ \gamma & \gamma' & v' & \partial_z V \\ \delta & \delta' & l' & \partial_t V \end{matrix} + \begin{matrix} \alpha & \alpha' & u & \partial_x^2 \ V \\ \beta & \beta' & v & \partial_x\partial_y V \\ \gamma & \gamma' & w & \partial_x\partial_z V \\ \delta & \delta' & k & \partial_x\partial_t V \end{matrix} \right\} = \begin{matrix} \alpha & \alpha' & p & \partial_x V \\ \beta & \beta' & q & \partial_y V \\ \gamma & \gamma' & r & \partial_z V \\ \delta & \delta' & s & \partial_t V \, ; \end{matrix}$$

or substituting θu for $\partial_x V, \ldots$,

$$\frac{3\mathrm{H}}{\theta u}\left\{ \begin{matrix} \alpha & \alpha' & u_1 & u \\ \beta & \beta' & u' & v \\ \gamma & \gamma' & v' & w \\ \delta & \delta' & l' & k \end{matrix} \ \theta + \begin{matrix} \alpha & \alpha' & u & \partial_x^2 \ V \\ \beta & \beta' & v & \partial_x\partial_y V \\ \gamma & \gamma' & w & \partial_x\partial_z V \\ \delta & \delta' & k & \partial_x\partial_t V \end{matrix} \right\} = \begin{matrix} \alpha & \alpha' & p & u \\ \beta & \beta' & q & v \\ \gamma & \gamma' & r & w \\ \delta & \delta' & s & k \, , \end{matrix}$$

or

$$\begin{matrix} \alpha & \alpha' & u & (up-3\mathrm{H}u_1)\partial_x V+3\mathrm{H}u\partial_x^2 \ V=0 \\ \beta & \beta' & v & (uq-3\mathrm{H}w')\partial_x V+3\mathrm{H}u\partial_x\partial_y V \\ \gamma & \gamma' & w & (ur-3\mathrm{H}v')\partial_x V+3\mathrm{H}u\partial_x\partial_z V \\ \delta & \delta' & k & (us-3\mathrm{H}l')\partial_x V+3\mathrm{H}u\partial_x\partial_t V \, ; \end{matrix} \quad . \ . \ . \ . \ . \ . \quad (27)$$

and if

$$\mathrm{V}=(\mathrm{a, b, c, d, f, g, h, l, m, n})(x, y, z, t)^2, \quad . \ . \ . \ . \ . \ . \ . \quad (28)$$

the expression (27) is equivalent to

$$\mathrm{a}\left\{ \begin{matrix} \alpha & \alpha' & u & (up-3\mathrm{H}u_1)x+3\mathrm{H}u \\ \beta & \beta' & v & (uq-3\mathrm{H}w')x \\ \gamma & \gamma' & w & (ur-3\mathrm{H}v')x \\ \delta & \delta' & k & (us-3\mathrm{H}l')x, \end{matrix} \right\} + \mathrm{h}\{..\} + \mathrm{g}\{..\} + \mathrm{l}\{..\} = 0 \quad . \ . \quad (29)$$

which contains only the four coefficients a, h, g, l. In this expression the coefficient of

$$\mathrm{a} = \begin{matrix} \varpi, & \varpi', & . & 2\mathrm{H}u \\ \beta, & \beta', & v\,, & uq-3\mathrm{H}w' \\ \gamma, & \gamma', & w, & ur-3\mathrm{H}v' \\ \delta\,, & \delta'\,, & k, & us-3\mathrm{H}l'. \end{matrix} \qquad \begin{matrix} =\mathrm{B}, & u, & uq-3\mathrm{H}w'-2\mathrm{H}u \\ \mathrm{C}, & w, & ur-3\mathrm{H}v' \\ \mathrm{D}, & k, & us-3\mathrm{H}l' \end{matrix} \quad \begin{matrix} \beta, & \beta', & v \\ \gamma, & \gamma', & w \\ \delta\,, & \delta', & k \end{matrix}$$

If, therefore,

$$\left.\begin{aligned} X,\ Y,\ Z,\ T &= \left\| \begin{matrix} A, & B, & C, & D \\ p, & q, & r, & s \\ u, & v, & w, & k \end{matrix} \right\| \\ P_1,\ P_2,\ P_3,\ P_4 &= \left\| \begin{matrix} A & B & C & D \\ u_1 & w' & v' & l' \\ u & v & w & k \end{matrix} \right\| + \tfrac{2}{3}u \left\| \begin{matrix} \alpha & \beta & \gamma & \delta \\ \alpha' & \beta' & \gamma' & \delta' \\ u & v & w & p \end{matrix} \right\| \end{aligned} \right\} \quad . \quad (30)$$

(29) will take the form

$$(uX-3HP_1)a+(uY-3HP_2)h+(uZ-3HP_3)g+(uT-3HP_4)l=0, \quad . \quad . \quad (31a)$$

to which might be added the analogous equations in h, b, f, m; g, f, c, n; l, m, n, d.

Another, and for some purposes a more convenient form may be given to these equations by the following transformation:

$H=(\mathfrak{A}', \ldots)(A, B, C, D)^2$, suppose;

$$\therefore\ \partial_x H=(\partial_x\mathfrak{A}', \ldots)(A, B, C, D)^2+\frac{2}{\varpi}(\mathfrak{A}', \ldots)(A, B, C, D)(\alpha A-\alpha A,\ \beta A-\alpha B,\ \gamma A-\alpha C,\ \delta A-\alpha I$$

$$=(\partial_x\mathfrak{A}', \ldots)(A, B, C, D)^2+\frac{2A}{\varpi}(\mathfrak{A}', \ldots)(A, B, C, D)(\alpha, \beta, \gamma, \delta)-\frac{2\alpha}{\varpi}H.$$

If, therefore, henceforward p, q, r, s represent the differential coefficients of H upon the supposition that A, B, C, D are regarded as constants, we shall have

$$X=\begin{matrix} B & C & D \\ q & r & s \\ v & w & k \end{matrix} +2H \begin{matrix} \beta & \gamma & \delta \\ \beta' & \gamma' & \delta' \\ v & w & k \end{matrix}$$

$$P_1=\begin{matrix} B & C & D \\ w' & v' & l' \\ v & w & k \end{matrix} +\tfrac{2}{3}u \begin{matrix} \beta & \gamma & \delta \\ \beta' & \gamma' & \delta' \\ v & w & k, \end{matrix}$$

so that

$$uX-3HP_1=\begin{matrix} . & B & C & D \\ 3H & q & r & s \\ u & w' & v' & l' \\ . & v & w & k \end{matrix} =(n-2)^{-1} \begin{matrix} Ax & B & C & D \\ px & q & r & s \\ -u+u_1x & w' & v' & l' \\ ux & v & w & k; \end{matrix}$$

and if P, Q, R, S represent the determinants formed from the first three together with the fourth, the fifth, the sixth, and the seventh columns respectively of the following matrix:

$$\begin{matrix} A & u & p & u_1 & w' & v' & l' \\ B & v & q & w' & v_1 & u' & m' \\ C & w & r & v' & u' & w_1 & n' \\ D & k & s & l' & m' & n' & k_1 \end{matrix}$$

and if, as was indicated above, H in the transformed expression is supposed to be differentiated without reference to A, B, C, D; then

$$\begin{aligned}(n-2)(uX-3HP_1)&=-uX+xP\\(n-2)(uY-3HP_2)&=-uY+yP\\(n-2)(uZ-3HP_3)&=-uZ+zP\\(n-2)(uT-3HP_4)&=-uT+tP,\end{aligned}$$

and the equations, of which (31 *a*) is one, take the form

$$\left.\begin{aligned}(uX-xP)\mathrm{a}+(uY-yP)\mathrm{h}+(uZ-zP)\mathrm{g}+(uT-tP)\mathrm{l}&=0\\(vX-xQ)\mathrm{h}+(vY-yQ)\mathrm{b}+(vZ-zQ)\mathrm{f}+(vT-tQ)\mathrm{m}&=0\\(wX-xR)\mathrm{g}+(wY-yR)\mathrm{f}+(wZ-zR)\mathrm{c}+(wT-tR)\mathrm{n}&=0\\(wX-xS)\mathrm{l}+(wY-yS)\mathrm{m}+(wZ-zS)\mathrm{n}+(wT-tS)\mathrm{d}&=0.\end{aligned}\right\}\quad . \quad . \quad . \quad (31\,b)$$

Representing any one of these equations, say, the first, by $W=0$, the equations $\Box^3V=0$, $\Box^4V=0$, $\Box^5V=0$ may be replaced by a system of the form (20); and writing them in the form $\partial_x W=\theta_1 u$, $\partial_y W=\theta_1 v$, . ., where θ_1 is indeterminate, we may from the five equations so written eliminate the five quantities a, h, g, l, θ_1; and the resulting equation takes the form

$$\begin{vmatrix}\partial_x(uX-xP) & \partial_x(uY-yP) & \partial_x(uZ-zP) & \partial_x(uT-tP) & u\\ \partial_y(uX-xP) & \partial_y(uY-yP) & \partial_y(uZ-zP) & \partial_y(uT-tP) & v\\ \partial_z(uX-xP) & \partial_z(uY-yP) & \partial_z(uZ-zP) & \partial_z(uT-tP) & w\\ \partial_t(uX-xP) & \partial_t(uY-yP) & \partial_t(uZ-zP) & \partial_t(uT-tP) & k\\ \Delta(uX-xP) & \Delta(uY-yP) & \Delta(uZ-zP) & \Delta(uT-tP) & \nu H,\end{vmatrix}=0 \quad . \quad . \quad (32)$$

where ν is a numerical factor.

§ 4. *Determination of the extraneous factors.*

The degree of the equation (32) in its present form is $23n-32$; but it admits of reduction, in the first place, as follows. Since the equation

$$A(uX-xP)+B(uY-yP)+C(uZ-zP)+D(uT-tP)=0$$

is identically satisfied, we have by differentiation, and by substitution of the values of $\partial_x A$, . . from (11),

$$\begin{aligned}&A\partial_x(uX-xP)+B\partial_x(uY-yP)+C\partial_x(uZ-zP)+D\partial_x(uT-tP)\\&=-(uX-xP)\partial_x A-(uY-yP)\partial_x B-(uZ-zP)\partial_x C-(uT-tP)\partial_x D\\&=-\frac{1}{\varpi}\{(uX-xP)(\alpha A-\alpha A)+(uY-yP)(\alpha B-\beta A)+(uZ-zP)(\alpha C-\gamma A)+(uT-tP)(\alpha D-\delta A\\&=\ \frac{A}{\varpi}\{\alpha(uX-xP)+\beta(uY-yP)+\gamma(uZ-zP)+\delta(uT-tP)\}\\&=\ \frac{A}{\varpi}K,\ \text{suppose.}\end{aligned}$$

From this we may deduce the following system:

$$\left.\begin{array}{l} A\partial_x(uX-xP)+B\partial_x(uY-yP)+\,..=A\frac{k}{\varpi} \\ A\partial_y(uX-xP)+B\partial_y(uY-yP)+\,..=B\frac{k}{\varpi} \\ \quad\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (33)$$

Again, since $\Delta A=0,\ \Delta B=0,\ \Delta C=0,\ \Delta D=0$, it follows that

$$\begin{array}{l} A\Delta(uX-xP)+B\Delta(uY-yP)+C\Delta(uZ-zP)+D\Delta(uT-tP) \\ =-\frac{2}{\varpi}\{(\mathfrak{A},\,..)(\alpha A-\alpha A,\ \alpha B-\beta A,\ \alpha C-\gamma A,\ \alpha D-\delta A)(\partial_x,\ \partial_y,\ \partial_x,\ \partial_t)(uX-xP) \\ \quad +(\mathfrak{A},\,..)(\beta A-\alpha B,\ \beta B-\beta B,\ \beta C-\gamma B,\ \beta D-\delta B)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uY-yP) \\ \quad +(\mathfrak{A},\,..)(\gamma A-\alpha C,\ \gamma B-\beta C,\ \gamma C-\gamma C,\ \gamma D-\delta C)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uZ-zP) \\ \quad +(\mathfrak{A},\,..)(\delta A-\alpha D,\ \delta B-\beta D,\ \delta C-\gamma D,\ \delta D-\delta D)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uT-tP)\}. \end{array}$$

But since

$$\left.\begin{array}{l} \mathfrak{A}A+\mathfrak{H}\,B+\mathfrak{G}C+\mathfrak{L}\,D=0 \\ \mathfrak{H}A+\mathfrak{B}\,B+\mathfrak{F}C+\mathfrak{M}D=0 \\ \mathfrak{G}A+\mathfrak{F}\,B+\mathfrak{C}C+\mathfrak{N}\,D=0 \\ \mathfrak{L}A+\mathfrak{M}B+\mathfrak{N}C+\mathfrak{D}\,D=0, \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

the expression above written reduces itself to

$$\begin{array}{l} \frac{2}{\varpi}\{A(\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uX-xP) \\ \quad +B(\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uY-yP) \\ \quad +C(\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uZ-zP) \\ \quad +D(\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)(uT-tP)\} \\ =\frac{2}{\varpi}\{\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)\Big[(uX-xP)\partial_xA+(uY-yP)\partial_xB+\,.. \\ \qquad\qquad\qquad\qquad (uX-xP)\partial_yA+(uY-yP)\partial_yB+\,.. \\ \qquad\qquad\qquad\qquad\quad \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \Big] \\ =-\frac{2k}{\varpi^2}(\mathfrak{A},\,..)(\alpha,\ \beta,\ \gamma,\ \delta)(A,\ B,\ C,\ D) \\ =0, \end{array}$$

i. e. $$A\Delta(uX-xP)+B\Delta(uY+yP)+C\Delta(uZ-zP)+D\Delta(uT-tP)=0. \quad . \; . \quad (35)$$

Hence, if we multiply the first three columns of (32) by A, B, C respectively, and add them to the fourth multiplied by D, the whole equation will be divisible by $\frac{K}{\varpi}$; and on the division being made the equation in question will take the form

$$\begin{vmatrix} \partial_x(uX-xP) & \partial_x(uY-yP) & \partial_x(uZ-zP) & A & u \\ \partial_y(uX-xP) & \partial_y(uY-yP) & \partial_y(uZ-zP) & B & v \\ \partial_z(uX-xP) & \partial_z(uY-yP) & \partial_z(uZ-zP) & C & w \\ \partial_t(uX-xP) & \partial_t(uY-yP) & \partial_t(uZ-zP) & D & k \\ \Delta(uX-xP) & \Delta(uY-yP) & \Delta(uZ-zP) & . & \nu H \end{vmatrix} =0 \quad . \; . \qquad . \quad (36)$$

which is of the degree $18n-24$.

Next it may be shown that H is a factor of this expression. For when $H=0$ the following equations subsist,

$$\begin{aligned} Ax+By+Cz+Dt&=0\\ px+qy+rz+st&=0\\ ux+vy+wz+kt&=0, \end{aligned}$$

whence by elimination

$$\frac{X}{x}=\frac{Y}{y}=\frac{Z}{z}=\frac{T}{t}=\frac{P}{(n-1)u}; \quad \ldots\ldots\ldots\ldots \quad (37)$$

so that, omitting a numerical factor, $uX-xP$ will be replaced by xP, and (36) will then become

$$\begin{array}{lllll} x\partial_x P+P & y\partial_x P & z\partial_x P & A & u\\ x\partial_y P & y\partial_y P+P & z\partial_y P & B & v\\ x\partial_z P & y\partial_z P & z\partial_z P+P & C & w\\ x\partial_t P & y\partial_t P & z\partial_t P & D & k\\ x\Delta P+2(\mathfrak{A}\partial_x+\,..)P & y\Delta P+2(\mathfrak{H}\partial_x+\,..)P & z\Delta P+2(\mathfrak{G}\partial_x+\,..)P & . & . \end{array}$$

But by adding x (line 1)$+y$ (line 2)$+z$ (line 3) to t (line 4), the whole of line 4 will be divisible by P and a numerical factor; so that the expression becomes

$$\begin{array}{lllll} x\partial_x P+P & y\partial_x P & z_x\partial P & A & u\\ x\partial_y P & y\partial_y P+P & z\partial_y P & B & v\\ x\partial_z P & y\partial_z P & z\partial_z P+P & C & w\\ x & y & z & . & .\\ x\Delta P+2(\mathfrak{A}\partial_x+\,..)P & y\Delta P+2(\mathfrak{H}\partial_x+\,..)P & z\Delta P+2(\mathfrak{G}\partial_x+\,..)P & & . \end{array}$$

Again, subtracting $\partial_x P$ line 4 from line 1
$\partial_y P$ line 4 from line 2
$\partial_z P$ line 4 from line 3
ΔP line 4 from line 5,

and dividing throughout by 2, the expression is reduced to

$$\begin{array}{ccccc} P & . & . & A & u\\ . & P & . & B & v\\ . & . & P & C & w\\ x & y & z & . & .\\ (\mathfrak{A}\partial_x\,..)P & (\mathfrak{H}\partial_x+\,..)P & (\mathfrak{G}\partial_x+\,..)P & . & . \end{array}$$

lastly, subtracting

A (column 1) $+$B (column 2) $+$C (column 3) from P (column 4),

and

u (column 1) $+v$ (column 2) $+w$ (column 3) from P (column 5), we have (remembering that $H=0$)

$$\begin{array}{ccccc}
P & . & . & . & . \\
. & P & . & . & . \\
. & . & P & . & . \\
x & y & z & Dt & kt \\
(\mathfrak{A}\partial_x+\ldots)P & (\mathfrak{H}\partial_x+\ldots)P & (\mathfrak{G}\partial_x+\ldots)P & D(\mathfrak{L}\partial_x+\ldots)P & k(\mathfrak{L}\partial_x+\ldots)P\,;
\end{array}$$

which vanishes identically. Hence H is a factor of (36); and on dividing it out the degree of the expression (36) is reduced to $15n-20$.

It remains now to be shown that u is likewise a factor of (36). Putting $u=0$, that equation becomes

$$\begin{array}{llccc}
u_1 & X-x\partial_xP+P & \ldots & A & u \\
w' & X-x\partial_yP & \ldots & B & u \\
v' & X-x\partial_zP & \ldots & C & w \\
l' & X-x\partial_tP & \ldots & D & p \\
\multicolumn{2}{l}{\Delta uX-x\Delta P+2H\partial_xX-2(\mathfrak{A}\partial_x+\ldots)P} & \ldots & 0 & \nu H
\end{array}$$

$$=\begin{array}{llccc}
1 & X & \ldots & 0 & 0 \\
u_1 & x\partial_xP+P & \ldots & A & u \\
w' & x\partial_yP & \ldots & B & v \\
v' & x\partial_zP & \ldots & C & w \\
l' & x\partial_tP & \ldots & D & p \\
\Delta u & z\Delta p+2H\partial_xX-2(\mathfrak{A}\partial_x+\ldots)P & \ldots & 0 & \nu H\,;
\end{array}$$

and following a process similar to that adopted in the former transformation, this may be reduced to

$$\begin{array}{llccc}
1 & X & \ldots & 0 & 0 \\
u_1 & P & \ldots & A & u \\
w' & . & \ldots & B & v \\
v' & . & \ldots & C & w \\
. & x & \ldots & 0 & 0 \\
\Delta u & 2H\partial_xX-2(\mathfrak{A}\partial_x+\ldots)P & \ldots & 0 & \nu H,
\end{array}$$

and thence to

$$\begin{array}{llccc}
t & tX-xT & \ldots & 0 & 0 \\
u_1 & P & \ldots & 0 & 0 \\
w' & . & \ldots & 0 & 0 \\
v' & . & \ldots & 0 & 0 \\
. & x & \ldots & Dt & kt \\
\Delta u & 2H\partial_xX-2(\mathfrak{A}\partial_x+\ldots)P & \ldots & \Theta & \Psi,
\end{array}$$

Θ and Ψ being two functions, with the exact forms of which we are not concerned. The expression then takes the form of the product of two factors, viz. $(\mathrm{D}\Psi-k\Theta)t$, and

$$\begin{array}{cccc} t & t\mathrm{X}-x\mathrm{T} & t\mathrm{Y}-y\mathrm{T} & t\mathrm{Z}-z\mathrm{T} \\ u_1 & \mathrm{P} & . & . \\ w' & . & \mathrm{P} & . \\ v' & . & . & \mathrm{P} \end{array}$$

$$=\mathrm{P}^2\{\mathrm{P}t-t(u_1\mathrm{X}+w'\mathrm{Y}+v'\mathrm{Z}+l'\mathrm{T})+\mathrm{T}(u_1x+w'y+v'z+l't)\}$$

$$=\mathrm{P}^2t(\mathrm{P}-\mathrm{P})$$

$$=0.$$

Hence u is a factor of (36). Now that equation as it stands is of the third degree in u: so that u being divided out it is reduced to the second degree, say, to the form $\lambda u^2+\mu u+\varrho=0$. Now referring to (31), and forming the equations in h, b, f, m; g, . .; l, . ., the equations corresponding to (36) may by a similar process be shown to be divisible by v, w, k; and those divisions having been effected, the equations in question will be reduced to the forms $\lambda v^2+\mu_1 v+\varrho_1=0$, . ., in which it is to be observed, from the symmetry of the expressions, that the coefficients of u^2, v^2, . . are the same. But as these equations (viz. those in u, v, w, k) all lead to the same result, namely the determination of the sextactic points, they can differ from one another only by factors. Hence, as in my memoir before quoted, equation (50), we must have identically

$$-\lambda=\frac{\mu u+\varrho}{u^2}=\frac{\mu_1 v+\varrho_1}{v^2}=\ldots, \quad \ldots\ldots\ldots \quad (38)$$

which can hold good in general, only in virtue of μ being divisible by u, and ϱ by u^2, μ_1 being divisible by v, and ϱ_1 by v^2, and so on. Hence (36) is divisible not only by u but by u^3; and that division having been effected, the degree of (36) will be reduced to $12n-17$.

This completes the enumeration of the extraneous factors; but although the degree of the equation (36) cannot in general be depressed below $12n-17$, we have yet to show that, as stated in the introduction, the variables enter to the degree 10 in the form of lineo-linear functions of the arbitrary quantities. From what has gone before, it is clear that the quantities X, Y, Z, P, H involved in (36), are all functions of x, y, z, t, A, B, C, D only, that is to say, not of α, β, . . α', β', . . ., except in so far as they are included in A, B, C, D. It remains to be proved that this is still the case after the differentiations and operation Δ involved in (36) have been performed. For this purpose let φ represent any function of x, y, z, t, A, B, C, D, and let ∂'_x, . . indicate differentiation with respect to x, . . so far as they appear explicitly in φ, irrespectively of A, B, C, D, then

$$\partial_x\varphi=\partial_{\mathrm{A}}\varphi\partial_x\mathrm{A}+\partial_{\mathrm{B}}\varphi\partial_x\mathrm{B}+\ldots+\partial'_x\varphi;$$

$$\therefore\ \varpi\partial_x\varphi=\mathrm{A}(\alpha\partial_{\mathrm{A}}+\beta\partial_{\mathrm{B}}+\ldots)\varphi-\alpha(\mathrm{A}\partial_{\mathrm{A}}+\mathrm{B}\partial_{\mathrm{B}}+\ldots)\varphi+\varpi\partial'_x\varphi;$$

and if ϕ be of the degree m in A, B, C, D, and $\nabla=\alpha\partial_A+\beta\partial_B+\ldots$, it follows that

$$\left.\begin{array}{l} \varpi\partial_x\phi=A\nabla\phi-m\alpha\phi+\varpi\partial'_x\phi \\ \varpi\partial_y\phi=B\nabla\phi-m\beta\phi+\varpi\partial'_y\phi \\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (39)$$

Again,

$$\begin{aligned}\varpi^2\partial_x^2\phi&=\varpi\partial_x\varpi\partial_x\phi-\alpha\varpi\partial_x\phi\\ &=(\varpi\partial_x-\alpha)\varpi\partial_x\phi,\end{aligned}$$

whence

$$\left.\begin{array}{l} \varpi^2\partial_x^2\phi\ \ =(A\nabla-\overline{m+1}\alpha+\varpi\partial'_x)(A\nabla-m\alpha+\varpi\partial'_x)\phi \\ \varpi^2\partial_y\partial_x\phi=(B\nabla-\overline{m+1}\beta+\varpi\partial'_y)(A\nabla-m\alpha+\varpi\partial'_x)\phi \\ \vdots \qquad\quad \vdots \qquad\qquad\qquad \vdots \end{array}\right\} \quad \ldots\ldots \quad (40)$$

whence, again,

$$\begin{array}{l}\varpi^2(\mathfrak{A}\partial_x^2\ \ +\mathfrak{H}\partial_y\partial_x+\ldots)\phi=\{-(m+1(\mathfrak{A}\alpha+\mathfrak{H}\beta+\ldots)+\varpi(\mathfrak{A}\partial'_x+\mathfrak{H}\partial'_y+\ldots)\}(A\nabla-m\alpha+\varpi\partial'_x)\phi\\ \varpi^2(\mathfrak{H}\partial_x\partial_y+\mathfrak{B}\partial_y^2\ \ +\ldots)\phi=\{-(m+1)\mathfrak{H}\alpha+\mathfrak{B}\beta+\ldots)+\varpi(\mathfrak{H}\partial'_x+\mathfrak{B}\partial'_y+\ldots)\}(B\nabla-m\beta+\varpi\partial'_y)\phi\,;\\ \quad\vdots \qquad\qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots\end{array}$$

and combining these expressions,

$$\varpi^2(\mathfrak{A},\ldots)(\partial_x,\ldots)^2\phi=m(m+1)(\mathfrak{A},\ldots)(\alpha,\ldots)^2\phi-(2m+1)\varpi(\mathfrak{A},\ldots)(\alpha,\ldots)(\partial'_x,\ldots)\phi+\varpi^2(\mathfrak{A},\ldots)(\partial'_x,\ldots)^2\phi.$$

Hence

$$\begin{array}{ccccc} \partial_x\phi & \partial_x\phi_1 & \partial_x\phi_2 & A & u \\ \partial_y\phi & \partial_y\phi_1 & \partial_y\phi_2 & B & v \\ \partial_z\phi & \partial_z\phi_1 & \partial_z\phi_2 & C & w \\ \partial_t\phi & \partial_t\phi_1 & \partial_t\phi_2 & D & k \\ \Delta\phi & \Delta\phi_1 & \Delta\phi_2 & . & \nu H \end{array}$$

$$=\varpi^{-4}\begin{array}{cccc} -m\alpha\phi+\varpi\partial'_x\phi & \cdot\cdot & A & u \\ -m\beta\phi+\varpi\partial'_y\phi & \cdot\cdot & B & v \\ -m\gamma\phi+\varpi\partial'_z\phi & \cdot\cdot & C & w \\ -m\delta\phi+\varpi\partial_t\phi & \cdot\cdot & D & k \\ \left(\begin{array}{l} \quad m(m+1)\ (\mathfrak{A},\ldots)(\alpha,\ldots)^2\phi \\ -(2m+1)\varpi(\mathfrak{A},\ldots)(\alpha,\ldots)(\partial'_x,\ldots)\phi \\ + \qquad\quad \varpi^2(\mathfrak{A},\ldots)(\partial'_x,\ldots)^2\phi \end{array}\right) & \cdot\cdot & 0 & \nu H, \end{array}$$

which, by a method precisely similar to that followed above in investigating the extraneous factors, may be reduced to

$$\begin{array}{ccccc} \partial'_x\phi & \partial'_x\phi_1 & \partial'_x\phi_2 & A & u \\ \partial'_y\phi & \partial'_y\phi_1 & \partial'_y\phi_2 & B & v \\ \partial'_z\phi & \partial'_z\phi_1 & \partial'_z\phi_2 & C & w \\ \partial'_t\phi & \partial'_t\phi_1 & \partial'_t\phi_2 & D & k \\ \Delta\phi & \Delta\phi_1 & \Delta\phi_2 & . & \nu H, \end{array}$$

which is the same as if A, B, C, D had not been affected by the differentiations or by the operation Δ. The expression is therefore a function of x, y, z, t, A, B, C, D, and not of $\alpha, \beta, \ldots, \alpha', \beta', \ldots$, excepting so far as they appear in A, B, C, D. If, therefore, we put $\varphi = uX - xP$, $\varphi_1 = uY - yP$, $\varphi_2 = uZ - zP$, we conclude that (36), when divested of its extraneous factors (say, Ω), is a function explicitly of the degree $12n-27$ in x, y, z, t, and of the degree 10 in A, B, C, D. But A, B, C, D are themselves linear homogeneous functions of ϖ, ϖ'; so that the expression in question may be regarded as explicitly of the degree $12n-27$ in x, y, z, t, and of the degree 10 in ϖ, ϖ'. This equation, solved for $k' = \varpi : \varpi'$, will consequently give ten positions of the cutting plane all passing through the point (x, y, z, t), for which the curve of section is sextactic at the point. Hence the theorem, "If L be any line through any point P of a surface, ten conics may be drawn in planes passing through L having six-pointic contact with the surface at the point."

It is to be observed that the plane of section is not necessarily normal; but if the line whose six coordinates are (a, b, c, f, g, h) be made to coincide with the normal at the point whose rectangular coordinates are x, y, z, it will follow that

$$a : b : c = u : v : w$$

$$Au + Bv + Cw = 0$$

$$fu + gv + hw = 0,$$

with other relations which would abbreviate, although not essentially simplify, some previous expressions. The results of placing the line in the tangent plane are noticed below.

§ 5. *Note on tangents of more than two-pointic contact.*

If V, instead of being as hitherto quadric, be linear, we shall have the case of tangents to the curve of section of U with the plane $\varpi - k'\varpi' = 0$. And if the contact be three-pointic, each of the ratios $\partial_x V : u$, $\partial_y V : v, \ldots$ will be, in virtue of (20) of § 2, equal to $\Delta V : H$. But since V is linear $\Delta V = 0$, and consequently the condition for a tangent of three-pointic contact is $H = 0$. Now

$$\begin{aligned} H &= (\mathfrak{A}, \ldots)(A, B, C, D)^2 \\ &= (\mathfrak{A}, \ldots)(\alpha'\varpi - \alpha\varpi', \beta'\varpi - \beta\varpi', \gamma'\varpi - \gamma\varpi', \delta'\varpi - \delta\varpi')^2 \\ &= \varpi'^2(\mathfrak{A}, \ldots)(\alpha'k' - \alpha, \beta'k' - \beta, \gamma'k' - \gamma, \delta'k' - \delta)^2 ; \end{aligned}$$

so that the condition $H = 0$ may be written

$$(\mathfrak{A}, \ldots)(\alpha', \beta', \gamma', \delta')^2 k'^2 - 2(\mathfrak{A}, \ldots)(\alpha', \beta', \gamma', \delta')(\alpha, \beta, \gamma, \delta)k + (\mathfrak{A}, \ldots)(\alpha, \beta, \gamma, \delta)^2 = 0, \quad . \quad (41)$$

which will determine two values of k, and consequently two positions of the cutting plane for which the tangent line has three-pointic contact.

It may be noticed that in the solution of the equation above written there occurs the following expression:

$$(\mathfrak{A}, \ldots)(\alpha', \beta', \gamma', \delta')^2 . (\mathfrak{A}, \ldots)(\alpha, \beta, \gamma, \delta)^2 - [(\mathfrak{A}. \ldots)(\alpha', \beta', \gamma', \delta')(\alpha, \beta, \gamma, \delta)]^2.$$

But if H_0 represent the Hessian of U, then $\mathfrak{B}\mathfrak{C}-\mathfrak{F}^2=H_0(v_1w_1-u^2), \ldots$; and the expression in question will be equal to the product of the Hessian into

$$(v_1w_1-u'^2,\ w_1u_1-v'^2,\ \ldots\ v'w'-u_1u',\ \ldots)(a,\ b,\ c,\ f,\ g,\ h)^2,$$

where $a, b, \ldots f, \ldots$ are the six coordinates of the line about which the cutting plane is supposed to revolve, as given in equations (9). The two positions of the cutting plane will coincide, if either the Hessian or this expression vanishes; the latter, regarded as a relation between the coordinates of the line, expresses the condition that the plane of section may contain both principal tangents, *i. e.* may be the tangent plane, as may be verified by the following considerations. If the plane of section coincide with the tangent plane, then

$$\mathrm{A}:\mathrm{B}:\mathrm{C}:\mathrm{D}=u:v:w:k;\ \text{or}\ \varpi\alpha'=\varpi'\alpha+\lambda u,\ \varpi\beta'=\varpi'\beta+\lambda v,\ \ldots,$$

which, being substituted in the expression in question, will cause it to vanish identically; or we may proceed otherwise, thus: regarding A, B, C, D as the constants of the cutting plane, the equation H=0 is

$$(\mathfrak{A}, \ldots)(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})^2=0; \quad \ldots\ldots\ldots\ldots \quad (42)$$

the equation of the plane itself may be written

$$\mathrm{A}\xi+\mathrm{B}\eta+\mathrm{C}\zeta+\mathrm{D}\vartheta=0; \quad \ldots\ldots\ldots\ldots \quad (43)$$

and since this plane passes through the point (x, y, z, t), we have also

$$\mathrm{A}x+\mathrm{B}y+\mathrm{C}z+\mathrm{D}t=0; \quad \ldots\ldots\ldots\ldots \quad (44)$$

by means of which three equations the ratios A : B : C : D may be determined. By a process quoted by Professor CAYLEY (Quarterly Journal of Mathematics, vol. vii. p. 1), the solution of these equations depends upon the square root of the quantity

$$\begin{vmatrix} \mathfrak{A}, & \mathfrak{H}, & \mathfrak{G}, & \mathfrak{L}, & x, & \xi \\ \mathfrak{H}, & \mathfrak{B}, & \mathfrak{F}, & \mathfrak{M}, & y, & \eta \\ \mathfrak{G}, & \mathfrak{F}, & \mathfrak{C}, & \mathfrak{N}, & z, & \zeta \\ \mathfrak{L}, & \mathfrak{M}, & \mathfrak{N}, & \mathfrak{D}, & t, & \vartheta \\ x, & y, & z, & t & . & . \\ \xi, & \eta, & \zeta, & \vartheta & . & . \end{vmatrix} = H_0 \begin{vmatrix} u_1, & w', & v', & l', & x, & \xi \\ w', & v_1, & u', & m', & y, & \eta \\ v', & u', & w_1, & n', & z, & \zeta \\ l', & m', & n', & k_1, & t, & \vartheta \\ x, & y, & z, & t, & . & . \\ \xi, & \eta, & \zeta, & \vartheta, & . & . \end{vmatrix} \quad \ldots \quad (45)$$

which is in fact identical with the expression (41).

The locus of the points for which one of the principal tangents meets a given line, say, the line $(a_1, b_1, c_1, f_1, g_1, h_1)$, will be found by eliminating A, B, C, D from the equations (42), (43), (44), combined with the following condition:

$$af_1+bf_1+ch_1+a_1f+b_1g+c_1h=0. \quad \ldots\ldots\ldots \quad (46)$$

But since the principal tangent is the intersection of the plane (A, B, C, D) with the tangent plane, we have

$$a, b, c, f, g, h, = \begin{Vmatrix} \mathrm{A}, & \mathrm{B}, & \mathrm{C}, & \mathrm{D} \\ u, & v, & w, & k \end{Vmatrix} \quad \ldots\ldots\ldots \quad (47)$$

Substituting these values, the condition above written becomes

$$\begin{aligned} &A(\ vh_1-wg_1+ka_1\,)\\ &+B(\ wf_1-uh_1+kb_1\,)\\ &+C(\ ug_1-vf_1+kc_1\,)\\ &+D(-ua_1-vb_1-wc_1)=0, \end{aligned}$$

and the ratios A : B : C : D will be expressed by the determinants of the matrix

$$\left\|\begin{matrix} vh_1-wg_1+ka_1, & wf_1-uh_1+kb_1, & ug_1-vf_1+kc_1, & -ua_1-vb_1-wc_1 \\ x & y & z & t \\ \xi & \eta & \zeta & \vartheta. \end{matrix}\right\|$$

But if by analogy with (11) we write

$$\begin{aligned} b_1z-c_1y-f_1t&=A_1,\\ c_1x-a_1z-g_1t&=B_1,\\ a_1y-b_1x-h_1t&=C_1,\\ f_1x+g_1y+h_1z&=D_1, \end{aligned}$$

it will be found that, putting

$$\begin{aligned} \xi\ u+\eta\ v+\zeta w+\vartheta\ k&=U_1,\\ A_1\xi+B_1\eta+C_1\zeta+D_1\vartheta&=E_1, \end{aligned}$$

the ratios A : B : C : D are equal to

$$A_1U_1-E_1u : B_1U_1-E_1v : C_1U_1-E_1w : D_1U_1-E_1u\,;$$

and consequently when these are substituted in H the terms having E_1 for a coefficient will vanish, and the equation of the locus resulting will be

$$(\mathfrak{A},\ldots)(A_1, B_1, C_1, D_1)^2=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (48)$$

To find the locus of points at which one of the principal tangents has a four-pointic contact with the surface, we must add to the equation H=0 the following, viz. $\square H=0$, which, as has been shown in a former part of this paper, may be replaced by any one of the group

$$\left\|\begin{matrix} A\ , & B\ , & C\ , & D \\ u\ , & v\ . & w\ , . & k \\ \partial_xH, & \partial_yH, & \partial_zH, & \partial_tH \end{matrix}\right\|=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

But, remembering that p, q, r, s represent the differential coefficients of H with respect to x, y, z, t, on the supposition that A, B, C, D are constant, and writing

$$(\mathfrak{A},\ldots)(A, B, C, D)(\alpha, \beta, \gamma, \delta)=H',$$

it is easy to deduce the following system:

$$\begin{aligned} \partial_xH&=p+\frac{2A}{\varpi}H'\\ \partial_yH&=q+\frac{2B}{\varpi}H'.\\ \vdots\quad&\ \ \vdots\quad\ \vdots \end{aligned}$$

But $\square H=0$ is equivalent to the system $\partial_x H=\theta_1 u$, $\partial_y H=\theta_1 v$, .. *i. e.* to

$$\begin{aligned} \varpi p+2AH'&=\theta_1 u \quad \text{or} \quad 2H'A=\theta_1 u-\varpi p \\ \varpi q+2BH'&=\theta_1 v \qquad\quad 2H'B=\theta_1 v-\varpi q \\ \varpi r+2CH'&=\theta_1 w \qquad\quad 2H'C=\theta_1 w-\varpi r \\ \varpi s+2DH'&=\theta_1 k \qquad\quad 2H'D=\theta_1 k-\varpi s. \end{aligned}$$

Substituting these values of A, B, C, D, in the equation $H=0$, the terms having θ_1 for a coefficient will vanish, and the equation will take the form

$$(\mathfrak{A}, \ldots)(p, q, r, s)^2=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

But from what has gone before it is clear that H when resolved into its factors is of the form $\chi^2-H_0\mu^2\psi^2$, where $H_0\mu^2$ represents the expression (45); hence putting $\chi^2=\mu^2\psi^2\phi^2$, $\frac{H}{\mu^2\psi^2}=\phi^2-H_0$; and consequently since $H=0$, $p=2\phi\partial_x\phi-\partial_x H_0$, .. But $\phi=\pm\sqrt{H_0}$, the upper or lower sign being taken according as one or other principal tangent is the subject of consideration. Substituting then in the equation (50), we have

$$(\mathfrak{A}, \ldots)(2\sqrt{H_0}\partial_x\phi\pm\partial_x H_0,\ 2\sqrt{H_0}\partial_y\phi\pm\partial_y H_0, \ldots)^2=0.$$

But since we are seeking the condition under which either one or the other principal tangent may have four-pointic contact, the terms of ambiguous sign must disappear; and the condition required will take the form

$$4H_0(\mathfrak{A}, \ldots)(\partial_x\phi, \partial_y\phi, \ldots)^2+(\mathfrak{A}, \ldots)(\partial_x H_0, \partial_y H_0, \ldots)^2=0,$$

which is of the degree $11n-24$, and may be compared with CLEBSCH's form, viz. $(\mathfrak{A}, \ldots)(\partial_x H_0, \ldots)^2-4H_0\Phi$; but the comparison of the terms in ϕ and Φ appears difficult.

The additional condition for a five-pointic contact on the part of one of the principal tangents will be $\square^2H=0$; or, having reference to (20) and to the consideration that $H=0$, the condition will be $\Delta H=0$. But the further discussion of this question I postpone to another occasion.

XVII. *On the Relation between the Sun's Altitude and the Chemical Intensity of Total Daylight in a Cloudless Sky.*

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The difficulty of securing in England a sufficient number of consecutive cloudless days to render it possible to determine with any degree of accuracy the relation existing between the sun's altitude and the chemical intensity of total daylight, induced us to undertake a series of measurements on the west coast of Portugal, where during the months of July and August the sky is generally cloudless. The method of measurement employed was that described by one of us in previous communications to the Royal Society, founded upon an exact estimation of the tint which standard sensitive paper assumes when exposed for a given time to the action of daylight*.

The observations, the results of which are given in the following communication, were made in the autumn of 1867, through the kindness of Thomas Creswell, Esq., at the Quinta do Estero Furado, situated on the flat table-land on the southern side of the Tagus, about 8½ miles to the south-east of Lisbon, lat. 38° 40′ N. and long. 9° W. The sensitive paper was exposed in the plane of the horizon, the instrument being placed upon a carefully levelled stand at the height of 4 feet 5 inches above the level of the ground in a sandy field having a clear horizon, the most considerable object in the neighbourhood being a house distant 130 paces to the west, the roof of which subtended an angle of 7° with the plane of the paper.

All the experiments were made in the following order:—

1. The chemical action of total daylight was observed in the ordinary manner.
2. The chemical action of the diffused daylight was then observed by throwing on to the exposed portion of the sensitive paper the shadow of a small blackened brass ball placed at such a distance that its apparent diameter seen from the position of the paper was slightly larger than that of the sun's disk.
3. The chemical action of total daylight was again determined.
4. That of the diffused daylight was a second time ascertained.

The means of Observations 1 and 3 and of 2 and 4 were then taken. The sun's altitude was determined immediately before and immediately after the foregoing observations of chemical intensity, the altitude at the time of observation being ascertained by interpolation. A box-sextant made by Worthington, of London, was employed with an artificial horizon of black glass, which was carefully levelled before each observation. The watch used could be read to quarter seconds, and the Lisbon mean time was obtained by regulating the watch by means of the time-ball of the Lisbon Naval Arsenal, which falls at 1 P.M., and afterwards ascertaining the amount of the watch's daily variation. In cases in which the altitude could not be determined by experiment, it has been calculated from the following formula,

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* Bakerian Lecture, Philosophical Transactions, 1865, Part II. p. 605.

$$\cos\varphi = \cos\delta \,.\, \cos t \,.\, \cos p + \sin\delta \sin p,$$

in which

δ represents the sun's declination,
p represents the latitude of the place $= 38^\circ\ 40'$,
and t represents the sun's hour-angle.

It was necessary, in the first place, to ascertain what error was introduced into the determination of diffused daylight by any accidental variation in the distance of the brass ball from the sensitive paper. A series of experiments were made with this object on August 8th, 1867, at 9 A.M., the blackened ball being placed at distances varying from 140 millims. to 230 millims. above the silvered paper. These experiments showed that the ball placed at

140 millims. gave mean reading	143·3	or Chemical Intensity	0·116
160 „ „ „	143·6	„ „	0·116
190 „ „ „	141·9	„ „	0·116
230 „ „ „	136·8	„ „	0·118

consequently the height of the blackened ball may safely vary between 140 and 190 millims. without producing any appreciable error.

The following gives the elements of an observation, and may serve as a sample of the others, 134 in number, which were all carried out in a manner exactly similar, and form the data from which are derived the conclusions drawn in this paper.

August 5th, 1867.

Determination of Altitude.—

	Observed time.	Calculated time.	Observed double altitude.	Sun's altitude.
	h m	h m	° ′	° ′
1st Observation.	3 37 P.M.	3 57 P.M.	71 42	35 51
2nd „	3 41 P.M.	4 1 P.M.	70 00	35 00

Determination of Chemical Intensity.—

Observed time $3^h\ 39^m$ to $3^h\ 40^m$. Calculated time $3^h\ 59^m$ to 4^h.

	Readings on calibrated strip.	Mean.
I. Total daylight	100, 97, 97, 105, 103, 106, 102	=101
II. Diffused . .	110, 108, 112, 116, 116, 117, 108, 110	=112
III. Total . . .	104, 100, 100, 97, 100, 102, 107, 105, 103	=102
IV. Diffused . .	108, 112, 113, 108, 115, 114, 107	=111

By reference to the calibration Table of the strip the following numbers were found to correspond to the above readings.

Time $3^h\ 59^m$ P.M. Sun's altitude 35° 25′.

Observed Chemical Intensity.—

Direct sunlight.	Diffused light.	Total daylight.
0·069	0·138	0·207

The meteorological conditions of the time of observation were also carefully observed.

Clouds 0·0. Fine breeze, S.S.W.	Dry-bulb thermometer 81°·0 F. Wet-bulb thermometer 69°·0 F.
Barometer at sea-level 763·9 millims.	Aqueous vapour in 1 cub. ft. 5·7 grains. Relative humidity=50.

One of the 134 sets of observations was made as nearly as possible every hour, and they thus naturally fall into seven groups, viz.—

(1) Six hours from noon, (2) five hours from noon, (3) four hours from noon, (4) three hours from noon, (5) two hours from noon, (6) one hour from noon, and (7) noon.

Each of the first six of these groups contains two separate sets of observations, viz. those made before noon marked A.M., and those made after noon and marked P.M.

One of us has already pointed out[*], from measurements made at Kew, that the mean chemical intensity of total daylight for hours equidistant from noon is the same. That this result is a general one is fully borne out by the inspection of the following Tables, giving the results of 134 series of observations; the single experiments made at the same hour being grouped together, and those of the hours equidistant from noon being placed side by side.

TABLE I. (1.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
11.	6	6 4	10 33	000	·049	·049	19.	6	6 0	11 31	·000	·056	·056
29.	9	6 6	10 15	·001	·033	·034	28.	8	6 2	10 43	·000	·041	·041
48.	12	6 9	10 20	·001	·040	·041	40.	9	6 0	10 58	·000	·034	·034
							47.	10	6 0	10 50	·002	·050	·052
			31 08	·002	·122	·124	57.	12	5 59	10 22	·000	·034	·034
							67.	21	5 51	10 02	·000	·031	·031
		Means	10 23	·000	·041	·041	76.	22	6 0	7 58	·000	·031	·031
							86.	23	6 0	7 20	·000	·026	·026
							97.	24	5 47	9 50	·002	·038	·040
							107.	26	5 49	8 37	·000	·031	·031
							118.	27	5 51	8 04	·000	·029	·029
							128.	28	5 53	7 17	·000	·022	·022
										113 32	·000	·423	·427
									Means	9 28	·000	·035	·035

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[*] Philosophical Transactions, 1867, p. 558.

TABLE I. (2.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
12.	6	7 1	20 59	·024	·095	·119	18.	6	5 0	23 15	·035	·069	·104
30.	9	7 2	21 10	·023	·062	·085	27.	8	5 2	22 10	·027	·072	·099
49.	12	7 1	20 20	·032	·063	·095	39.	9	5 6	21 26	·026	·060	·086
87.	24	6 54	17 29	·017	·058	·075	46.	10	5 0	22 15	·045	·069	·114
108.	27	7 6	19 17	·017	·057	·074	66.	21	5 4	19 07	·013	·051	·064
119.	28	7 1	18 18	·016	·059	·075	75.	22	5 1	19 15	·023	·060	·083
129.	29	7 1	18 24	·016	·047	·063	85.	23	4 58	19 24	·024	·068	·092
132.	30	6 59	17 56	·017	·050	·067	96.	24	5 0	18 50	·022	·060	·082
							106.	26	5 4	17 22	·017	·058	·075
			153 53	·162	·491	·653	117.	27	4 58	18 23	·015	·059	·074
		Means	19 14	·020	·061	·082				201 27	·247	·626	·873
									Means	20 09	·025	·062	·087

(3.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
13.	6	8 0	32 25	·081	·111	·192	10.	5	4 0	35 21	·069	·138	·207
31.	9	7 59	31 58	·053	·101	·154	17.	6	4 0	34 49	·049	·116	·165
50.	12	8 0	31 49	·079	·103	·182	26.	8	4 1	34 11	·060	·112	·172
58.	21	8 2	30 43	·050	·097	·147	38.	9	4 12	31 50	·049	·096	·145
77.	23	8 0	30 23	·044	·097	·151	45.	10	4 0	34 04	·081	·109	·190
88.	24	7 59	29 58	·051	·087	·138	65.	21	4 10	29 29	·041	·087	·128
98.	26	8 0	29 58	·051	·098	·149	74.	22	3 58	31 21	·036	·118	·154
109.	27	8 1	29 57	·045	·096	·141	84.	23	3 59	30 53	·043	·077	·120
120.	28	8 0	29 41	·047	·093	·140	95.	24	4 0	30 36	·051	·100	·151
130.	29	7 59	29 38	·032	·080	·112	105.	26	4 2	29 38	·053	·092	·145
133.	30	8 0	29 38	·036	·084	·120	116.	27	4 5	28 40	·053	·097	·150
			336 08	·569	1·047	1·606				350 52	·585	1·143	1·727
		Means	30 33	·052	·097	·146			Means	31 54	·053	·104	·157

(4.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
14.	6	9 0	44 13	·102	·124	·226	9.	5	3 2	46 25	·105	·158	·263
51.	12	9 2	43 33	·154	·129	·283	25.	8	3 1	45 46	·142	·136	·278
59.	21	8 59	41 16	·144	·129	·273	44.	10	3 3	44 59	·139	·128	·267
68.	22	9 0	41 38	·089	·124	·213	56.	12	2 59	45 02	·069	·083	·152
78.	23	8 59	41 20	·091	·109	·200	64.	21	3 1	42 35	·086	·114	·200
89.	24	8 58	40 58	·079	·104	·183	73.	22	3 0	42 34	·114	·116	·230
99.	26	9 1	41 21	·083	·114	·197	83.	23	2 58	42 20	·103	·104	·207
110.	27	9 7	41 09	·108	·120	·228	104.	26	3 2	40 42	·094	·116	·210
121.	28	9 5	41 00	·090	·112	·202	115.	27	3 0	40 45	·091	·112	·203
131.	29	9 0	40 48	·098	·098	·196	127.	28	3 0	40 23	·075	·110	·185
134.	30	9 5	41 35	·082	·096	·178	137.	30	3 6	38 25	·070	·089	·159
			458 51	1·120	1·259	2·379				469 56	1·088	1·266	2·354
		Means	41 43	·102	·114	·216			Means	42 43	·099	·115	·214

TABLE I. (5.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
4.	5	10 0	54 50	·109	·180	·289	8.	5	2 5	56 26	·143	·150	·293
15.	6	10 0	54 58	·182	·128	·310	37.	9	2 0	56 01	·095	·118	·213
21.	8	10 2	54 59	·157	·130	·287	55.	12	1 59	55 36	·091	·106	·197
33.	9	9 57	53 55	·122	·121	·243	94.	24	1 57	52 25	·149	·114	·263
41.	10	10 5	54 54	·215	·168	·383	126.	28	2 2	50 12	·091	·112	·203
52.	12	10 0	53 42	·152	·104	·256	136.	30	2 2	49 09	·080	·118	·198
60.	21	10 1	52 12	·142	·158	·300							
69.	22	9 58	51 40	·157	·116	·273				319 49	·649	·718	1·367
79.	23	9 58	51 27	·155	·112	·267							
90.	24	10 5	52 25	·107	·112	·219			Means	53 18	·108	·120	·228
100.	26	10 2	51 27	·161	·120	·281							
111.	27	10 3	51 26	·155	·118	·273							
122.	28	10 3	51 21	·133	·120	·253							
			689 16	1·947	1·687	3·634							
		Means	53 01	·149	·130	·279							

(6.)

A.M.							P.M.						
Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.	Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′						h m	° ′			
5.	5	11 00	64 01	·173	·182	·355	7.	5	1 0	65 22	·130	·180	·310
16.	6	11 07	64 29	·215	·128	·343	24.	8	1 1	64 35	·167	·148	·315
22.	8	11 00	63 25	·149	·124	·273	36.	9	12 59	64 22	·110	·124	·234
34.	9	10 58	62 56	·153	·127	·280	54.	12	1 0	63 28	·162	·114	·276
42.	10	11 00	62 55	·237	·156	·393	63.	21	12 58	60 51	·258	·126	·384
53.	12	11 01	62 24	·212	·114	·326	72.	22	12 59	60 27	·288	·144	·432
61.	21	11 00	60 04	·201	·140	·341	82.	23	12 59	60 08	·178	·106	·284
70.	22	11 05	60 15	·263	·125	·388	93.	24	1 0	59 39	·223	·121	·344
80.	23	11 02	59 45	·263	·115	·378	114.	27	12 59	58 39	·250	·166	·416
91.	24	11 00	59 15	·186	·128	·314	125.	28	1 11	57 06	·158	·114	·272
101.	26	10 59	58 35	·236	·126	·362	135.	30	1 0	57 36	·099	·124	·223
112.	27	11 00	58 18	·178	·118	·296							
123.	28	11 05	58 32	·187	·118	·305				672 13	2·023	1·467	3·490
			794 54	2·653	1·701	4·354			Means	61 07	·184	·133	·317
		Means	61 09	·204	·131	·335							

NOON. (7.)

Expt.	Date.	Hour.	Altitude.	Sun.	Sky.	Total.
		h m	° ′			
6.	5	12 0	68 21	·186	·188	·374
23.	8	12 0	67 31	·213	·177	·390
35.	9	12 0	67 14	·218	·126	·344
43.	10	11 59	66 58	·248	·172	·420
62.	21	12 0	63 32	·278	·132	·410
71.	22	12 0	63 12	·225	·120	·345
81.	23	11 59	62 51	·254	·114	·368
92.	24	11 58	62 31	·199	·118	·317
102.	26	12 2	61 50	·210	·128	·338
113.	27	12 2	61 29	·213	·122	·335
124.	28	12 7	61 06	·191	·116	·307
			707 35	2·435	1·513	3·948
		Means	64 14	·221	·138	·359

The results are more clearly seen in the following Table.

TABLE II.

Time.	Mean altitude.	Mean chemical intensity.			Number of observations.	Diff.
		Sun	Diffused.	Total.		
h m						
6 0 A.M.	10° 23′	0·000	0·041	0·041	3	+ ·003
6 0 P.M.	9 28	0·000	0·035	0·035	12	
7 0 A.M.	19 14	0·020	0·061	0·082	8	—0·025
5 0 P.M.	20 09	0·025	0·062	0·087	10	
8 0 A.M.	30 33	0·052	0·097	0·146	11	—0·005
4 0 P.M.	31 54	0·053	0·104	0·157	11	
9 0 A.M.	41 43	0·102	0·114	0·216	11	+0·001
3 0 P.M.	42 43	0·099	0·115	0·214	11	
10 0 A.M.	53 01	0·149	0·130	0·279	13	+0·025
2 0 P.M.	53 18	0·108	0·120	0·228	6	
11 0 A.M.	61 09	0·204	0·131	0·335	13	+0·009
1 0 P.M.	61 07	0·184	0·133	0·317	11	
12 0 Noon.	64 14	0·221	0·138	0·359	11	

The numbers contained in columns 3 and 4 of Table II. give the relation between direct sunlight and diffuse daylight. These sets of numbers, expressed graphically in terms of the altitude, are seen on Plate XXX. fig. 3; the dotted curve indicating the intensity of the diffused light, the black curve that of the direct sunlight. These two curves intersect at an altitude of 50°, at which elevation, therefore, the place of equal chemical illumination occurs for a surface placed in the plane of the horizon.

The fact that the curve of direct sunlight cuts the base line at 10° bears out the conclusion which one of us has already announced, namely, that at altitudes below 10° direct sunlight is robbed of almost all its chemically active rays.

The curves (fig. 1) show the daily march of chemical intensity at Lisbon, as a mean of all the observations, compared with that at Kew for the preceding August, and at Pará for the preceding April. The number representing the mean chemical intensity at Kew is 94·5, at Lisbon 110, and at Pará 313·3; light of the intensity 1·0 acting for twenty-four hours being taken as 1000.

If we now arrange the observations according to the sun's altitude, we have:—

TABLE III.

Number of observations	Mean altitude	Chemical intensity		
		Sun	Sky.	Total.
15.	9° 51′	0·000	0·038	0·038
18.	19 41	0·023	0·062	0·085
22.	31 14	0·052	0·100	0·152
22.	42 13	0·100	0·115	0·215
19.	53 09	0·136	0·126	0·262
24.	61 08	0·195	0·132	0·327
11.	64 14	0·221	0·138	0·359

Curves shewing March of Total Chemical Intensity for Sunshine. Pará April 1866 Kew and Lisbon August 1866 & 1867.

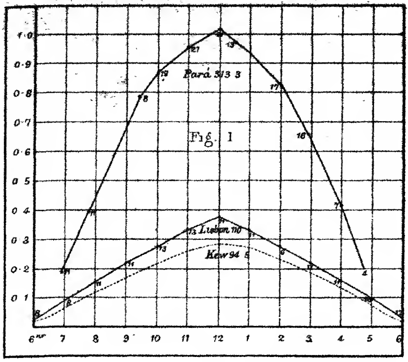
Fig. 1

Curve shewing Relation of Total Chemical Intensity to Sun's Altitude. Lisbon August 1867.

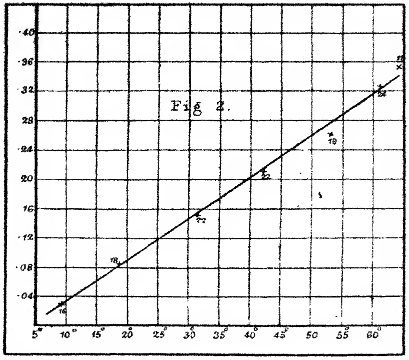
Fig 2.

Curves shewing Relation of direct & diffused Chemical Intensity to Sun's Altitude.— Lisbon August 1867.

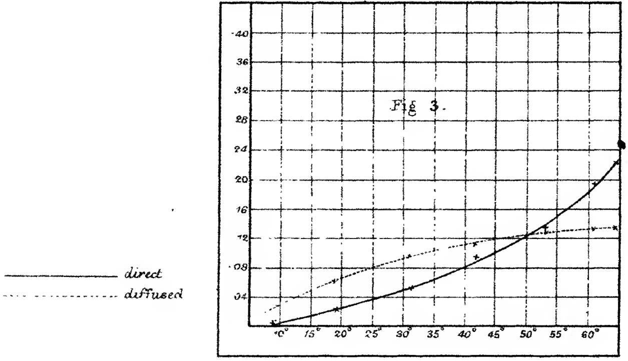
Fig 3.

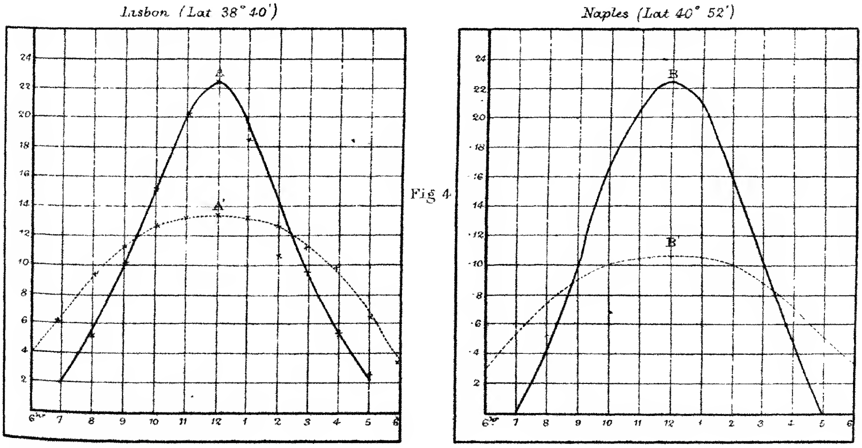
Fig 4.

Lisbon (Lat 38° 40′)

Naples (Lat 40° 52′)

Thorpe delt

Fig. [illegible] gives the graphical representation of the relation of the total chemical intensity as ordinate to the sun's altitude as abscissa. The relation between the altitude and chemical intensity for altitude above 10° is here seen to be accurately represented by a straight line. The position of the experimentally determined points are noted, and serve to show how closely they lie to the straight line.

In former communications* it has been shown that a similar relation between altitude and chemical intensity of total daylight has been found to hold good at Heidelberg, Kew, and Pará; and that although the chemical intensity for the same altitude at different places and at different times of the year may greatly vary according to the varying transparency of the atmosphere, yet that the relation at the same place between altitude and intensity is always represented by a straight line. Thus the mean intensities at Lisbon and Pará for 30° are 0·15 and 0·44 respectively, whilst for 60° they are 0·32 and 0·80. That this variation in the direction of the straight line expressed by the constant in the formula, given in the paper last referred to, is due to the opalescence of the atmosphere, we have evidence of in the fact that, for equal altitudes, the higher intensity is always found where the mean temperature of the air is greater, as in summer, when we compare the same place at different seasons, or as we approach the equator, when we compare different places. The first of these conditions of variation is clearly seen if we compare the Kew observations for the same altitudes, but for different times of the year. The following Table clearly shows that the altitude in the warmer half of the year is invariably accompanied by a higher chemical action than that in the colder, and this is attributable to the varying opalescence, which is certainly a function of the atmospheric temperature, and is less marked as we approach the summer solstice or pass towards the equator.

Comparison of Chemical Intensities at Kew, 1866.

Month.		Time of observation.	Corresponding altitude.	Chemical intensity of total daylight.
		h m		
I.	October	2 30 P.M.	23° 10′	0·059
	August	4 42 „	23 58	0·115
II.	November ...	2 27	14 52	0·035
	September ...	4 43	14 14	0·058
III.	March	2 30	28 36	0·075
	June............	4 43	29 52	0·106
IV.	April	2 30	38 06	0·116
	July	4 39	30 05	0·141

It is interesting to observe the close agreement which exists between the measurements made at Lisbon with sensitive paper, and the luminous intensities calculated from observations made at Heidelberg by a totally different method. In 1859 Professor Bunsen, and one of us in Part IV. of 'Photochemical Researches,' gave curves of the chemical action effected, (1) by sunlight, (2) by diffused daylight on the horizontal unit of surface situated at various localities on the earth's surface.

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* Roscoe, Philosophical Transactions, 1867, p. 555.

A comparison of the unit of measure there taken can unfortunately not be made with that used in the Lisbon experiments; but if we reduce the observational results for Lisbon (lat. 38° 40′ N.) and the calculated one for Naples (lat. 40° 52′ N.) to a common measure by assuming that the action of the direct sunlight at noon is equal in both cases, and reduce the other points in the Naples curve in the same ratio, we obtain the two curves B, B′, fig. 4, whilst the corresponding curves for Lisbon are given on fig. 4, A, A′.

The correspondence between the results of the measurements above described and those made by a totally different method is further shown by the close coincidence of the "phases of equal chemical illumination" for sun- and diffuse-light as determined by both methods. In a former communication* it has been shown that in all places where the sun rises to a height of more than 20° 56′ above the horizon the chemical action effected by the diffuse daylight exceeds that of the direct sunlight at first; and that as the sun gradually rises a point is reached at which both sunlight and diffuse daylight produce exactly the same amount of chemical action, whilst beyond this point the effect of the sunshine is more powerful. This phase of equal illumination was calculated from theoretical considerations, and the result was confirmed by experiment, the difference between the calculated and the experimental points of equality amounting in mean to about thirty-five minutes. By reducing the chemical intensity of the direct sunlight in Table III. to that which the sunlight would produce on a plane perpendicular to the incident rays, we find that the phase of equal chemical intensity in reality is one which lasts for some time, that it begins near the calculated altitude of 18° 48′, but that it continues for about an hour. When, however, the sun reaches an elevation of 35°, the intensity of the sun's perpendicular rays becomes greater than that of the total diffuse light acting on a horizontal surface.

* Bunsen and Roscoe, "Photochemical Researches.—Part IV.," Phil. Trans. 1859, p. 915.

XVIII. *Researches on Vanadium.*—Part III.

By Henry E. Roscoe, *B.A.*, *Ph.D.*, *F.R.S.*

Received and Read April 7, 1870.

I. METALLIC VANADIUM.

In the second Part of my "Researches on Vanadium," communicated to the Royal Society (Phil. Trans. 1869, p. 691), I stated that metallic vanadium absorbs hydrogen. This conclusion has been fully borne out by subsequent experiment; and it appears that the amount of absorbed or combined hydrogen taken up by the metal varies according to the state of division, first, of the chloride (VCl_2) from which the metal is prepared, and secondly, and especially, of the metallic powder itself. The metal containing absorbed hydrogen on exposure to dry air slowly takes up oxygen, water being formed and the metal undergoing oxidation to a substance which resembles the metal in its appearance, but possesses a darker grey colour, and has a less brilliant metallic lustre than vanadium itself. At this point the oxidation stops, although in moist air it proceeds still further. Thus a portion of pure dichloride was reduced in hydrogen; of the reduced substance, free from chlorine, 0·2666 grm. yielded on complete oxidation 0·4441 of V_2O_5, corresponding to a percentage of 93·6 of pure metal. On exposure to the air for some days this substance absorbed oxygen, losing its brilliant metallic lustre; and when burnt in a current of dry oxygen, water was given off, thus:—

(1) 0·4232 grm. gave 0·0502 grm. of water and 0·6615 grm. V_2O_5,
(2) 0·2695 „ „ 0·0315 „ „ 0·414 „ „

or

(1) gives 87·8 p c. vanadium; 1·3 p. c. hydrogen; 10·9 p. c. oxygen.
(2) „ 86·7 p. c. vanadium; 1·3 p. c. hydrogen; 12·0 p. c. oxygen.

The difficulty of obtaining metallic vanadium perfectly free from admixture of oxide was again rendered evident. Pure tetrachloride was prepared in quantity, and from this the dichloride was made. On heating this dichloride to whiteness for forty-eight hours a substance was obtained which gained on oxidation 70·7 per cent., and therefore still contained a slight admixture of oxide. The reducing action of sodium on the solid chlorides was next examined; in this case the reduction takes place rapidly but quietly in an atmosphere of hydrogen at a red heat, and may be best conducted in strong iron tubes proved air-tight under hydraulic pressure of 200 lbs. on the square inch. Explosions occur when the tetrachloride is heated with sodium. The substance thus obtained by the action of sodium was found on lixiviation to be free from chlorine, and on washing it was found to separate into two parts—(1) a light and finely divided black powder (trioxide), soluble in hydrochloric acid, which remains in suspension, and (2) a heavier grey powder (insoluble in hydrochloric acid), which is deposited, and which by repeated washing may be entirely freed from the lighter trioxide. This bright grey powder consists of metallic vanadium mixed with more or less oxide. If the finely divided metallic powder, after drying *in vacuo*, be reduced at a low red heat in a current of pure hydrogen, it takes fire spontaneously, when cold, on exposure to air or oxygen, a distinct flame being seen playing on the surface whilst water is formed. In one experiment a product thus prepared contained 91·1 per cent. of metallic vanadium (0·354 substance gave 0·574 grm. V_2O_5). This substance, exposed for some weeks to dry air, slowly absorbed oxygen, and on roasting gave a percentage increase of 53·75 (0·453 grm. yielded 0·6965 V_2O_5), whilst 0·034 grm. or 7·5 per cent. of water was at the same time formed. This shows that the point of oxidation at which the metal containing hydrogen becomes stable in dry air nearly corresponds to the oxide V_2O.

A similar slow change in the appearance of the metal has been noticed in some of the portions of the metallic powder placed on microscopic slides.

II. VANADIUM AND BROMINE.

1. *Vanadium Oxytribromide or Vanadyl Tribromide*, $VOBr_3$, molec. wt.=307·3.—When the vapour of perfectly dry and pure bromine is passed over vanadium trioxide (V_2O_3) heated to redness, dense yellowish-white fumes of the oxytribromide are formed in the heated portion of the tube, and these condense together with the excess of bromine to form a dark red transparent liquid. In order to free the oxytribromide from excess of bromine, the mixed liquids must be rectified *in vacuo*, as the temperature of decomposition of the oxybromide lies (under the ordinary atmospheric pressure) below its boiling-point. By distilling under a pressure of 106 millims. of mercury in a current of perfectly dry air the whole of the bromine was got rid of before the thermometer rose to 45° C. The transparent liquid remaining in the retort had a dark red colour, gave off white fumes on exposure to moist air, and when thrown into water produced a light yellow-coloured solution of a vanadic salt. It is possible to distil the oxybromide under diminished pressure with but slight decomposition occurring, although when heated under the atmospheric pressure it suddenly solidifies at 180° C., splitting up into the oxydibromide and bromine. Under a pressure of 100 millims. of mercury the oxytribromide volatilizes without decomposition between 130° to 136° C.

The following analytical results were obtained:—Analysis No. 1 was made from a portion of oxytribromide which had not been distilled, No. 2 from a portion of the same substance, after further treatment with dry air at 63°, No. 3 from another preparation which had been distilled *in vacuo*, and in which the bromine determination is too high owing to traces of free bromine.

	Oxybromide taken.	Silver bromide found.	Vanadium pentoxide found.	Percentages of bromine.	Percentages of vanadium.
1.	0·7173	——	0·2110	——	16·52
	0·8022	1·5013	——	79·62	——
2.	0·9150	——	0·2748	——	16·87
	0·5580	1·038	——	79·10	——
3.	0·4850	——	0·1435	——	16·62
	0·3745	0·7085	——	80·48	——

Hence we have as the composition of the oxytribromide:

	Calculated.		Found. I.	II.	III.	Mean.
V =	51·3	16·69	16·52	16·87	16·62	16·67
Br_3 =	240·0	78·10	79·62	79·10	80·48	79·36
O =	16·0	5·21	——	——	——	——
	307·3	100·00				

The colour of the oxytribromide is somewhat redder than that of bromine, and it is more transparent in thin films, and much more translucent than bromine.

The oxytribromide slowly decomposes at the ordinary atmospheric temperature into bromine and oxydibromide; it is very deliquescent and hydroscopic, and cannot be formed in presence of moisture. The specific gravity of the oxytribromide at 0° is 2·9673, and at 14°·5 it is 2·9325.

2. *Vanadium Oxydibromide or Vanadyl Bromide*, $VOBr_2$, molec. wt. =227·3.—This substance forms suddenly when the oxytribromide is heated to temperatures above 180°, and it is slowly produced by the same decomposition at lower temperatures. The oxydibromide is a yellowish-brown solid body, in appearance resembling ochre; it is very deliquescent, and on heating in the air it loses all its bromine and is converted into the pentoxide. Thrown into water it dissolves, furnishing a blue solution of hypo-vanadic (V_2O_4) salt.

The following analyses were made from oxydibromide prepared on different occasions.

	Oxydibromide taken.	Vanadium pentoxide found.	Silver bromide found.	Percentages of vanadium.	Percentages of bromine.
(1)	0·7260	0·3025	1·2245	23·40	71·75
(2)	0·5910	——	0·9738	——	70·11
	1·1259	0·4308	——	21·50	——

Hence we have

	Calculated.		Found. (1).	(2).	Mean.
V . .	51·3	22·57	23·40	21·50	22·45
Br_2 . .	160·0	70·39	71·75	70·11	70·93
O . .	16·0	7·04	——	——	——
	227·3	100·00			

3. *Vanadium Tribromide*, VBr_3, molec. wt. =291·3.—This body condenses as a greyish-black opake amorphous sublimate, when dry bromine vapour is passed in excess over vanadium nitride heated to redness. Brown vapours are given off, which soon condense in the cooler portions of the tube. The tribromide is a very unstable compound, losing bromine even at the ordinary temperature in dry air, and being converted into V_2O_3 when gently heated. It deliquesces rapidly on exposure to moist air, giving rise to a brown-coloured liquid, in this respect resembling the trichloride, but on addition of a few drops of hydrochloric acid the brown liquid changes to the green colour characteristic of a solution of a vanadous salt (V_2O_3). No free bromine is evolved when the tribromide dissolves in water. In order to prepare the tribromide, pure nitride of vanadium, contained in a porcelain boat, was introduced into a combustion-tube, and after all the air had been displaced by dry carbonic acid, the part of the tube containing the nitride was heated to redness, the other part of the tube being kept at such a temperature as to volatilize any excess of bromine which might pass over. After all the nitride had burnt away, the bulb containing the bromine was sealed off, and a current of dry carbonic acid passed over the solid bromide to displace all traces of free bromine. A second method of preparing the tribromide is to pass bromine vapour over a mixture of vanadium trioxide and charcoal; in this reaction the oxytribromide is first formed, then the oxydibromide, and lastly, the tribromide, VBr_3; but this plan is not to be recommended, as the tube soon becomes stopped up by the formation of these solid compounds. The bromide thus prepared was not analyzed, but it presented exactly the same appearance and properties as the tribromide obtained by the first method.

No higher compound of bromine and vanadium than the tribromide could be obtained. The volatile liquid passing into the bulb in the first preparation was carefully rectified, and it was all found to distil over at the boiling-point of bromine, leaving only a small quantity of the tribromide in the bulb. Some difficulty was experienced in obtaining satisfactory analytical results with the tribromide, owing to the fact, already observed by Stas[*], that bromide of silver, when boiled with excess of nitrate of silver, carries down with it some of this latter salt inclosed in the bromide, and that this impurity cannot be got rid of by washing. Owing to this admixture of nitrate of silver the bromine determinations usually come out about two per cent. too high, whilst the vanadium determinations gave constant numbers, agreeing much more nearly with the calculated results. Thus in four analyses of the tribromide prepared on different occasions the mean percentage of bromine was found to be 84·15, the calculated percentage being 82·4; whilst the vanadium determinations of the same portions gave 18·57 per cent. instead of 17·6 per cent. In order to lessen as much as possible this error, the precipitated bromide of silver was reduced in hydrogen until no further diminution of weight occurred, and the percentage of bromine calculated from this loss.

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[*] Stas, Recherches sur les Lois des Proportions chimiques, p. 156.

	Weight of tribromide taken.	Vanadium pentoxide found.	Loss of bromine.	Percentages	
				of vanadium.	of bromine.
(1)	0·8004	0·2630	0·6500	18·46	81·21
(2)	0·3462	0·1160	0·2790	18·80	80·58
(3)	0·5960	0·1965	0·4815	18·52	80·85

Hence we have:—

	Calculated.		Found.			
			(1).	(2).	(3).	Mean.
$V =$ . .	51·3	17·6	18·46	18·80	18·52	18·59
$Br_3 =$ . .	240·0	82·4	81·21	80·58	80·78	80·86
	291·3	100·0	99·67	99·38	99·30	99·45

Experiments made with the bromine employed, which had been rectified over potassium bromide, and was carefully tested for chlorine and iodine and showed to be pure, proved that a similar excess of weight occurred on precipitation with nitrate of silver. In one experiment the percentage of bromine thus found was 100·96, and in a second experiment 101·41. It will also be seen that the bromine determinations of the oxybromides are similarly all too high from the same cause.

III. VANADIUM AND IODINE.

When the vapour of iodine is passed over the red-hot nitride of vanadium contained in a tube no action whatever takes place, the nitride after the operation remaining perfectly unchanged. Vanadium trioxide is likewise unacted upon by iodine at all temperatures.

IV. METALLIC VANADATES.

In the first Part of these researches* I pointed out (1) that the vanadates analyzed by Berzelius, prepared by boiling vanadic acid with the alkaline hydroxides and by double decomposition, must be considered as meta- or monobasic vanadates, (2) that the so-called bi-vanadates analyzed by Von Hauer†, and prepared by acting on the metavanadates with acids are anhydro-salts, and (3) that the naturally occurring vanadates are tribasic salts, and that sodium ortho-vanadate is formed when one molecule of vanadium pentoxide is fused with three molecules of carbonate of soda, three molecules of carbon dioxide being expelled. I have now to describe the preparation and properties of some characteristic members of these three classes of vanadates, as well as those of a fourth new class, viz. the tetrabasic or pyro-vanadates.

Determination of vanadium in the soluble vanadates.—The separation of vanadic acid from the metals of the alkalies by means of chloride of ammonium, as proposed by Von Hauer, is apt to give too low results, both as regards the vanadium and the alkali. It is almost impossible to prevent traces of ammonium metavanadate from dissolving, and on ignition, even with the greatest care, some portions of the finely divided vanadium pentoxide are invariably carried off when the ammonia escapes. On the other hand, the volatilization of the comparatively large quantities of sal-ammoniac which must be employed in order to ensure the complete precipitation of the vanadium, almost always entails a considerable loss of the fixed alkaline chlorides. A far more accurate plan for the separation of vanadium is the precipitation of the soluble vanadate by acetate of lead, when basic lead vanadate is precipitated, which is so insoluble that a portion when finely powdered and boiled in water did not dissolve in sufficient quantity to enable the lead reaction with sulphuretted hydrogen to be detected in the filtrate. This salt is also insoluble in acetic, but it dissolves readily in nitric acid, liberating vanadic acid, which separates out, but dissolves completely when the liquid is warmed. In the analysis of a soluble vanadate this insoluble lead salt is collected on a filter, dried at 100° C. and weighed; a given quantity of the dried salt is then dissolved in nitric acid, the lead precipitated by pure sulphuric acid, and the lead sulphate determined with the usual precautions of evaporation with addition of alcohol, &c. The lead sulphate thus obtained is (contrary to BERZELIUS'S statement) quite free from vanadium, whilst the vanadic acid in the filtrate is obtained perfectly pure, and well crystallized on evaporation and ignition. The filtrate from the lead vanadate, freed from excess of lead by means of sulphuric acid and evaporated, yields the alkaline sulphate not containing a trace of vanadium.

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* Philosophical Transactions, 1868 (Bakerian Lecture).

† Journ. Prac. Chem. Bd. lxix. p. 388, 1856.

Sodium Vanadates.

1. *Sodium Orthovanadate*, $Na_3VO_4 + 16H_2O$.—When a mixture of three molecules of sodium carbonate and one molecule of vanadium pentoxide is fused until no further evolution of carbon dioxide is observed, three molecules of CO_2 have been expelled and a tribasic vanadate remains as a white crystalline mass.

In one experiment in which a slight excess of sodium carbonate was taken 0·5785 grm. V_2O_5 liberated on fusion 0·4185 grm. CO_2. According to the equation

$$V_2O_5 + 3Na_2CO_3 = 2Na_3VO_4 + 3CO_2,$$

the weight of CO_2 liberated by this quantity of vanadium pentoxide is 0·4182 grm.

The mixture is easily fusible at first, but becomes less so as the reaction proceeds; whilst to begin with the heat of a Bunsen's burner is sufficient to melt the mass, it is necessary to apply the heat of a blowpipe-flame to keep up the fusion when the decomposition becomes more nearly complete. On cooling, the solidified mass acquires first a dark green colour, and then passes through yellow, until when cold it becomes perfectly white, and is found to possess a crystalline appearance. It dissolves easily in cold water, but is insoluble in alcohol. Hot water must *not* be employed for dissolving the fused mass, and as little cold water as possible. The cold strong aqueous solution must be instantly mixed with excess of strong alcohol; two layers of liquid are then formed, the upper one consisting of dilute alcohol, the lower one of the saline solution. After standing for a few hours the lower layer of liquid solidifies, forming an aggregate of colourless needle-shaped crystals. These crystals, which possess a strong alkaline reaction, are washed with small quantities of alcohol, then placed on a porous plate over sulphuric acid *in vacuo*, and after remaining for a short time they may be taken out for analysis. The following analytical results were obtained:—

Water determination.—0·8077 grm. of the crystals which first fuse in their water of crystallization lost on careful ignition in platinum 0·4882 grm. H_2O; corresponding to 60·44 per cent.

Vanadium determination.—The residual anhydrous salt left in the crucible after the previous experiment, gave, on precipitation with lead acetate, a precipitate (dried at 100° C.) weighing 0·7472 grm.; 0·7245 grm. of this precipitate was dissolved in nitric acid, and the lead precipitated with slight excess of sulphuric acid, the usual precautions being taken. The filtrate from the lead sulphate yielded on evaporation 0·1515 grm. of finely crystallized V_2O_5. Consequently the whole lead precipitate contained 0·1565 grm. V_2O_5, corresponding to 19·34 per cent., or to 10·86 per cent. of vanadium, on the sodium salt taken.

Sodium determination.—The liquid filtered off from the lead precipitate, and freed from excess of lead, left on evaporation and ignition 0·3440 grm. of Na_2SO_4, corresponding to 0·1502 Na_2O, or 13·8 per cent. of sodium in the salt.

These numbers correspond to the formula

$$Na_3VO_4 + 16H_2O.$$

	Calculated.		Found.
Na_3 . . .	69·0	14·61	13·80 per cent.
V	51·3	10·86	10·86 " "
O_4	64·0	13·56	——
$16H_2O$ . .	288·0	60·97	60·44 " "
	472·3	100·00	

Sodium orthovanadate is an extremely unstable compound. Its aqueous solution slowly undergoes decomposition on standing at the ordinary temperature of the air out of contact with atmospheric carbonic acid, whilst at higher temperatures the same change takes place quickly. This decomposition consists in the formation of a new salt, sodium tetravanadate, the liquid becomes strongly alkaline, whilst caustic soda is liberated, according to the equation

$$2(Na_3VO_4) + H_2O = Na_4V_2O_7 + 2NaHO.$$

This remarkable reaction was carefully investigated, as is seen in the sequel.

I have not been successful in several attempts to prepare a tribasic sodium vanadate containing basic hydrogen. All the reactions which with the corresponding phosphate yield hydrogen-sodium salts give with the vanadate the tetrabasic compound above mentioned. The orthovanadates of most of the metals are insoluble compounds obtained by precipitating neutral solutions of the soluble metallic salts with a solution of orthovanadate of sodium. The reactions of the more important metals are as follows:—

Reactions of the Orthovanadates.

1. Ferric salt . . .	Gelatinous precipitate of a light brownish-yellow colour, soluble in hydrochloric, insoluble in acetic acid.
2. Ferrous salt . .	Dark grey precipitate.
3. Manganous salt .	Brownish-yellow crystalline precipitate.
4. Zinc salt . . .	White gelatinous precipitate.
5. Cobalt salt . . .	Brown-grey gelatinous precipitate.
6. Nickel salt . . .	Canary-yellow crystalline precipitate.
7. Aluminium salt .	Bright yellow gelatinous precipitate, soluble in excess of both reagents; on boiling a white precipitate is again thrown down.
8. Copper salt . . .	Apple-green coloured precipitate.
9. Mercuric salt . .	Orange-yellow precipitate.

The reaction which serves best to distinguish the ortho- from the metavanadates is that of the corresponding copper salts. Orthovanadate of copper possesses a bright apple-green colour, whilst the metavanadate falls down a light yellow crystalline powder.

2. *Tetrasodium Vanadate, or Pyrovanadate*, $Na_4V_2O_7+18H_2O$.—This salt crystallizes in beautiful six-sided tables. It is easily soluble in water, insoluble in alcohol, and is precipitated from aqueous solutions by the latter liquid in the form of white scales of a pearly lustre. The pyrovanadate can be readily obtained by fusing one molecule of vanadium pentoxide (V_2O_5) with two molecules of sodium carbonate (Na_2CO_3), dissolving and crystallizing. It can also be obtained by the decomposition of the orthovanadate in aqueous solutions. As long as the tetrabasic salt contains free alkali, from the decomposition of the orthovanadate, the precipitate with alcohol forms oily drops, which only solidify after some time, whilst the pure salt is at once thrown down in the form of silky scales. If the fusion of vanadium pentoxide with three molecules of carbonate of soda be not completed at a very high temperature, the carbonate is not fully decomposed, and the fused mass when dissolved in water crystallizes at once in six-sided tables, or, if the solution be very concentrated, in nodular groups of needle-shaped crystals. The tetrabasic salt is more easily fusible than the tribasic salt, and on cooling from fusion it also forms a crystalline mass.

Decomposition of Tribasic into Tetrabasic Vanadates by boiling the aqueous solution.—That a decomposition of the above nature takes place is seen by the analyses of the different samples of 4 basic sodium salt which follow, not one of which was prepared by fusing two molecules of the carbonate with only one of vanadic acid, but by repeatedly recrystallizing the tribasic salt. The decomposition is not brought about by atmospheric carbonic acid; for in the following experiments the solution of the salts and the filtration of their solutions were effected in an apparatus from which all access of carbonic acid was so completely excluded that the alkaline mother liquor from the tetrabasic salt did not effervesce on the addition of excess of acid.

This apparatus consisted of two flasks connected with each other in such a way that the liquid contained in one of them could be passed into the other by compressing an india-rubber ball, and before the liquid entered the second flask it passed through a wide piece of glass-tubing in which a small filter was placed. The apertures were then closed by tubes containing solid caustic soda, and the salt could thus be dissolved in alcohol or water without any fear of entrance of carbonic acid.

2·0393 grms. pure V_2O_5 and 3·7 grms. pure carbonate of soda were mixed and fused until no further effervescence occurred. The loss of weight amounted to 1·441 grm., whilst the theoretical loss for three molecules of CO_2 is 1·474 grm. The fused mass was dissolved in water and boiled for some time in an atmosphere free from carbonic acid, and finally precipitated with an excess of strong alcohol. After filtration the clear solution containing the caustic soda formed by the decomposition was neutralized by standard hydrochloric acid; it required 4 cub. centims. for saturation (1 cub. centim. $=0·0366$ grm. HCl), corresponding to 0·124 grm. Na_2O. The precipitate was again dissolved in a small quantity of water and reprecipitated by alcohol; this second solution needed 4·6 cub. centims. of acid for saturation, corresponding to 0·1429 grm. Na_2O. A third repetition of the process showed that 2 cub. centims. of acid was needed, or 0·062 grm. Na_2O. Thus altogether 0·3289 grm. of Na_2O was obtained, or nearly half the amount required by the formula $2Na_3VO_4=Na_4V_2O_7+Na_2O$, namely 0·692 grm. The alcoholic solutions were free from carbonic acid, and they yielded, on addition of silver nitrate, a brown precipitate of silver oxide. They were likewise proved, by evaporation and treatment with oxalic acid, to be free from any trace of vanadic acid.

From the above experiment it is seen that the decomposition of the tetrasodium salt goes on by degrees, a fresh portion of free soda being found in solution each time the salt is dissolved and precipitated; it is even possible that the precipitation with alcohol causes a partial recombination of the caustic soda with the tribasic salt.

Analyses of Tetrasodium Vanadate. Vanadium determinations:—

(1) 1·0325 grm. of crystallized pyro-salt was dissolved in a small quantity of water and mixed with pure chloride of ammonium. After standing for twelve hours the insoluble metavanadate of ammonium* filtered off, washed first with a mixture of a saturated aqueous solution of sal-ammoniac and alcohol, and lastly with pure alcohol. On ignition 0·2995 grm. V_2O_5 was left, corresponding to 29·00 per cent. of V_2O_5, or 16·29 per cent. of vanadium.

(2) 1·3155 grm. of crystallized salt from another preparation, precipitated as above with sal-ammoniac, gave 0·3680 V_2O_5, corresponding to 27·97 per cent. of V_2O_5, or to 15·71 per cent. of vanadium.

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* This reaction shows that the pyrovanadate of ammonium is not formed by double decomposition, but that the meta-salt is precipitated whilst the solution becomes alkaline. Thus:—

$$Na_4V_2O_7+4NH_4Cl=4NaCl+2(NH_4VO_3)+2NH_3+H_2O.$$

(3) 0·5227 grm. of substance was dissolved in water and reduced by means of zinc and sulphuric acid until the liquid acquired the permanent lavender tint of hypovanadous salt. A standard permanganate solution (1 cub. centim.=0·00066 grm. oxygen) was then added until the oxidation was complete; cub. centims. needed, 57·8=0·03815 grm. oxygen, corresponding to 0·1451 V_2O_5, or 15·60 per cent. of vanadium.

(4) 0·5858 grm. substance was precipitated with lead acetate, and yielded 0·6845 grm. of $2(Pb_2V_2O_7)+PbO$ (see lead salts) dried at 100°. Hence the pyro-salt contained 0·09487 grm. of vanadium, or 16·19 per cent.

(5) 0·5285 grm. of a third preparation yielded on precipitation with sal-ammoniac 0·1506 V_2O_5, corresponding to 16·06 per cent. of vanadium.

Sodium determinations:—

(1) 0·5858 grm. substance (Analysis No. 4) yielded in the filtrate from the lead precipitate 0·2655 grm. Na_2SO_4, corresponding to 14·67 per cent. of sodium.

(2) 0·5415 substance of another preparation yielded in a similar way, on evaporation and ignition, 0·2470 grm. Na_2SO_4, corresponding to 14·79 per cent. of sodium in the crystallized salt.

(3) 0·5285 grm. of the crystals (Analysis No. 5) gave 0·1925 grm. NaCl, corresponding to 14·57 per cent. of sodium.

Water Determinations.—The crystals lose at 100° C. seventeen molecules of water; at 140° C. eighteen molecules are driven off:—

(1) 0·572 grm. crystals dried at 140° lost 0·290 grm., or 50·69 per cent. of water.
(2) 0·5415 grm. lost on ignition 0·2893, or 53·4 per cent. of water.
(3) 0·5285 grm. lost on ignition 0·2719, or 51·44 per cent. of water.

In vacuo over sulphuric acid or at 100° C.:—

(1) 0·6535 grm. *in vacuo* lost 0·3185=48·73 per cent.
(2) 0·499 grm. of another preparation lost *in vacuo* 0·234=46·89 per cent.
(3) 1·0755 grm. dried at 100° lost 0·5205=48·39 per cent.
(4) 0·572 grm. dried at 100° lost 0·279 =48·77 per cent.

The numbers obtained from the foregoing analyses agree with those calculated from the formula

$$Na_4V_2O_7+18H_2O.$$

	Calculated.		Found.					Mean.
Na_4 . . .	92·0	14·58	14·67	14·79	14·57	——	——	14·68
V_2 . . .	102·6	16·27	16·29	15·71	[15·60]	16·19	16·06	16·06
O_7 . . .	112·0	17·77	——	——	——	——	——	——
$18H_2O$ . .	324·0	51·38	50·69	53·40	51·44	——	——	51·84
	630·6	100·00						

When a solution of tetrasodium vanadate is treated with carbonic acid the salt is decomposed into sodium carbonate, which crystallizes out, and sodium metavanadate, which, being the more soluble salt, remains in solution; thus:—

$$Na_4V_2O_7 + CO_2 = 2NaVO_3 + Na_2CO_3.$$

The insoluble pyrovanadates precipitated in solutions of the various metals possess properties generally similar to those of the corresponding tribasic vanadates.

Calcium Vanadates.

If to a freshly prepared solution of trisodium vanadate a solution of chloride of calcium be added, a white precipitate falls down, whilst the liquid possesses a strongly alkaline reaction and absorbs carbonic acid from the air. The precipitate is a mixture of calcium pyrovanadate and calcium hydroxide; the tribasic calcium salt, therefore, cannot thus be obtained, as it at once decomposes as follows:—

$$Ca_3V_2O_8 + H_2O = Ca_2V_2O_7 + CaH_2O_2.$$

Calcium Pyrovanadate, $Ca_2V_2O_7 + 2\frac{1}{2}H_2O$.—This compound is precipitated as a white amorphous powder when a solution of chloride of calcium is added to one of the tetrabasic sodium salt. The salt was dried at 100° C., and of this dry salt 0·132 grm. lost on ignition 0·0270 grm., corresponding to 12·63 per cent. of water. For the calcium determination 0·3366 grm. was dissolved in acetic acid and precipitated by oxalic acid, the liquid being warmed until all the vanadic acid was reduced; the precipitation of the oxalate had to be twice repeated in order to free the precipitate completely from vanadium. 0·1956 grm. $CaCO_3$ was thus obtained, corresponding to 23·23 per cent. of calcium, and the filtrate yielded on evaporation 0·1802 grm. V_2O_5, or the salt contained 30·16 per cent. of vanadium.

These numbers correspond to the formula

$$Ca_2V_2O_7 + 2\tfrac{1}{2}H_2O.$$

	Calculated.		Found.
Ca_2	80·0	23·56	23·23
V_2	102·6	30·21	30·16
O_7	112·0	32·98	——
$2\frac{1}{2}H_2O$ . . .	45·0	13·25	12·63
	339·6	100·00	

Barium Pyrovanadate, $Ba_2V_2O_7$.—The dibasic barium salt is anhydrous, but otherwise it closely resembles the corresponding calcium compound. It is slightly soluble in water. For analysis it was dried at 100° C. 0·438 grm. dissolved in hydrochloric acid and precipitated with sulphuric acid yielded 0·4097 grm. $BaSO_4$, corresponding to 54·69 per cent. of barium. The filtrate from the barium precipitate left on evaporation 0·1678 grm. V_2O_5, corresponding to 21·5 per cent. of vanadium.

	Calculated.		Found.
Ba_2	274·0	56·08	54·69
V_2	102·6	20·99	21·50
O_7	112·0	22·93	——
	488·6	100·00	

Lead Vanadates.

Three native lead vanadates are known.

(*a*) Lead metavanadate, $Pb(VO_3)_2$, occurs as Dechenite.

(*b*) Lead pyrovanadate, $Pb_2V_2O_7$, occurs as Descloizite.

(*c*) Lead orthovanadate and lead chloride, $3(Pb_3(VO_4)_2)+PbCl_2$, occurs as vanadinite.

1. *Basic Pyrovanadate of Lead*, $2(Pb_2V_2O_7)+PbO$.—When a solution of the tetrasodium vanadate is mixed with a solution of lead acetate, a pale yellow precipitate is thrown down, and the liquid acquires an acid reaction.

The properties of this salt have already been described.

For analysis the salt was dissolved in nitric acid, and (with the exception of No. 1) the lead precipitated by sulphuric acid. The sulphate of lead was found to be quite free from vanadium, and the vanadic acid contained no lead provided care had been taken to remove all nitric acid by evaporation, and if the liquid was mixed with alcohol before the lead sulphate was filtered.

(1) From a tetrasodium salt which had been only once recrystallized, substance taken 0·3935 grm.; fused with bisulphate of potash, weight of $PbSO_4=0·4084$ grm., corresponding to 70·92 per cent. of lead.

(2) 0·373 grm. of the same salt gave 0·0902 V_2O_5, or 13·55 per cent. of vanadium.

(3) 0·365 grm. substance gave 0·3732 grm. $PbSO_4$, or 69·85 per cent. of lead and 0·086 V_2O_5, or 12·98 per cent. of vanadium.

(4) 0·5195 substance gave 0·5311 $PbSO_4$ or 69·83 per cent. of lead, and 0·125 V_2O_5, or 13·52 per cent. of vanadium.

(5) 0·681 substance gave 0·7036 $PbSO_4$ or 70·57 per cent. of lead, and 0·158 V_2O_5, or 13·03 per cent. of vanadium.

These numbers correspond to the formula $2(Pb_2V_2O_7)+PbO$.

	Calculated.		Found.				Mean.
Pb_5	1035·0	69·92	70·92	69·85	69·83	70·57	70·29
V_4	205·2	13·86	13·55	12·98	13·52	13·03	13·27
O_{15}	240·0	16·22	——	——	——	——	——
	1480·2	100·00					

2. *Lead Orthovanadate*, $Pb_3(VO_4)_2$.—The tribasic vanadate of lead falls as an insoluble nearly white powder when tribasic sodium salt is precipitated by lead acetate. 0·7245 of the substance, when decomposed by nitric acid and precipitated by sulphuric

acid, yielded 0·1515 of V_2O_5, or contained 11·75 per cent. of vanadium, the percentage required by the formula being 12·04.

3. *Lead Orthovanadate and Lead Chloride, artificial Vanadinite,* $3(Pb_3(VO_4)_2)PbCl_2$, or lead trivanado-chlorhydine, $\left.\begin{matrix}3VO'''\\ Pb_5\\ Cl\end{matrix}\right\}O_9$.

If oxide of lead, vanadic acid, and chloride of lead be fused together for a few hours in the proportions in which they are contained in the above formula, the mass after slowly cooling is found to consist of a greyish-yellow crystalline substance, in the interstices of which groups of needle-shaped crystals occur. The fused mass on boiling in water is soon reduced to a powder entirely consisting of fine crystals. This crystalline powder is boiled with water until no further trace of chlorine can be detected in the washings when it is dried ready for analysis. The crystals obtained were too small for measurement; they were, however, seen to consist of hexagonal prisms; the faces of the hexagonal pyramid could not be identified. The crystals have a yellow colour, and possess the waxy lustre characteristic of the natural mineral.

(1) 0·738 substance fused with Na_2CO_3 gave 0·0525 AgCl and 0·0122 Ag, or 2·33 per cent. of chlorine.

(2) 0·4135 substance dissolved in nitric acid and precipitated with silver nitrate, gave 0·0347 AgCl and 0·0003 Ag, or 2·17 per cent. of chlorine.

(3) 0·5582 substance dissolved in nitric acid and precipitated with sulphuric acid, gave 0·5881 $PbSO_4$, or 71·96 per cent. of lead, and 0·1104 of V_2O_5, corresponding to 11·11 per cent. of vanadium.

(4) 0·5443 substance of another preparation, dissolved in nitric acid and precipitated with silver nitrate, gave 0·045 AgCl and 0·0035 Ag, corresponding to 2·17 per cent. chlorine, and 0·5705 $PbSO_4$, or 71·57 per cent. of lead.

The following gives the composition of various specimens of natural vanadinites compared with that of the artificial mineral:—

	Calculated $3(Pb_3(VO_4)_2)PbCl_2$.	Natural vanadinites.				Artificial vanadinites.	
		Zimapan (Berzelius).	Windischkappel (Rammelsberg).	Wieklow.	Beresesowsk (Struve).	(1)	(2)
Lead	73·08	70·40	71·20	68·72	73·76	71·96	71·57
Vanadium	10·86	——	9·77	13·15	9·54	11·11	——
Phosphorus	——	——	——	——	1·34	——	——
Chlorine	2·56	2·54	2·23	2·44	2·46	2·33	2·17
Oxygen	13·55	——	——	——	——	——	——

The specific gravity of the artificial vanadinite at 12° C. is 6·707, that of the natural mineral varies from 6·66 to 7·2*.

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* Pyromorphite and apatite were prepared artificially for the first time in 1852 by Manross (Ann. Ch. Pharm. lxxxii. p. 348), and afterwards by Deville and Caron, and Debray. Mimetesite has also been recently artificially prepared by Lechartier (Comptes Rendus, 1867, lxv. p. 172).

Silver Vanadates.

1. *Silver Orthovanadate, or Tribasic Silver Vanadate,* Ag_3VO_4, is precipitated as a deep orange-coloured powder when a freshly prepared solution of tribasic sodium salt is mixed with a perfectly neutral solution of silver nitrate. If the precaution of neutralizing the silver solution with carbonate of soda, filtering, and boiling be not adopted, a salt is precipitated which consists of a mixture of tribasic and tetrabasic silver salt. The colour of this mixed salt is lighter than that of the tribasic compound, and it gives on analysis a percentage of silver and vanadium intermediate between the two salts.

Silver orthovanadate is easily soluble in nitric acid and ammonia. For analysis it was dissolved in nitric acid, the silver being precipitated as chloride, and the vanadium estimated in the filtrate.

(1) 0·385 substance gave 0·369 $AgCl$ and 0·0076 Ag, or 74·12 per cent. of silver, and 0·0795 V_2O_5, or 11·59 per cent. of vanadium.

(2) 0·522 substance, of another preparation, gave 0·508 $AgCl$ and 0·0016 Ag, or 73·54 per cent. of silver, and 0·111 V_2O_5, or 11·94 per cent. of vanadium.

Hence we have:—

	Calculated.		Found.		
			(1)	(2)	Mean.
Ag_3 . .	324·0	73·75	74·12	73·54	73·83
V . .	51·3	11·67	11·59	11·94	11·86
O_4 . .	64·0	14·58	——	——	
	439·3	100·00			

2. *Tetrabasic Silver Vanadate*=$Ag_4V_2O_7$.—This salt is prepared by precipitating a solution of pure tetrasodium salt with a neutral solution of silver nitrate. It is a dense yellow precipitate, settling very easily when the liquid is warmed, and resembling in its appearance ordinary tribasic phosphate of silver.

(1) 0·4725 substance gave 0·4105 $AgCl$ and 0·0051 Ag, or 66·45 per cent. of silver, and 0·1345 V_2O_5, corresponding to 15·99 per cent. vanadium.

Hence we have:—

	Calculated.		Found.
Ag_4 . .	432·0	66·81	66·45
V_2 . .	102·6	15·87	15·99
O_7 . .	112·0	17·32	——
	646·6	100·00	

From the foregoing experiments on the vanadates it appears:

(1) That the soluble tribasic salts are less stable at the ordinary temperature than the tetrabasic compounds, Na_3VO_4, splitting up in solution into free caustic soda and the pyro-salt.

(2) That at a high temperature, on the other hand, the tribasic form is the most

stable, V_2O_5 liberating three molecules of CO_2 when fused with carbonate of soda, but forming a monobasic (meta)salt when boiled with a solution of alkaline carbonate.

(3) That as the majority of the naturally occurring vanadates are tribasic compounds, we may assume that these have been produced at a high temperature.

(4) That in aqueous solutions the soluble pyrovanadates are easily decomposed by carbonic acid into an alkaline carbonate and a monobasic or metavanadate.

Hence the order of stability of the different vanadates at the ordinary temperatures is as follows:—

(1) Monobasic or metavanadates.
(2) Tetrabasic or pyrovanadates.
(3) Tribasic or orthovanadates.

In the phosphorus series the order of stability is (as is well known) exactly the reverse of this, the tribasic phosphoric acid and soluble orthophosphates being most stable, and being formed from the other two classes of acids and soluble salts, either by ebullition alone or in presence of weak acids.

I have much pleasure in acknowledging the able assistance which I have received from Messrs. Œlhofer and Finkelstein in carrying out the above investigation.

XIX. *On the Action of Rays of high Refrangibility upon Gaseous Matter.*

By JOHN TYNDALL, *LL.D., F.R.S.*

Received December 4, 1869,—Read January 27, 1870.

§ I.

Introduction.

WITHIN the last ten years I have had the honour of submitting to the Royal Society a series of investigations the principal aim of which was to render the less refrangible rays of the spectrum interpreters and expositors of the molecular condition of matter.

Unlike the beautiful researches of MELLONI and KNOBLAUCH, these inquiries made radiant heat a means to an end. My thoughts were fixed on it in relation to the matter through which it passed. Placing before my mind such images of molecules and their constituents as modern science justifies or renders probable, such images of the luminiferous ether and its motions as the undulatory theory enables us to form, I endeavoured to fashion and execute experiments founded upon these conceptions which should give us a surer hold upon molecular constitution.

Thus, definite physical ideas have accompanied and guided the whole course of these researches. That matter is constituted of atoms and molecules has been accepted as a verity throughout. The phenomena under examination rendered it impossible for me to halt at the law of multiple proportions, which so many chemists of the present day appear inclined to make their intellectual bourne. In following up a train of ether waves, in idea, to their source, I could not place at that source a multiple proportion; the waves could not be connected physically with such a multiple; I was forced to put there a bit of matter, in other words, a *molecule,* which bore the same relation to the ether as a vibrating string does to the air which accepts its motions and transmits them as waves of sound.

One result among many others which these researches established will, I think, play an important part in the chemistry of the future. I refer to the proved change of relation between the luminiferous ether and ordinary matter which accompanies the act of chemical combination. Here, without any alteration in the quantity, or in the ultimate quality of the medium traversed by the ethereal waves, vast changes may occur in the amount of wave-motion intercepted. Let pure nitrogen and ordinary oxygen be mixed mechanically together in the proportion by weight of 14 : 8. Radiant heat, it is now known, will pass through the mixture as through a vacuum. No doubt a certain amount of heat is intercepted; but it is so small an amount as to be practically insensible. At all events it is multiplied by hundreds, if not by thousands, the moment the oxygen and nitrogen combine to form nitrous oxide. Or let nitrogen and hydrogen be mixed mechanically together in the proportion of 14:3; the amount of radiant heat which they then absorb is augmented more than a thousandfold* the moment they build themselves together into molecules of ammonia. Neither the quantity nor the ultimate quality of the matter is here changed; the act of chemical union is the sole cause of the enormous alteration in the amount of heat intercepted. The converse of these statements is of course also true; dissolve the chemical bond, either of the nitrous oxide or of the ammonia, and you instantly destroy the absorption. As a proof that our atmosphere is a mixture, and not a compound, no experiment with which I am acquainted matches in point of conclusiveness that which demonstrates the deportment of dry air to radiant heat.

But the molecules which can thus intercept the waves of ether must be shaken by those waves, possibly shaken asunder. That ordinary thermometric heat provokes chemical actions is one of the commonest facts of observation. These actions, considered from a physical point of view, are changes of molecular position and arrangement consequent on the acceptance of motion from the source of heat. Radiant heat also, if sufficiently intense and if absorbed with sufficient avidity, could produce all the effects of ordinary thermometric heat. The dark rays, for example, which can make platinum white-hot, could also, if absorbed, produce the chemical effects of white-hot platinum. They could decompose water, as now in a moment they can boil water. But the decomposition in this case would be effected by the virtual conversion of the radiant heat into thermometric heat. There would be nothing in the act characteristic of *radiation*, or demanding it as an essential element in the decomposition.

The dark calorific rays are powerfully absorbed by various bodies, but, as a general rule, they do not appear competent to set up that particular motion among the constituents of a molecule which breaks the tie of chemical affinity. All the rays of the spectrum exercise no doubt chemical powers. We should have scant vegetation upon the earth's surface if the red and ultra-red rays of the sun were abolished. But the chemical actions in which the *radiant form* comes into play, are mainly produced by the least energetic rays of the spectrum. The photographer has his heat focus in advance of the chemical focus; which latter, though potent for his special purposes, possesses almost infinitely less mechanical energy than its neighbour. Some special relation must, therefore, as a general rule, subsist between chemical molecules and the more refrangible rays; we arrive at the conclusion that chemical decomposition by *rays*, to keep to the ordinary term, is less a matter of *amplitude* on the part of the vibrating ether particles than of *time of vibration*.

The decomposition of a molecule must result from the internal strain of its parts; to them, therefore, and not to the molecule as a whole, the vibrations which produce chemical change must be imparted. The question remains an outstanding one in molecular physics, why it is that the longer and more powerful ether waves are generally incompetent to set up the motion which results in decomposition. The influence of synchronism here suggests itself. These shorter waves are effectual because their motion is *stored up*. Their infinitesimal impulses, because imparted at the proper intervals, *accumulate* and finally become intense enough to jerk asunder the atoms with whose periods they are in accordance.

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* It may be a millionfold; for we do not yet know how small the absorption of the absolutely pure mixture really is.

§ II.

The investigation which I have now the honour to offer to the Royal Society is in a certain sense complementary to those referred to at the outset of this paper. It deals with the relations of gaseous matter to the most refrangible rays of the spectrum. It treats of the chemical energies of such rays as exerted upon such matter. If we except the combination of chlorine and hydrogen by light, and the decomposition of carbonic acid by the solar rays in the leaves of plants, which latter, however, may not be the decomposition of *a gas*, no fact I believe has hitherto been known to exist in which light, or heat in the radiant form, acts chemically upon a gas or vapour*. By this inquiry the range of radiant energy as a chemical agent is considerably extended; the phenomena resulting from that energy are exhibited in a new and exceedingly impressive form, and they prompt reflections regarding the possible influence of solar radiation on the gases, vapours, and effluvia of our atmosphere which could not previously be entertained.

The inquiry was started thus:—It is known to the Society that the experiments on radiant heat already referred to, were for the most part performed in tubes of brass or glass, called for the sake of distinction "experimental tubes." It is also known to the physical members of the Society that a difference† exists between my eminent friend Professor Magnus‡ and myself with regard to the deportment of aqueous vapour towards radiant heat. Last autumn, and in reference to the reasons assigned by him for this difference, I scrutinized the appearance of my experimental tubes during the entrance into them of various gases and vapours. The vapours were carried into the tubes by dry air which had been permitted to bubble through their liquids. I watched carefully, and with the aid of magnifying-lenses, for any signs of the precipitation of moisture either upon the surface of the experimental tube itself or upon the plates of rock-salt employed to close it, keeping at the same time my eyes open to any other action which the intensely concentrated beam employed in the inquiry might reveal.

On the 9th of October, 1868, while thus engaged upon the vapour of the nitrite of amyl, I observed a curious cloudiness in the experimental tube when the beam was sent through the vapour. For a moment this appearance troubled me; for it required a little reflection to assure me that in my previous publications I had not sometimes ascribed to pure cloudless vapour actions which were really due to such nebulous matter as was then before me. The appearance, however, immediately declared itself to my mind as a product of chemical action then and there exerted on the vapour.

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* Professor Stokes reminds me that Phosgen gas derives its name from its formation under the influence of light.—[J. T., July 1870.]

† To be still cleared up.

‡ Unhappily lost to science since these words were written.—[J. T., July 1870.]

The nitrite vapour was then intentionally subjected to the action of the light. The beam employed was convergent. As the vapour reached the point of greatest concentration of the beam cloudy matter was there precipitated, which was afterwards whirled by the moving air into the more distant parts of the tube. The cloud thus carried away was incessantly renewed, and after the mixed air and vapour had ceased to enter, precipitation occurred all along the cone of rays in front of the focus.

The lamp was then extinguished, and the mixture of air and nitrite vapour permitted to enter the tube in the dark. When the tube was full the condensed beam was sent through it. For a moment the light seemed to pass through air only; but after a moment's pause a white cloud fell suddenly upon the conical portion of the beam, causing it to flash forth almost like an illuminated solid.

When the beam, previous to allowing it to enter the vapour, was caused to pass through a red or yellow glass, the action though visible was feeble; it was much more energetic when the beam passed through a blue glass. I sent a convergent beam through a red glass and observed the feeble effect. A blue glass was then added, and by the concert of both the light was completely cut off. On withdrawing the red glass, a very beautiful blue cloud came down upon the conical beam. The experiment proved that in this case, as in so many others, the blue rays are the "chemical rays."

Solar light, as might be expected, produces all the effects of the electric light, and in regions more favoured than London may be employed in continuous researches of this nature. When the parallel beams of the sun are duly concentrated, the precipitation which they invoke in passing through nitrite-of-amyl vapour is copious and immediate.

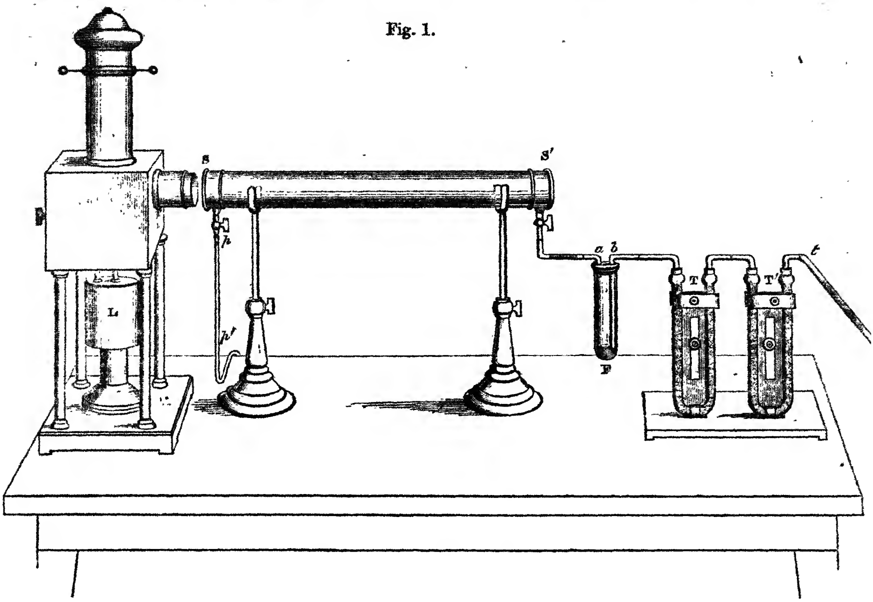
§ III.

The simple apparatus employed in these experiments will be at once understood by reference to fig. 1. S S′ is the glass experimental tube which has varied in length from 1 to 5 feet, and which may be from 2 to 3 inches in diameter. From the end S the pipe *p p′* passes to an air-pump. Connected with the other end we have the flask F, containing the liquid whose vapour is to be examined; then follows a U-tube, T, filled with fragments of clean glass wetted with sulphuric acid; then a second U-tube, T′, containing fragments of marble wetted with caustic potash; and finally a narrow straight tube *t t′*, containing a tolerably tightly fitting plug of cotton-wool. To save the air-pump gauge from the attack of such vapours as act on mercury, as also to facilitate observation, a separate barometer tube was employed.

Through the cork which stops the flask F two glass tubes, *a* and *b*, pass air-tight. The tube *a* ends immediately under the cork; the tube *b*, on the contrary, descends to the bottom of the flask and dips into the liquid. The end of the tube *b* is drawn out so as to render very small the orifice through which the air escapes into the liquid.

The experimental tube S S′ being exhausted, a cock at the end S′ is carefully turned on. The air passes slowly through the cotton-wool, the caustic potash, and the sulphuric acid in succession. Thus purified it enters the flask F and bubbles through the liquid. Charged with vapour it finally passes into the experimental tube, where it is submitted to examination. The electric lamp L placed at the end of the experimental tube furnished the necessary beam.

Fig. 1.



§ IV.

The floating Matter of the Air.

Prior to the discovery of the foregoing action, and also during the experiments just referred to, the nature of my work compelled me to aim at obtaining experimental tubes absolutely clean upon the surface, and absolutely empty within. Neither condition is, however, easily attained.

For however well the tubes might be washed and polished, and however bright and pure they might appear in ordinary daylight, the electric beam infallibly revealed signs and tokens of dirt. The air was always present, and it was sure to deposit some impurity. All chemical processes, not conducted in a vacuum, are open to this disturbance. When the experimental tube was exhausted it exhibited no trace of floating matter, but on admitting the air through the U-tubes containing caustic potash and sulphuric acid, a *dust-cone* more or less distinct was always revealed by the powerfully condensed electric beam.

The floating motes resembled minute particles of liquid which had been carried mechanically into the experimental tube. Precautions were therefore taken to prevent any such transfer. They produced little or no mitigation. I did not imagine at the time that the dust of the external air could find such free passage through the caustic potash and the sulphuric-acid tubes. But the motes really came from without. They also passed with freedom through a variety of ethers and alcohols placed in the flask F. In fact, it requires long-continued action on the part of an acid first to *wet* the motes and afterwards to destroy them. By carefully passing the air through the flame of a spirit-lamp or through a platinum tube heated to bright redness, the floating matter was sensibly destroyed. It was therefore combustible, in other words, *organic* matter*. I tried to intercept it by a large respirator of cotton-wool tied round the end of the tube $t\,t'$. Close pressure was necessary to render the wool effective. A plug of the wool rammed pretty tightly into the tube $t\,t'$ was finally found competent to hold back the motes. They appeared from time to time afterwards and gave me much trouble; but they were invariably traced in the end to some defect in the purifying-apparatus,—to some crack or flaw in the sealing-wax used to render the tubes air-tight. Without due care, moreover, liquid particles may also be carried mechanically over. To prevent the entrance of such into the experimental tube, the narrow conduit which connects it with the flask F is plugged with clean asbestos. Thus through proper care, but not without a great deal of searching out of disturbances, the experimental tube, even when filled with pure air or vapour, contains nothing competent to scatter the light. The space within it has the aspect of an absolute vacuum.

An experimental tube in this condition I call *optically empty*.

Here follows one of the numerous experiments executed in relation to this subject. A platinum tube 9 inches long, 0·4 of an inch wide, and having within it a roll of platinum gauze, was placed in a gas-furnace where it could be intensely heated. One end of this tube was connected with the entry stopcock of the experimental tube S S′, fig. 1, the other end was open to the air of the laboratory. The air was sent first through the platinum tube cold, then through the same tube heated to various degrees of redness, into the experimental tube, where it was subjected to the scrutiny of the concentrated electric beam. Here are the results.

Quantity of air.	State of platinum tube.	State of experimental tube.
15 in. of mercury.	Cold.	Full of floating particles.
15 ,,	Red-hot.	Optically empty.
15 ,,	Cold.	Full of floating particles.
15 ,,	Red-hot.	Optically empty.
15 ,,	Intensely heated.	An exceedingly fine cloud, which discharged perfectly polarized blue light in a direction at right angles to the illuminating beam.

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* Mr. Dancer has recently examined microscopically the dust of Manchester, and found it to consist almost wholly of organic particles.

Here the character of the floating matter is very clearly shown. The non-combustible or inorganic part of it was too minute to be revealed by the concentrated beam. The tube after its combustion appeared "optically empty."

The "blue cloud" just referred to was due to the too rapid passage of the air through the intensely heated tube, which prevented the perfect combustion of the floating matter. It was to all intents and purposes the *smoke* of the particles*. The subject is further illustrated by the following series of experiments:—

Quantity of air.	State of platinum tube.	State of experimental tube.
15 inches.	Cold.	Full of particles.
15 „	Dull red.	Optically empty.
15 „	Intensely heated.	Optically empty.
30 „	Intensely heated.	Optically empty.
15 „ (admitted quickly).	„	A perfectly polarized blue cloud.
15 „ (quickly).	Barely visible redness.	Particles.
15 „ (quickly).	Intensely heated.	Blue cloud.
15 „ (slower).	„	A very fine blue cloud.
15 „ (very slow).	„	Optically empty.
15 „	Cold.	Full of particles.
15 „ (quickly).	Red-hot.	Blue cloud.

The polarization of light by such clouds as the blue ones here mentioned will receive due attention subsequently.

A remarkably fine experiment may be thus made:—Placing a spirit-lamp underneath the cylindrical beam of the electric lamp as it marks its track upon the illuminated dust of the atmosphere, torrents of what would be infallibly mistaken for black smoke rise from the flame into the beam. A Bunsen's flame produces the same effect. But the action of a red-hot poker placed underneath the beam is precisely similar; the action of a hydrogen flame, moreover, where smoke is out of the question, is not to be distinguished from that of the spirit-lamp flame. The apparent smoke rises even when the flame or the poker is placed at a good distance below the beam. The action is really due to the destruction of the floating matter by contact with the heated body. It sends upwards streams of air from which every thing competent to scatter the light has been removed. This air, in passing through the beam, jostles aside the illuminated particles, the space it occupies being *black* in contrast with the adjacent luminosity. The experiment is capable of various instructive modifications, and may of course be executed with sunlight.

It is needless to dwell upon the possible influence of the floating organic matter of the air upon health. Its quantity, when illuminated by a powerful and strongly concentrated beam, appears sometimes to be enormous. One recoils from the idea of placing the mouth at the intensely illuminated focus and inhaling the swimming dirt revealed there. Nor is the disgust removed by the reflection that at a distance from the focus, though we do not see the dirt, we are breathing precisely the same air. The difficulty of wetting it before referred to, may render this suspended matter comparatively harmless to the lungs, but where these are fragile its mere mechanical irritation must go for something. Perhaps a respirator of cotton-wool might in some cases be found useful*.

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* In subsequent experiments I found that this "smoke" arose in great part from the action of the heated air upon the india-rubber joint which connected the platinum tube with the experimental tube.

§ V.

I now return to the nitrite of amyl. The action of light upon the vapour of this substance is exceedingly prompt and energetic. It may be illustrated by simply blowing the vapour into a concentrated sunbeam. Or the experiment may be made to take the following form:—Connecting the tube *b* of the flask F with the pipe of a bellows, after inflating the latter a sharp tap upon its board sends a puff of vapour through the tube *a* into the air. In a moderately lighted space nothing is seen; but when the puff is projected into a concentrated sunbeam, or into the beam from the electric lamp, on crossing the limiting boundary of light and shade it is instantly precipitated as a *white ring*. The ring has of course the same mechanical course as the smoke-rings puffed from the mouth of a cannon, but it is *latent* until revealed by actinic precipitation†.

In every one of the numerous experiments made with the nitrite of amyl, the chemical energy appeared to exhaust itself in the frontal portion of the experimental tube. A dense white cloud would fall for a distance of 12 or 15 inches upon the beam, while beyond this distance the tube would appear almost empty. This absence of action might naturally be ascribed to the diffusion of the beam beyond the focus; but when the light was so converged as to bring the focus near the distant end of the tube the effect was the same. When, moreover, a concave mirror received a parallel beam which had traversed the tube, and returned it into the vapour in a high state of luminous concentration, the light was ineffectual. The passage of the beam through a comparatively small depth of the vapour appeared to extract from it those constituents which produced decomposition. That the vapour was present at the distant end of the tube, was proved by the fact that both with the sun and with the electric light the reversal of the tube instantly brought down a heavy cloud. As regards the chemical rays nitrite of amyl is the *blackest* substance that I have yet encountered. It rapidly extinguishes them, leaving behind a beam of sensibly undiminished photometric intensity, but powerless as a chemical agent as far as the nitrite is concerned.

In these experiments air was employed as the vehicle of the nitrite-of-amyl vapour. By varying the quantity sent into the experimental tube, it was possible to vary in a remarkable manner the character of the resulting decomposition. The most splendid diffraction colours could be thus produced, and the finest texture could be imparted to the clouds. When pure oxygen or pure nitrogen was used, the effect was almost the same as with air. With hydrogen the clouds appeared more delicate and lustrous; and they sometimes fell immediately after their formation in nebulous festoons to the bottom of the tube. This doubtless is to be ascribed to the attenuation of the atmosphere in which they floated. In many cases, however, the particles remained suspended, and some of them continued to float even after the tube had been so far exhausted as to produce a tolerably good air-pump vacuum.

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* Since this paper was forwarded to the Royal Society these experiments have been greatly extended. See Proceedings of the Royal Institution, January 1870.—[J. T., July 1870].

† By a special arrangement it is easy to obtain such rings 2 inches and more in diameter.

An additional effect of considerable beauty and interest is obtained in the following way. Permitting the convergent beam to play for a time upon the mixture of air and nitrite-of-amyl vapour, or, better still, upon mixture of hydrogen and vapour, a coarse cloud is formed. Suspending the action of the lamp for a minute or so, a new distribution of the vapour appears to occur; for, on reigniting the lamp, along its convergent beam, and *within* the old cloud, a new cloud is precipitated. The tint of this new cloud is a delicate bluish white, and its texture is of exquisite fineness. This precipitation of one cloud within another may be obtained a dozen times in succession. Or, permitting a parallel beam to pass for a time through the coarser cloud, on pushing out the lens so as to concentrate the light, the fine cloud comes suddenly down upon the beam about its place of greatest concentration. This effect also may be obtained several times with the same charge of vapour.

No phenomena of the kind thus far described have, I believe, been hitherto observed. The necessary conditions for their production are, first, that the light should decompose the vapour, and secondly, that one or more of the products of decomposition should either be a solid, or should possess a boiling-point so high as to ensure its precipitation when set free.

For though chemical action might occur, and be even energetic, if the products of decomposition be vaporous and colourless they will remain unseen. In the case just considered, the *nitrate* of amyl is in all probability a product of the decomposition of the *nitrite*. The boiling-point of the latter is estimated at from 91° to 96° C., that of the former being 149° C. The nitrite, therefore, can maintain itself as true vapour in a space where the nitrate, at the moment of its liberation, must fall as a cloud.

§ VI.

An exceedingly fine example of actinic action is furnished by the vapour of the iodide of allyl. The effect of light upon this substance was observed on the 7th of October, 1868, but I did not then know the meaning of the "thin cloud like a kind of smoke" which showed itself in the experimental tube. On satisfying myself regarding the deportment of nitrite of amyl, the iodide of allyl occurred to me, and on it experiments were immediately made.

The decomposition of this vapour was slower than that of the nitrite of amyl. The slowness, moreover, augmented rapidly as the quantity of vapour was diminished. When only a few inches of the mixed air and vapour were in the experimental tube the action was very slow. The clouds were formed both in oxygen and in air. After the action had been continued for some time, the fine purple colour of iodine exhibited itself at the end of the tube most distant from the source of light. When hydrogen was the vehicle, the clouds were particularly lustrous and beautiful. Here and there also, amid the white and coarser sections of a cloud, spaces of delicate blue would reveal themselves, reminding one of the colour of a pure sky. The words "wonderful," "beautiful," "lustrous," and others of a similar nature, occur frequently and naturally in my notes of this period; for in those earlier experiments the cloud-forms obtained were so amazing, and their colours and textures so fine, as to rivet attention upon them alone.

With long-continued action the colour due to the discharge of iodine became very intense. It was strong enough to empurple the beam which passed through the air of the laboratory after its transmission through the experimental tube, and to colour deeply a white screen on which the beam was permitted to fall. In what condition was this iodine? It could be liberated by a beam deprived almost wholly of its calorific rays. The temperature of the experimental tube was indeed so moderate that a quantity of iodine placed within it and permitted to saturate the space with its vapour, produced a barely perceptible flush on a piece of white paper placed there expressly to detect it. The far more deeply coloured iodine revealed by the beam in the actinic cloud must, I think, have been for the most part liquid, and not vaporous iodine. I say liquid, because the substance was probably dissolved by the particles of the cloud with which it was so intimately mixed. Di-allyl, for example, is a powerful solvent of iodine, and it was probably one of the products of decomposition.

The iodide of isopropyl also capitally illustrates the action of light upon vapours. It is more slowly acted upon than either the nitrite of amyl or the iodide of allyl; nevertheless, in sufficient quantity, its decomposition is very brisk and energetic. Purified air which had bubbled through the liquid iodide was conducted into the experimental tube. When the pressure was 1 inch of mercury, the light playing upon the vapour for five minutes produced no action; but when it was 10 inches a blue cloud made its appearance in two minutes, and in ten minutes it had almost filled the tube. When the pressure was 20 inches, the action commenced more quickly, and the cloud generated was more dense. The whirling motions of this cloud appeared to be more brisk than that of the others examined. With 30 inches of the mixed air and iodide the action began in a quarter of a minute, and in five minutes a dense cloud was formed throughout the tube. The purple of the discharged iodine was also very plain in this cloud.

§ VII.

In the preliminary notice of these experiments laid before the Royal Society in June 1868, considerable stress is laid upon the fact that the same rays are absorbed by the nitrite of amyl in the liquid and in the vaporous state. A layer of the liquid not more than one-eighth of an inch in thickness was found competent to withdraw from a powerful beam all the constituents which could effect the decomposition of its vapour. The action of the nitrite resembles in this respect that of the sulphate of quinine on the rays which cause it to fluoresce. Both substances quench the effectual rays close to the surface at which they enter.

I endeavoured at the time to apply this fact to the solution of the question whether the absorption of chemical energy was the act of the molecule as a whole, or of its constituent atoms. I tried to show that on the first of these assumptions it is impossible for the self-same rays to be absorbed by a liquid and its vapour. For absorption depends upon the rate of molecular vibration, and reaches its maximum when this rate synchronizes perfectly with the rate of succession of the ethereal waves. Now as the rate of molecular vibration depends upon the elastic forces exerted between the molecules, and as it could hardly be imagined that these forces would remain undisturbed during the passage of a vapour to the liquid condition, the fact of the liquid nitrite of amyl and its vapour absorbing the same rays indicated that the absorption was not molecular. We were thus driven to conclude that it was atomic*; and this conclusion was fortified by the consideration already adverted to,—that were the absorption the act of the molecule as a whole, no mechanical ground could be assigned for the falling asunder of its atoms. Thus actinic action itself pointed out the seat of the absorption.

A wide, if not entire generality was anticipated for the proposition that the same rays are absorbed by a liquid and its vapour. I have now no reason to retract this anticipation; but when it was expressed I believed that liquids in general would be found so destructive of the effectual rays as to render transmission through moderate depths of them sufficient to rob a beam of all power to act upon their vapours. This idea, entertained though not expressed, has not been verified, and the deportment of iodide of allyl may be taken as representative of a class of facts which contradict it.

Glass cells were employed varying from one-eighth of an inch to an inch in width. Filled with the transparent iodide, these cells were placed between the electric lamp and the experimental tube charged with the iodide vapour. The rays after traversing an inch of the liquid produced copious decomposition in the tube. A marked distinction was thus proved to exist between the liquid iodide of allyl and the liquid nitrite of amyl.

But the same distinction extends to their vapours. The exceeding absorbent avidity of the nitrite-of-amyl vapour, and the rapidity with which it deprives a powerful beam of its effective constituents, have been already noticed. It is quite different with the iodide of allyl. A tube 5 feet long was charged with the iodide vapour, and after it, in the same line, was placed another tube 3 feet long charged with the same vapour. On sending a beam through both tubes in succession, the 5-foot tube, through which the light first passed, was filled immediately with an actinic cloud; but a similar cloud was at the [illegible] time falling in the second tube. A transmission through 5 feet did not seem to diminish very materially the power of the beam. A passage through 1 foot of the nitrite of amyl would have been far more destructive.

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* When I use the word "atomic" in contrast with "molecular," I by no means pledge myself to an absolute limit of divisibility. The molecule may resemble a house, the atoms the hard bricks composing that house. But while it is both convenient and correct to regard the house as constituted of bricks definitely bounded, it is by no means essential to regard the bricks themselves as absolutely indivisible. The divisibility or non-divisibility of the atoms does not in the least affect the atomic theory as a *working conception*.

As these actions are representative and, I believe, important, I will here sum up some recent confirmatory experiments executed with the two substances now under consideration.

1°. The vaporous nitrite of amyl absorbs with such avidity the rays competent to decompose it, that a very small depth of the vapour quenches the efficient rays of a powerful beam of solar or electric light.

2°. The vaporous iodide of allyl, on the contrary, permits a beam to traverse it for long distances without very materially diminishing the chemical power of the beam.

3°. The liquid nitrite of amyl, in a stratum one quarter of an inch thick, quenches all the rays which could act chemically upon its vapour.

4°. The liquid iodide of allyl, on the contrary, in a stratum of four times the thickness just mentioned, does not materially diminish the power of the beam to act upon its vapour.

5°. A very marked difference exists between the deportment of the nitrite of amyl alone, and its deportment when mixed with hydrochloric acid. The *chemical penetrability* of the mixture is far greater than that of the pure vapour. The actinic cloud, which with the vapour alone is confined to the anterior portion of the experimental tube, extends in the case of the mixture through the entire tube.

6°. A beam, moreover, which has been transmitted through a quarter of an inch of the liquid nitrite is also competent to act chemically upon the mixture, and to produce in it dense actinic clouds.

The action in this last case, though not stopped by the liquid nitrite, is retarded. Employing first the liquid screen, it was interesting to observe the sudden development of a fine-grained luminous cloud, and its violent tumbling about by the decomposing beam the moment the liquid was withdrawn. The action of a solution of the yellow chromate of potash is substantially the same as that of the liquid nitrite. By the successive introduction and removal of a cell containing either substance, successive flashes of actinic energy may be produced a dozen times and more in the same vapour.

The molecular relationship of a liquid and its vapour receives new illustration from these experiments. Whatever alters the action of the one appears to change in a proportionate degree the action of the other.

§ VIII.

Carbonic acid is decomposed by the solar beams in the leaves of plants; but here it is in presence of a substance, chlorophyll, ready, as it were, to take advantage of the loosening of the atoms by the solar rays. The present investigation has furnished numerous cases of a similar mode of action. All the vapours examined may be more or less powerfully affected in their actinic relations by the presence of a second body with which they can interact. The presence, for example, of nitric acid, or of hydrochloric acid, may either greatly intensify or greatly diminish the visible action of the light on many vapours decomposable alone or when mixed with air; while the presence of the one or the other of the same acids may provoke energetic actions in substances which are wholly inactive when left to themselves.

We need not go beyond the nitrite of amyl for an example of this kind. For, prompt and copious as the decomposition of this substance is when mixed with air, the energy and brilliancy of the action are materially augmented by the presence of hydrochloric acid. Let a quantity of the nitrite vapour mixed with air be sent into the experimental tube till the mercury column sinks, say, 8 inches. Then let the flask containing the nitrite be removed and one containing strong hydrochloric acid be put in its place. Let purified air which has bubbled through the acid be carried into the experimental tube until a further depression of 8 inches is obtained. On allowing the convergent beam to play upon this mixture a cloud of extraordinary density and brilliance is precipitated. The beam appears to pierce like a shining sword the nebulous mass of its own creation, tossing the precipitated particles in heaps right and left of it. This experiment is very easily made, and nothing could more finely or forcibly illustrate the phenomena here under consideration.

By varying the proportions of the vapour to the acid we vary the effects. For example, the proportion of 1 inch of the nitrite vapour to 15 inches of the hydrochloric acid did not produce so brilliant an effect as the proportion 8 : 8. The same is true of the proportion 15 inches of nitrite vapour to 1 inch of hydrochloric acid. But in this latter case, though the general action was less intense than in the case of 8 : 8, the iridescences due to diffraction were much finer. No doubt for each particular substance a determinable proportion exists which corresponds to the maximum of actinic action*.

The nitrite of butyl affords another striking example of the influence of a second body in the experimental tube. With air, or alone, it was not visibly affected by the light; there was no cloud formed by its exposure. It was also mixed with nitric acid in various proportions, but no visible effect was produced by the beam.

It was then tried with air which had been permitted to bubble through pure hydrochloric acid in the following proportions:—

1. 1 inch of air and vapour to 15 inches of air and acid.
2. 8 inches ,, ,, 8 ,, ,,
3. 15 inches ,, ,, 1 inch ,, ,,

In the first case a dense and brilliant cloud was immediately precipitated. In the second case the precipitation of the fine white cloud was confined to the convergent luminous cone, coarser particles being scattered through the rest of the tube. In the third case the cloud was very coarse and very scanty. The experiment indicates that the best effect is obtained when a small quantity of the vapour is mixed with a considerable quantity of the acid.

Benzol is also a good example of a substance which, when alone, defies the power of the light, but which in the presence of other substances is readily decomposed. During the earlier stages of this inquiry a vast number of experiments were made with benzol and *commercial* hydrochloric acid. The results well illustrate actinic action, but they are not to be accepted as indicative of the action of *pure* hydrochloric acid. Indeed with the pure acid and benzol vapour there is no visible action.

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* This might form the subject of an interesting inquiry.

On the 16th of November, 1868, 2 inches of air and benzol vapour were sent into the experimental tube, and afterwards the tube was filled with air which had bubbled through the commercial acid. My notes, written at the time, describe the action of light upon the mixture as producing a cloud of an exquisite sky-blue colour, only more luminous and ethereal than the sky. The figure of the cloud was also very wonderful.

This cloud was permitted to remain for fifteen hours in the experimental tube uninfluenced by light. After this interval it was found still floating, being composed of curiously shaped granular sections joined together by others of more delicate hue and texture. The renewed light set the cloud immediately in motion, the granular parts disappeared, and the whole for a length of 18 inches resumed its primitive delicate hue and texture. In some portions it became white or whitish grey, but at others it was a pure firmamental blue. It became very dense as the light continued to act, and finally developed itself into a form of astonishing complexity and beauty.

The experimental tube had then a current of dry air swept through it, and it was afterwards exhausted. 2 inches of the benzol vapour were admitted as before, and dry air was added until the tube was full. It required five minutes' action of the light to develope the faintest visible cloud; even after ten minutes' action the cloud was very faint*. The tube was again cleansed and exhausted, 2 inches of the benzol vapour were admitted, followed by air and hydrochloric acid until the tube was full. On starting the light chemical action began almost immediately, and ended by the formation of a cloud throughout the tube. The influence of the commercial hydrochloric acid is here demonstrated. The interaction of nitric acid and benzol will be immediately referred to.

Bisulphide of carbon is also an illustration in point. Alone or mixed with air it resists the action of the light; in the presence of hydrochloric or of nitric acid it is responsive to that action. On the 17th of November, 1868, for example, the pure vapour was admitted into the experimental tube until a depression of 2 inches of the mercury column was observed. A powerful light was permitted to act for twelve minutes upon the vapour, but no action was observed. A quantity of air which had passed through aqueous hydrochloric acid was then admitted into the tube. Six minutes subsequent action of the light developed a cloud of considerable density. Toluol and other substances might here be mentioned in further illustration of this mode of decomposition. But I pass over hundreds of these earlier experiments which were made chiefly to instruct myself and to secure me from error. Some definite results will be given further on.

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* It was certainly due to a residue of the previous charge.

§ IX.

I have now to introduce, though only for partial treatment, a subject which might with advantage be kept isolated, but which is so mixed up with my notes of 1868 as to be inseparable from the descriptions of chemical action which they contain. I refer to the blue colour always exhibited at the birth of clouds obtained from small quantities of vapour in the case of active substances, and often from large quantities in the case of substances of slow decomposition. The first distinct record of this appearance occurs in my notes for the 10th of October, 1868. On the 9th I had been engaged upon the iodide of allyl with reference to its interaction with hydrochloric acid. Small quantities only of the vapour had been employed; and it was found that when the acid was fresh and strong the action was vigorous, that it declined in energy as successive charges of dry air were sent through the acid, becoming vanishingly feeble on the fifth filling of the experimental tube.

On the morning of the 10th the tube used on the preceding day was washed with distilled water, and swept out by a current of dry air. A mixture of air and hydrochloric acid was then sent into it, no vapour of any kind being employed. When the light first passed through it, and for some time afterwards, the experimental tube appeared perfectly empty. Slowly and gradually, however, upon the condensed beam a cloud was formed which passed in colour from the deepest violet, through blue, to whiteness. To this record of my note-book the remark is added, "connect this blue with the colour of the sky."

In fact it was impossible to avoid seeing the relationship of both. Previous to this entry the blue had attracted my attention. It was unfailing it its appearance when the action was slow. The blue colour was in all cases the herald of the denser actinic cloud. I took a pleasure in developing it in connexion with general actinic action, and in determining whether in all its bearings and phenomena the blue light was not identical with the light of the sky. This to the most minute detail appears to be the case. The incipient actinic clouds are to all intents and purposes pieces of artificial sky, and they furnish an experimental demonstration of the constitution of the real one.

Reserving the fuller discussion of the subject for a subsequent paper, it may be stated in a general way that all the phenomena of polarization observed in the case of skylight are manifested by these blue actinic clouds; and that they exhibit additional phenomena which it would be neither convenient to pursue, nor perhaps possible to detect, upon the actual firmament. They enable us, for example, to follow the growth and modification of the phenomena of polarization from their first appearance in the barely visible blue, to their final extinction when the cloud has become so coarsely granular as no longer to scatter polarized light.

These changes, as far as it is now necessary to refer to them, may be thus described.

1°. The incipient cloud, as long as it continues blue, discharges polarized light in all directions, but the direction of *maximum* polarization is at right angles to the direction of the illuminating beam.

2°. As long as the colour of the cloud remains distinctly blue, the light discharged from it normally is *perfectly polarized;* this light may be utterly quenched by a Nicol's prism, the cloud from which it issues being caused to disappear. Any deviation of the line of vision from the normal enables a portion of the light to reach the eye in all positions of the prism.

3°. The plane of polarization of the perfectly polarized light is parallel to the direction of the illuminating beam. Hence a plate of tourmaline with its axis parallel to the beam stops the light, and with its axis perpendicular to the beam transmits it.

4°. A plate of selenite placed between the Nicol and the cloud shows the colours of polarized light, and as long as the cloud continues blue these colours are most vivid in the direction of the normal.

5°. The particles of the incipient cloud are immeasurably small, but they gradually grow in size, and at a certain period of their growth cease to discharge perfectly polarized light. For some time afterwards the light that reaches the eye, when the Nicol is in its position of minimum transmission, is of a magnificent blue colour. It is called in the following pages the *residual blue.*

6°. Thus the waves *that first feel the influence of size,* both at the minor and major polarizing limits of the growing particles, are the smallest waves of the spectrum. These waves are the first to accept polarization and the first to escape from it.

7°. As the actinic cloud grows coarser in texture the direction of maximum polarization changes from the normal, enclosing an angle more or less acute with the axis of the illuminating beam.

8°. In passing from section to section of the same cloud the plane of polarization often undergoes a rotation of 90°. In the following pages this is designated as a change from positive to negative polarization, or the reverse.

§ X.

The experiments on benzol vapour and hydrochloric acid now to be described are of interest on optical rather than on chemical grounds. They were preceded by other experiments in which the vapour was mixed with nitric acid, and a minute residue of the latter lingering in the experimental tube may have influenced the results. The hydrochloric acid employed, moreover, was the commercial acid, and could not be regarded as pure. Thus though the decomposition of *a* vapour was certain, that it was not the pure vapour of benzol mixed with pure hydrochloric acid gas may be taken for granted. Indeed other experiments executed with the pure acid reduced the action to nil.

Dry air charged with the benzol vapour was permitted to enter the tube till a depression of *one inch* of the mercurial column was obtained; half an atmosphere of air charged with hydrochloric acid was then added. The action of light on this mixture was very powerful. The tube was for a moment optically empty, but its transparent contents were immediately shaken into a dense and luminous cloud. The normal polarization was here feeble, the oblique strong; the selenite colours in the former case were weak, in the latter brilliant. When the line of vision was transverse, the colours seemed mainly limited to red and green.

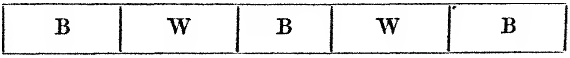
The tube was swept with dry air and exhausted. *Half an inch* of air and benzol vapour was admitted, and after it half an atmosphere of air and hydrochloric acid. A fine blue colour soon appeared, and as long as it continued the direction of maximum polarization was along the normal. But a luminous white cloud was rapidly generated, the normal polarization becoming feeble and the oblique strong. The distant end of the cloud, however, continued blue, and in passing from it to the white cloud the plane of polarization changed 90°.

The tube was again exhausted, and *a quarter of an inch* of air and benzol vapour was permitted to enter it, followed by a quarter of an atmosphere of air and hydrochloric acid. The incipient cloud showed an exceedingly fine blue, the polarization along the normal being a maximum. The cloud gradually thickened at the centre, and finally the polarization there disappeared. As before, when the normal polarization became feeble the oblique became strong.

The tube was once more cleansed and *one-tenth of an inch* of air and vapour was admitted, followed by one-tenth of an atmosphere of hydrochloric acid and air. The blue of the incipient cloud was here superb, and it lasted longer than in the last case. The selenite tints produced by the normally polarized light were exceedingly brilliant; but they faded gradually as the cloud passed from blue to whitish blue. At the centre of the cloud the normal polarization first fell to nil and then reappeared, having changed, however, from positive to negative, the two ends remaining as before. The influence of attenuation on the production of the blue colour is here strikingly exemplified.

The tube containing the benzol vapour was again cleansed and exhausted, and the last experiment was repeated. That is to say, one-tenth of an atmosphere of the air and vapour was mixed with one-tenth of an atmosphere of hydrochloric acid. After ten minutes' action the actinic cloud was found divided into five segments, alternately blue and white. Every two adjacent segments of the cloud were oppositely polarized, being divided from each other by a section of no polarization. The rectangle (fig. 2) represents the several divisions of the cloud; the letters B and W denoting the blue and white segments respectively. The transverse lines represent the neutral sections.

Fig. 2.



On the 9th of December, 1868, some experiments were made with the nitrite of butyl which merit a passing notice.

Atmospheric air was permitted to bubble through the nitrite until the experimental tube was quite filled with the mixture. Fifteen minutes' exposure produced a very slight action, an exceedingly scanty and coarse precipitate being formed. When due care is taken the action entirely disappears.

1 inch of the mixed air and vapour was now admitted into the experimental tube, and after it half an atmosphere of air which had bubbled through aqueous hydrochloric acid. The instant the beam passed through the experimental tube an intensely white cloud was precipitated.

The tube being cleansed, one-tenth of an inch of the nitrite and air, followed by one-tenth of an atmosphere of air and hydrochloric acid, was sent into it. The blue of the incipient cloud was in this instance perfectly superb. The polarization at right angles to the beam was perfect, and the selenite colours exceedingly vivid. As the cloud thickened the polarization along the normal disappeared, but it became strong obliquely. Two neutral points were observed *by oblique vision* in the case of this cloud. This effect is not uncommon.

The tube was withdrawn from the light for six minutes; on reexamination the cloud was found to have lost its beauty of form; and now the cloud-centre, by normal vision, polarized the light in a plane opposite to that of the two ends.

Twelve bubbles of the air and nitrite vapour were then sent into the exhausted experimental tube, and after them thirty-six bubbles of air and hydrochloric acid; several minutes' exposure produced no action. 3 inches of hydrochloric acid were then added, and the same superb blue as that noticed in the last experiment soon made itself manifest. It faded gradually as the cloud became more dense, and finally merged into whiteness.

The mixture of nitrite of amyl and hydrochloric acid was also examined in small quantities; but though the blue was fine, it had not the splendid depth and purity of the colour obtained with the nitrite of butyl.

§ XI.

The whole of the autumn of 1868 was devoted to the investigation from which I have taken the foregoing brief extracts. During this period 100 different substances must, I think, have been subjected to examination, and in the case of many of them the experimental tube must have been exhausted and refilled from 50 to 100 times. In some instances, indeed, the largest of these numbers falls considerably short of the truth. For a time I had no notion of the delicacy of the inquiry, nor of the caution required to prevent the action of infinitesimal residues and impurities from being mistaken for the decomposition of substances really inert. The necessity of thoroughly cleansing, or renewing, every tube and every stopcock, on passing from one substance to another, became gradually apparent. Water, alcohol, caustic potash, and acids were successively employed to cleanse the experimental tubes; but the method found most convenient, and that finally adopted, consists in the thorough lathering and sponging out of the tubes with soft soap and hot water, and the flooding of them with pure water afterwards. They are then dried with clean towels, and finally polished by the passing to and fro within them, by means of a ramrod, of a clean silk handkerchief. The stopcocks are cleansed by suitable brushes; fresh cocks, a fresh tube, and a fresh plug of asbestos being employed for each fresh substance.

From the draft of the present memoir, written last February, I take a few notes indicative of the difficulties caused by small impurities. Wishing to set my mind at rest with regard to nitric acid and hydrochloric acid, I operated for a time upon these substances unmixed with any vapour. 15 inches of air which had been permitted to bubble through aqueous nitric acid were sent into the experimental tube. The decomposing beam was first sent through a stratum of the liquid acid an inch in thickness. It screened the vapour effectually; no visible decomposition was produced. In this case, at the beginning of the experiment, there were a few scattered particles in the tube.

The cell containing the liquid acid was removed, and a minute afterwards a delicate blue colour began to shed itself among the floating particles. It augmented in intensity for five minutes, but during that time it could be entirely quenched by the Nicol, the particles floating in the blue being left intact.

These floating particles (mechanically carried in) extended only about 6 inches down the experimental tube. Beyond them was a streak of fine actinic blue perfectly polarized, and beyond this again a dusky grey cloud, which showed no trace of polarization.

After ten minutes' action the cloud had assumed a fair density, but it suggested doubts whether it was due purely to the nitric acid or to the interaction of the acid and some accidental impurity. The experiment was repeated four times with substantially the same result. In all cases the beam when passed through the liquid acid proved powerless; but always on the removal of this screen, or on displacing it by a cell of water, an action was manifested. To all appearance the nitric acid alone generated an actinic cloud.

The experiments, however, did not quite set my mind at rest. The tube was cleansed and the stopcocks heated to redness. When subsequently exposed the nitric acid required a much longer time to develope a cloud. After five minutes' exposure with no cell interposed the faintest blue cloud was visible. After ten minutes' exposure the cloud, at first seen with difficulty, was evident for some distance down the tube. By the complete removal of residues and by strict attention to the cerate employed to make the tubes air-tight, the action thus lessened was caused finally to disappear. In each of the experiments with nitric acid recorded in the following pages the acid itself was first tried, and not until its perfect visible inertness had been proved was it permitted to mix with the vapour.

I also wished to set my mind at rest regarding the action of hydrochloric acid. Several experimental tubes were sponged with soap and hot water, washed with alcohol, and finally flooded with hot water. They were then thoroughly dried and mounted. On a first trial most of them showed a feeble actinic action, which on a second trial usually disappeared. In one case the light generated a fine blue cloud which stretched throughout the entire length of an experimental tube 3 feet long. One whitish spot only of the cloud discharged imperfectly polarized light. The cloud could be utterly quenched by the Nicol, with the exception of a small patch of residual blue about 2 inches long, which was left curiously suspended in the general darkness of the tube.

On thoroughly cleansing with dry air the tube containing the cloud, and trying the acid a second time, an exposure of twenty minutes was found to produce no action. This and many other similar experiments demonstrate the inertness of pure hydrochloric acid. The inert acid of the foregoing experiment was permitted to remain in the experimental tube all night. Next morning, when the beam was permitted to play upon it, a blue streak became visible in less than a minute. In ten minutes the tube was filled with a delicate cloud. This was an almost every-day occurrence at the time here referred to. There must have been something in this tube in the morning which was not there on the preceding night. An infinitesimal residue had crept out of the stopcocks, or the hydrochloric acid had acted on the cerate employed to render the tube air-tight.

And here I would allude in passing to an effect which at a future stage of this inquiry will be found suggestive of the mechanism by which the complex cloud-forms are produced. I touched the top and bottom of the experimental tube for a moment with my two fingers; the cloud, which was of exceeding lightness, immediately showed responsive convection. It was wonderfully sensitive to the slightest local change of temperature. Once started in this simple way the motions of the cloud went on, and ended in the development of a splendid cloud-figure.

The influence of a minute residue is also strikingly illustrated by the following fact. 15 inches of mixed hydrochloric acid and air, exposed for fifteen minutes to a powerful beam, showed not the slightest trace of action. A small pellet of bibulous paper, not half the size of a pea, was moistened with the iodide of allyl. I held the pellet between my fingers till it became almost dry, then inserted it into a connecting piece, and sent a little air over it into the experimental tube. On stopping the flow of air a blue cloud began to form immediately, and in five minutes the rich colour had extended quite through the experimental tube. This cloud was 3 feet long and discharged a good body of light, but for some minutes it could be completely quenched by the Nicol. At the end of fifteen minutes a white massive cloud filled the experimental tube. Considering the amount of matter concerned in the production of this nebula, it seemed like the development of a cloud-world out of nothing.

But this is not all. The pellet of bibulous paper was removed, and the experimental tube was cleansed by allowing a current of dry air to sweep through it. *The current passed through the connecting piece in which the pellet of bibulous paper had rested.* The supply of air was at length cut off and the experimental tube exhausted. 15 inches of hydrochloric acid were sent into the tube through the same connecting piece. It is here to be noted, 1°, that the whole quantity of iodide of allyl absorbed by the pellet was exceedingly small; 2°, that I had allowed almost the whole of this small quantity to evaporate; 3°, that the pellet had been cast away and the tube in which it had rested had been rendered the conduit of a strong current of pure air. It was such a residue as could linger after all this in the connecting piece that was carried by the hydrochloric acid into the tube, and there acted on by the light.

A minute after the ignition of the lamp chemical action declared itself by the formation of a faint cloud. It appeared first at the focus. In a couple of minutes more a faint blue, perfectly polarized along the normal, filled the anterior portion of the tube. The blue also extended from the place of most vigorous action down the tube. An amorphous cylinder of cloud soon filled the first 10 inches of the tube, and pushed gradually down it. It was followed by a complicated cloud-figure, and it again by a vase-shaped nebula fainter than either. At the end of fifteen minutes a body of light, which, considering the amount of matter involved, was simply astonishing, was discharged from the cloud. In one position of the Nicol this cloud was a salmon-colour, in the other a blue-green. When a plate of tourmaline, with its axis parallel to the beam, was passed along in front of the cloud, at some places it showed a particularly vivid blue-green. When placed perpendicular at these places, the field of the crystal was a yellow-green.

I doubt whether spectrum analysis itself is competent to deal with more minute traces of matter than those revealed by actinic decomposition. I think it probable that if the weight of the cloud formed in this experiment were multiplied by trillions it would not amount to a single grain. Bodies placed behind it were seen undimmed through the cloud. The flame of a candle suffered no sensible diminution of its light. It was easy to read through the cloud a page which the cloud itself illuminated. In fact the cloud was a comet's tail on a small scale. It proved that matter of almost infinite tenuity is competent to shed forth light of similar quality, and in far greater quantity than that discharged by the tails of comets*.

These facts render the statement intelligible that even when all reasonable precautions appear to have been taken it is not easy to escape every trace of chemical action on first charging the experimental tube even with an inert substance. In my earlier experiments, when distilled water only was employed to cleanse the tube, the first experiment with air alone was sure to develope an actinic cloud of a beautiful fern-leaf pattern. And even now, after the most careful employment of the soft soap and hot water, the first charge of pure nitric, or of pure hydrochloric acid often developes a blue and exceedingly delicate actinic cloud. As regards the optical question, these irregular clouds exhibit some of the finest effects. One additional fact will illustrate a class of disturbances already touched upon. Pure nitric acid had been proved over and over again to exhibit no visible action; but after having demonstrated its inertness, a case occurred where it produced rather dense actinic clouds five times in succession. Indeed there seemed to be no end to their possible development. The only thing to which this change from inertness to activity could be ascribed, was a change in the cerate used to render the ends of the tube air-tight. On examination it was found that the infinitesimal effluvia yielded by the new cerate to the nitric acid was the sole cause of the anomaly. Nitric acid, then, produces no actinic cloud; hydrochloric acid produces no actinic cloud; air passed throughout potash and sulphuric acid produces no actinic cloud, no matter how powerful or how long-continued the action of the light may be.

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* The action here referred to has been since developed into a formal hypothesis of cometary phenomena. I shall return to the subject.

I hoped during the present year to be able to go over again a vast amount of ground rendered debateable by the discovery of such irregular actions as those here recorded. An accident in the Alps has unfortunately disqualified me from doing this. But as I find that ardent workers have already entered this new field of inquiry, I think it right to lay before the Society this first part of my researches. I omit not only descriptions of the deportment but even the names of the vast majority of the substances with which I have experimented; confining myself to eight or ten closely examined and well-established cases of actinic decomposition, and putting aside for reconsideration all such matters as might vainly occupy the attention of the Society.

§ XII.

The vapours of the substances mentioned in this section were sent into the tube in the manner described in § III. They were mixed, in the proportions stated, with air which had been permitted to bubble through aqueous nitric acid, and the effect produced by exposure to the condensed beam of the electric lamp is in each case described.

TOLUOL ($C_7 H_8$):—A transparent colourless liquid.

Contents of experimental tube.

I. Air with toluol vapour . . . 1 inch; then
Air with aqueous nitric acid . 15 inches.

On igniting the lamp the experimental tube was optically empty.

After thirty seconds the track of the beam through the experimental tube became blue; the blue was about as pure as that of an ordinary cloudless sky in England. After two minutes the colour began to change to a whitish blue.

The light discharged normally by the blue cloud continued to be perfectly polarized for four minutes after the first appearance of the cloud. A rich residual blue was afterwards observed when the Nicol was in its position of minimum transmission.

At the end of ten minutes the residual colour was no longer blue, but bluish white. Hence the light which first exhibited perfect polarization, and which first escaped from perfect polarization, was blue.

At the end of fifteen minutes a very beautiful cloud-figure was developed. The denser portions of the cloud were very luminous.

II. Air and toluol vapour . . . 8 inches; then
Air and aqueous nitric acid . . 8 inches.

The experimental tube was optically empty for a moment at starting, but the action was so rapid that in two or three seconds the tube was filled with a heavy cloud. At the beginning the colour of the cloud was blue. The incipient cloud which whirled round the beam discharged for two or three seconds perfectly polarized light; but the perfection of the polarization ceased almost immediately.

The cloud for a time was divided from beginning to end into two longitudinal lobes, separated from each other by an apparently empty space about a quarter of an inch wide. When the cloud was looked at *obliquely* in a vertical plane, one of these lobes was found to polarize the light positively, the other negatively. In passing from the one to the other the selenite tints were reversed.

The quantity of light scattered by this cloud was very considerable; it brightly illuminated the walls and ceiling of the laboratory. As the cloud became denser, the central empty space, which at first divided it into two lobes, gradually disappeared.

Looked at *normally* the polarization of the one half of this cloud was positive, and that of the other negative. Between the two a neutral point existed. The oblique polarization of the dense cloud was strong.

III. Air and aqueous nitric acid . . 1 inch; then
Air and toluol vapour . . . . 15 inches.

The action here was not so prompt as in the last case, nor was the cloud generated so dense. The cloud-particles, moreover, were coarser, and showed iridescent colours. Still the chemical action of the light was distinct and copious.

Looked at normally, a portion of this light was salmon-coloured. The selenite bands appeared to be of this colour, and its complementary greenish tint.

BISULPHIDE OF CARBON (CS_2):—A transparent colourless liquid.

Contents of experimental tube.

I. Air and bisulphide-of-carbon vapour . 1 inch; then
Air and aqueous nitric acid . . . . 15 inches.

On starting the experimental tube was optically empty; but in a minute afterwards the track of the beam became blue, which was particularly deep and rich in the middle portion of the beam.

The blue light discharged normally was perfectly polarized, but the least deviation of the line of vision from the normal caused a portion of the light to pass through the Nicol.

The growth of this cloud and the gradual brightening and subsequent whitening of the blue were very instructive.

The light discharged normally remained perfectly polarized for seven minutes after the first appearance of the blue colour. A faint but rich residual blue was seen for some time afterwards.

The selenite colours were exceedingly vivid with this cloud. When, moreover, a plate of tourmaline was placed with the crystallographic axes parallel to the beam it was black; placed at right angles to the beam, a large portion of the light of the cloud was transmitted.

After ten minutes' exposure the cloud itself still showed a distinct trace of blue. The residual blue was then particularly rich and pure. After fifteen minutes the selenite colours were still vivid, though the cloud had then become greyish white.

II. Air and bisulphide-of-carbon vapour . . . . 8 inches; then
Air and aqueous nitric acid . . . . 8 inches.

When the lamp was ignited the experimental tube was found optically empty; but the chemical action commenced three-quarters of a minute afterwards, the convergent beam assuming the appearance of a fine blue spear. The action was more energetic than in the last case, though the battery was sensibly sinking in power.

The light discharged normally remained perfectly polarized for two minutes after its first appearance. The selenite colours were rich and vivid, and the tourmaline in its two characteristic positions showed the same striking contrast observed in the last experiment.

In five or six minutes the entire tube was filled with cloud, the residual blue being then perfectly gorgeous.

III. Air and aqueous nitric acid . . . . 1 inch; then
Air and bisulphide-of-carbon vapour . 15 inches.

The tube was optically empty when the lamp was ignited. The chemical action soon commenced, a series of layers of blue cloud stretching through the entire tube. The action was less energetic than in the former cases, this being due in part to the sinking of the battery. The light discharged normally remained perfectly polarized for ten minutes.

CYANIDE OF ETHYL (C_2H_5Cn):—A transparent colourless liquid.

Contents of experimental tube.

I. Air and cyanide-of-ethyl vapour . . . . 1 inch; then
Air and aqueous nitric acid . . . . 15 inches.

The tube was optically empty when the lamp was ignited. In a minute and a half the track of the beam became distinctly blue. The blue light was at first perfectly polarized.

The beam was crossed by a series of disks, which were denser and more whitish than the general mass of the cloud. The extinction of these disks by the Nicol was curious and interesting.

The growth of the particles in this case was so slow that the light emitted normally continued perfectly polarized for thirteen minutes after the first appearance of the cloud. A faint residual blue was afterwards developed.

II. Air and cyanide-of-ethyl vapour . . 8 inches; then
Air and aqueous nitric acid . . . 8 inches.

The experimental tube was optically empty for two seconds after the starting of the lamp; a fine blue colour was then observed upon the upper boundary of the convergent beam. The light emitted normally did not remain perfectly polarized for more than half a minute. In two minutes the tube was filled with cloud, the anterior portion being white, and the posterior portion bluish. The posterior portion could be utterly extinguished by the Nicol long after the anterior portion had began to show a residual blue. Passing with the Nicol from the densest to the least dense portion of the cloud, the residual colour changed from a bright blue through a gorgeous Alpine skyblue to absolute extinction.

Looked at obliquely in a vertical plane, the two semicylinders into which the cloud was longitudinally divided were found in opposite states of polarization.

This was a truly splendid action. The chemical effect was exceedingly vigorous, and the cloud-form fine.

III. Air and aqueous nitric acid . . . . . 1 inch; then
Air and cyanide-of-ethyl vapour . . . . 15 inches.

On starting the light the experimental tube was found optically empty. In a quarter of a minute, however, the track of the beam, which previously had been invisible, was coloured blue. The chemical action appeared to exert itself with almost the same intensity throughout the entire length of the experimental tube.

For a brief interval the whole of the light emitted normally was polarized. Then for a time about three-fourths of the length of the cloud could be quenched by the Nicol, the remainder showing a fine residual blue. This sank from a brilliant azure at the densest portion of the cloud through deep rich blue to entire extinction.

The selenite bands were exceedingly vivid long after this cloud had ceased to be blue. An immense quantity of polarized light was discharged normally, even after the cloud had become white. Placed between the cloud and the eye, a plate of tourmaline with its axis parallel to the beam was practically black, while when placed across the beam a bright green light was copiously transmitted.

In one position of the Nicol this cloud was yellow, in the rectangular position it was blue. Here also the chemical action was very vigorous, and the cloud-form very fine.

BENZOL (C_6H_6):—A transparent colourless liquid.

Contents of experimental tube.

I. Air and benzol vapour . . . . . . 1 inch; then
Air and aqueous nitric acid . . . 15 inches.

Nitric acid is known to form with benzol nitro-benzol, a liquid possessing a high boiling-point. But though the mixed vapours were allowed to remain together for ten minutes before starting the lamp, when the beam passed through the experimental tube it was optically empty.

Chemical action commenced a quarter of a minute after the ignition of the lamp; a very delicate blue light was then discharged from the beam, the centre of which was particularly bright and transparent. The light emitted normally remained perfectly polarized for one minute.

I looked through the Nicol towards the cloud. For a minute it was absolutely extinguished. Continuing to look in the same direction the residual colour appeared, and passed from a rich deep violet to a hard whitish blue. It was exceedingly interesting to watch the growth and change of the residual colour. At a certain period of its existence it rivalled the richest blue of the spectrum.

In two or three minutes the anterior portion of the tube was filled by a thick cloud generated by the beam. The cloud rapidly diminished in density as the more distant end of the tube was approached. It was composed of two longitudinal lobes, which, looked at obliquely in a vertical plane, discharged light polarized in planes at right angles to each other.

When the cloud was looked at normally, the line of vision being horizontal, on one side of the centre the polarization was positive, on the other side negative. Moved to and fro across the neutral section, the sudden expansion and contraction of the selenite bands was very curious.

After twenty minutes' action the neutral section was abolished, and the normal polarization (now feeble) became the same throughout the entire length of the cloud.

II. Air and benzol vapour . . . . . . 8 inches; then
Air and aqueous nitric acid . . . 8 inches.

On starting the light the tube was not optically empty, but crowded with particles. Through them the beam appeared to force its way like a spear, bringing down upon itself a finer cloud, which soon swathed and masked the coarser spherules.

This experiment was many times repeated, but it was found impossible to bring the benzol and nitric acid together in the quantities here employed without the formation of a crowd (cloud would hardly be the word) of coarse particles. Chemical action had manifestly set in without the intervention of the light.

The chemical action without light appearing to depend on the quantity of benzol vapour and nitric acid present, I varied that quantity. When 2 inches of each were admitted into the experimental tube, no particles were seen when the lamp was ignited. A quarter of a minute after the starting of the lamp the track of the beam became blue. This light remained perfectly polarized for a minute. In three minutes a dense cloud had filled the tube. In the two rectangular positions of the Nicol the cloud exhibited a salmon-colour and a hard bluish greenish white.

When the quantities of the two vapours were 4 inches each, there were no particles in the tube when the lamp was ignited. No doubt the substances were ready to attack each other, and in less than a quarter of a minute the beam precipitated the attack. The action was exceedingly vigorous. For a moment, and only for a moment, the polarization was perfect. In less than a minute the rapid thickening of the cloud and the quick growth of its particles abolished almost all traces of polarization.

When the quantities were 5 inches to 5, particles were found in the experimental tube on starting; and the same occurred with all greater quantities. When, for example, the quantities were 6 inches to 6, 10 inches to 10, or 15 inches to 15, there were invariably particles. In some of the experiments it seemed as if the chemical attractions were satisfied before the light started, the subsequent action being very feeble. In other instances this did not seem to be the case; for though the particles existed, the spaces between them became immediately filled by a fine dense cloud when the beam passed among them. In some instances the precipitation was exceedingly sudden and copious. Mr. COTTRELL, who has assisted me with zealous intelligence in these experiments, thus describes one result. "Some coarse particles were in the tube on commencing, and these, when the light was started, remained perfectly tranquil for a moment; but after an instant's pause the beam appeared to pierce like a ploughshare the cloud it had formed, throwing right and left of it heaps of precipitated particles. This cloud filled the tube almost instantaneously."

To give the benzol and nitric acid more time to act upon each other, on Tuesday evening, the 16th of February, 2 inches of each were admitted into the experimental tube, and allowed to remain there through the night. Sixteen hours subsequently the beam was permitted to act upon the mixture. The tube which contained it was to all appearance absolutely empty; no particles whatever had formed during the night. In a quarter of a minute after starting the lamp chemical action began, and in five minutes the beam had filled the tube with a dense cloud.

The deportment of benzol may be thus summed up:—

Benzol.	Nitric acid.	
2 inches.	2 inches.	No particles; strong actinic action.
4 ,,	4 ,,	No particles; very strong actinic action.
5 ,,	5 ,,	Particles; dense actinic cloud precipitated among them.
6 ,,	6 ,,	,, sometimes ,, ,,
10 ,,	10 ,,	,, sometimes ,, ,,
15 ,,	15 ,,	,, sometimes ,, ,,
1 ,,	15 ,,	No particles; strong actinic action.
15 ,,	1 ,,	Particles.

IODIDE OF ALLYL (C_3H_5I):—A transparent yellowish liquid.

Contents of experimental tube.

I. Air and iodide-of-allyl vapour . . . 1 inch; then
Air and nitric acid . . . . 15 inches.

The beam traversed the tube for an instant as if the space within it were a vacuum, but in the fraction of a second a brilliant shower of particles fell upon the beam. The cloud became coarse immediately. The action occurred in the anterior part of the tube, the most distant part being apparently free from action. This is quite different from the deportment of iodide of allyl and hydrochloric acid. On reversing the tube another cloud, of finer texture than the first, was precipitated. The cloud assumed, beautiful and curious forms. The inner portions of its two longitudinal lobes were shaped like screws; they moreover rotated like screws, moving as if they were pushed mechanically into the mass of cloud in front of them. The whole effect was very fine, and the action extremely vigorous. As might be expected from the density of the cloud, the normal polarization was almost *nil.*

II. Air and iodide-of-allyl vapour . . 8 inches; then
Air and nitric acid . . . . 8 inches.

The tube was optically empty at first, but the action, though not so brilliant as in the last case, was very prompt and energetic. A very coarse cloud was rapidly formed throughout the entire tube, upon the bottom of which the particles appeared to fall in showers.

The cloud having apparently ceased to thicken, the lamp was suspended. On its accidental reignition a fine cloud, dense and luminous, was suddenly precipitated among the coarser particles. On suspending the lamp the finer cloud vanished, but the coarser particles remained. On reignition the fine white cloud was precipitated as before, entirely masking the coarser one by its superior density and closeness of texture. This action was repeated several times in succession.

Allowing the *parallel* beam from the lamp to act for a time upon the cloud, on changing it to a convergent one the superior intensity of the light immediately caused a fine, dense, and luminous precipitation. By rendering the beam alternately parallel and convergent, this action could be reproduced several times in succession.

III. Air and nitric acid . . . . 1 inch; then
Air and iodide-of-allyl vapour . . 15 inches.

Immediately after the starting of the lamp the action commenced, and spread through the entire tube in less than two minutes. The falling of the particles in vertical showers occurred here also.

After it had acted for a time the lamp was extinguished, and the tube was permitted to remain quiescent for an hour. On reigniting the lamp the tube appeared to be quite empty. The cloud that had previously filled it had entirely disappeared. Half a minute's action of the beam brought down upon it copious precipitation, a revival of the action occurring afterwards throughout the entire tube.

IODIDE OF ISOPROPYL ($CH(CH_3)_2I$).

Contents of experimental tube.

I. Air and iodide-of-isopropyl vapour . . 1 inch; then
Air and nitric acid . . . . 15 inches.

After a moment of apparent emptiness a very splendid action set in. A cloud of exceeding brightness suddenly filled the space occupied by the convergent beam. The light scattered by this anterior cloud was very powerful. At the distant end of the tube the action was feeble. I reversed the tube; but the precipitation here was by no means so prompt and copious as at the other end, into which the vapour had been evidently swept by the air and nitric acid.

The lamp was suspended for about five minutes; on reigniting it a coarse cloud was found within the tube; but instantly through this coarseness a finer cloud of exquisite colour, luminousness, and texture was shed. A violent whirling motion was set up at the same time. The longitudinal lobes in this case were very curiously found.

II. Air and iodide-of-isopropyl vapour . . . 8 inches; then
Air and nitric acid . . . . 8 inches.

Tube optically empty, but in the fraction of a minute a shower of very coarse particles had fallen upon the beam. They augmented up to a certain point and then appeared to diminish. The reversal of the tube caused fresh precipitation. The rendering of the beam more convergent also caused augmented precipitation, but nothing so fine as in the last experiment. The action, indeed, was altogether inferior to the last in point both of beauty and of energy.

I suspended the lamp for a few minutes; on restarting it the tube appeared empty, but in a moment a cloud much finer than that at first obtained was precipitated on the beam. Curious masses of particles gushed at irregular intervals upon the beam. On reversing the tube the action was decidedly finer than at first.

Thus, suspending the lamp after it has been acting for a time, the vapour during the period of suspension undergoes a change which enables it to fall as a finer and more visibly copious cloud than at the beginning of the action.

III. Air and nitric acid . . . . 1 inch; then
Air and iodide-of-isopropyl vapour . . . 15 inches.

Action commenced immediately, and in less than a minute the beam had filled the tube with an unbroken cloud. The beam was rendered parallel, and the action continued for eight minutes. The end nearest the light became rapidly empty, while in the distant half of the tube the particles fell in heavy showers. The whole tube subsequently became almost empty; the disappearance of the dense cloud first generated was very striking. It would appear as if after the first sudden precipitation evaporation had set in and restored the particles to the gaseous condition.

NITRITE OF AMYL ($C_5H_{11}ONO$):—A transparent yellowish liquid.

Contents of experimental tube.

I. Air and nitrite-of-amyl vapour . . . . . 1 inch; then
Air and nitric acid . . . . 15 inches.

The tube was optically empty at starting; action commenced in half a minute, the cloud particles formed being very coarse. In four minutes the anterior two-thirds of the tube were filled with a very coarse cloud, the remaining third with a finer one. The whole rotated round a longitudinal axis, and the finer portion was rolled into a curious spiral form, and was tinged throughout with iridescent colours. The normal polarization was almost *nil*, except in the finer part of the cloud, which was slightly blue.

II. Air and nitrite-of-amyl vapour . . . . 8 inches; then
Air and nitric acid . . . . 8 inches.

The tube was optically empty for an instant only, a dense precipitation occurring immediately upon the concentrated beam. The distant part of the tube, however, was but scantily filled, showing the sifting action of the nitrite vapour. On reversing the tube copious precipitation occurred. After ten minutes' exposure the particles tended to settle at the bottom of the tube.

III. Air and nitric acid . . . . 1 inch; then
Air and nitrite-of-amyl vapour . . . . 15 inches.

The tube was optically empty only for an instant; as in the last experiment, a dense cloud was immediately precipitated on the cone of rays. Here also the distant end of the tube was protected by the vapour in front.

In all these cases the action was distinctly less energetic than when the nitrite vapour mixed with air alone was exposed to the light; and very much less energetic than when hydrochloric acid was mixed with the vapour.

NITRITE OF BUTYL ($C_4 H_9 ONO$):—A transparent yellowish liquid.

This substance gives no sensible action with nitric acid; but with hydrochloric, as already mentioned, the action is vigorous and brilliant. Here are a few of the results.

Contents of experimental tube.

I. Air and nitrite-of-butyl vapour . . . . . 1 inch; then
Air and hydrochloric acid . . . . 15 inches.

The action began a quarter of a minute after starting, a very white and brilliant cloud forming upon the concentrated beam and quickly spreading throughout the tube.

II. Air and nitrite-of-butyl vapour . . . . 8 inches; then
Air and hydrochloric acid . . . . 8 inches.

The action began about half a minute after starting, a cloud of comparatively fine particles being precipitated in the cone of rays, while the distant part of the tube was filled with coarse particles. The cloud was coarser, and the action less energetic than in the last experiment.

III. Air and hydrochloric acid. . . . . . . 1 inch; then
Air and nitrite-of-butyl vapour . . . . 15 inches.

After four minutes' action a number of coarse particles had formed in the tube together with a faint scroll of cloud. The action was very feeble. For vigorous action with the nitrite of butyl the proportion of the acid to the vapour must be large.

The hydrochloric acid here employed was that ordinarily used by chemists in quantitative analysis. The same series of experiments was executed with *commercial* hydrochloric acid; the action in this case was distinctly more energetic than when the pure acid was employed.

HYDRIDE OF CAPROYL ($C_6 H_{11} O, H$):—A transparent colourless liquid.

Contents of experimental tube.

Air and hydride of caproyl	8 inches.
Air and nitric acid	8 inches.

The tube was optically empty at starting. In three quarters of a minute a blue cloud had formed throughout the tube. It remained perfectly polarized for three minutes; then became gradually white, discharging imperfectly polarized light. At the end of ten minutes a dense white cloud filled the tube.

§ XIII.

I thought it worth while, for the sake of bringing out the influence of vibrating period, to contrast the action of powerful foci of dark rays with the feeble foci produced by the convergence of the more refrangible rays of the spectrum. It is known that the dark calorific rays pass freely through a solution of iodine in bisulphide of carbon; such a solution was employed to hold back the luminous part of the electric beam. A cell containing ammonia sulphate of copper was employed to hold back the rays of low refrangibility and allow those of high refrangibility transmission. The destructive action of the ammonia sulphate in the calorific rays is well known. Its depth in the present case was such as to quench completely the red, orange, and yellow of the spectrum, but it allowed transmission to the violet and blue, and a small portion of the green. The vapours employed were mixed with the various acids mentioned.

Nitrite of amyl	8 inches.
Pure hydrochloric acid . .	8 inches.

The convergent beam of the lamp was sent through the cell containing the solution of iodine, and was permitted to act upon the mixed acid and vapour for ten minutes. The ammonia-sulphate cell was then introduced and the opaque solution removed. For an instant afterwards the tube was optically empty. Then a dense cloud was precipitated, which advanced like a moving share towards the most distant end of the tube. Within half a minute after the withdrawal of the opaque solution the tube was filled with cloud, which augmented in density for five minutes, when the experiment ceased. A repetition of the experiment yielded the same result.

Iodide of allyl	8 inches.
Nitric acid	8 inches.

Looked at for an instant after the vapour and acid had entered, with the white light of the electric lamp, the experimental tube was seen to be optically empty. The opaque solution was immediately introduced, and the vapour was subjected to the action of the dark rays for ten minutes.

The opaque solution was then removed for an instant, and the tube was seen to be optically empty. The strong calorific rays had produced no action.

The cell containing the blue liquid was then introduced; in less than half a minute the action became visible, and augmented rapidly. In three minutes a cloud stretched quite through the tube from end to end. The scattering of the blue light by the coarse particles of this cloud produced a very pretty effect.

Benzol	4 inches.
Nitric acid	4 inches.

Looked at for an instant after the admission of the vapour and acid the tube was optically empty. The opaque solution was introduced, and the invisible rays permitted to act for ten minutes. The solution was then removed, and the tube was examined for a moment with white light. It was optically empty. The blue liquid being interposed, visible action commenced $2\frac{1}{2}$ minutes afterwards*, and in ten minutes a cloud was formed throughout the tube. A repetition of this experiment confirmed the inaction of the calorific rays, and showed the action of the blue rays to be visible a minute after the introduction of the ammonia-sulphate cell.

Toluol	8 inches.
Nitric acid	8 inches.

Looked at for an instant after the admission of the vapour and acid, the tube was found optically empty. Ten minutes' action of the calorific rays produced no effect. The blue liquid was then interposed, and in two minutes a cloud was visible upon the feeble blue beam. At the end of ten minutes this cloud stretched throughout the tube.

Iodide of β propyl . . .	8 inches.
Nitric acid	8 inches.

The tube was optically empty at the commencement. At the end of ten minutes' exposure to the calorific rays the tube was also empty. The blue cell was introduced, but in two minutes after its introduction, no cloud appearing, the cell was removed for an instant. The action had begun, though the coarse particles of the actinic cloud were too sparsely distributed to be seen by the weak blue light. The experiment was repeated. As before, ten minutes' action of the calorific rays proved quite ineffectual. In one minute after the introduction of the blue liquid, no cloud being visible in the tube, the cell was removed. A crowd of particles were then seen upon the cone of light.

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* No doubt it had previously commenced, but it was invisible in the feeble light.

The cell was again introduced, and after three minutes again withdrawn. The particles had increased considerably. Seven minutes' action rendered them sufficiently numerous to be visible in the blue light. After ten minutes the coarse cloud was very plainly seen. The action was continued with white light after the removal of the blue liquid; it was scarcely more energetic than that produced by the blue rays.

Nitrite of butyl	1 inch.
Hydrochloric acid	15 inches.

Examined for a moment by white light the tube was optically empty. After ten minutes' exposure to the dark rays the tube was again examined by the white beam; it was still optically empty. The blue liquid was then introduced, and in a $\frac{1}{4}$ of a minute a long streak of cloud had formed. In $2\frac{1}{2}$ minutes a dense cloud had formed throughout the entire tube. An exceedingly delicate blue light, and at some parts a deep violet, was scattered by this cloud. After five minutes' exposure to the blue rays an intensely white cloud had formed, which completely filled the tube. The action here was very fine.

Bisulphide of carbon	8 inches.
Nitric acid	8 inches.

The tube was optically empty when the opaque solution was introduced; but after ten minutes' exposure to the calorific rays a faint blue tinge was observed, when the opaque solution was removed*. The experiment was abandoned, and the mixed vapour and acid were again introduced. At the beginning the tube was optically empty; after ten minutes' exposure to the calorific rays it was also empty. In two minutes after the introduction of the blue cell, a cloud became visible; it quickly increased, and after four minutes extended throughout the tube. After ten minutes' action a dense whitish-blue cloud filled the entire tube. The experiment was repeated twice with the bisulphide, with substantially the same result.

These experiments are quite conclusive as to the inability of the calorific rays to produce actinic clouds; they are the product of the more refrangible rays of the spectrum.

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* It is sometimes difficult to get the bisulphide into the tube without this blue tinge. It is certainly due to some impurity. With care it can be caused to disappear wholly.

XX. *Tables of the Numerical Values of the Sine-integral, Cosine-integral, and Exponential-integral. By* J. W. L. GLAISHER, *Trinity College, Cambridge. Communicated by Professor* CAYLEY, *F.R.S.*

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IT has for a long time been evident that the extension of the Integral Calculus would require the introduction of new functions; or, rather, that certain functions should be regarded as primary, so that forms reduced to dependence on them might be considered known.

Thus, in the evaluation of Definite Integrals, the three transcendents

$$\int_0^x \frac{\sin u}{u}\,du,\quad \int_\infty^x \frac{\cos u}{u}\,du,\quad \int_\infty^{-x} \frac{e^{-u}}{u}\,du,$$

called the sine-integral, the cosine-integral, and the exponential-integral, have become recognized elementary functions, and great use has been made of them to express the values of more complicated forms. They were introduced by SCHLÖMILCH to evaluate the integral $\int_0^\infty \frac{a\sin x\theta}{a^2-\theta^2}\,d\theta$, and several allied forms*, and denoted by him $\operatorname{Si} x$, $\operatorname{Ci} x$, $\operatorname{Ei} x$. ARNDT also employed them in a similar manner about the same time.

The first two functions had, however, previously received some attention from BRETSCHNEIDER, who appears to have been led to their consideration by their analogy with the logarithm-integral, as in a paper in the 17th volume of CRELLE's Journal he announces his intention of not only tabulating this integral, but also of forming "tabulas aliarum quarundam functionum, cum logarithmo integrali arcte junctarum." The Tables are published in the third volume of GRUNERT's 'Archiv der Mathematik und Physik' for 1843, and contain ten positive and negative values of the logarithm-integral, and ten values of the sine-integral and cosine-integral, besides tables of several other functions.

The exponential-integral was introduced in its present form by SCHLÖMILCH, though for all real values it is the same as the logarithm-integral $\operatorname{li} x = \int_0^x \frac{du}{\log u}$, the relation between the two forms being

$$\operatorname{li} e^x = \operatorname{Ei} x.$$

The logarithm-integral appears to have been first discussed by MASCHERONI†, and in 1809 a work was published by SOLDNER at Munich concerning its theory, which also contained a Table of its values. This Table is reprinted in DE MORGAN's 'Differential and Integral Calculus,' p. 662.

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* CRELLE's Journal, vol. xxxiii. p. 316.

† Referred to by BRETSCHNEIDER, CRELLE's Journal, vol. xvii. p. 257.

The sine-integral and cosine-integral occur in a Memoir by BIDONE in the Turin Transactions for 1812, where they are expanded in series, the same as those marked (1) on the next page.

From the moment of its introduction the logarithm-integral excited considerable interest, but it is only in the last twenty-five years that the other functions have become of importance. A complete list of all the memoirs in which these functions are considered is given by Professor BIERENS DE HAAN, on page 83 of his 'Supplément aux tables d'intégrales définies,' published in the tenth volume of the Transactions of the Royal Academy of Amsterdam; and in the second volume reference is made to several other works in a memoir by the same author.

Since 1845 the three integrals have been practically regarded as primary functions in the integral calculus; and how well suited they are to this purpose is evident from the success which has attended the labours of those analysts who have sought to reduce more complicated integrals to dependence on them.

Professor DE HAAN, in the fifth volume of the Amsterdam Transactions, has evaluated a very large number of integrals by means of them; and in the great Tables* of the same author there are given nearly 450 functions dependent for their evaluation on that of these integrals. Considering therefore their extreme importance as a means of extending the Integral Calculus, and the probable value of many of the integrals evaluated in physical inquiries, it seemed very desirable that they should be systematically tabulated, so as to be known, not only by convention, but in reality; and on this subject Professor DE HAAN has strongly expressed his opinion of the value of such Tables.

BRETSCHNEIDER, in the memoir previously cited, has computed $\text{Si}\,x$, $\text{Ci}\,x$, $\text{Ei}\,x$, $\text{Ei}(-x)$ for the values 1, 2, 3 . . . 10 of the argument to 20 places of decimals (except the values for $x=1$, which are extended to 35 places). This Table is reprinted by SCHLÖMILCH at the end of his 'Analytische Studien,' and a portion of it is quoted by the same author at the end of a paper in the thirty-third volume of CRELLE'S Journal.

The Tables given in the present paper are the following:—

Tables I., II., III., IV.—$\text{Si}\,x$, $\text{Ci}\,x$, $\text{Ei}\,x$, $\text{Ei}(-x)$ from $x=0$ to $x=1$ at intervals of ·01 to 18 places of decimals, with differences to the third order.

Tables V., VI., VII., VIII.—$\text{Si}\,x$, $\text{Ci}\,x$, $\text{Ei}\,x$, $\text{Ei}(-x)$ from $x=1$ to $x=5$ at intervals of 0·1 to 11 places of decimals, with differences to the third order.

Table IX.—$\text{Si}\,x$, $\text{Ci}\,x$, $\text{Ei}\,x$, and $\text{Ei}(-x)$ from $x=5$ to $x=15$ at intervals of unity, to 11 places of decimals.

Table X.—$\text{Si}\,x$ and $\text{Ci}\,x$ from $x=20$ to $x=100$ at intervals of 5, from $x=100$ to $x=200$ at intervals of 10, from $x=200$ to $x=1000$ at intervals of 100, and for several higher values of x to 7 places of decimals.

Table XI.—Maxima and minima values of $\text{Si}\,x$ to 7 places of decimals.

Table XII.—Maxima and minima values of $\text{Ci}\,x$ to 7 places of decimals.

In the course of the work BRETSCHNEIDER'S values have been verified as far as they coincided with those in the present paper, though from the great care he used, and the mode of verification he adopted, there was little doubt of their accuracy. One error was detected in Ei(−5), which should be ·00114 .... instead of ·00144 .... This has doubtless arisen in the final copying or printing.

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* Nouvelles tables d'intégrales définies, Leyden, 1867.

Expressed in series the functions are:

$$\left.\begin{aligned}
\text{Si}\,x&=x-\frac{1}{3}\cdot\frac{x^3}{1\,.\,2\,.\,3}+\frac{1}{5}\cdot\frac{x^5}{1\,.\,2\,.\,3\,.\,4\,.\,5}-\frac{1}{7}\cdot\frac{x^7}{1\,.\,2\ldots7}+\ldots\\
\text{Ci}\,x&=\gamma+\frac{1}{4}\log_e(x^4)-\frac{1}{2}\cdot\frac{x^2}{1\,.\,2}+\frac{1}{4}\cdot\frac{x^4}{1\,.\,2\,.\,3\,.\,4}-\frac{1}{6}\cdot\frac{x^6}{1\,.\,2\ldots6}+\ldots\\
\text{Ei}\,x&=\gamma+\frac{1}{4}\log_e(x^4)+x+\frac{1}{2}\cdot\frac{x^2}{1\,.\,2}+\frac{1}{3}\cdot\frac{x^3}{1\,.\,2\,.\,3}+\frac{1}{4}\cdot\frac{x^4}{1\,.\,2\,.\,3\,.\,4}+\ldots
\end{aligned}\right\}\quad .\ .\ (1)$$

γ being EULER's constant 0·5772156 ...

From these expressions it is evident that Si x, Ci x, and Ei x are connected by the relation

$$\text{Ei}(x\sqrt{-1})=\text{Ci}\,x+\sqrt{-1}\,\text{Si}\,x.$$

The logarithms are written as above to indicate that they are real when x is negative, or of the form $a\sqrt{-1}$. The logarithm-integral differs in this respect only from the exponential-integral, for

$$\text{li}\,e^x=\gamma+\frac{1}{2}\log_e(x^2)+x+\frac{1}{2}\cdot\frac{x^2}{1\,.\,2}+\frac{1}{3}\cdot\frac{x^3}{1\,.\,2\,.\,3}+\frac{1}{4}\cdot\frac{x^4}{1\,.\,2\,.\,3\,.\,4}+\ldots$$

The series in (1) are clearly always convergent, however large x may be.

The following series are easily obtained by integration by parts:

$$\left.\begin{aligned}
\text{Si}\,x=\frac{\pi}{2}&-\cos x\left\{\frac{1}{x}-\frac{1\,.\,2}{x^3}+\frac{1\,.\,2\,.\,3\,.\,4}{x^5}-\frac{1\,.\,2\ldots6}{x^7}+\ldots\right\}\\
&-\sin x\left\{\frac{1}{x^2}-\frac{1\,.\,2\,.\,3}{x^4}+\frac{1\,.\,2\,.\,3\,.\,4\,.\,5}{x^6}-\frac{1\,.\,2\ldots7}{x^8}+\ldots\right\}\\
\text{Ci}\,x=\ &\ \ \sin x\left\{\frac{1}{x}-\frac{1\,.\,2}{x^3}+\frac{1\,.\,2\,.\,3\,.\,4}{x^5}-\frac{1\,.\,2\ldots6}{x^7}+\ldots\right\}\\
&-\cos x\left\{\frac{1}{x^2}-\frac{1\,.\,2\,.\,3}{x^4}+\frac{1\,.\,2\,.\,3\,.\,4\,.\,5}{x^6}-\frac{1\,.\,2\ldots7}{x^8}+\ldots\right\}\\
\text{Ei}\,x=\ &\ \ e^x\left\{\frac{1}{x}+\frac{1}{x^2}+\frac{1\,.\,2}{x^3}+\frac{1\,.\,2\,.\,3}{x^4}+\frac{1\,.\,2\,.\,3\,.\,4}{x^5}+\ldots\right\}
\end{aligned}\right\}\quad .\ .\ .\ (2)$$

These series are ultimately divergent, though for values of x greater than unity they begin by converging, and when $x=17$, seven places of decimals are obtainable from them for Si x and Ci x.

Formulæ (1) were used for values where the argument was less than 16, formulæ (2) where it was greater.

Tables I. to IX. were calculated in the following manner. The denominators of the terms in the series (1) were first computed, the first 20 figures of which, as far as the 71st power, and the logarithms of their reciprocals, are given in the following Table.

Table of the Constants.

x.	First twenty figures of $x\Gamma(x+1)$.	$-\log\left\{x\Gamma(x+1)\right\}$.	x.	First twenty figures of $x\Gamma(x+1)$.	$-\log\left\{x\Gamma(x+1)\right\}$.
2	4	$\overline{1}$·397 940 008 7	37	509 258 864 375 374 766 71	$\overline{45}$·293 061 402 7
3	18	$\overline{2}$·744 727 494 9	38	198 748 594 637 308 422 46	$\overline{47}$·701 695 933 6
4	96	$\overline{2}$·017 728 767 0	39	795 517 401 166 700 290 98	$\overline{48}$·099 350 316 1
5	600	$\overline{3}$·221 848 749 6	40	326 366 113 299 159 093 73	$\overline{50}$·486 294 940 5
6	432 0	$\overline{4}$·364 516 253 2	41	137 155 359 113 971 609 14	$\overline{52}$·862 787 218 4
7	352 80	$\overline{5}$·452 471 423 5	42	590 102 569 456 209 557 38	$\overline{53}$·229 072 494 3
8	322 560	$\overline{6}$·491 389 489 6	43	259 785 631 172 507 493 24	$\overline{55}$·585 384 873 6
9	326 592 0	$\overline{7}$·485 994 457 7	44	116 963 949 290 691 745 79	$\overline{57}$·931 947 976 2
10	362 880 00	$\overline{8}$·440 236 967 1	45	538 299 993 894 660 875 52	$\overline{58}$·268 975 625 1
11	439 084 800	$\overline{9}$·357 451 596 8	46	253 120 619 351 356 091 69	$\overline{60}$·596 672 475 5
12	574 801 920 0	$\overline{10}$·240 481 789 9	47	121 552 923 510 249 044 90	$\overline{62}$·915 234 591 3
13	809 512 704 00	$\overline{11}$·091 776 331 3	48	595 867 948 441 731 488 20	$\overline{63}$·224 849 974 5
14	122 049 607 680 0	$\overline{13}$·913 463 612 3	49	298 058 113 376 791 104 82	$\overline{65}$·525 699 051 8
15	196 151 155 200 00	$\overline{14}$·707 409 129 8	50	152 070 466 008 566 890 21	$\overline{67}$·817 955 123 2
16	334 764 638 208 000	$\overline{15}$·475 260 423 6	51	791 070 564 176 564 962 91	$\overline{68}$·101 784 775 3
17	604 668 627 763 200 0	$\overline{16}$·218 482 563 5	52	419 422 510 888 908 168 57	$\overline{70}$·377 348 264 2
18	115 242 726 703 104 000	$\overline{18}$·938 386 474 6	53	226 568 814 055 181 354 90	$\overline{72}$·644 799 868 6
19	231 125 690 776 780 800 0	$\overline{19}$·636 151 777 8	54	124 655 596 563 190 345 45	$\overline{74}$·904 288 218 5
20	486 580 401 635 328 000 00	$\overline{20}$·312 845 387 5	55	698 302 184 451 205 175 92	$\overline{75}$·155 956 599 4
21	107 290 978 560 589 824 00	$\overline{22}$·969 436 793 7	56	398 159 209 170 723 533 03	$\overline{77}$·399 943 234 9
22	247 280 160 111 073 689 60	$\overline{23}$·606 810 726 7	57	231 003 441 177 800 135 50	$\overline{79}$·636 381 550 5
23	594 596 384 994 354 462 72	$\overline{24}$·225 777 735 5	58	136 332 557 214 406 957 16	$\overline{81}$·865 400 419 1
24	148 907 616 415 977 465 44	$\overline{26}$·827 083 088 1	59	818 230 309 419 570 030 85	$\overline{82}$·087 124 389 4
25	387 780 251 083 274 649 60	$\overline{27}$·411 414 312 5	60	499 259 226 764 483 408 65	$\overline{84}$·301 673 900 2
26	104 855 779 892 917 465 25	$\overline{29}$·979 407 625 2	61	309 623 930 465 107 127 26	$\overline{86}$·509 165 480 6
27	293 999 475 161 295 508 34	$\overline{30}$·531 653 444 9	62	195 113 834 214 405 212 65	$\overline{88}$·709 711 936 6
28	853 687 364 912 798 809 40	$\overline{31}$·068 701 146 4	63	124 904 323 870 479 724 03	$\overline{90}$·903 422 527 2
29	256 411 097 818 451 356 68	$\overline{33}$·591 063 181 9	64	812 076 365 989 658 650 26	$\overline{91}$·090 403 128 7
30	795 758 579 436 573 175 90	$\overline{34}$·099 218 670 4	65	536 097 288 485 360 593 33	$\overline{93}$·270 736 389 4
31	254 907 998 279 515 607 34	$\overline{36}$·593 616 537 4	66	359 267 659 791 112 422 24	$\overline{95}$·444 581 874 9
32	842 018 678 187 819 296 53	$\overline{37}$·074 678 274 6	67	244 356 443 151 864 191 43	$\overline{97}$·611 976 205 1
33	286 549 481 420 792 254 35	$\overline{39}$·542 800 373 2	68	168 642 416 885 704 480 77	$\overline{99}$·773 033 182 4
34	100 379 151 673 465 407 88	$\overline{41}$·998 356 479 0	69	118 074 492 175 417 504 84	$\overline{101}$·927 843 913 6
35	361 660 178 823 515 072 53	$\overline{42}$·441 699 307 3	70	838 500 016 897 892 425 72	$\overline{102}$·076 496 924 3
36	133 917 597 644 364 438 28	$\overline{44}$·873 162 350 1	71	603 839 797 883 182 245 44	$\overline{104}$·219 078 266 9

The powers of all numbers from 1 to 100 were then formed up to the 20th, and in some extreme cases up to the 40th, the numbers under each power being entered in Tables*. The divisions were then made, and the values of the following four series,

$$x+\frac{1}{5}\cdot\frac{x^5}{1.2.3.4.5}+\frac{1}{9}\cdot\frac{x^9}{1.2\ldots9}+\ldots$$

$$\frac{1}{2}\cdot\frac{x^2}{1.2}+\frac{1}{6}\cdot\frac{x^6}{1.2.3.4.5.6}+\frac{1}{10}\cdot\frac{x^{10}}{1.2\ldots10}+\ldots$$

---

* The powers of numbers from 1 to 100 as far as the tenth are given in HUTTON'S 'Powers of Numbers,' published by the Commissioners of Longitude in 1781. In this Table, which was only used as a check, the fifth power of 81 and the seventh power of 98 were found to be inaccurate.

$$\frac{1}{3}\cdot\frac{x^3}{1.2.3}+\frac{1}{7}\cdot\frac{x^7}{1.2\ldots7}+\frac{1}{11}\cdot\frac{x^{11}}{1.2\ldots11}+\ldots$$

$$\gamma+\log x+\frac{1}{4}\cdot\frac{x^4}{1.2.3.4}+\frac{1}{8}\cdot\frac{x^8}{1.2\ldots8}+\ldots$$

were formed, which, when suitably combined by additions and subtractions, gave the values of the functions.

In Tables I. to IV. the highest power of x included was x^{20}.

In Table IX. x^{63} was required for the value 15. In the intermediate Tables x^{29} was the highest power included.

The values of the functions when the argument was above 16 were calculated from formulæ (2), the log sines and log cosines being taken from TAYLOR's logarithms.

As the formulæ (2) are divergent, to remove every shade of doubt that might attach to their use, the functions for $x=20$ were computed from both formulæ, and the agreement was perfect to the eighth place, which was as far as the second formulæ could give correct results for this value of x.

The calculation of the values from $x=10$ to $x=20$ was extremely difficult and laborious, owing to the great number of terms and high powers necessary to be included. The term involving x^{76} was the first one rejected in the calculation for $x=20$; and to show how extremely unmanageable the formulæ (1) had become, it may be stated that this value, calculated as before described, required the formation of about 22,000 figures exclusive of verifications. Great confidence may, however, be placed in the truth of these results (from $x=10$ to $x=20$), as they were also calculated entirely independently by deducing each term from its predecessor, and in addition the value of Ei $(-x)$, which, on account of its extreme smallness, admitted of being obtained from formulæ (2), served as a rigorous verification of the whole process, excepting the final additions and subtractions.

The functions for $x=20$ were obtained correct to the 12th place, the values being

$$\begin{array}{ll}
\text{Si}20 & =+1\cdot548\ 241\ 701\ 043 \\
\text{Ci}20 & =+0\cdot044\ 419\ 820\ 845 \\
\text{Ei}20 & =+\ 256\ 156\ 52\cdot664\ 056\ 588\ 820 \\
\text{Ei}(-20) & =-0\cdot000\ 000\ 000\ 098.
\end{array}$$

Having obtained the values for $x=20$, it was a matter of comparative ease to give the values of the functions for $x=2$ to a great many places; they are to 43 places as follows:

$$\begin{array}{ll}
\text{Si}2 & =+1\cdot605\ 412\ 976\ 802\ 694\ 848\ 576\ 720\ 148\ 198\ 588\ 940\ 848\ 5834 \\
\text{Ci}2 & =+0\cdot422\ 980\ 828\ 774\ 864\ 995\ 698\ 565\ 153\ 198\ 255\ 894\ 135\ 7378 \\
\text{Ei}2 & =+4\cdot954\ 234\ 356\ 001\ 890\ 163\ 379\ 505\ 130\ 227\ 035\ 275\ 518\ 0536 \\
\text{Ei}(-2) & =-0\cdot048\ 900\ 510\ 708\ 061\ 119\ 567\ 239\ 835\ 228\ 049\ 522\ 314\ 4922
\end{array}$$

agreeing with BRETSCHNEIDER's values to the first 20 places, which is as far as he has computed them. The value of γ was taken from a paper by Mr. SHANKS in No. 114 of the 'Proceedings of the Royal Society.' BRETSCHNEIDER has calculated the functions for $x=1$ to 35 places.

The maxima and minima values of the sine-integral correspond to multiples of π. Those above 4π were calculated by formulæ (2), the others were deduced by TAYLOR'S theorem from other values, Si π from Si 3 and Si 3·1; Si 2π from Si 6; Si 3π from Si 9; and Si 4π from Si 13.

The cosine-integral has its maxima and minima values for odd multiples of $\frac{1}{2}\pi$. Those above $\frac{9}{2}\pi$ were calculated from formulæ (2), the others were deduced from previously calculated values, Ci $\frac{1}{2}\pi$ from Ci 1·6; Ci $\frac{3}{2}\pi$ from Ci 4·7; Ci $\frac{5}{2}\pi$ from Ci 8; Ci $\frac{7}{2}\pi$ from Ci 11; and Ci $\frac{9}{2}\pi$ from Ci 14. The difference formulæ in the form best adapted for logarithmic computation are:

$$
\begin{aligned}
&\text{Si }(x+h)-\text{Si }x\\
&\quad=\frac{h\sin x}{x}\biggl\{\quad 1-\frac{h}{2x}\\
&\qquad\qquad+h^2\left(\frac{1}{3}-\frac{h}{4x}\right)\left(\frac{1}{x^2}-\frac{1}{1\,.\,2}\right)\\
&\qquad\qquad+h^4\left(\frac{1}{5}-\frac{h}{6x}\right)\left(\frac{1}{x^4}-\frac{1}{1\,.\,2x^2}+\frac{1}{1\,.\,2\,.\,3\,.\,4}\right)\\
&\qquad\qquad+h^6\left(\frac{1}{7}-\frac{h}{8x}\right)\left(\frac{1}{x^6}-\frac{1}{1\,.\,2\,x^4}+\frac{1}{1\,.\,2\,.\,3\,.\,4x^2}-\frac{1}{1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6}\right)\\
&\qquad\qquad+\ldots\ldots\biggr\}\\
&\quad+\frac{h^2\cos x}{x}\biggl\{\quad \frac{1}{2}-\frac{h}{3x}\\
&\qquad\qquad+h^2\left(\frac{1}{4}-\frac{h}{5x}\right)\left(\frac{1}{x^2}-\frac{1}{1\,.\,2\,.\,3}\right)\\
&\qquad\qquad+h^4\left(\frac{1}{6}-\frac{h}{7x}\right)\left(\frac{1}{x^4}-\frac{1}{1\,.\,2\,.\,3x^2}+\frac{1}{1\,.\,2\,.\,3\,.\,4\,.\,5}\right)\\
&\qquad\qquad+\ldots\ldots\biggr\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{Ci }(x+h)-\text{Ci }x\\
&\quad=\frac{h\cos x}{x}\biggl\{\quad 1-\frac{h}{2x}\\
&\qquad\qquad+h^2\left(\frac{1}{3}-\frac{h}{4x}\right)\left(\frac{1}{x^2}-\frac{1}{1\,.\,2}\right)\\
&\qquad\qquad+h^4\left(\frac{1}{5}-\frac{h}{6x}\right)\left(\frac{1}{x^4}-\frac{1}{1\,.\,2\,.\,x^2}+\frac{1}{1\,.\,2\,.\,3\,.\,4}\right)\\
&\qquad\qquad+\ldots\ldots\biggr\}\\
&\quad-\frac{h^2\sin x}{x}\biggl\{\quad \frac{1}{2}-\frac{h}{3x}\\
&\qquad\qquad+h^2\left(\frac{1}{4}-\frac{h}{5x}\right)\left(\frac{1}{x^2}-\frac{1}{1\,.\,2\,.\,3}\right)\\
&\qquad\qquad+\ldots\ldots\biggr\}
\end{aligned}
$$

Si π was calculated both from Si 3 and Si 3.1, thus verifying the formula as well as the value of Si π.

In all cases the values calculated from the difference formulæ on the preceding page were verified to as many places as could be obtained from formulæ (2).

In Table XI. Si $(x\pi)-\frac{1}{2}\pi$ is tabulated in preference to Si $x\pi$, as the changes in the function are thus rendered more apparent.

The curves $y=\text{Si}\ x$, $y=\text{Ci}\ x$, $y=\text{Ei}\ x$ were easily drawn from the values in the Tables. It is thus seen how rapidly the two former curves become flattened, and it is worth notice that the radius of curvature at any maximum or minimum point is equal to the abscissa of that point.

The point where the exponential-integral curve cuts the axis of x has the abscissa 0·37249680 . . ., in other words, this is the only real root of the equation Ei $x=0$.

It is my intention to determine the points where the sine-integral and cosine-integral curves cut the lines with which they ultimately coincide, and with this view I have already determined the ordinates corresponding to points midway between the maxima and minima values as data for a first approximation. This will amount to finding the roots of the equations Si $x=\frac{1}{2}\pi$; Ci $x=0$.

In Tables I. to IV., the work has been performed *correct* to the 20th place, and the last two figures have been finally rejected; in Tables V. to IX., the twelfth place has been corrected throughout the work, and the last figure only rejected. In the other Tables two figures have been generally rejected.

In Tables I. to VIII., all the results were verified as far as 7-figure logarithms were available, and in some cases 10-figure logarithms have been used. A very large portion of the work was done in duplicate, and every figure has been carefully examined either by my father or myself; in all cases great pains were taken to ensure accuracy, by independent methods as by logarithms, or in the way by which the values from 10 to 20 were verified.

The Exponential-integral Curve, $y=\text{Ei}\ x$.

It may be mentioned also that differences as far as the ninth order have been taken of the numbers in Tables I. to IV., thus affording a rigid test of the accuracy of more than the first ten, and in the sine-integral of the whole eighteen figures.

Table I.

Values of the Sine-integral from 0 to 1 at intervals of ·01.

x.	Si *x*.	Δ	Δ²	Δ³
·00	+0·000 000 000 000 000 000	+·009 999 944 444 611 111	−·000 000 333 328 333 369	−·000 000 333 308 333 845
·01	·009 999 944 444 611 111	9 999 611 116 277 742	00 666 636 667 214	333 268 336 226
·02	·019 999 555 560 888 853	9 998 944 479 610 527	00 999 905 003 440	333 208 343 369
·03	·029 998 500 040 499 380	9 997 944 574 607 087	01 333 113 346 809	333 128 359 558
·04	·039 996 444 615 106 467	9 996 611 461 260 278	01 666 241 706 367	333 028 390 507
·05	·049 993 056 076 366 745	9 994 945 219 553 911	01 999 270 096 874	332 908 443 356
·06	·059 988 001 295 920 656	9 992 945 949 457 037	02 332 178 540 231	332 768 526 673
·07	·069 980 947 245 377 693	9 990 613 770 916 806	02 664 947 066 904	332 608 650 451
·08	·079 971 561 016 294 499	9 987 948 823 849 903	02 997 555 717 354	332 428 826 108
·09	·089 959 509 840 144 402	9 984 951 268 132 548	03 329 984 543 462	332 229 066 489
·10	·099 944 461 108 276 950	9 981 621 283 589 086	03 662 213 609 951	332 009 385 859
·11	·109 926 082 391 866 036	9 977 959 069 979 135	03 994 222 995 810	331 769 799 909
·12	·119 904 041 461 845 172	9 973 964 846 983 325	04 325 992 795 719	331 510 325 749
·13	·129 878 006 308 828 497	9 969 638 854 187 606	04 657 503 121 468	331 230 981 910
·14	·139 847 645 163 016 103	9 964 981 351 066 138	04 988 734 103 378	330 931 788 339
·15	·149 812 626 514 082 241	9 959 992 616 962 760	05 319 665 891 717	330 612 766 404
·16	·159 772 619 131 045 001	9 954 672 951 071 043	05 650 278 658 121	330 273 938 884
·17	·169 727 292 082 116 044	9 949 022 672 412 922	05 980 552 597 005	329 915 329 975
·18	·179 676 314 754 528 966	9 943 042 119 815 917	06 310 467 926 980	329 536 965 282
·19	·189 619 356 874 344 884	9 936 731 651 888 938	06 640 004 892 261	329 138 871 820
·20	·199 556 088 526 233 821	9 930 091 646 996 676	06 969 143 764 081	328 721 078 013
·21	·209 486 180 173 230 498	9 923 122 503 232 595	07 297 864 842 095	328 283 613 689
·22	·219 409 302 676 463 093	9 915 824 638 390 501	07 626 148 455 784	327 826 510 079
·23	·229 325 127 314 853 593	9 908 198 489 934 717	07 953 974 965 863	327 349 799 815
·24	·239 233 325 804 788 310	9 900 244 514 968 854	08 281 324 765 678	326 853 516 926
·25	·249 133 570 319 757 164	9 891 963 190 203 176	08 608 178 282 604	326 337 696 837
·26	·259 025 533 509 960 340	9 883 355 011 920 572	08 934 515 979 441	325 802 376 367
·27	·268 908 888 521 880 913	9 874 420 495 941 131	09 260 318 355 808	325 247 593 721
·28	·278 783 309 017 822 044	9 865 160 177 585 323	09 585 565 949 529	324 673 388 495
·29	·288 648 469 195 407 367	9 855 574 611 635 794	09 910 239 338 024	324 079 801 666
·30	·298 504 043 807 043 161	9 845 664 372 297 770	10 234 319 139 690	323 466 875 591
·31	·308 349 708 179 340 931	9 835 430 053 158 079	10 557 786 015 282	322 834 654 007
·32	·318 185 138 232 499 010	9 824 872 267 142 798	10 880 620 669 289	322 183 182 023
·33	·328 010 010 499 641 808	9 813 991 646 473 509	11 202 803 851 312	321 512 506 117
·34	·337 824 002 146 115 317	9 802 788 842 622 197	11 524 316 357 429	320 822 674 137
·35	·347 626 790 988 737 514	9 791 264 526 264 768	11 845 139 031 566	320 113 735 290
·36	·357 418 055 515 002 282	9 779 419 387 233 201	12 165 252 766 857	319 385 740 146
·37	·367 197 474 902 235 484	9 767 254 134 466 345	12 484 638 507 003	318 638 740 627
·38	·376 964 729 036 701 829	9 754 769 495 959 342	12 803 277 247 630	317 872 790 009
·39	·386 719 498 532 661 171	9 741 966 218 711 712	13 121 150 037 639	317 087 942 912
·40	·396 461 464 751 372 883	9 728 845 068 674 073	13 438 237 980 550	316 284 255 300
·41	·406 190 309 820 046 956	9 715 406 830 693 523	13 754 522 235 851	315 461 784 478
·42	·415 905 716 650 740 480	9 701 652 308 457 673	14 069 984 020 328	314 620 589 081
·43	·425 607 368 959 198 152	9 687 582 324 437 344	14 384 604 609 409	313 760 729 075
·44	·435 294 951 283 635 496	9 673 197 719 827 935	14 698 365 338 484	312 882 265 752
·45	·444 968 149 003 463 431	9 658 499 354 489 450	15 011 247 604 237	311 985 261 722
·46	·454 626 648 357 952 882	9 643 488 106 885 214	15 323 232 865 959	311 069 780 912
·47	·464 270 136 464 838 095	9 628 164 874 019 255	15 634 302 646 871	310 135 888 557
·48	·473 898 301 338 857 350	9 612 530 571 372 384	15 944 438 535 429	309 183 651 200
·49	·483 510 831 910 229 734	9 596 586 132 836 955	16 253 622 186 629	308 213 136 681
·50	+0·493 107 418 043 066 689	+·009 580 332 510 650 327	−·000 016 561 835 323 309	−·000 000 307 224 414 136

Table I. (continued).

Values of the Sine-integral from 0 to 1 at intervals of ·01.

x.	Si *x*.	Δ	Δ²	Δ³
·50	+0·493 107 418 043 066 689	+·009 580 332 510 650 327	−·000 016 561 835 323 309	−·000 000 307 224 414 136
·51	502 687 750 553 717 016	9 563 770 675 327 017	16 869 059 737 445	306 217 553 991
·52	512 251 521 229 044 033	9 546 901 615 589 572	17 175 277 291 436	305 192 627 954
·53	521 798 422 844 633 606	9 529 726 338 298 137	17 480 469 919 389	304 149 709 013
·54	531 328 149 182 931 742	9 512 245 868 378 747	17 784 619 628 403	303 088 871 429
·55	540 840 395 051 310 490	9 494 461 248 750 345	18 087 708 499 831	302 010 190 727
·56	550 334 856 300 060 834	9 476 373 540 250 513	18 389 718 690 559	300 913 743 696
·57	559 811 229 840 311 348	9 457 983 821 559 955	18 690 632 434 255	299 799 608 379
·58	569 269 213 661 871 302	9 439 293 189 125 706	18 990 432 042 634	298 667 864 067
·59	578 708 506 850 997 002	9 420 302 757 083 065	19 289 099 906 702	297 518 591 296
·60	588 128 809 608 080 067	9 401 013 657 176 364	19 586 618 497 997	296 351 871 835
·61	597 529 823 265 256 430	9 381 427 038 678 366	19 882 970 369 832	295 167 788 687
·62	606 911 250 303 934 797	9 361 544 068 308 534	20 178 138 158 519	293 966 426 077
·63	616 272 794 372 243 331	9 341 365 930 150 015	20 472 104 584 596	292 747 869 447
·64	625 614 160 302 393 346	9 320 893 825 565 419	20 764 852 454 043	291 512 205 450
·65	634 935 054 127 958 765	9 300 128 973 111 377	21 056 364 659 493	290 259 521 943
·66	644 235 183 101 070 142	9 279 072 608 451 884	21 346 624 181 436	288 989 907 981
·67	653 514 255 709 522 026	9 257 725 984 270 448	21 635 614 089 417	287 703 453 807
·68	662 771 981 693 792 474	9 236 090 370 181 031	21 923 317 543 225	286 400 250 849
·69	672 008 072 063 973 505	9 214 167 052 637 806	22 209 717 794 074	285 080 391 711
·70	681 222 239 116 611 311	9 191 957 334 843 732	22 494 798 185 785	283 743 970 164
·71	690 414 196 451 455 043	9 169 462 536 657 948	22 778 542 155 948	282 391 081 143
·72	699 583 658 988 112 991	9 146 683 994 501 999	23 060 933 237 091	281 021 820 736
·73	708 730 342 982 614 990	9 123 623 061 264 908	23 341 955 057 827	279 636 286 178
·74	717 853 966 043 879 898	9 100 281 106 207 081	23 621 591 344 005	278 234 575 845
·75	726 954 247 150 086 979	9 076 659 514 863 075	23 899 825 919 850	276 816 789 241
·76	736 030 906 664 950 054	9 052 759 688 943 226	24 176 642 709 091	275 383 026 997
·77	745 083 666 353 893 280	9 028 583 046 234 135	24 452 025 736 088	273 933 390 860
·78	754 112 249 400 127 415	9 004 131 020 498 047	24 725 959 126 918	272 467 983 684
·79	763 116 380 420 625 462	8 979 405 061 371 099	24 998 427 110 633	270 986 909 425
·80	772 095 785 481 996 560	8 954 406 634 260 466	25 269 414 020 057	269 490 273 128
·81	781 050 192 116 257 026	8 929 137 220 240 409	25 538 904 293 186	267 978 180 927
·82	789 979 329 336 497 435	8 903 598 315 947 223	25 806 882 474 112	266 450 740 027
·83	798 882 927 652 444 658	8 877 791 433 473 111	26 073 333 214 140	264 908 058 705
·84	807 760 719 085 917 769	8 851 718 100 258 971	26 338 241 272 845	263 350 246 295
·85	816 612 437 186 176 740	8 825 379 858 986 126	26 601 591 519 140	261 777 413 180
·86	825 437 817 045 162 866	8 798 778 267 466 986	26 863 368 932 320	260 189 670 790
·87	834 236 595 312 629 852	8 771 914 898 534 665	27 123 558 603 110	258 587 131 584
·88	843 008 510 211 164 517	8 744 791 339 931 555	27 382 145 734 694	256 969 909 049
·89	851 753 301 551 096 072	8 717 409 194 196 861	27 639 115 643 743	255 338 117 686
·90	860 470 710 745 292 933	8 689 770 078 553 117	27 894 453 761 429	253 691 873 005
·91	869 160 480 823 846 050	8 661 875 624 791 688	28 148 145 634 434	252 031 291 514
·92	877 822 356 448 637 739	8 633 727 479 157 254	28 400 176 925 948	250 356 490 709
·93	886 456 083 927 794 993	8 605 327 302 231 306	28 650 533 416 657	248 667 589 068
·94	895 061 411 230 026 299	8 576 676 768 814 649	28 899 201 005 725	246 964 706 039
·95	903 638 087 998 840 948	8 547 777 567 808 925	29 146 165 711 764	245 247 962 033
·96	912 185 865 566 649 873	8 518 631 402 097 161	29 391 413 673 797	243 517 478 412
·97	920 704 496 968 747 033	8 489 239 988 423 364	29 634 931 152 209	−·000 000 241 773 377 483
·98	929 193 736 957 170 397	8 459 605 057 271 155	−·000 029 876 704 529 692	
·99	937 653 342 014 441 552	+·008 429 728 352 741 463		
1·00	+0·946 083 070 367 183 015			

Table II.

Values of the Cosine-integral from 0 to 1 at intervals of ·01.

x.	Ci x.	Δ	Δ^2	Δ^3
·00	$-\infty$			
·01	−4·027 979 520 982 392 072	+·693 072 182 122 430 726	−·287 732 067 243 586 944	+·169 899 043 044 911 376
·02	3·334 907 338 859 961 346	·405 340 114 878 843 782	·117 833 024 198 675 568	·053 244 523 267 548 460
·03	2·929 567 223 981 117 564	·287 507 090 680 168 214	·064 588 500 931 127 108	·023 716 537 864 691 258
·04	2·642 060 133 300 949 350	·222 918 589 749 041 107	·010 871 963 066 435 849	·012 651 131 298 823 265
·05	2·419 141 543 551 908 243	·182 046 626 682 605 257	·028 220 831 767 612 585	·007 551 606 006 198 032
·06	2·237 094 916 869 302 986	·153 825 794 914 992 673	·020 669 225 761 414 552	·004 870 948 972 724 095
·07	2 083 269 121 954 310 313	133 156 569 153 578 120	·015 798 276 788 690 458	·003 325 858 202 350 686
·08	1·950 112 552 800 732 193	117 358 292 364 887 663	·012 472 418 586 339 772	·002 372 207 871 050 106
·09	1·832 754 260 435 844 530	·104 885 873 778 547 891	010 100 210 715 289 665	·001 751 559 256 444 941
·10	1·727 868 386 657 296 639	094 785 663 063 258 226	·008 348 651 458 844 724	·001 330 162 206 136 986
·11	1·633 082 723 594 038 413	·086 437 011 604 413 502	·007 018 489 252 707 738	·001 033 964 994 797 193
·12	1 546 645 711 989 624 911	079 418 522 351 705 763	005 984 524 260 910 545	000 819 668 534 466 454
·13	1·467 227 189 637 919 148	073 433 998 090 795 218	005 161 855 726 444 091	000 660 786 482 471 572
·14	1 393 793 191 547 123 930	068 269 142 364 351 127	·001 504 069 243 972 519	000 540 489 674 574 183
·15	1·325 524 049 182 772 803	·063 765 073 120 378 608	·003 963 579 569 398 336	000 447 732 469 653 596
·16	1·261 758 976 062 394 196	·059 801 493 550 980 272	·003 515 847 099 741 740	·000 375 059 007 749 541
·17	1·201 957 482 511 413 924	056 285 646 451 235 532	003 140 788 091 995 199	·000 317 311 760 910 951
18	1·145 671 836 060 178 393	·053 144 858 359 240 333	·002 823 476 331 084 248	·000 270 845 208 527 120
·19	1·092 526 977 700 938 060	050 321 382 028 156 085	·002 552 631 122 537 128	·000 233 032 694 222 349
·20	1·042 205 595 672 781 975	·047 768 730 905 598 956	·002 319 598 428 334 780	000 201 948 944 416 945
·21	0 994 436 844 767 183 019	·045 449 152 477 264 177	·002 117 649 483 887 835	000 176 160 845 749 172
·22	0·948 987 692 289 918 843	·043 331 502 993 376 342	·001 941 488 638 138 663	·000 154 586 643 260 153
23	0 905 656 189 296 542 501	·041 390 014 355 237 679	·001 786 901 994 878 510	000 136 399 373 475 365
·24	0 864 266 174 941 304 822	·039 603 112 360 359 169	·001 650 502 621 403 115	·000 120 959 486 540 614
·25	0·824 663 062 580 945 653	·037 952 609 738 956 024	·001 529 543 134 862 531	·000 107 767 092 340 306
·26	0 786 710 452 841 989 629	·036 423 066 604 093 493	·001 421 776 042 522 225	·000 096 427 625 744 883
27	0·750 287 386 237 896 135	035 001 290 561 571 269	·001 325 348 416 777 342	·000 086 626 832 352 608
·28	0 715 286 095 676 324 866	·033 675 942 144 793 927	001 238 721 584 124 733	000 078 112 321 492 205
·29	0·681 610 153 531 530 939	·032 437 220 560 369 194	·001 160 609 262 932 529	·000 070 679 808 267 854
·30	0·649 172 932 971 161 745	·031 276 611 297 436 665	·001 089 929 454 661 675	·000 064 162 744 941 555
·31	0·617 896 321 673 725 080	030 186 681 842 771 991	001 025 766 709 723 119	·000 058 424 430 296 270
·32	0·587 709 659 830 953 089	·029 160 915 133 048 871	·000 967 342 279 426 849	000 053 351 950 025 087
33	0·558 548 721 697 904 217	·028 193 572 853 622 022	·000 913 990 329 401 763	000 048 851 483 588 263
·34	0 530 355 151 844 282 195	·027 279 582 524 220 260	·000 865 158 845 813 500	·000 044 844 640 358 635
·35	0·503 075 569 320 061 936	026 414 443 678 406 760	000 820 294 205 454 862	000 041 265 577 858 653
·36	0 476 661 125 641 655 175	·025 594 149 472 951 898	·000 779 028 627 596 209	·000 038 058 719 145 214
·37	0·451 066 976 168 703 278	024 815 120 845 355 689	·000 740 969 908 450 995	·000 035 176 932 744 862
·38	0·426 251 855 323 347 588	024 074 150 936 904 695	·000 705 792 975 706 133	000 032 580 072 291 218
39	0 402 177 704 386 442 894	·023 368 357 961 198 562	·000 673 212 903 411 915	000 030 233 797 814 381
·40	0·378 809 346 425 244 332	022 695 145 057 783 646	·000 642 979 105 600 535	·000 028 108 619 007 410
·41	0 356 114 201 367 460 686	·022 052 165 952 183 112	·000 614 870 486 593 125	·000 026 179 114 520 912
·42	0 334 062 035 415 277 574	·021 437 295 465 589 987	·000 588 691 372 072 213	·000 024 423 291 667 800
43	0 312 624 739 949 687 587	·020 848 604 093 517 774	·000 564 268 080 404 413	·000 022 822 058 752 324
·44	0 291 776 155 856 169 812	·020 284 336 013 113 361	·000 541 446 021 652 089	·000 021 358 788 215 773
·45	0 271 491 799 843 056 452	·019 742 889 991 461 271	·000 520 087 233 436 316	·000 020 018 953 384 443
·46	0 251 748 909 851 595 180	019 222 802 758 024 955	·000 500 068 280 051 873	·000 018 789 825 156 349
·47	0·232 526 107 093 570 225	018 722 734 477 973 082	·000 481 278 454 895 524	·000 017 660 217 724 651
·48	0 213 803 372 615 597 143	·018 241 456 023 077 558	·000 463 618 237 170 873	·000 016 620 274 595 416
·49	0 195 561 916 592 519 586	·017 777 837 785 906 685	·000 446 997 962 575 458	·000 015 661 287 855 561
·50	−0·177 784 078 806 612 901	+·017 330 839 823 331 227	−·000 431 336 674 719 896	+·000 014 775 544 989 064

TABLE II. (continued).

Values of the Cosine-integral from 0 to 1 at intervals of ·01.

x.	Ci x.	Δ	Δ^2	Δ^3
·50	−0·177 784 078 806 612 901	+·017 330 839 823 331 227	−·000 431 336 674 719 896	+·000 014 775 544 989 064
·51	·160 453 238 983 281 674	16 899 503 148 611 331	416 561 129 730 832	13 956 198 605 652
·52	·143 553 735 834 670 344	16 482 942 018 860 499	402 604 931 125 180	13 197 155 297 124
·53	·127 070 793 815 789 845	16 080 337 087 755 318	389 407 775 828 056	12 492 980 518 590
·54	·110 990 456 728 034 527	15 690 929 311 927 262	376 914 795 309 466	11 838 816 941 275
·55	·095 299 527 416 107 265	15 314 014 516 617 796	365 075 978 368 191	11 230 314 167 701
·56	·079 985 512 899 489 469	14 948 938 538 249 605	353 845 664 200 490	10 663 568 060 746
·57	·065 036 574 361 239 864	14 595 092 874 049 115	343 182 096 139 744	10 135 068 231 990
·58	·050 441 481 487 190 749	14 251 910 777 909 371	333 047 027 907 755	09 641 652 475 284
·59	·036 189 570 709 281 378	13 918 863 750 001 616	323 405 375 432 471	09 180 467 128 916
·60	·022 270 706 959 279 763	13 595 458 374 569 145	314 224 908 303 555	08 748 932 512 471
·61	− ·008 675 248 584 710 618	13 281 233 466 265 590	305 475 975 791 084	08 344 712 718 958
·62	+ ·004 605 984 881 554 972	12 975 757 490 474 506	297 131 263 072 126	07 965 689 154 386
·63	·017 581 742 372 029 478	12 678 626 227 402 380	289 165 573 917 741	07 609 937 309 740
·64	·030 260 368 599 431 858	12 389 460 653 484 639	281 555 636 608 000	07 275 706 327 822
·65	·042 649 829 252 916 497	12 107 905 016 876 639	274 279 930 280 178	06 961 400 992 214
·66	·054 757 734 269 793 136	11 833 625 086 596 461	267 318 529 287 964	06 665 565 820 091
·67	·066 591 259 356 389 596	11 566 306 557 308 496	260 652 963 467 874	06 386 870 986 369
·68	·078 157 665 913 698 093	11 305 653 593 840 623	254 266 092 481 505	06 124 099 845 379
·69	·089 463 319 507 538 715	11 051 387 501 359 118	248 141 992 636 126	05 876 137 848 932
·70	·100 514 707 008 897 833	10 803 245 508 722 992	242 265 854 787 194	05 641 962 687 398
·71	·111 317 952 517 620 825	10 560 979 653 935 798	236 623 892 099 796	05 420 635 504 037
·72	·121 878 932 171 556 622	10 324 355 761 836 002	231 203 256 595 758	05 211 293 052 883
·73	·132 203 287 933 392 625	10 093 152 505 240 244	225 991 963 542 875	05 013 140 687 697
·74	·142 296 440 438 632 868	09 867 160 541 697 368	220 978 822 855 178	04 825 446 084 212
·75	·152 163 600 980 330 237	09 646 181 718 842 190	216 153 376 770 966	04 647 533 610 477
·76	·161 809 782 699 172 427	09 430 028 342 071 224	211 505 843 160 490	04 478 779 271 008
·77	·171 239 811 041 243 651	09 218 522 498 910 734	207 027 063 889 481	04 318 606 159 767
·78	·180 458 333 540 154 385	09 011 495 435 021 253	202 708 457 729 714	04 166 480 365 071
·79	·189 469 828 975 175 638	08 808 786 977 291 539	198 541 977 364 643	04 021 907 276 523
·80	·198 278 615 952 467 177	08 610 244 999 926 896	194 520 070 088 120	03 884 428 250 105
·81	·206 888 860 952 394 073	08 415 724 929 838 776	190 635 641 838 016	03 753 617 592 816
·82	·215 304 585 882 232 849	08 225 089 288 000 760	186 882 024 245 200	03 629 079 832 820
·83	·223 529 675 170 233 609	08 038 207 263 755 560	183 252 944 412 379	03 510 447 245 054
·84	·231 567 882 433 989 169	07 854 954 319 343 181	179 742 497 167 326	03 397 377 605 691
·85	·239 422 836 753 332 350	07 675 211 822 175 855	176 345 119 561 634	03 289 552 151 954
·86	·247 098 048 575 508 205	07 498 866 702 614 221	173 055 567 409 680	03 186 673 726 348
·87	·254 596 915 278 122 425	07 325 811 135 204 540	169 868 893 683 332	03 088 465 086 809
·88	·261 922 726 413 326 966	07 155 942 241 521 209	166 780 428 596 523	02 994 667 366 229
·89	·269 078 668 654 848 175	06 989 161 812 924 686	163 785 761 230 294	02 905 038 666 700
·90	·276 067 830 467 772 860	06 825 376 051 694 392	160 880 722 563 594	02 819 352 775 327
·91	·282 893 206 519 467 252	06 664 495 329 130 798	158 061 369 788 267	02 737 397 989 950
·92	·289 557 701 848 598 049	06 506 433 959 342 531	155 323 971 798 317	02 658 976 044 266
·93	·296 064 135 807 940 580	06 351 109 987 544 214	152 664 995 754 051	02 583 901 123 017
·94	·302 415 245 795 484 794	06 198 444 991 790 162	150 081 094 631 034	02 511 998 958 817
·95	·308 613 690 787 274 956	06 048 363 897 159 128	147 569 095 672 217	02 443 106 003 102
·96	·314 662 054 684 434 084	05 900 794 801 486 911	145 125 989 669 116	02 377 068 664 402
·97	·320 562 849 485 920 995	05 755 668 811 817 795	142 748 921 004 714	+·000 002 313 742 607 896
·98	·326 318 518 297 738 790	05 612 919 890 813 081	− ·000 140 435 178 396 818	
·99	·331 931 438 188 551 871	+·005 472 484 712 416 263		
1·00	+0·337 403 922 900 968 135			

TABLE III.

Values of the Exponential-integral from 0 to 1 at intervals of ·01.

x.	Ei x.	Δ	Δ^2	Δ^3
·00	$-\infty$			
·01	−4·017 929 465 426 669 387	+·703 222 571 016 515 484	−·287 631 400 546 641 026	+·169 899 376 444 219 825
·02	3·314 706 894 410 153 902	·415 591 170 469 874 458	·117 732 024 102 421 201	·053 244 856 728 419 607
·03	2·899 115 723 940 279 444	·297 859 146 367 453 257	·064 487 167 374 001 594	·023 716 871 408 300 820
·04	2·601 256 577 572 826 187	·233 371 978 993 451 663	·040 770 295 965 700 774	·012 651 464 946 686 004
·05	2·367 884 598 579 374 524	·192 601 683 027 750 888	·028 118 831 019 014 771	·007 551 939 780 169 192
·06	2·175 282 915 551 623 636	·164 482 852 008 736 117	·020 566 891 238 845 579	·004 871 282 895 000 827
·07	2·010 800 063 542 887 518	·143 915 960 769 890 539	·015 695 608 343 844 752	·003 326 192 295 473 483
·08	1·866 884 102 772 996 980	·128 220 352 426 045 787	·012 369 416 048 371 269	·002 372 512 157 804 231
·09	1·738 663 750 346 951 193	·115 850 936 377 674 518	·009 996 873 890 567 038	·001 751 893 760 561 864
·10	1·622 812 813 969 276 675	·105 854 062 487 107 480	·008 244 980 130 005 174	·001 330 496 950 195 812
·11	1·516 958 751 482 169 195	·097 609 082 357 102 306	·006 914 483 179 809 362	·001 034 300 000 726 105
·12	1·419 349 669 125 066 889	·090 694 599 177 292 944	·005 880 183 179 083 257	·000 820 003 832 044 144
·13	1·328 655 069 947 773 945	·084 814 415 998 209 687	·005 060 179 347 039 113	·000 661 122 093 428 678
·14	1·243 840 653 949 564 258	·079 754 236 651 170 574	·004 399 057 253 610 435	·000 540 825 623 894 734
·15	1·164 086 417 298 393 684	·075 355 179 397 560 139	·003 858 231 629 715 700	·000 448 068 782 676 449
·16	1·088 731 237 900 833 545	·071 496 947 767 844 459	·003 410 162 847 039 252	·000 375 395 710 169 824
·17	1·017 234 290 132 989 106	·068 086 784 920 805 187	·003 034 767 136 869 428	·000 317 648 878 781 511
·18	0·949 147 505 212 183 919	·065 052 017 783 935 760	·002 717 118 258 087 916	·000 271 182 768 259 970
·19	0·884 095 487 428 248 159	·062 334 899 525 847 843	·002 445 935 489 827 947	·000 233 370 722 590 115
·20	0·821 760 587 902 400 316	·059 888 964 036 019 897	·002 212 564 767 237 832	·000 202 287 468 584 321
·21	0·761 871 623 866 380 419	·057 676 399 268 782 065	·002 010 277 298 653 511	·000 176 499 893 154 373
·22	0·704 195 224 597 598 354	·055 666 121 970 128 553	·001 833 777 405 499 139	·000 154 926 241 796 379
·23	0·648 529 102 627 469 801	·053 832 344 564 629 415	·001 678 851 163 702 760	·000 136 739 551 372 245
·24	0·594 696 758 062 840 385	·052 153 493 400 926 655	·001 542 111 612 330 515	·000 121 300 272 395 693
·25	0·542 543 264 661 913 730	·050 611 381 788 596 140	·001 420 811 339 934 822	·000 108 108 515 120 495
·26	0·491 931 882 873 317 589	·049 190 570 448 661 319	·001 312 702 824 814 327	·000 096 769 714 787 934
·27	0·442 741 312 424 656 270	·047 877 867 623 846 992	·001 215 933 110 026 393	·000 086 969 617 368 590
·28	0·394 863 444 800 809 278	·046 661 934 513 820 599	·001 128 963 492 657 804	·000 078 455 832 564 978
·29	0·348 201 510 286 988 679	·045 532 971 021 162 795	·001 050 507 660 092 825	·000 071 024 075 856 557
·30	0·302 668 539 265 825 884	·044 482 463 361 069 970	·000 979 483 584 236 269	·000 064 507 799 882 085
·31	0·258 186 075 904 755 915	·043 502 979 776 833 701	·000 914 975 784 354 183	·000 058 770 303 802 776
·32	0·214 683 096 127 922 214	·042 588 003 992 479 517	·000 856 205 480 551 408	·000 053 698 673 691 462
·33	0·172 095 092 135 442 697	·041 731 798 511 928 110	·000 802 506 806 859 945	·000 049 199 089 389 648
·34	0·130 363 293 623 514 587	·040 929 291 705 068 165	·000 753 307 717 470 297	·000 045 193 160 652 919
·35	0·089 434 001 918 446 422	·940 175 983 987 597 867	·000 708 114 556 817 378	·000 041 615 045 387 977
·36	0·049 258 017 930 848 555	·039 467 869 430 780 489	·000 666 499 511 429 402	·000 038 409 167 037 497
·37	−0·009 790 148 500 068 066	·038 801 369 919 351 087	·000 628 090 344 391 905	·000 035 528 394 515 312
·38	+0·029 011 221 419 283 021	·038 173 279 574 959 182	·000 592 561 949 876 593	·000 032 932 581 843 860
·39	0·067 184 500 994 242 204	·037 580 717 625 082 589	·000 559 629 368 032 733	·000 030 587 389 443 588
·40	0·104 765 218 619 324 793	·037 021 088 257 049 856	·000 529 041 978 589 145	·000 028 463 327 399 437
·41	0·141 786 306 876 374 650	·036 492 046 278 460 711	·000 500 578 651 189 708	·000 026 534 974 755 443
·42	0·178 278 353 154 835 360	·035 991 467 627 271 003	·000 474 043 676 434 266	·000 024 780 339 219 494
·43	0·214 269 820 782 106 363	·035 517 423 950 836 737	·000 449 263 337 214 772	·000 023 180 329 492 372
·44	0·249 787 244 732 943 100	·035 068 160 613 621 966	·000 426 083 007 722 400	·000 021 718 318 413 462
·45	0·284 855 405 346 565 066	·034 642 077 605 899 566	·000 404 364 689 308 938	·000 020 379 779 708 722
·46	0·319 497 482 952 464 632	·034 237 712 916 590 628	·000 383 984 909 600 216	·000 019 151 984 677 409
·47	0·353 735 195 869 055 260	·033 853 728 606 990 412	·000 364 832 924 922 807	·000 018 023 747 915 507
·48	0·387 588 923 876 045 672	·033 488 895 082 067 604	·000 346 809 177 007 300	·000 016 985 213 333 501
·49	0·421 077 818 958 113 276	·033 142 085 905 060 304	·000 329 823 963 673 799	·000 016 027 673 424 323
·50	+0·454 219 904 863 173 580	+·032 812 261 941 386 505	−·000 313 796 290 249 476	+·000 015 143 416 079 574

TABLE III. (continued).

Values of the Exponential-integral from 0 to 1 at intervals of ·01.

x.	Ei *x*.	Δ	Δ²	Δ³
·50	+0·454 219 904 863 173 580	+·032 812 261 941 386 505	−·000 313 796 290 249 476	+·000 015 143 416 079 574
·51	0·487 032 166 804 560 085	32 498 465 651 137 028	298 652 874 169 903	14 325 594 318 221
·52	0·519 530 632 455 697 113	32 199 812 776 967 126	284 327 279 851 682	13 568 115 142 926
·53	0·551 730 445 232 664 239	31 915 485 497 115 444	270 759 164 708 756	12 865 544 421 295
·54	0·583 645 930 729 779 683	31 644 726 332 406 688	257 893 620 287 461	12 213 025 238 691
·55	0·615 290 657 062 186 371	31 386 832 712 119 227	245 680 595 048 770	11 606 207 613 426
·56	0·646 677 489 774 305 598	31 141 152 117 070 458	234 074 387 435 344	11 041 187 825 826
·57	0·677 818 641 891 376 055	30 907 077 729 635 114	223 033 199 609 517	10 514 455 906 593
·58	0·708 725 719 621 011 169	30 684 044 530 025 596	212 518 743 702 925	10 022 850 070 374
·59	0·739 409 764 151 036 766	30 471 525 786 322 671	202 495 893 632 551	09 563 517 077 949
·60	0·769 881 289 937 359 437	30 269 029 892 690 120	192 932 376 554 602	09 133 877 673 091
·61	0·800 150 319 830 049 557	30 076 097 516 135 518	183 798 498 881 512	08 731 596 374 710
·62	0·830 226 417 346 185 075	29 892 299 017 254 006	175 066 902 506 802	08 354 555 016 438
·63	0·860 118 716 363 439 081	29 717 232 114 747 205	166 712 347 490 363	08 000 829 518 616
·64	0·889 835 948 478 186 286	29 550 519 767 256 811	158 711 517 971 748	07 668 669 455 143
·65	0·919 386 468 245 443 127	29 391 808 249 285 094	151 042 818 516 605	07 356 480 042 458
·66	0·948 778 276 494 728 220	29 240 765 400 768 489	143 686 368 474 146	07 062 806 232 359
·67	0·978 019 041 895 496 709	29 097 079 032 294 342	136 623 562 241 787	06 786 318 636 166
·68	1·007 116 120 927 791 051	28 960 455 470 052 555	129 837 243 605 621	06 525 801 046 404
·69	1·036 076 576 397 843 607	28 830 618 226 446 934	123 311 442 559 217	06 280 139 354 884
·70	1·064 907 194 624 290 541	28 707 306 783 887 717	117 031 303 204 332	06 048 311 693 796
·71	1·093 614 501 408 178 258	28 590 275 480 683 385	110 982 991 510 537	05 829 379 650 047
·72	1·122 204 776 888 861 642	28 479 292 489 172 848	105 153 611 860 489	05 622 480 423 165
·73	1·150 684 069 378 034 490	28 374 138 877 312 358	099 531 131 437 324	05 426 819 814 266
·74	1·179 058 208 255 346 848	28 274 607 745 875 034	094 104 311 623 059	05 241 605 948 303
·75	1·207 332 816 091 221 883	28 180 503 434 251 976	088 862 645 674 756	05 056 343 644 439
·76	1·235 513 319 435 473 858	28 091 649 788 577 220	083 796 302 030 316	04 900 229 360 204
·77	1·263 604 960 224 051 079	28 007 844 486 546 904	078 896 072 670 113	04 742 746 644 486
·78	1·291 612 804 710 597 983	27 928 948 413 876 791	074 153 326 025 627	04 593 362 042 462
·79	1·319 541 753 124 474 774	27 854 795 087 851 164	069 559 963 983 165	04 451 581 402 545
·80	1·347 396 548 212 325 938	27 785 235 123 867 999	065 108 382 580 620	04 316 946 541 485
·81	1·375 181 783 336 193 938	27 720 126 741 287 379	060 791 436 039 136	04 189 032 229 030
·82	1·402 901 910 077 481 317	27 659 335 305 248 244	056 602 403 810 106	04 067 443 458 089
·83	1·430 561 245 382 729 560	27 602 732 901 438 138	052 534 960 352 017	03 951 812 970 353
·84	1·458 163 978 284 167 699	27 550 197 941 086 122	048 583 147 381 664	03 841 799 010 783
·85	1·485 714 176 225 253 820	27 501 614 793 704 458	044 741 348 370 881	03 737 083 287 429
·86	1·513 215 791 018 958 278	27 456 873 445 333 577	041 004 265 083 452	03 637 369 115 694
·87	1·540 672 664 464 291 855	27 415 869 180 250 125	037 366 895 967 759	03 542 379 728 489
·88	1·568 088 533 644 541 980	27 378 502 284 282 366	033 824 516 239 270	03 451 836 735 785
·89	1·595 467 035 928 824 346	27 344 677 768 043 096	030 372 659 503 485	03 365 558 718 867
·90	1·622 811 713 696 867 441	27 314 305 108 539 610	027 007 100 784 619	03 283 259 946 176
·91	1·650 126 018 805 407 052	27 287 298 007 754 992	023 723 840 838 442	03 204 749 199 040
·92	1·677 413 316 813 162 043	27 263 574 166 916 550	020 519 091 639 402	03 129 828 696 825
·93	1·704 676 890 980 078 593	27 243 055 075 277 148	017 389 262 942 577	03 058 313 112 134
·94	1·731 919 946 055 355 741	27 225 665 812 334 571	014 330 949 830 443	02 990 028 667 662
·95	1·759 145 611 867 690 311	27 211 334 862 504 128	011 340 921 162 780	02 924 812 307 161
·96	1·786 356 946 730 194 439	27 199 993 941 341 348	008 416 108 855 619	02 862 510 933 738
·97	1·813 556 940 671 535 787	27 191 577 832 485 729	005 553 597 921 881	+000 002 802 980 709 425
·98	1·840 748 518 504 021 516	27 186 024 234 563 848	−·000 002 750 617 212 456	
·99	1·867 934 542 738 585 363	+·027 183 273 617 351 392		
1·00	+1·895 117 816 355 936 755			

TABLE IV.

Values of the Exponential-integral from 0 to 1 at intervals of ·01.

x.	Ei $(-x)$.	Δ	Δ²	Δ³
·00	$-\infty$			
·01	−4·037 929 576 538 113 832	+·683 221 793 228 404 301	−·287 632 733 939 975 455	+·169 898 709 647 547 372
·02	3·354 707 783 309 709 531	395 589 059 288 428 847	117 734 024 292 428 082	053 244 189 811 732 869
·03	2·959 118 724 021 280 684	277 855 034 996 000 764	064 489 834 480 695 214	023 716 204 331 581 697
·04	2·681 263 689 025 279 920	213 365 200 515 305 550	040 773 630 149 113 517	012 650 797 669 904 970
·05	2·467 898 488 509 974 370	172 591 570 366 192 034	028 122 832 479 208 546	007 551 272 263 282 429
·06	2·295 306 918 143 782 336	144 468 737 886 983 487	020 571 560 215 926 118	004 870 615 097 947 363
·07	2·150 838 180 256 798 849	123 897 177 671 057 370	015 700 945 117 978 754	003 325 524 178 172 337
·08	2·026 941 002 585 741 479	108 196 232 553 078 615	012 375 420 939 806 418	002 371 873 680 151 542
·09	1·918 744 770 032 662 864	095 820 811 613 272 198	010 003 547 259 654 875	001 751 224 882 428 031
·10	1·822 923 958 419 390 666	085 817 264 353 617 322	008 252 322 377 226 844	001 329 827 631 422 625
·11	1·737 106 694 065 773 344	077 564 941 976 390 478	006 922 494 745 804 219	001 033 630 201 123 876
·12	1·659 541 752 089 382 866	070 642 447 230 586 259	005 888 864 544 680 343	000 819 333 511 388 831
·13	1·588 899 304 858 796 606	064 753 582 685 905 916	005 069 531 033 291 512	000 660 451 211 459 020
·14	1·524 145 722 172 890 691	059 684 051 652 614 404	004 409 079 821 832 492	000 540 154 140 309 362
·15	1·464 461 670 520 276 287	055 274 971 830 781 912	003 868 925 681 523 131	000 447 396 657 131 013
·16	1·409 186 698 689 494 375	051 406 046 149 258 781	003 421 529 024 392 117	000 374 722 902 274 108
·17	1·357 780 652 540 235 594	047 984 517 124 866 664	003 046 806 122 118 010	000 316 975 348 096 544
·18	1·309 796 135 415 368 931	044 937 711 002 748 654	002 729 830 774 021 466	000 270 508 474 295 132
·19	1·264 858 424 412 620 276	042 207 880 228 727 188	002 459 322 299 726 333	000 232 695 624 800 250
·20	1·222 650 544 183 893 088	039 748 557 929 000 855	002 226 626 674 926 083	000 201 611 526 366 829
·21	1·182 901 986 254 892 234	037 521 931 251 074 772	002 025 015 148 559 254	000 175 823 065 846 310
·22	1·145 380 055 000 817 462	035 496 916 105 515 517	001 849 192 082 712 944	000 154 218 488 671 547
·23	1·109 883 138 895 301 945	033 647 724 022 802 573	001 694 943 594 041 397	000 136 060 831 638 280
·24	1·076 235 414 872 499 371	031 952 780 428 761 176	001 558 882 762 403 116	000 120 620 545 191 144
·25	1·044 282 634 443 738 195	030 393 897 666 358 060	001 438 262 217 211 973	000 107 427 739 511 901
·26	1·013 888 736 777 380 135	028 955 635 449 146 087	001 330 834 477 700 072	000 096 087 849 766 893
·27	0·984 933 101 328 234 048	027 624 800 971 446 015	001 234 746 627 933 179	000 086 286 621 848 827
·28	0·957 308 300 356 788 033	026 390 054 343 512 836	001 148 460 006 084 352	000 077 771 665 379 399
·29	0·930 918 246 013 275 197	025 241 594 337 428 484	001 070 688 340 704 953	000 070 338 695 754 303
·30	0·905 676 651 675 846 712	024 170 905 996 723 531	001 000 349 644 950 650	000 063 821 165 525 578
·31	0·881 505 745 679 123 181	023 170 556 351 772 882	000 936 528 479 425 072	000 058 082 373 764 756
·32	0·858 335 189 327 350 299	022 234 027 872 347 810	000 878 446 105 660 316	000 053 009 406 452 021
·33	0·836 101 161 455 002 489	021 355 581 766 687 494	000 825 436 699 208 295	000 048 508 443 333 252
·34	0·814 745 579 688 314 995	020 530 145 067 479 200	000 776 928 255 875 042	000 044 501 094 065 429
·35	0·794 215 434 620 835 795	019 753 216 811 604 158	000 732 427 161 809 613	000 040 921 516 453 654
·36	0·774 462 217 809 231 637	019 020 789 649 794 545	000 691 505 645 355 959	000 037 714 133 836 006
·37	0·755 441 428 159 437 092	018 329 284 004 438 586	000 653 791 511 519 953	000 034 831 815 018 716
·38	0·737 112 144 154 998 507	017 675 492 492 918 632	000 618 959 696 501 237	000 032 234 413 913 604
·39	0·719 436 651 662 079 874	017 056 532 796 417 396	000 586 725 282 587 633	000 029 887 590 827 474
·40	0·702 380 118 865 662 479	016 469 807 513 829 763	000 556 837 691 760 159	000 027 761 855 728 595
·41	0·685 910 311 351 832 716	015 912 969 822 069 604	000 529 075 836 031 564	000 025 831 787 541 285
·42	0·669 997 341 529 763 112	015 383 893 986 038 040	000 503 244 048 490 279	000 024 075 393 850 669
·43	0·654 613 447 543 725 072	014 880 649 937 547 761	000 479 168 654 639 610	000 022 473 583 231 700
·44	0·639 732 797 606 177 312	014 401 481 282 908 150	000 456 695 071 407 910	000 021 009 728 394 867
·45	0·625 331 316 323 269 161	013 944 786 211 500 240	000 435 685 343 013 043	000 019 669 302 934 150
·46	0 611 386 530 111 768 921	013 509 100 868 487 197	000 416 016 040 078 893	000 018 439 578 013 741
·47	0·597 877 429 243 281 723	013 093 084 828 408 305	000 397 576 462 065 152	000 017 309 368 091 453
·48	0·584 781 344 414 873 419	012 695 508 366 343 153	000 380 267 093 973 699	000 016 268 816 936 490
·49	0·572 088 836 048 530 266	012 315 241 272 369 454	000 363 998 277 037 209	000 015 309 216 897 384
·50	−0·559 773 594 776 160 812	+·011 951 242 995 332 245	−·000 348 689 060 139 825	+·000 014 422 855 718 195

TABLE IV. (continued).

Values of the Exponential-integral from 0 to 1 at intervals of ·01.

x.	Ei $(-x)$.	Δ	Δ^2	Δ^3
·50	−0·559 773 594 776 160 812	+·011 951 242 995 332 245	−·000 348 689 060 139 825	+·000 014 422 855 718 195
·51	547 822 351 780 828 566	11 602 553 935 192 420	334 266 204 421 630	13 602 886 267 210
·52	536 219 797 845 636 146	11 268 287 730 770 790	320 663 318 154 421	12 843 215 393 249
·53	524 951 510 114 865 356	10 947 624 412 616 369	307 820 102 761 171	12 138 408 806 911
·54	514 003 885 702 248 987	10 639 804 309 855 198	295 681 693 954 260	11 483 609 433 366
·55	503 364 081 392 393 789	10 344 122 615 900 938	284 198 084 520 894	10 874 467 127 542
·56	493 019 958 776 492 851	10 059 924 531 380 014	273 323 617 393 351	10 307 078 003 175
·57	482 960 034 245 112 807	09 786 600 913 986 693	263 016 539 390 177	09 777 931 921 152
·58	473 173 433 331 126 115	09 523 584 374 596 516	253 238 607 469 025	09 283 866 923 079
·59	463 649 848 956 529 599	09 270 345 767 127 491	243 954 740 545 946	08 822 029 593 446
·60	454 379 503 189 402 108	09 026 391 026 581 545	235 132 710 952 500	08 389 840 496 472
·61	445 353 112 162 820 563	08 791 258 315 629 045	226 742 870 456 028	07 984 963 968 245
·62	436 561 853 847 191 518	08 564 515 445 173 017	218 757 906 487 783	07 605 281 656 282
·63	427 997 338 402 018 501	08 345 757 538 685 234	211 152 624 831 501	07 248 869 291 509
·64	419 651 580 863 333 267	08 134 604 913 853 733	203 903 755 539 992	06 913 976 255 089
·65	411 516 975 949 479 534	07 930 701 158 313 741	196 989 779 284 903	06 599 007 567 397
·66	403 586 274 791 165 793	07 733 711 379 028 838	190 390 771 717 505	06 302 507 980 815
·67	395 852 563 412 136 955	07 543 320 607 311 333	184 088 263 736 691	06 023 147 903 883
·68	388 309 242 804 825 622	07 359 232 343 574 642	178 065 115 832 807	05 759 710 922 972
·69	380 950 010 461 250 979	07 181 167 227 741 835	172 305 404 909 835	05 511 082 720 336
·70	373 768 843 233 509 144	07 008 861 822 832 000	166 794 322 189 499	05 276 241 215 202
·71	366 759 981 410 677 144	06 842 067 500 612 501	161 518 080 974 298	05 054 247 778 083
·72	359 917 913 910 034 644	06 680 549 419 668 203	156 463 833 196 214	04 844 239 388 669
·73	353 237 364 490 366 441	06 524 085 586 471 989	151 619 593 807 545	04 645 421 624 773
·74	346 713 278 903 894 452	06 372 465 992 664 444	146 974 172 182 773	04 457 062 384 569
75	340 340 812 911 230 008	06 225 491 820 481 671	142 517 109 798 204	04 278 486 256 939
·76	334 115 321 090 748 336	06 082 974 710 683 467	138 238 623 541 265	04 109 069 465 619
·77	328 032 346 380 064 869	05 944 736 087 142 202	134 129 554 075 647	03 948 235 322 169
·78	322 087 610 292 922 667	05 810 606 533 066 555	130 181 318 753 478	03 795 450 130 887
·79	316 277 003 759 856 112	05 680 425 214 313 077	126 385 868 622 591	03 650 219 495 731
·80	310 596 578 545 543 035	05 554 039 345 690 486	122 735 649 126 860	03 512 084 985 411
·81	305 042 539 199 852 548	05 431 303 696 563 626	119 223 564 141 449	03 380 621 118 022
·82	299 611 235 503 288 922	05 312 080 132 422 176	115 842 943 023 428	03 255 432 631 193
·83	294 299 155 370 866 746	05 196 237 189 398 748	112 587 510 392 235	03 136 152 007 684
·84	289 102 918 181 467 998	05 083 649 679 006 514	109 451 358 384 551	03 022 437 229 855
·85	284 019 268 502 461 484	04 974 198 320 621 963	106 428 921 154 696	02 913 969 739 468
·86	279 045 070 181 839 521	04 867 769 399 467 267	103 514 951 415 228	02 810 452 581 925
·87	274 177 300 782 372 254	04 764 254 448 052 039	100 704 498 833 302	02 711 608 716 403
·88	269 413 046 334 320 215	04 663 549 949 218 737	097 992 890 116 899	02 617 179 475 390
·89	264 749 496 385 101 478	04 565 557 059 101 838	095 375 710 641 509	02 526 923 158 907
·90	260 183 939 325 999 641	04 470 181 348 460 328	092 848 787 182 603	02 440 613 750 339
·91	255 713 757 977 539 312	04 377 332 560 977 725	090 408 173 732 264	02 358 039 742 136
·92	251 336 425 416 561 587	04 286 924 387 245 461	088 050 133 990 129	02 279 003 060 944
·93	247 049 501 029 316 126	04 198 874 253 255 333	085 771 130 929 185	02 203 318 082 785
·94	242 850 626 776 060 793	04 113 103 122 326 148	083 567 812 846 400	02 130 810 729 881
·95	238 737 523 653 734 645	04 029 535 309 479 748	081 437 002 116 519	02 061 317 641 600
·96	234 707 988 344 254 897	03 948 098 307 363 229	079 375 684 474 919	01 994 685 412 731
·97	230 759 890 036 891 668	03 868 722 622 888 310	077 380 999 062 188	+·000 001 930 769 893 030
·98	226 891 167 414 003 358	03 791 341 623 826 121	−·000 075 450 229 169 158	
·99	223 099 825 790 177 237	+·003 715 891 394 656 963		
1·00	−0·219 383 934 395 520 274			

TABLE V.

Values of the Sine-integral from 1 to 5 at intervals of 0·1.

x.	Si x.	Δ	Δ²	Δ³
1·0	+0·946 083 070 37	+·082 602 148 31	−·003 240 167 97	−·000 211 002 31
1·1	1·028 685 218 67	79 361 980 34	3 451 170 28	190 916 06
1·2	1·108 047 199 01	75 910 810 06	3 642 086 34	169 839 01
1·3	1·183 958 009 08	72 268 723 72	3 811 925 35	147 918 33
1·4	1·256 226 732 80	68 456 798 37	3 959 843 68	125 307 21
1·5	1·324 683 531 17	64 496 954 70	4 085 150 88	102 162 36
1·6	1·389 180 485 87	60 411 803 81	4 187 313 24	078 643 74
1·7	1·449 592 289 68	56 224 490 57	4 265 957 08	054 913 36
1·8	1·505 816 780 26	51 958 533 49	4 320 870 44	031 133 17
1·9	1·557 775 313 75	47 637 663 05	4 352 003 61	−·000 007 464 83
2·0	1·605 412 976 80	43 285 659 44	4 359 468 45	+·000 015 932 01
2·1	1·648 698 636 24	38 926 191 00	4 343 536 43	038 900 83
2·2	1·687 624 827 24	34 582 654 56	4 304 635 61	061 289 33
2·3	1·722 207 481 81	30 278 018 96	4 243 346 28	082 950 68
2·4	1·752 485 500 76	26 034 672 68	4 160 395 60	103 744 46
2·5	1·778 520 173 44	21 874 277 09	4 056 651 14	123 537 81
2·6	1·800 394 450 53	17 817 625 95	3 933 113 33	142 206 29
2·7	1·818 212 076 47	13 884 512 61	3 790 907 05	159 634 84
2·8	1·832 096 589 08	10 093 605 56	3 631 272 21	175 718 53
2·9	1·842 190 194 65	06 462 333 35	3 455 553 68	190 363 35
3·0	1·848 652 528 00	+·003 006 779 67	3 265 190 33	203 486 76
3·1	1·851 659 307 67	−·000 258 410 65	3 061 703 57	215 018 33
3·2	1·851 400 897 02	03 320 114 22	2 846 685 25	224 900 05
3·3	1·848 080 782 80	06 166 799 47	2 621 785 19	233 086 83
3·4	1·841 913 983 33	08 788 584 66	2 388 698 36	239 546 60
3·5	1·833 125 398 67	11 177 283 02	2 149 151 75	244 260 50
3·6	1·821 948 115 65	13 326 434 77	1 904 891 26	247 222 88
3·7	1·808 621 680 88	15 231 326 03	1 657 668 38	248 441 28
3·8	1·793 390 354 85	16 888 994 40	1 409 227 10	247 936 14
3·9	1·776 501 360 45	18 298 221 50	1 161 290 96	245 740 62
4·0	1·758 203 138 95	19 459 512 46	0 915 550 34	241 900 13
4·1	1·738 743 626 49	20 375 062 80	0 673 650 21	236 471 94
4·2	1·718 368 563 69	21 048 713 01	0 437 178 27	229 524 51
4·3	1·697 319 850 68	21 485 891 28	−·000 207 653 75	221 136 90
4·4	1·675 833 959 41	21 693 545 03	+·000 013 483 15	211 398 04
4·5	1·654 140 414 38	21 680 061 88	224 881 19	200 405 87
4·6	1·632 460 352 50	21 455 180 69	425 287 06	188 266 51
4·7	1·611 005 171 81	21 029 893 64	613 553 56	+·000 175 093 36
4·8	1·589 975 278 17	20 416 340 07	+·000 788 646 92	
4·9	1·569 558 938 10	−·019 627 693 15		
5·0	+1·549 931 244 94			

Table VI.

Values of the Cosine-integral from 1 to 5 at intervals of 0·1.

x.	Ci *x*.	Δ	Δ^2	Δ^3
1·0	+0·337 403 922 90	+·047 469 454 52	—·011 883 649 05	+·001 577 228 22
1·1	384 873 377 42	35 585 805 47	10 306 420 83	1 295 053 76
1·2	420 459 182 89	25 279 384 64	09 011 367 07	1 093 081 60
1·3	445 738 567 53	16 268 017 57	07 918 285 47	0 944 753 11
1·4	462 006 585 10	08 349 732 10	06 973 532 36	0 833 179 87
1·5	470 356 317 19	+·001 376 199 74	06 140 352 50	0 747 270 49
1·6	471 732 516 93	—·004 764 152 76	05 393 082 00	0 679 537 03
1·7	466 968 364 18	10 157 234 76	04 713 544 98	0 624 803 80
1·8	456 811 129 42	14 870 779 74	04 088 741 17	0 579 421 15
1·9	441 940 349 68	18 959 520 91	03 509 320 02	0 540 772 16
2·0	422 980 828 77	22 468 840 93	02 968 547 86	0 506 955 15
2·1	400 511 987 84	25 437 388 79	02 461 592 72	0 476 574 11
2·2	375 074 599 05	27 898 981 51	01 985 018 60	0 448 597 64
2·3	347 175 617 54	29 884 000 11	01 536 420 96	0 422 261 73
2·4	317 291 617 43	31 420 421 07	01 114 159 23	0 397 002 10
2·5	285 871 196 36	32 534 580 30	00 717 157 13	0 372 405 58
2·6	253 336 616 06	33 251 737 43	—·000 344 751 55	0 348 175 05
2·7	220 084 878 63	33 596 488 99	+·000 003 423 50	0 324 103 84
2·8	186 488 389 64	33 593 065 49	00 327 527 34	0 300 056 00
2·9	152 895 324 16	33 265 538 15	00 627 583 34	0 275 952 00
3·0	119 629 786 01	32 637 954 81	00 903 535 34	0 251 757 02
3·1	086 991 831 20	31 734 419 48	01 155 292 36	0 227 472 21
3·2	055 257 411 72	30 579 127 11	01 382 764 58	0 203 127 38
3·3	+0·024 678 284 61	29 196 362 54	01 585 891 95	0 178 775 28
3·4	−0·004 518 077 93	27 610 470 58	01 764 667 24	0 154 486 69
3·5	032 128 548 51	25 845 803 35	01 919 153 92	0 130 346 43
3·6	057 974 351 86	23 926 649 42	02 049 500 35	0 106 449 88
3·7	081 901 001 28	21 877 149 07	02 155 950 22	0 082 899 95
3.8	103 778 150 36	19 721 198 85	02 238 850 17	0 059 804 56
3·9	123 499 349 21	17 482 348 68	02 298 654 73	0 037 274 27
4·0	140 981 697 89	15 183 693 94	02 335 929 01	+·000 015 420 19
4·1	156 165 391 83	12 847 764 94	02 351 349 19	—·000 005 647 73
4·2	169 013 156 77	10 496 415 75	02 345 701 46	025 822 48
4·3	179 509 572 51	08 150 714 29	02 319 878 98	045 001 30
4·4	187 660 286 80	05 830 835 30	02 274 877 68	063 087 18
4·5	193 491 122 10	03 555 957 62	02 211 790 51	079 989 86
4·6	197 047 079 73	—·001 344 167 12	02 131 800 65	095 626 89
4·7	198 391 246 84	+·000 787 633 53	02 036 173 75	—·000 109 924 67
4·8	197 603 613 31	02 823 807 29	+·001 926 249 08	
4·9	194 779 806 02	+·004 750 056 37		
5·0	−0·190 029 749 66			

TABLE VII.

Values of the Exponential-integral from 1 to 5 at intervals of 0·1.

x.	Ei x.	Δ	Δ^2	Δ^3
1·0	+ 1·89 511 781 636	+·272 260 463 21	+·002 453 542 42	+·002 139 046 99
1·1	2·16 737 827 956	·274 714 005 63	·004 592 589 41	01 909 399 47
1·2	2·44 209 228 519	·279 306 595 04	·006 501 988 88	01 767 412 19
1·3	2·72 139 888 023	·285 808 583 92	·008 269 401 06	01 687 113 85
1·4	3·00 720 746 415	·294 077 984 98	·009 956 514 91	01 652 237 53
1·5	3·30 128 544 913	·304 034 499 89	·011 608 752 44	01 652 351 36
1·6	3·60 531 994 902	·315 643 252 33	·013 261 103 80	01 680 669 53
1·7	3·92 096 320 135	·328 904 356 13	·014 941 773 33	01 732 766 26
1·8	4·24 986 755 749	·343 846 129 46	·016 674 539 59	01 805 795 01
1·9	4·59 371 368 695	·360 520 669 05	·018 480 334 60	01 898 001 91
2·0	4·95 423 435 600	·379 001 003 65	·020 378 336 51	02 008 414 85
2·1	5·33 323 535 965	·399 379 340 16	·022 386 751 37	02 136 642 15
2·2	5·73 261 469 981	·421 766 091 52	·024 523 393 51	02 282 739 71
2·3	6·15 438 079 133	·446 289 485 02	·026 806 133 21	02 447 123 76
2·4	6·60 067 027 635	·473 095 618 23	·029 253 256 97	02 630 513 26
2·5	7·07 376 589 458	·502 348 875 21	·031 883 770 23	02 833 892 91
2·6	7·57 611 476 979	·534 232 645 44	·034 717 663 15	03 058 491 02
2·7	8·11 034 741 522	·568 950 308 59	·037 776 154 17	03 305 767 14
2·8	8·67 929 772 381	·606 726 462 75	·041 081 921 31	03 577 408 12
2·9	9·28 602 418 656	·647 808 384 06	·044 659 329 43	03 875 329 94
3·0	9·93 383 257 062	·692 467 713 49	·048 534 659 37	04 201 684 43
3·1	10·62 630 028 411	·741 002 372 86	·052 736 343 80	04 558 870 07
3·2	11·36 730 265 697	·793 738 716 65	·057 295 213 87	04 949 546 62
3·3	12·16 104 137 362	·851 033 930 52	·062 244 760 49	05 376 652 89
3·4	13·01 207 530 414	·913 278 691 01	·067 621 413 38	05 843 427 61
3·5	13·92 535 399 515	·980 900 104 39	·073 464 841 00	06 353 433 87
3·6	14·90 625 409 954	1·054 364 945 38	·079 818 274 86	06 910 586 18
3·7	15·96 061 904 492	1·134 183 220 24	·086 728 861 04	07 519 181 29
3·8	17·09 480 226 516	1·220 912 081 28	·094 248 042 33	08 183 932 32
3·9	18·31 571 434 644	1·315 160 123 61	·102 431 974 65	08 910 006 54
4·0	19·63 087 447 006	1·417 592 098 27	·111 341 981 19	09 703 067 21
4·1	21·04 846 656 832	1·528 934 079 46	·121 045 048 40	10 569 319 70
4·2	22·57 740 064 778	1·649 979 127 86	·131 614 368 10	11 515 562 27
4·3	24·22 737 977 564	1·781 593 495 96	·143 129 930 37	12 549 241 83
4·4	26·00 897 327 160	1·924 723 426 34	·155 679 172 20	13 678 515 39
4·5	27·93 369 669 794	2·080 402 598 54	·169 357 687 60	14 912 317 43
4·6	30·01 409 929 648	2·249 760 286 13	·184 270 005 03	16 260 433 83
4·7	32·26 385 958 261	2·434 030 291 16	·200 530 438 85	+·017 733 583 15
4·8	34·69 788 987 377	2·634 560 730 01	+·218 264 022 01	
4·9	37·33 245 060 378	+2·852 824 752 02		
5·0	+40·18 527 535 580			

TABLE VIII.

Values of the Exponential-integral from 1 to 5 at intervals of 0·1.

x.	Ei $(-x)$.	Δ	Δ^2	Δ^3
1·0	−0·219 383 934 40	+·033 393 029 86	−·005 810 562 18	+·001 185 573 49
1·1	185 990 904 54	27 582 467 68	4 624 988 69	0 899 154 96
1·2	158 408 436 85	22 957 479 00	3 725 833 72	0 693 918 60
1·3	135 450 957 85	19 231 645 28	3 031 915 12	0 543 433 67
1·4	116 219 312 57	16 199 730 16	2 488 481 45	0 430 922 04
1·5	100 019 582 41	13 711 248 71	2 057 559 41	0 345 385 15
1·6	086 308 333 70	11 653 689 30	1 712 174 26	0 279 410 41
1·7	074 654 644 40	09 941 515 04	1 432 763 85	0 227 880 13
1·8	064 713 129 36	08 508 751 19	1 204 883 72	0 187 185 44
1·9	056 204 378 18	07 303 867 47	1 017 698 27	0 154 733 53
2·0	048 900 510 71	06 286 169 20	0 862 964 75	0 128 630 17
2·1	042 614 341 51	05 423 204 45	0 734 334 57	0 107 470 93
2·2	037 191 137 06	04 688 869 88	0 626 863 64	0 090 200 47
2·3	032 502 267 18	04 062 006 24	0 536 663 18	0 076 016 18
2·4	028 440 260 94	03 525 343 06	0 460 647 00	0 064 300 91
2·5	024 914 917 88	03 064 696 07	0 396 346 09	0 054 575 48
2·6	021 850 221 81	02 668 349 98	0 341 770 61	0 046 464 45
2·7	019 181 871 82	02 326 579 38	0 295 306 15	0 039 671 06
2·8	016 855 292 45	02 031 273 22	0 255 635 09	0 033 959 29
2·9	014 824 019 23	01 775 638 13	0 221 675 80	0 029 139 74
3·0	013 048 381 09	01 553 962 33	0 192 536 06	0 025 059 75
3·1	011 494 418 76	01 361 426 27	0 167 476 31	0 021 595 39
3·2	010 132 992 50	01 193 949 96	0 145 880 92	0 018 645 54
3·3	008 939 042 54	01 048 069 04	0 127 235 39	0 016 127 26
3·4	007 890 973 51	00 920 833 65	0 111 108 12	0 013 972 27
3·5	006 970 139 86	00 809 725 53	0 097 135 85	0 012 124 02
3·6	006 160 414 33	00 712 589 67	0 085 011 83	0 010 535 52
3·7	005 447 824 66	00 627 577 84	0 074 476 31	0 009 167 65
3·8	004 820 246 82	00 553 101 53	0 065 308 66	0 007 987 56
3·9	004 267 145 28	00 487 792 87	0 057 321 10	0 006 967 78
4·0	003 779 352 41	00 430 471 77	0 050 353 32	0 006 085 09
4·1	003 348 880 64	00 380 118 45	0 044 268 23	0 005 319 92
4·2	002 968 762 18	00 335 850 23	0 038 948 31	0 004 655 68
4·3	002 632 911 96	00 296 901 92	0 034 292 63	0 004 078 28
4·4	002 336 010 04	00 262 609 29	0 030 214 35	0 003 575 74
4·5	002 073 400 76	00 232 394 94	0 026 638 61	0 003 137 83
4·6	001 841 005 82	00 205 756 33	0 023 500 78	0 002 755 80
4·7	001 635 249 49	00 182 255 55	0 020 744 98	+·000 002 422 18
4·8	001 452 993 94	00 161 510 57	−·000 018 322 80	
4·9	001 291 483 36	+·000 143 187 77		
5·0	−0·001 148 295 59			

TABLE IX.

Values of Si x, Ci x, Ei x, and Ei$(-x)$ from 6 to 15 at intervals of unity.

x.	Si x.	Ci x.	Ei x.	Ei$(-x)$.
6	+1·424 687 551 28	−0·068 057 243 89	+ 85·989 762 142 44	−0·000 360 082 45
7	1·454 596 614 25	+0·076 695 278 48	191·504 743 335 50	115 481 73
8	1·574 186 821 71	+0·122 433 882 53	440·379 899 534 84	037 665 62
9	1·665 040 075 83	+0·055 347 531 33	1037·878 290 717 09	012 447 35
10	1·658 347 594 22	−0·045 456 433 00	2492·228 976 241 88	004 156 97
11	1·578 306 806 95	−0·089 563 135 49	6071·406 374 098 61	001 400 30
12	1·504 971 241 53	−0·049 780 006 88	14959·532 666 397 53	000 475 11
13	1·499 361 722 87	+0·026 764 125 57	37197·688 490 689 03	000 162 19
14	1·556 211 050 08	+0·069 396 355 93	93192·513 633 965 37	000 055 66
15	+1·618 194 443 70	+0·046 278 677 67	+234955·852 490 768 32	−0·000 000 019 18

TABLE X.

Table of the Sine-integral and Cosine-integral for values of the argument above 20.

x.	Si x.	Ci x.	x.	Si x.	Ci x.
20	+1·54 824 17	+0·04 441 98	190	+1·57 041 97	+0·00 524 95
25	1·53 148 26	− 0 684 86	200	1·56 838 23	− 437 84
30	1·56 675 65	− 3 303 24	300	1·57 088 11	− 333 22
35	1·59 692 22	− 1 147 99	400	1·57 211 49	− 212 40
40	1·58 698 51	+ 1 902 00	500	1·57 256 59	− 093 20
45	1·55 871 50	+ 1 863 17	600	1·57 246 12	+ 007 64
50	1·55 161 71	− 0 562 84	700	1·57 199 39	+ 077 88
55	1·57 072 41	− 1 817 26	800	1·57 135 51	+ 111 82
60	1·58 674 56	− 0 481 32	900	1·57 072 15	+ 110 86
65	1·57 747 11	+ 1 284 74	1000	1·57 023 31	+ 082 63
70	1·56 159 49	+ 1 092 20	2000	1·57 097 98	+ 046 51
75	1·55 857 96	− 0 533 23	3000	1·57 112 15	+ 007 32
80	1·57 233 09	− 1 240 25	4000	1·57 097 89	− 017 08
85	1·58 239 84	− 0 193 48	5000	1·57 076 54	− 019 76
90	1·57 566 34	+ 0 998 61	6000	1·57 064 57	− 007 13
95	1·56 303 63	+ 0 710 98	7000	1·57 067 32	+ 007 24
100	1·56 222 55	− 0 514 88	8000	1·57 078 81	+ 012 47
110	1·57 988 05	− 0 031 96	9000	1·57 088 39	+ 006 84
120	1·56 397 22	+ 0 478 12	10,000	1·57 089 15	− 003 06
130	1·57 367 63	− 0 713 21	11,000	1·57 082 20	− 008 72
140	1·57 215 91	+ 0 701 11	100,000	1·57 080 63	+ 000 04
150	1·56 616 68	− 0 479 65	1,000,000	1·57 079 54	− 000 04
160	1·57 688 50	+ 0 140 94	10,000,000	1·57 079 64	+ 000 00
170	1·56 526 71	+ 0 200 65	100,000,000	+1·57 079 63	+0·00 000 00
180	+1·57 414 56	−0·00 443 21	∞	$\frac{1}{2}\pi$	0·0

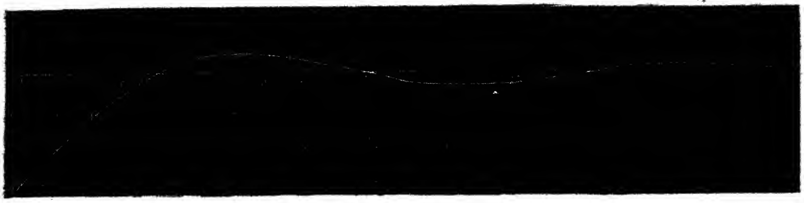
TABLE XI.—Maxima and Minima Values of the Sine-integral.

x.	$\mathrm{Si}\,(x\pi)-\frac{\pi}{2}$.	x.	$\mathrm{Si}\,(x\pi)-\frac{\pi}{2}$.	x.	$\mathrm{Si}\,(x\pi)-\frac{\pi}{2}$.	x.	$\mathrm{Si}\,(x\pi)-\frac{\pi}{2}$.
1	+0·28 114 07	21	+0·01 515 07	50	−0·00 636 57	350	−0·00 090 95
2	− 15 264 47	22	− 01 446 26	51	+ 624 09	351	+ 90 69
3	+ 10 396 56	23	+ 01 383 43	60	− 530 49	400	− 79 58
4	− 07 863 54	24	− 01 325 83	61	+ 521 79	401	+ 79 38
5	+ 06 316 85	25	+ 01 272 83	70	− 454 71	450	− 70 74
6	− 05 276 24	26	− 01 223 90	71	+ 448 31	451	+ 70 58
7	+ 04 528 92	27	+ 01 178 60	80	− 397 87	500	− 63 66
8	− 03 966 50	28	− 01 136 53	81	+ 392 96	501	+ 63 53
9	+ 03 528 06	29	+ 01 097 36	90	− 353 67	600	− 53 05
10	− 03 176 72	30	− 01 060 79	91	+ 349 78	601	+ 52 96
11	+ 02 888 93	31	+ 01 026 59	100	− 318 30	700	− 45 47
12	− 02 648 88	32	− 00 994 52	101	+ 315 15	701	+ 45 41
13	+ 02 445 62	33	+ 00 964 40	150	− 212 20	800	− 39 79
14	− 02 271 31	34	− 00 936 04	151	+ 210 80	801	+ 39 74
15	+ 02 120 16	35	+ 00 909 31	200	− 159 15	900	− 35 37
16	− 01 987 87	36	− 00 884 06	201	+ 158 36	901	+ 35 33
17	+ 01 871 10	37	+ 00 860 17	250	− 127 32	1000	− 31 83
18	− 01 767 29	38	− 00 837 54	251	+ 126 82	1001	+ 31 80
19	+ 01 674 38	39	+ 00 816 07	300	− 106 10	2000	− 15 92
20	−0·01 590 75	40	−0·00 795 67	301	+0·00 105 75	2001	+ 0·00 15 91

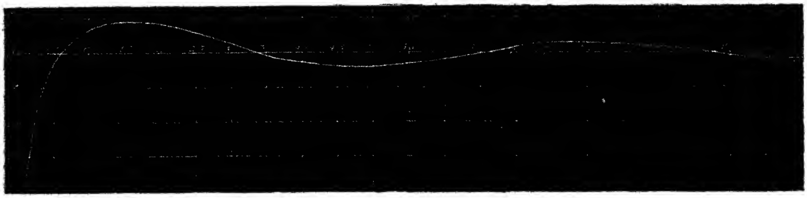
TABLE XII.—Maxima and Minima Values of the Cosine-integral.

x.	$\mathrm{Ci}\,\frac{x\pi}{2}$.	x.	$\mathrm{Ci}\,\frac{x\pi}{2}$.	x	$\mathrm{Ci}\,\frac{x\pi}{2}$.	x.	$\mathrm{Ci}\,\frac{x\pi}{2}$.
1	+0·4 720 007	39	−0·0 163 149	77	+0·0 082 667	501	+0·0 012 707
3	− ·1 984 076	41	+ 155 198	79	− 80 574	599	− 10 628
5	+ ·1 237 723	43	− 147 986	99	− 64 300	601	+ 10 593
7	− ·0 895 640	45	+ 141 415	101	+ 63 027	699	− 09 108
9	+ ·0 700 653	47	− 135 401	119	− 53 494	701	+ 09 082
11	− ·0 575 011	49	+ 129 879	121	+ 52 610	799	− 07 968
13	+ ·0 487 422	51	− 124 789	139	− 45 798	801	+ 07 948
15	− ·0 422 916	53	+ 120 082	141	+ 45 148	899	− 07 081
17	+ ·0 373 449	55	− 115 718	159	− 40 038	901	+ 07 066
19	− ·0 334 321	57	+ 111 660	161	+ 39 540	999	− 06 373
21	+ ·0 302 601	59	− 107 877	179	− 35 564	1001	+ 06 360
23	− ·0 276 371	61	+ 104 341	181	+ 35 171	1099	− 05 793
25	+ ·0 254 320	63	− 101 030	199	− 31 990	1101	+ 05 782
27	− ·0 235 525	65	+ 097 923	201	+ 31 672	1199	− 05 310
29	+ ·0 219 314	67	− 095 001	299	− 21 291	1201	+ 05 301
31	− ·0 205 189	69	+ 092 248	301	+ 21 150	1299	− 04 901
33	+ ·0 192 772	71	− 089 650	399	− 15 955	1301	+ 04 893
35	− ·0 181 771	73	+ 087 195	401	+ 15 876	1399	− 04 551
37	+0·0 171 958	75	−0·0 084 870	499	−0·0 012 758	1401	+0·0 004 544

The Sine-integral Curve, $y=\mathrm{Si}\,x$, for positive abscissæ.



The Cosine-integral Curve, $y=\mathrm{Ci}\,x$.



Note added July 30, 1870.

Professor OPPERMANN, of Copenhagen, who was present at the reading of this paper, shortly afterwards presented to the Royal Society two pamphlets, "Tabulæ logarithmi integralis, auctore L. STENBERG, Malmogiæ, Pars I. 1861, Pars II. 1867," containing values of $\mathrm{li}\,10^x$ from $x=-15$ to $x=3\cdot5$ at intervals of ·01 to 18 places of decimals; the arguments differ therefore from those in this paper by the modulus of the common logarithms as a factor. From a reference in the second of these tracts the author found that Tables of $\mathrm{Si}\,x$, $\mathrm{Ci}\,x$, $\mathrm{Ei}\,x$, and $\mathrm{Ei}(-x)$ from 0 to 1 at intervals of ·01 and from 1 to 7·5 at intervals of 0·1, had been computed by BRETSCHNEIDER, and published in the 6th volume of SCHLÖMILCH'S 'Zeitschrift für Mathematik und Physik.' The referees recommended the comparison of the parts common to these Tables and those given in this paper; this has been made, and the following errors have been found in BRETSCHNEIDER'S values :—

$\mathrm{li}\,e^a$ for $a=0\cdot34$ should be $-0\cdot13036\ 32936$ instead of $-0\cdot13030\ 32936$

$\mathrm{li}\,e^{-a}$ for $a=1\cdot9$ should be $-0\cdot05620\ 43781$: instead of $-0\cdot05620\ 43780$:

a	$\mathrm{li}\ e^a$	$\mathrm{li}\ e^{-a}$	$\mathfrak{S}\mathrm{i}\ a$	$\mathrm{si}\ a$
4·9	37·33237 06037:	−0·00121 14833.	18·66679 10435:	1·56963 89381
should be				
4·9	37·33245 06037:	−0·00129 14833.	18·66687 10435:	1·56955 89381

the error previouly alluded to in $\mathrm{Ei}(-5)$ is corrected in this paper.

BRETSCHNEIDER has indicated by dots certain limits between which the eleventh figure must lie, and the agreement between these and the eleventh figure in the Tables V. to VIII. was so close, that it seemed worth while to retain this figure, on the understanding that it may be in error to the extent of a unit.

XXI. *Researches on Solar Physics.*—No. II. *The Positions and Areas of the Spots observed at Kew during the years* 1864, 1865, 1866, *also the Spotted Area of the Sun's visible disk from the commencement of* 1832 *up to May* 1868. *By* WARREN DE LA RUE, *Esq., D.C.L, V.P.R.S., F.R.A.S.,* BALFOUR STEWART, *Esq., LL.D., F.R.S., F.R.A.S., Superintendent of the Kew Observatory, and* BENJAMIN LOEWY, *Esq., F.R.A.S.*

Received February 15,—Read March 10, 1870.

20. IN a paper presented to the Society and published in the Philosophical Transactions for 1869 (vol. clix. p. 1), we have given a full description of the method adopted by us for ascertaining the positions and areas of the various sun-spots observed at Kew, and we have likewise, in Tables II. and III. of that paper, given the areas and the positions determined after the method described by us, for the various sun-spots of the years 1862 and 1863. In the present paper we give the same elements for the years 1864, 1865, and 1866, forming Tables II. and III., so that these Tables in our present paper form a continuation of the Tables bearing the same number in our previous paper.

We have stated elsewhere that HOFRATH SCHWABE, of Dessau, had very generously put into our hands the valuable collection of drawings of the solar disk made by him during the course of about forty years, and thus it became an object of importance to us to fix upon some method of testing the value of these drawings, and of extracting from them what information they might contain.

Method of examination of SCHWABE'S *drawings.*

21. A cursory inspection of these drawings revealed the existence of several progressive stages in accuracy of delineation, from the time of their commencement in 1825 to that of their termination in 1867. By the commencement of the year 1832 the indefatigable observer had evidently matured his system to such an extent as to give (no doubt, with considerable precision) the shape and area of each group, although it was not until the commencement of 1840 that he had finally fixed upon that exceedingly good system of delineation which he thenceforth pursued up to the time when he discontinued his observations.

Between the latter part of 1832 and the beginning of 1840 the circle representing the solar disk had in these drawings a diameter equal to 2·08 inches nearly, while after 1840 this was very slightly increased, being sometimes as great as 2·15 inches. There was thus no material alteration in the scale from the latter part of 1832 up to the end of the series; and although in the beginning of 1832 the size of the disk was slightly less, being about 1·80 inch, we have thought it unnecessary to take account of this. We have therefore supposed that for all the observations of SCHWABE discussed in this research the scale of delineation remained unaltered.

Previous to 1840 the disk was divided into four quarters by a vertical and horizontal diameter, but after 1840 there were drawn in addition four other lines forming a square inscribed in the circle, and having horizontal and vertical sides, and this square was by means of the two diameters cut into four smaller squares. It is possible that by this arrangement the position of a group on the solar disk might be more accurately ascertained; nevertheless we have not hesitated to commence our research from the year 1832.

22. From 1832 to 1854 SCHWABE has the merit of being the only systematic observer of the solar disk. In November 1853 CARRINGTON began his observations, while in February 1862 the Kew Heliograph was in regular operation under the direction of Mr. WARREN DE LA RUE. Adopting the photographic pictures taken with the Kew instrument as the standard of accuracy, we have already shown that CARRINGTON'S results are almost equally trustworthy. It will therefore be necessary to compare SCHWABE'S results with those of CARRINGTON and with the Kew series in order to test their accuracy.

23. In reducing SCHWABE'S drawings we proceeded in the following manner. Selecting an arbitrary scale, the same individual measured in terms of the scale the area of every group given by SCHWABE, and occasionally during the operation (which occupied some time) he took pains to ascertain that his mode of estimation remained unaltered. We were thus furnished with a series of results which were probably affected throughout with the same personal peculiarity, but which, being founded upon an arbitrary scale, it was necessary to connect by means of a proper multiplier with that scale which we have hitherto adopted, of which the unit is one-millionth of the sun's visible hemisphere.

24. The following Table will elucidate this step in the process of reduction.

TABLE IV.—Comparison of results derived from SCHWABE by the arbitrary scale, with simultaneous results derived from CARRINGTON'S observations and from the Kew series.

Date.	Schwabe.	Carrington.	Date.	Schwabe.	Kew.
1854.			1862.		
Dec. 1–15	2·7	105	April 20 to May 1	11·9	698
Nov. 16–30	9·5	565	1863.		
Sept. 1–15	3·4	215	Jan. 1–15	21·6	1511
Sept. 16–30	2·3	138	Jan. 16–31	15·7	901
1859.			May 1–15	13·5	622
Dec. 1–15	18·2	1222	May 16–31	13·3	812
Dec. 16–31	16·4	1060			
	52·5	3305		76·0	4544

From the left-hand series we derive 65 as the multiplier to be applied to SCHWABE'S results, while from the Kew series we derive 60. On the whole, we have adopted 60 as the most probable multiplier, and by this number the measurements deduced from SCHWABE, according to the arbitrary scale, have been multiplied in order to bring them to agree with the Kew scale, of which the unit is one-millionth of the sun's visible hemisphere.

25. Applying this multiplier to each of the fortnightly results from SCHWABE exhibited in Table IV., and considering CARRINGTON and KEW as absolutely accurate, we obtain the following Table, which gives a general idea of the trustworthiness of a fortnight's observation of SCHWABE.

TABLE V.

No. of series.	Date.	Schwabe × 60.	Standard.
	1854.		
1.	Dec. 1–15	162	105
2.	Nov. 16–30	570	565
3.	Sept. 1–15	204	215
4.	Sept. 16–30	138	138
	1859.		
5.	Dec. 1–15	1092	1222
6.	Dec. 16–31..............	984	1060
	1862.		
7.	April 20 to May 1......	714	698
	1863.		
8.	Jan. 1–15	1296	1511
9.	Jan. 16–31	942	901
10.	May 1–15	810	622
11.	May 16–31..............	798	812

This is we think a very satisfactory result, showing that the mean spotted area derived from a single fortnight of SCHWABE is never probably far from the truth.

If instead of a single fortnight we take the mean of six fortnights, we derive the following Table:—

TABLE VI.

Series.	Schwabe, mean of 6 fortnights.	Standard, mean of 6 fortnights.
1–6	525	551
2–7	617	649
3–8	738	807
4–9	861	921
9–10	973	1002
10–11	924	934

which exhibits the very great trustworthiness of the means of six fortnights, or three months of SCHWABE's observations.

26. We ought to add that, in estimating the spotted area of the various groups from SCHWABE's pictures, we applied an approximate multiplier on account of foreshortening when the group was near the limb; but this was a part of the process on which we

esteemed it superfluous to spend very great labour, inasmuch as we came to the conclusion that the distances from the centre of the various groups, as recorded by SCHWABE, were not sufficiently precise to warrant very great refinement in the mode of estimating the foreshortening. We have not therefore attempted to make use of SCHWABE's observations for the purpose of determining whether there be any law regulating the behaviour with regard to increase or diminution of the groups as they pass across the disk of the sun, such as we have elsewhere attempted to deduce from CARRINGTON's observations, but we have confined ourselves to estimations of the spotted area of the whole visible disk.

We ought also to add that, in preparing our fortnightly mean values, those for the days in which the sun's disk was not observed, were obtained by interpolation, so that in Table V. the differences between the fortnights of SCHWABE's series and simultaneous fortnights of CARRINGTON's and of the Kew series may be partly accounted for by the fact that the days of observations at Dessau were not the same as those in this country. In fine, the differences registered in the Table include the uncertainty occasioned by interpolation, as well as that occasioned by the method of observation.

Description of Tables and Plate.

27. In addition to fortnightly values we have given three-monthly values for every fortnight, each of the latter being the mean of the three fortnightly values which precede, and of the three which follow it.

We have given these three-monthly means in addition to the fortnightly means, inasmuch as our experience has led us to conclude that in the former while we get rid of fluctuations of extremely small period, we yet preserve the most striking peculiarities which characterize the progress of solar disturbance.

These three-monthly means form the curve in black which accompanies this paper.

The dotted curve is obtained from the black by a simple process of equalization analogous to that described by the Astronomer Royal (Phil. Trans. 1863, p. 619), that is to say, the middle points of the various inclined black lines were joined together forming a curve of a smooth character, which by a repetition of the same process gave us the dotted line which indicates the larger or so-called eleven-yearly period of solar disturbance.

In Table VII. we have the various fortnightly means forming the first column for each year, and the three-monthly means forming the second column. For convenience' sake, in the margin of this Table each month is supposed to be capable of division, without much error, into four weeks ending on the 7th, the 14th, the 21st, and the 28th days of the month. The single fortnightly values are assumed to correspond in epoch to the 7th and 21st days, while the three-monthly values on the other hand correspond to the 14th and 28th. All those fortnightly values inclosed in brackets without an asterisk are interpolated. Those inclosed in brackets, and having an asterisk, are derived from SCHWABE's observations; they occur during the time when CARRINGTON had left off and before the Kew series had regularly commenced, and also during the time when the Kew photoheliograph was in the process of removal from Mr. De La Rue's observatory at Cranford to the Kew Observatory.

The observations up to the end of 1853 are those of Schwabe, those from the beginning of 1854 to the end of 1860 are those of Carrington, while the remainder constitute the Kew series, with the exception of those bracketed and having an asterisk, which are derived from Schwabe.

In Table VIII. we have for every fortnight (beginning with January 14th of each year) the readings of the equalized curve or that exhibited by a dotted line. It is proper to remark that, in order to obtain these readings, this curve was first laid down on a very much larger scale than that exhibited in the accompanying Plate.

Results of Reduction.

28. From Table VIII. we obtain the following epochs for the minima and maxima of the longer period of solar disturbances:—

Minimum	Nov. 28, 1833	
Maximum	. . . .	Dec. 21, 1836
Minimum	Sept. 21, 1843	
Maximum	. . . .	Nov. 14, 1847
Minimum	April 21, 1856	
Maximum	. . . .	Sept. 7, 1859
Minimum	Feb. 14, 1867	

We deduce from these numbers, in the first instance, the fact of the variability of the length of the whole period

Thus we have from first to second minimum	9·81 years.
from second to third ,, ...	12·58 years.
from third to fourth ,, ...	10·81 years.
Mean of the three periods . . .	11·07 years.

Secondly, we see that in all of the three cases the time from a minimum to the next maximum is less than that from a maximum to the next minimum,—a fact which has, we think, been previously noticed by Sir J. Herschel. Thus the time from the first minimum to the next maximum is 3·06 years, and from the first maximum to the second minimum 6·75 years. In like manner that from the second minimum to the next maximum is 4·14 years, and from the second maximum to the third minimum 8·44 years. Finally, the time from the third minimum to the third maximum is 3·37 years, and from the third maximum to the fourth minimum 7·44 years. We see again that the second curve, which was longer in period as a whole than either of the other two, manifests this excess in each of its branches, that is to say, its left or ascending branch is larger as a whole than the same branch of the other two curves, and the same takes place for the second or descending branch. On the other hand, the maximum of this curve is not so high as that of either of the other two—in fact the curve has the appearance as if it were pressed down from above, and pressed out laterally so as to lose in elevation what it gains in time.

Lastly, after undergoing the process of equalization which has been described, there is still the appearance in all the three curves of a secondary maximum in the second branch; and this peculiarity has induced us to compare the solar curves with the curves of brightness of two variable stars, the peculiarities of which have been well determined. We annex with this purpose the light-curve of R. Sagittæ (Plate XXXI. fig. 1) determined by BAXENDELL, showing like the sun's spot-curve a secondary maximum in the second branch of the curve; and we also exhibit the light-curve of β Lyræ (fig. 2) determined by POGSON from the observations of ARGELANDER, exhibiting two maxima, which are, however, both of the same value as well as can be estimated.

29. We were induced to imagine from our preliminary researches that the amount of spotted area may possibly be influenced by the positions of the planets in such a way as to exhibit excessive solar action when two influential planets are together at the same ecliptical longitude; we therefore resolved to test this hypothesis by the series of results which we have just described. As the two most influential combinations, we have selected that of Venus and Jupiter, and that of Venus and Mercury. With regard to the former combination, or that of Venus and Jupiter, as the period is about eight months, the influence, if any, would not be materially equalized by the three-monthly means, and it might therefore be expected to appear in the black curve which represents these means, separated to a great extent from any influence of shorter period, such as that due to conjunctions of Mercury and Venus.

Any influence due to the relative positions of Jupiter and Venus might therefore be supposed to show itself in the curve of three-monthly means, and any due to the relative positions of Venus and Mercury in the fortnightly curves.

Viewing, therefore, the dotted curve as the normal line which equalizes all the inequalities of short period, including that which may be due to the relative positions of Jupiter and Venus, we have laid down the departures of the black curve from the dotted one for every fortnight of the whole series of observations; and joining all these into a curve, we have found from it the united values of the departures corresponding to those times when Venus and Jupiter were between 0° and 30° of one another, or between 30° and 60°, between 60° and 90°, and so on.

There are in all fifty-four periods.

30. In like manner, if we take the three-monthly curves as equalizing any influence due to the relative positions of Venus and Mercury, and lay down the departures of the fortnightly means from the corresponding three-monthly means, we are thus furnished with another set of departures which we may suppose likely to exhibit any influence due to the relative positions of Venus and Mercury. But these departures of very short period will no doubt embody, as well as inequalities due to physical causes, others

Solar Spotted Area from 1832 to 1868 in Millionths of the Sun's Visible Hemisphere

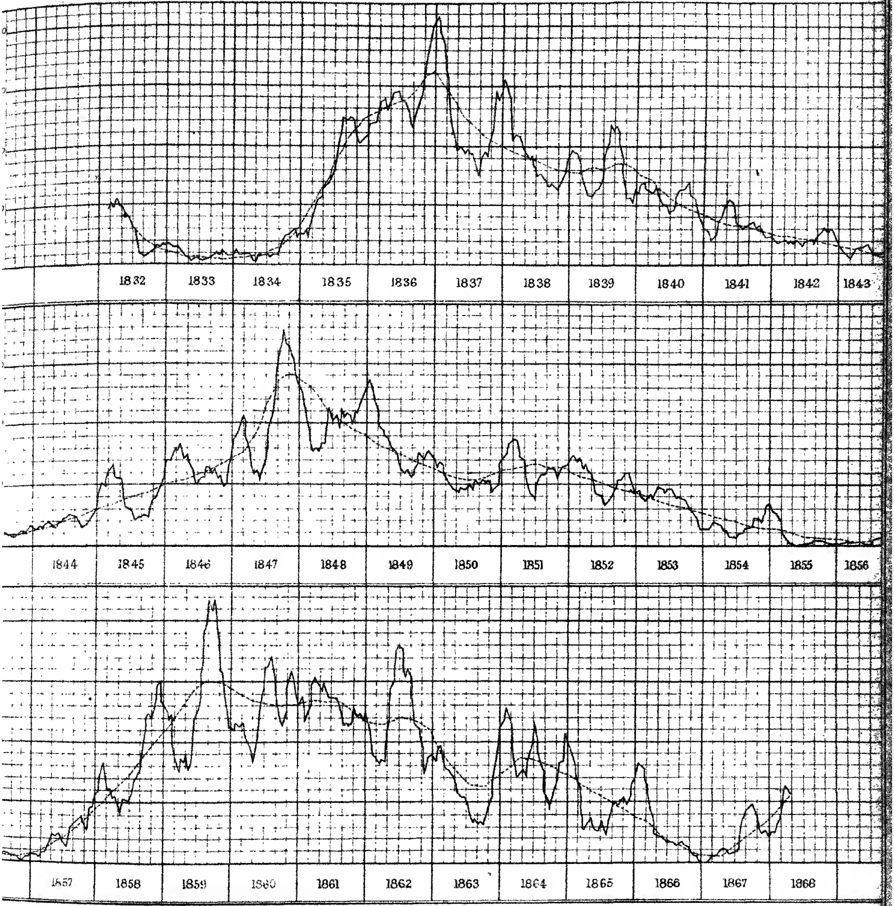
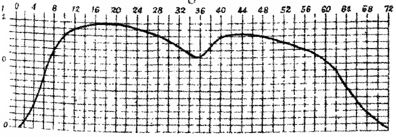
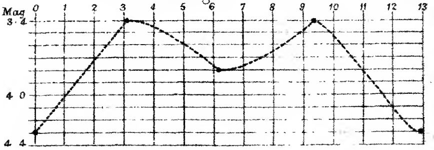


Fig. 1.



Light curve of R. Sagittæ, Period = 70·88 days determined from 8 periods, by Joseph Baxendell

Fig. 2.



Light curve of β Lyrae – from the observations recorded by Argelander —— Period 12·91 days.

ppl. del.

W West imp.

due to the fact that large groups come abruptly into view, and depart abruptly from the disk of the sun, so that a high positive departure for one fortnight is sometimes followed by a negative one for the next.

We have endeavoured to eliminate the effect of this source of inequality by the following means. Suppose, for instance, that the departures for the beginning of 1850 are as follows:—

(1)	January	7 . . .	+194
(2)		21 . . .	− 66
(3)	February	7 . . .	− 68
(4)		21 . . .	+ 36
(5)	March	7 . . .	+228
(6)		21 . . .	− 13

we should, by taking the mean of (1) and (2) of (2) and (3), and so on, form a second series as follows:—

January	14	+ 64
	28	− 67
February	14	− 16
	28	+132
March	14	+107

If this operation be repeated once more, we obtain results which, without any further attempt at equalization, may be formed into a series of short-period curves, which may be taken to be due to some physical cause, and from which we may hope to discover traces of the action of Mercury if such exist.

We have, as in the previous case, taking this curve as our basis, found the united departures for those times when Mercury and Venus are between 0° and 30° of one another, between 30° and 60°, and so on. There are in all ninety periods of this nature.

31. The results of our investigation regarding planetary influence are exhibited in the following Table:—

TABLE VI *a*.

Angular separation between	Excess or deficiency.	
	Jupiter and Venus.	Venus and Mercury.
0° and 30°	+ 881	+1675
30 „ 60	− 60	− 139
60 „ 90	− 452	−1665
90 „ 120	− 579	−2355
120 „ 150	− 705	−2318
150 „ 180	− 759	−1604
180 „ 210	− 893	− 481
210 „ 240	− 752	+ 547
240 „ 270	− 263	+ 431
270 „ 300	+ 70	+ 228
300 „ 330	+ 480	+1318
330 „ 360	+1134	+2283

From this Table it would appear that these observations so treated exhibit for Jupiter and Venus an excess of solar activity when these two planets are together, and a deficiency when they are apart, and that the same kind of influence, slightly modified in form, is exhibited in the case of Venus and Mercury.

Postscript (added 5th March).—Received March 10, 1870.

Since writing the above we have thought it worth while to investigate, after the manner we have described, the influence, if any, upon solar disturbance of the relative positions of Mercury and Jupiter, and also that of Mercury alone in its varying distances from the sun, and we have obtained the following results:—

Table VI *b*.

Angular separation* between	Mercury and his Perihelion (Perihelion =0).	Mercury and Jupiter.
0° and 30°	− 380	−227
30 ,, 60	−1188	−317
60 ,, 90	−1287	−594
90 ,, 120	−1262	−714
120 ,, 150	−1208	−508
150 ,, 180	−1027	−491
180 ,, 210	− 519	−416
210 ,, 240	+ 430	−189
240 ,, 270	+1082	− 25
270 ,, 300	+1436	+154
300 ,, 330	+1282	+164
330 ,, 360	+ 586	− 45

The numbers in the above Table are smaller than those in the Tables already given; but this may be owing to the method of equalization we have adopted, which will necessarily tell very greatly in small periods.

Nevertheless there appears to be a certain amount of likeness between the march of the numbers in the four periods which we have investigated, namely, those of Venus and Jupiter, Venus and Mercury, Mercury and his Perihelion, and Mercury and Jupiter.

We desire to record this rather as a result brought out by a certain specified method of treating the material at our disposal, than as a fact from which we are at present prepared to draw conclusions. As the investigation of these and similar phenomena proceeds it may be hoped that much light will be thrown upon the causes of sun-spot periodicity.

July 11, 1870.—Sir J. Herschel has kindly permitted us to append the following remarks which he has made upon the foregoing paper:—

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* In all the above Tables equal angles have been supposed to be described in equal times; there will be a slight correction on this account for Mercury and his Perihelion.

"The curves, as delineated for the three complete periods embraced, are highly interesting and instructive. They place in a very clear and unmistakable light the anticipation in point of time of the occurrence of the maximum of maculiferous excitement before the middle of the total period which the present rapidly increasing number of spots actually in progress seems to promise fully to confirm for the period (1866·6–1877·7); and here I cannot help observing that although the lengths of the three periods here embraced vary between 9·81 and 12·68 years, yet the mean of the three is almost exactly 11·1, and this agrees with the whole course of the solar history since 1800, which was a year of maximum. Another point of much interest is the evaluation of the spotted area in numerical aliquots of the solar surface, showing to how very small an extent the sun is entitled to the character of a variable star (in so far at least as diminution and increase of illuminated area are to be regarded as causes of variability). The total fluctuation arising from this cause it appears does not exceed about 2000 parts in a million, or about $\frac{1}{500}$ part of the total light, which in estimating the brightness of a star would be quite inappreciable*. The tendency to a double maximum, an earlier and a later, pointed out as analogous to those observed in β Lyræ and R. Sagittæ, it may be worth noticing, has a similar though less strongly marked analogous feature in the light curve of η Argûs, as laid down for a periodic variation sixty-seven years in duration by Professor LOOMIS, which presents a subordinate maximum (followed by a rather marked depression) at about the twenty-sixth year, the principal one being deferred to the forty-fourth of the total period; and this law is observed in three successive periods."

It has also been suggested to us by Sir W. THOMSON, in connexion with that part of our paper which refers to the possible connexion between sun-spots and planetary position, to take arbitrary periods, viz. three-fourths of the planetary period, which we have taken, and to compare the results of these with the results derived in our paper. We have done this in the case of Venus and Jupiter, and also in that of Venus and Mercury, and have obtained the following numbers:—

Three-fourths of the period of Venus and Jupiter.	Three-fourths of the period of Venus and Mercury.
−117	− 184
−287	− 754
−694	−1621
−631	−1196
−461	+ 567
−140	+1764
+441	+ 848
+310	− 466
−139	− 605

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* We have not lost sight of the very small amount of variation in the brightness of the solar surface attributable

It will be seen from these numbers that the curves derived from these arbitrary periods are both less marked than the corresponding curves derived from the full periods, although the number of series summed together has been increased in the proportion of four to three.

In conclusion, we may state that when we proposed that the present series of observations should terminate in February 1872, we were under the impression that the period was nearer ten than eleven years. The results of this paper would tend to show that the period is over eleven years; nevertheless we do not propose an extension of our series, for we think that the maximum will probably be reached before the proposed termination.

We ought also to state that in all probability the disturbances which we have hitherto registered are only those which occur in the lower regions of the sun's atmosphere. It is therefore our intention to supplement our research on sun-spots with an evaluation of the faculæ or bright patches, which are also signs of solar disturbance, but which occur in the higher and not in the lower regions of the solar atmosphere. The best method of doing this is at present engaging our attention.

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to the variation in the number of spots, and we have had it in contemplation to make direct observations of variability in the actinism of the sun in order to ascertain whether the period of greatest brightness is or is not coincident with that of the maximum of spotted area; for it is not improbable that the sun may be the most brilliant when most spotted.

A thorough examination of the solar photographs will no doubt do much to elucidate the subject, but these have the disadvantage of only representing with the greatest distinctness the faculæ which are nearest the limb; on the other hand direct actinic observations are interfered with by the ever varying conditions of the earth's atmosphere, and also by the variations of the sun's meridian altitude, so that it will be necessary to combine both methods of observation.

TABLE VII.—Solar Spotted Area (in millionths).

Column I. contains the fortnightly means, Column II. contains the means of the three fortnights preceding, and the three following the date.

	1832.		1833.		1834.		1835.		1836.		1837.	
	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
January 7.	72	......	12	......	18	......	132	......	930	......	2124	......
14.	......	......	......	183	......	135	......	249	......	1195	......	2081
21.	318	......	294	......	6	......	66	......	1134	......	2412	......
28.	......	......	......	165	......	95	......	298	......	1193	......	1983
February 7.	210	......	102	......	276	......	354	......	1350	......	2028	......
14.	......	450	......	152	......	96	......	222	......	1205	......	1836
21.	726	......	282	......	258	......	246	......	1944	......	2052	......
28.	......	509	......	153	......	98	......	329	......	1314	......	1693
March ... 7.	666	......	132	......	6	......	486	......	954	......	1272	......
14.	......	477	......	106	......	97	......	445	......	1380	......	1436
21.	708	......	90	......	12	......	48	......	918	......	1128	......
28.	......	543	......	92	......	51	......	539	......	1390	......	1203
April...... 7.	426	......	18	......	30	......	774	......	1584	......	1266	......
14.	......	530	......	55	......	27	......	563	......	1290	......	1027
21.	126	......	12	......	0	......	762	......	1530	......	870	......
28.	......	489	......	33	......	38	......	633	......	1356	......	926
May ...... 7.	606	......	18	......	0	......	918	......	1410	......	630	......
14.	......	434	......	18	......	68	......	684	......	1444	......	984
21.	648	......	60	......	114	......	390	......	1344	......	996	......
28.	......	395	......	36	......	72	......	683	......	1448	......	982
June ...... 7.	420	......	0	......	72	......	906	......	1350	......	966	......
14.	......	388	......	37	......	86	......	723	......	1465	......	956
21.	378	......	0	......	192	......	354	......	1446	......	1176	......
28.	......	304	......	36	......	87	......	771	......	1400	......	972
July ...... 7.	192	......	126	......	54	......	768	......	1608	......	1254	......
14.	......	199	......	31	......	72	......	941	......	1360	......	930
21.	84	......	18	......	84	......	1002	......	1632	......	714	......
28.	......	150	......	47	......	85	......	1058	......	1317	......	935
August ... 7.	102	......	12	......	6	......	[1206]	......	1020	......	[726]	......
14.	......	88	......	65	......	91	......	1233	......	1214	......	862
21.	18	......	30	......	24	......	[1410]	......	1104	......	744	......
28.	......	72	......	75	......	87	......	1220	......	1153	......	754
September 7.	126	......	96	......	150	......	1608	......	1092	......	996	......
14.	......	79	......	79	......	166	......	1224	......	1208	......	903
21.	6	......	108	......	228	......	1404	......	828	......	738	......
28.	......	96	......	99	......	197	......	1235	......	1267	......	953
October 7.	96	......	186	......	30	......	690	......	1242	......	606	......
14.	......	118	......	98	......	241	......	1212	......	1369	......	911
21.	126	......	42	......	558	......	1026	......	1962	......	1608	......
28.	......	137	......	123	......	248	......	1105	......	1497	......	1011
November 7.	204	......	132	......	192	......	1272	......	1374	......	1026	......
14.	......	164	......	106	......	294	......	1012	......	1694	......	1113
21.	150	......	24	......	288	......	1272	......	1716	......	492	......
28.	......	150	......	78	......	311	......	1052	......	1841	......	1466
December 7.	240	......	246	......	192	......	966	......	[1860]	......	1596	......
14.	......	178	......	72	......	229	......	1070	......	1916	......	1375
21.	168	......	6	......	504	......	846	......	2010	......	1350	......
28.	......	161	......	96	......	256	......	1083	......	2025	......	1418

TABLE VII. (continued).

	1838.		1839.		1840.		1841.		1842.		1843.	
	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
January 7.	2724	......	732	......	726	......	276	......	78	......	150	......
14.	......	1561	......	965	......	600	......	290	......	219	......	118
21.	1062	......	1362	......	612	......	168	......	186	......	126	......
28.	......	1488	......	941	......	658	......	277	......	185	......	99
February 7.	1284	......	1212	......	960	......	204	......	228	......	102	......
14.	......	1410	......	941	......	685	......	211	......	172	......	76
21.	1350	......	1104	......	408	......	222	......	72	......	6	......
28.	......	1138	......	895	......	648	......	241	......	208	......	53
March ... 7.	1158	......	678	......	774	......	246	......	294	......	60	......
14.	......	1103	......	774	......	635	......	275	......	215	......	70
21.	882	......	558	......	630	......	150	......	174	......	12	......
28.	......	1082	......	670	......	621	......	382	......	193	......	104
April...... 7.	1092	......	456	......	504	......	456	......	294	......	12	......
14.	......	1032	......	591	......	636	......	440	......	199	......	140
21.	852	......	636	......	534	......	372	......	228	......	228	......
28.	......	983	......	585	......	607	......	483	......	191	......	151
May ...... 7.	1158	......	588	......	876	......	846	......	96	......	306	......
14.	......	945	......	581	......	549	......	533	......	206	......	159
21.	1050	......	630	......	498	......	570	......	108	......	222	......
28.	......	883	......	640	......	513	......	518	......	183	......	161
June ...... 7.	864	......	642	......	600	......	504	......	246	......	126	......
14.	......	923	......	632	......	506	......	474	......	155	......	139
21.	654	......	534	......	282	......	450	......	264	......	60	......
28.	......	820	......	705	......	462	......	261	......	195	......	103
July ...... 7.	720	......	810	......	288	......	366	......	156	......	24	......
14.	......	747	......	898	......	466	......	333	......	202	......	90
21.	1092	......	588	......	492	......	108	......	60	......	96	......
28.	......	769	......	1003	......	543	......	305	......	212	......	69
August ... 7.	540	......	1026	......	612	......	168	......	336	......	90	......
14.	......	729	......	1159	......	608	......	323	......	176	......	61
21.	612	......	1788	......	522	......	402	......	150	......	144	......
28.	......	701	......	1133	......	618	......	327	......	197	......	73
September 7.	996	......	1272	......	1062	......	336	......	306	......	0	......
14.	......	663	......	1162	......	635	......	336	......	257	......	57
21.	414	......	1470	......	672	......	558	......	48	......	12	......
28.	......	682	......	1076	......	665	......	351	......	282	......	76
October... 7.	552	......	654	......	348	......	390	......	282	......	96	......
14.	......	667	......	855	......	690	......	304	......	306	......	92
21.	864	......	762	......	594	......	162	......	420	......	0	......
28.	......	638	......	714	......	567	......	331	......	284	......	111
November 7.	654	......	510	......	792	......	258	......	486	......	204	......
14.	......	662	......	547	......	546	......	280	......	301	......	118
21.	522	......	462	......	672	......	120	......	294	......	240	......
28.	......	692	......	559	......	534	......	228	......	279	......	124
December 7.	822	......	426	......	324	......	498	......	174	......	114	......
14.	......	775	......	534	......	463	......	232	......	230	......	148
21.	558	......	468	......	546	......	252	......	150	......	54	......
28.	......	868	......	609	......	365	......	227	......	166	......	132

TABLE VII. (continued).

	1844.		1845.		1846.		1847.		1848.	
	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
January 7.	132	......	432	......	300	......	438	......	1650	......
14.	......	125	......	433	......	700	......	881	......	1270
21.	144	......	402	......	948	......	1038	......	1098	......
28.	......	122	......	517	......	696	......	936	......	1206
February 7.	108	......	354	......	414	......	1146	......	1158	......
14.	......	137	......	493	......	734	......	1033	......	1058
21.	198	......	924	......	1260	......	1278	......	912	......
28.	......	120	......	597	......	786	......	1070	......	867
March ... 7.	96	......	576	......	618	......	1332	......	834	......
14.	......	162	......	651	......	792	......	961	......	770
21.	144	......	270	......	864	......	966	......	696	......
28.	......	177	......	662	......	846	......	852	......	814
April...... 7.	30	......	1056	......	612	......	660	......	504	......
14.	......	172	......	559	......	724	......	714	......	796
21.	396	......	726	......	984	......	384	......	516	......
28.	......	158	......	547	......	762	......	597	......	835
May ...... 7.	198	......	420	......	738	......	492	......	1422	......
14.	......	139	......	539	......	677	......	622	......	844
21.	168	......	306	......	528	......	450	......	804	......
28.	......	191	......	389	......	647	......	536	......	1014
June ...... 7.	12	......	504	......	846	......	430	......	1068	......
14.	......	179	......	307	......	569	......	612	......	1141
21.	30	......	222	......	354	......	1116	......	750	......
28.	......	219	......	288	......	504	......	677	......	1035
July ...... 7.	342	......	156	......	432	......	144	......	1524	......
14.	......	233	......	276	......	548	......	929	......	1094
21.	324	......	234	......	516	......	840	......	1278	......
28.	......	245	......	201	......	552	......	1040	......	984
August ... 7.	438	......	306	......	348	......	882	......	786	......
14.	......	246	......	231	......	654	......	1195	......	1150
21.	252	......	234	......	792	......	1962	......	1158	......
28.	......	239	......	228	......	637	......	1500	......	1073
September 7.	84	......	54	......	870	......	1296	......	408	......
14.	......	210	......	262	......	602	......	1634	......	1098
21.	36	......	402	......	966	......	2046	......	1746	......
28.	......	153	......	254	......	634	......	1776	......	1083
October... 7.	300	......	138	......	330	......	1974	......	1062	......
14.	......	129	......	244	......	562	......	1687	......	1016
21.	150	......	438	......	306	......	1644	......	1428	......
28.	......	127	......	342	......	584	......	1674	......	1106
November 7.	96	......	258	......	540	......	1734	......	696	......
14.	......	190	......	381	......	487	......	1597	......	1102
21.	108	......	174	......	360	......	1428	......	756	......
28.	......	212	......	408	......	505	......	1543	......	1175
December 7.	72	......	642	......	1002	......	1218	......	948	......
14.	......	254	......	493	......	627	......	1452	......	1244
21.	414	......	636	......	384	......	1584	......	1722	......
28.	......	297	......	519	......	728	......	1356	......	1315

TABLE VII. (continued).

	1849.		1850.		1851.		1852.		1853.	
	I.	II.	I.	II	I.	II.	I	II.	I.	II.
January 7.	1500	......	864	......	624	......	744	......	420	......
14.	......	1372	......	633	......	739	......	728	......	455
21.	1842	......	600	......	600	......	828	......	348	......
28.	......	1334	......	698	......	780	......	754	......	406
February 7.	1122	......	618	......	1368	......	498	......	282	......
14.	......	1215	......	675	......	863	......	696	......	386
21.	1098	......	666	......	966	......	852	......	564	......
28.	......	1151	......	584	......	876	......	718	......	400
March ... 7.	720	......	792	......	786	......	528	......	174	......
14.	......	989	......	545	......	884	......	654	......	412
21.	1008	......	510	......	834	......	726	......	528	......
28.	......	947	......	501	......	837	......	657	......	425
April...... 7.	1116	......	318	......	702	......	876	......	504	......
14.	......	899	......	451	......	746	......	618	......	404
21.	870	......	366	......	648	......	444	......	420	......
28.	......	896	......	446	......	714	......	597	......	484
May ...... 7.	868	......	354	......	1086	......	516	......	360	......
14.	......	816	......	438	......	681	......	529	......	485
21.	810	......	366	......	420	......	618	......	438	......
28.	......	778	......	473	......	637	......	425	......	485
June ...... 7.	702	......	762	......	594	......	402	......	654	......
14.	......	743	......	456	......	585	......	423	......	473
21.	528	......	462	......	636	......	318	......	534	......
28.	......	682	......	478	......	488	......	405	......	483
July ...... 7.	888	......	528	......	438	......	252	......	504	......
14.	......	615	......	495	......	523	......	351	......	485
21.	660	......	264	......	336	......	432	......	348	......
28.	......	610	......	453	......	567	......	362	......	446
August ... 7.	504	......	486	......	504	......	408	......	420	......
14.	......	624	......	532	......	597	......	379	......	401
21.	408	......	468	......	630	......	294	......	450	......
28.	......	573	......	514	......	596	......	405	......	408
September 7.	672	......	510	......	858	......	468	......	420	......
14.	......	594	......	545	......	647	......	467	......	412
21.	612	......	936	......	816	......	420	......	264	......
28.	......	651	......	513	......	635	......	519	......	404
October 7.	582	......	420	......	432	......	408	......	546	......
14.	......	769	......	529	......	630	......	562	......	347
21.	786	......	450	......	642	......	804	......	372	......
28.	......	724	......	534	......	549	......	562	......	310
November 7.	846	......	294	......	432	......	720	......	372	......
14.	......	730	......	434	......	592	......	600	......	287
21.	1116	......	564	......	600	......	552	......	108	......
28.	......	777	......	468	......	644	......	602	......	209
December 7.	402	......	540	......	372	......	468	......	198	......
14.	......	746	......	493	......	675	......	526	......	150
21.	648	......	336	......	1074	......	648	......	126	......
28.	......	708	......	672	......	686	......	453	......	130

Table VII. (continued).

	1854.		1855.		1856.		1857.		1858.	
	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
January 7.	77	......	373	......	0	......	158	......	1404	......
14.	......	136	......	269	......	23	......	61	......	591
21.	19	......	625	......	0	......	19	......	378	......
28.	......	137	......	269	......	9	......	61	......	782
February 7.	252	......	339	......	35	......	137	......	741	......
14.	......	143	......	264	......	9	......	55	......	837
21.	142	......	133	......	17	......	5	......	385	......
28.	......	185	......	202	......	13	......	73	......	621
March ... 7.	208	......	103	......	0	......	2	......	1279	......
14.	......	200	......	99	......	15	......	76	......	618
21.	159	......	10	......	0	......	6	......	837	......
28.	......	177	......	47	......	9	......	123	......	545
April...... 7.	330	......	2	......	23	......	266	......	106	......
14.	......	156	......	25	......	6	......	187	......	564
21.	111	......	5	......	12	......	38	......	358	......
28.	......	125	......	9	......	11	......	210	......	484
May ...... 7.	114	......	26	......	0	......	419	......	305	......
14.	......	122	......	8	......	11	......	220	......	411
21.	12	......	1	......	0	......	389	......	501	......
28.	......	70	......	7	......	7	......	212	......	528
June ...... 7.	26	......	11	......	28	......	144	......	796	......
14.	......	68	......	7	......	14	......	246	......	526
21.	138	......	1	......	0	......	66	......	399	......
28.	......	76	......	5	......	14	......	184	......	523
July ...... 7.	19	......	0	......	0	......	213	......	810	......
14.	......	78	......	5	......	62	......	141	......	588
21.	97	......	0	......	58	......	245	......	342	......
28.	......	109	......	3	......	61	......	218	......	696
August ... 7.	165	......	18	......	0	......	46	......	289	......
14.	......	109	......	3	......	62	......	297	......	722
21.	22	......	0	......	285	......	132	......	889	......
28.	......	135	......	7	......	62	......	316	......	815
September 7.	215	......	0	......	25	......	603	......	1448	......
14.	......	130	......	20	......	53	......	338	......	977
21.	138	......	1	......	2	......	540	......	556	......
28.	......	140	......	19	......	53	......	379	......	1205
October 7.	171	......	20	......	0	......	330	......	1364	......
14.	......	230	......	20	......	11	......	374	......	1241
21.	70	......	82	......	3	......	375	......	1313	......
28.	......	212	......	34	......	8	......	295	......	1219
November 7.	223	......	12	......	2	......	295	......	1657	......
14.	......	196	......	34	......	14	......	289	......	1309
21.	565	......	4	......	36	......	101	......	1110	......
28.	......	229	......	30	......	41	......	468	......	1489
December 7.	105	......	82	......	2	......	129	......	1313	......
14.	......	322	......	17	......	43	......	469	......	1510
21.	40	......	2	......	42	......	506	......	1098	......
28.	......	341	......	21	......	66	......	543	......	1337

TABLE VII. (continued).

	1859.		1860.		1861.		1862.		1863.	
	I.	II.	I.	II.	I.	II	I.	II.	I.	II.
January 7.	2442	......	442	......	[1140*]	......	[936*]	......	1511	......
14.	......	1308	......	1109	......	1377	......	1188	......	883
21.	1440	......	1097	......	[810*]	......	[948*]	......	901	......
28.	......	1199	......	1157	......	1181	......	1038	......	959
February 7.	620	......	1834	......	[588*]	......	1209	......	[654*]	......
14.	......	1244	......	1128	......	1239	......	879	......	969
21.	934	......	1003	......	[1722*]	......	1215	......	[960*]	......
28.	......	888	......	1165	... ..	1332	......	858	......	820
March ... 7.	661	.. ...	1474	......	[1494*]	......	750	......	[858*]	......
14.	......	768	......	1071	......	1479	......	823	......	763
21.	1367	......	917	.....	[1680*]	......	220	......	[930*]	.....
28.	......	869	......	980	......	1551	......	848	......	758
April...... 7.	306	......	666	......	[1698*]	......	805	......	[618*]	......
14.	... ..	861	......	961	......	1504	......	843	......	733
21.	718	. . ..	531	......	[1692*]	......	742	......	[558*]	......
28.	......	904	......	833	......	1427	......	1110	......	689
May ...... 7.	1230	......	1289	......	[1020*]	......	1356	......	622	......
14.	......	772	......	895	......	1492	......	1440	......	573
21.	884	......	880	......	[1440*]	......	1187	......	812	......
28.	......	820	......	1203	......	1465	......	1501	......	589
June ...... 7.	918	......	708	.....	[1032*]	......	2351	......	593	......
14.	......	1210	......	1423	......	1368	......	1776	......	546
21.	576	......	1286	......	[2070*]	......	2199	......	233	.. ...
28.	......	1237	......	1631	......	1370	......	1814	......	540
July ...... 7.	592	......	2518	......	[1536*]	......	1170	......	720	......
14.	......	1497	......	1624	......	1368	......	1795	......	436
21.	3062	......	1852	......	[1110*]	......	2396	......	298	......
28.	......	1904	......	1706	......	1330	......	1544	.. ...	348
August ... 7.	1387	......	2538	......	[1032*]	......	1580	......	587	......
14.	......	1990	......	1581	......	1256	......	1627	......	358
21.	2444	......	842	......	[1428*]	......	1073	......	183	......
28.	......	2186	......	1401	......	1143	......	1558	......	338
September 7.	3365	......	1200	......	[804*]	......	849	......	69	......
14.	......	2078	......	1301	......	1162	......	1276	......	343
21.	1090	......	534	......	[1626*]	......	2692	......	292	......
28.	......	2170	......	1144	......	1139	......	1131	......	316
October... 7.	1765	......	1443	......	[858*]	......	759	......	598	......
14.	......	1951	......	1213	......	1158	......	1123	......	356
21.	2414	......	1251	......	[1224*]	......	705	......	331	......
28.	......	1594	......	1458	......	1299	......	1049	......	440
November 7.	1941	......	1597	......	[894*]	......	708	......	422	......
14.	......	1589	......	1591	......	1223	......	745	......	610
21.	1131	......	1255	......	[1542*]	......	1023	.....	427	......
28.	......	1368	......	1541	......	1236	......	870	......	766
December 7.	1222	......	2670	......	[1650*]	......	405	......	573	......
14.	......	1152	......	1467	......	1190	......	903	......	1004
21.	1060	......	1332	......	[1170*]	......	869	......	1312	......
28.	......	1131	......	1297	......	1242	......	894	......	1090

Table VII. (continued).

	1864.		1865.		1866.		1867.		1868.	
	I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
January 7.	[1533]	......	341	...	964	......	0	......	587	......
14.	......	1163	......	916	......	815	......	2	......	261
21.	1755	......	1777	......	767	......	0	......	25	......
28.	......	1293	......	821	......	804	......	2	......	266
February 7.	941	......	980	......	1108	......	0	......	76	......
14.	......	1222	......	689	......	779	......	23	......	309
21.	863	......	254	......	1239	......	0	......	433	......
28.	......	1075	......	669	......	655	......	49	......	342
March ... 7.	1352	......	350	......	283	......	12	......	242	......
14.	......	802	......	404	......	553	......	51	......	537
21.	888	......	437	......	318	......	126	......	490	......
28.	......	706	......	276	......	396	......	54	......	618
April...... 7.	654	......	215	......	218	......	160	......	787	......
14.	......	751	......	321	......	221	......	101	......	578
21.	113	......	187	......	155	......	6	......	1197	
28.	......	777	......	362	......	214	......	107	......	
May ...... 7.	364	......	216	......	162	......	19	......	559	
14.	......	740	......	295	......	203	......	88	......	
21.	1136	......	519	......	187	......	282	......	193	
28.	......	855	......	277	......	189	......	74		
June ...... 7.	1505	......	598	......	247	......	52	......		
14.	......	945	......	274	......	172	......	73		
21.	671	......	36	......	248	......	8	......		
28.	......	1169	......	316	......	171	......	79		
July ...... 7.	1342	......	105	......	137	......	76	......		
14.	......	1041	......	295	......	183	......	108		
21.	655	......	172	......	47	......	2	......		
28.	......	859	......	239	......	160	......	258		
August ... 7.	1708	......	465	......	158	......	55	......		
14.	......	763	......	353	......	153	......	362		
21.	364	......	395	......	261	......	[457]	......		
28.	......	566	......	480	......	146	......	433		
September 7.	411	......	259	......	109	......	951	......		
14.	......	532	......	499	......	169	......	449		
21.	96	......	723	......	207	......	630	......		
28.	......	446	......	477	......	144	......	477		
October... 7.	164	......	868	......	92	......	504	......		
14.	......	557	......	514	......	119	......	417		
21.	447	......	285	......	190	......	96	......		
28.	......	642	......	529	......	103	......	293		
November 7.	1192	......	330	......	8	......	226	......		
14.	......	829	......	487	......	68	......	228		
21.	1034	......	622	......	108	......	97	......		
28.	......	859	......	503	......	53	......	241		
December 7.	920	......	350	......	11	......	207	......		
14.	......	1081	......	583	......	21	......	229		
21.	1222	......	465	......	0	......	236	......		
28.	......	1046	......	713	......	19	......	204		

TABLE VIII.—Table exhibiting the eleven-yearly period of Solar spot activity, equalized for shorter periods.

	1832.	1833.	1834.	1835.	1836.	1837.	1838.	1839.	1840.	1841.	1842.	1843.	1844.	1845.	1846.	1847.	1848.	1849.
Jan. 14.	......	120	55	375	1250	1580	930	790	775	415	265	155	115	295	500	730	1375	890
28.	......	110	55	415	1265	1555	975	790	760	405	255	150	120	305	510	730	1365	875
Feb. 14.	......	105	60	450	1280	1515	965	785	745	395	250	140	125	315	515	765	1350	860
28.	......	100	60	490	1290	1475	955	785	725	390	245	130	130	325	525	780	1320	850
Mar. 14.	......	95	65	525	1300	1435	945	785	705	385	240	120	140	340	535	815	1290	840
28.	......	95	70	565	1315	1400	935	790	685	380	235	115	150	350	545	845	1260	825
Apr. 14.	......	90	75	605	1325	1365	925	800	665	370	230	115	155	355	550	880	1220	810
28.	......	85	80	645	1335	1330	915	805	645	360	225	110	160	365	555	920	1190	800
May 14.	......	85	85	690	1345	1300	905	805	620	355	220	110	170	375	565	965	1165	795
28.	......	80	90	730	1355	1270	900	805	600	350	215	105	180	380	570	1020	1145	785
June 14.	......	75	100	780	1360	1240	890	810	580	345	210	100	185	390	580	1080	1115	775
28.	305	70	110	830	1370	1215	880	815	560	340	205	100	190	395	590	1140	1095	760
July 14.	285	70	120	875	1380	1185	870	820	545	335	205	95	200	400	600	1200	1070	750
28.	260	65	130	925	1390	1170	860	825	530	330	200	90	205	410	610	1250	1060	740
Aug. 14.	240	60	140	970	1405	1140	850	830	520	325	200	90	215	420	615	1300	1040	730
28.	220	60	150	1005	1425	1120	840	840	500	320	195	90	220	425	625	1330	1020	715
Sept. 14.	205	55	165	1045	1455	1100	830	840	490	315	195	85	230	430	635	1365	1000	705
28.	185	50	180	1080	1480	1080	815	845	480	310	190	85	240	440	645	1390	985	695
Oct. 14.	170	50	200	1120	1510	1065	805	845	470	305	185	90	245	445	655	1405	970	680
28.	155	50	225	1145	1550	1050	800	840	460	300	180	95	255	450	665	1410	960	670
Nov. 14.	145	50	250	1170	1575	1035	800	830	450	295	175	95	265	460	675	1415	945	660
28.	135	45	280	1195	1600	1025	795	820	440	285	170	100	270	475	690	1410	930	650
Dec. 14.	125	50	315	1220	1610	1015	795	805	435	280	165	105	280	480	700	1405	915	640
28.	120	50	350	1240	1610	1000	790	795	425	270	160	115	290	490	710	1400	900	630

TABLE VIII.—Table exhibiting the eleven-yearly period of Solar spot activity, equalized for shorter periods.

1850.	1851.	1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	1866.	1867.	1868.
625	630	620	430	270	135	35	80	465	1115	1440	1325	1180	1000	765	710	370	55	375
620	640	610	425	265	135	35	90	485	1150	1425	1330	1170	955	780	695	355	50	400
610	645	600	415	255	130	30	100	505	1175	1410	1335	1160	920	800	680	350	50	420
600	650	590	405	250	125	25	105	535	1200	1400	1340	1160	890	815	660	340	50	445
595	655	580	400	245	120	25	115	555	1250	1390	1345	1160	855	835	640	315	55	475
590	660	575	395	235	115	25	125	580	1275	1380	1350	1160	825	855	630	295	55	500
580	660	565	390	230	110	20	135	600	1305	1370	1350	1165	795	870	615	280	60	525
575	665	555	380	225	100	20	115	630	1340	1355	1350	1175	770	870	600	265	65	
570	670	550	370	220	95	25	155	660	1360	1350	1350	1185	745	870	590	250	75	
565	675	540	365	215	90	25	165	695	1400	1345	1350	1190	725	865	570	230	90	
565	680	530	360	205	85	30	180	720	1425	1335	1350	1195	705	865	560	215	105	
560	685	525	355	200	80	30	195	750	1450	1325	1350	1205	685	860	550	200	120	
560	680	515	350	195	75	30	205	775	1475	1320	1350	1205	670	850	540	180	145	
555	675	505	345	190	70	35	225	800	1500	1320	1345	1205	655	845	520	165	160	
555	670	500	335	180	65	40	250	830	1505	1315	1335	1205	655	835	505	150	175	
555	665	495	325	175	60	40	270	850	1510	1305	1325	1195	655	820	495	140	200	
560	660	485	315	165	55	45	285	885	1510	1300	1310	1180	650	805	475	125	210	
565	655	475	310	160	50	50	305	905	1505	1300	1290	1175	650	800	465	105	225	
575	655	465	305	155	50	55	330	915	1505	1300	1275	1170	655	790	455	95	255	
585	655	460	300	150	50	60	350	975	1500	1300	1255	1155	670	775	445	85	275	
595	650	455	290	150	45	65	375	1000	1490	1305	1230	1135	680	765	425	75	300	
605	640	450	285	145	45	70	400	1030	1480	1305	1215	1110	700	750	415	65	320	
615	635	445	280	140	40	70	420	1055	1475	1310	1200	1080	720	740	400	60	340	
620	630	440	275	140	40	75	445	1090	1455	1320	1190	1040	745	725	375	55	355	

TABLE II. (continued).—Showing the areas of all Sun-spots observed at the Kew Observatory from January 1, 1864 to December 31, 1866.

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. Jan. 24.	526	0·974	189	......	189	1458	405	1866
	525	0·869	503	230	733			
	524	0·156	17	4	21			
	523	0·078	145	21	167			
	521	0·562	208	61	370			
	520	0·687	123	23	146			
	519	0·713	108	36	145			
	522	0·693	65	30	95			
26.	526	0·677	35	17	52			
	525	0·531	888	301	1190	1811	606	2418
	523	0·500	118	25	142			
	521	0·885	408	136	545			
	519	0·963	170	31	201			
	520	0·948	192	96	288			
28.	526	0·286	446	89	535	1224	313	1537
	525	0·167	649	194	843			
	523	0·833	129	30	159			
30.	527	0·594	120	26	147	729	288	1020
	526	0·250	51	8	60			
	525	0·443	558	122	681			
	523	0·974	......	132	132			
Feb. 4.	529	0·635	75	16	92	386	73	461
	528	0·479	228	57	286			
	527	0·573	83	......	83			
5.	529	0·401	190	41	233	734	119	857
	528	0·271	424	66	491			
	527	0·739	120	12	133			
6.	530	0·521	108	19	128	891	188	1081
	529	0·255	316	44	360			
	528	0·156	391	103	495			
	527	0·833	76	22	98			
10.	530	0·437	198	61	260	934	226	1163
	529	0·740	362	63	426			
	528	0·864	374	102	477			
17.	534	0·635	59	10	75	332	70	410
	533	0·040	67	12	80			
	532	0·390	59	4	64			
	531	0·495	147	44	191			
Mar. 2.	540	0·869	289	93	382	1206	323	1527
	539	0·791	110	20	131			
	538	0·375	55	13	69			
	537	0·312	134	52	187			
	536	0·729	355	93	443			
	535	0·875	263	52	315			
4.	540	0·573	286	78	364	996	309	1307
	539	0·438	65	9	75			
	538	0·192	56	13	69			
	537	0·568	197	78	275			
	536	0·954	392	131	524			
10.	543	0·938	136	37	173			
	542	0·563	132	35	169			
	541	0·599	336	37	373			

Table II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. Mar. 10.	540	0·815	266	88	356	870	197	1071
11.	543	0·863	110	25	136			
	542	0·384	92	27	119			
	541	0·526	411	90	502			
	540	0·920	327	109	436	940	251	1193
12.	543	0·668	91	28	119			
	542	0·168	154	21	177			
	541	0·394	671	138	810			
	540	0·984	490	122	612	1406	309	1718
16.	544	0·984	441	73	514			
	543	0·210	87	21	108			
	542	0·736	137	25	161			
	541	0·768	523	66	589	1188	185	1372
17.	546	0·973	264	37	302			
	545	0·947	221	39	261			
	544	0·852	200	32	232			
	543	0·447	95	19	114			
	542	0·894	130	28	158			
	541	0·899	426	58	486	1336	223	1555
18.	546	0·904	170	50	221			
	545	0·847	136	56	192			
	544	0·752	201	45	247			
	543	0·605	101	21	122			
	541	0·973	662	37	699			
	542	0·968	193	35	228	1463	244	1709
19.	546	0·799	176	33	212			
	545	0·694	94	29	123			
	544	0·584	173	42	215			
	543	0·778	73	27	100	516	133	650
23.	548	0·762	211	66	278			
	547	0·589	136	31	168			
	546	0·247	73	17	91			
	545	0·168	17	...	17			
	544	0·247	104	35	139	541	149	693
29.	550	0·884	171	36	208			
	549	0·342	99	36	135			
	548	[illegible]	171	44	216			
	547	0·789	62	20	82	503	136	641
30.	550	0·736	74	31	105			
	549	0·368	128	46	174			
	548	0·826	311	76	388			
	547	0·931	95	11	107	608	164	774
31.	550	0·647	[illegible]	33	182			
	549	0·431	70	...	70			
	548	0·899	241	80	322	460	113	574
April 1.	550	0·468	120	24	144			
	549	0·631	97	10	109			
	548	0·973	264	56	321	481	90	574
2.	550	0·326	108	27	135			
	549	0·799	56	21	77	164	48	212
8.	554	0·979	348	61	410			
	553	0·394	295	49	347			
	552	0·868	494	76	570			
	551	0·973	151	56	197	1288	242	1524
9.	554	0·894	158	56	215			
	553	0·147	227	60	288			

TABLE II. (continued).

Date	Group	Mean distance from centre, radius=1.	Area of Penumbra	Area of Umbra.	Area of whole Spot	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. April 9.	552	0·973	226	113	340	611	229	843
11.	554	0·647	127	44	171	318	101	420
	553	0·289	191	57	249			
12.	554	0·684	92	23	116	280	72	353
	553	0·515	188	49	237			
13.	554	0·396	59	13	73	183	47	233
	553	0·159	124	34	159			
14.	554	0·370	64	18	82	176	36	212
	553	0·344	112	18	130			
15.	554	0·460	61	14	75	307	155	462
	553	0·952	219	29	247			
	555	0·634	27	21	49			
	556	0·989	...	91	91			
18.	554	0·989	183	91	275	279	120	401
	556	0·820	96	29	126			
19.	556	0·661	61	33	95	61	33	95
20.	556	0·492	68	24	93	68	24	93
21.	556	0·238	60	8	69	60	8	69
22.	556	0·021	46	12	59	46	12	59
23.	556	0·196	13	8	21	13	8	21
25.	556	0·635	21	5	27	21	5	27
26. 29.	No spots.							
May 3.	557	0·346	58	9	67	131	33	165
	558	0·985	73	24	98			
5.	557	0·772	80	27	107	218	68	286
	559	0·772	40	27	67			
	558	0·495	98	14	112			
6.	559	0 638	50	...	50	163	16	181
	558	0 309	70	8	79			
	557	0·872	43	8	52			
7.	560	0·888	215	46	262	406	82	490
	558	0 187	130	21	151			
	557	0·938	61	15	77			
10.	560	0·442	33	9	42	104	22	126
	558	0 756	71	13	84			
12.	560	0·356	22	9	31	170	79	249
	561	0·872	148	70	218			
13.	560	0·495	9	4	14	434	122	557
	561	0·729	425	118	543			
14.	561	0·548	603	106	711	705	123	830
	560	0·665	102	17	119			
15.	562	0·979	123	82	205	1028	318	1347
	561	0·346	764	208	973			
	560	0·798	141	28	169			
16.	562	0·453	70	14	85	992	235	1230
	561	0·176	804	150	956			
	560	0·931	118	71	189			
17.	562	0·293	111	66	178	943	362	1306
	561	0·203	832	174	1006			
	560	0 990	...	122	122			
18.	562	0·266	320	88	409	908	239	1149
	561	0·383	588	151	740			
19.	563	0·979	185	82	266	1006	290	1295
	562	0·426	263	98	361			
	561	0·585	558	110	668			

Table II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. May 20.	563	0·878	87	35	122			
	564	0·426	18	18	37			
	562	0·638	349	82	433			
	561	0·756	630	130	760	1084	265	1352
24.	564	0·920	316	43	359	316	43	359
27.	566	0·969	176	35	211			
	565	0·479	131	24	155	307	59	366
28.	566	0·888	139	28	168			
	565	0·642	111	77	188	250	105	356
30.	570	0·979	123	...	123			
	569	0·941	223	74	298			
	568	0·969	1390	457	1849			
	566	0·601	95	37	132			
	567	0·883	81	27	108	1912	595	2510
June 4.	571	0·638	105	33	138			
	568	0·282	894	332	1227			
	570	0·351	145	31	177			
	569	0·245	188	35	223			
	566	0·638	99	33	133	1431	464	1898
7.	572	0·638	311	55	366			
	571	0·256	237	22	259			
	568	0·506	993	196	1189			
	570	0·346	31	...	31			
	569	0·649	144	16	161			
	566	0·958	159	14	174	1875	303	2180
8.	572	0·453	200	23	224			
	571	0·372	216	18	234			
	568	0·692	833	236	1070			
	569	0·809	130	29	159	1379	306	1687
10.	573	0·957	14	43	58			
	572	0·294	178	40	218			
	571	0·952	116	...	116			
	568	0·936	547	320	867			
	569	0·979	61	...	61	915	403	1320
11.	573	0·888	111	46	158			
	572	0·420	141	37	178			
	568	0·990	275	306	581	527	389	917
13.	573	0·134	107	47	154			
	574	0·484	189	52	243			
	572	0·780	135	40	175	431	139	572
14.	574	0·296	106	17	124			
	573	0·291	66	31	97			
	572	0·888	149	37	187	321	85	408
15.	575	0·979	...	205	205			
	573	0·511	78	19	99			
	574	0·269	190	96	287			
	572	0·984	...	245	245	268	565	836
16.	575	0·915	324	157	482			
	574	0·672	91	34	126			
	573	0·619	384	119	503	799	310	1111
17.	575	0·807	493	231	725			
	574	0·753	117	19	135			
	573	0·312	228	89	317	838	339	1177
18.	575	0·672	406	136	543			
	573	0·915	303	94	397	709	230	940
20.	575	0·307	416	106	523	416	106	523

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. June 21.	575	0·215	405	82	487	405	82	487
23.	575	0·457	449	114	563	449	114	563
24.	575	0·645	416	111	527	416	111	527
30.	578	0·990	367	91	459			
	577	0·796	42	28	70			
	576	0·247	39	17	56			
	575	0·937	23	11	35	471	147	620
July 1.	577	0·457	85	19	104	85	19	104
2.	578	0·942	148	37	186			
	577	0·645	133	27	161			
	576	0·134	21	38	60	302	102	407
4.	579	0·990	244	91	336			
	578	0·688	92	29	123			
	577	0·242	43	8	51			
	576	0·457	19	9	28	398	137	538
5.	579	0·899	417	126	544			
	578	0·538	111	20	131			
	577	0·150	68	8	77	596	154	752
6.	579	0·753	494	104	598			
	578	0·371	110	32	142			
	577	0·296	35	4	40			
	576	0·845	32	16	48	671	156	828
7.	579	0·619	382	97	470			
	578	0·296	75	26	102			
	576	0·937	83	59	141	540	182	713
9.	582	0·990	826	826	1653			
	581	0·420	296	103	398			
	579	0·258	211	57	268			
	578	0·532	65	20	85	1398	1006	2404
11.	582	0·829	1226	212	1439			
	581	0·145	270	34	305			
	579	0·269	225	70	295			
	578	0·834	60	15	76	1781	331	2115
13.	582	0·430	1071	206	1278			
	581	0·468	24	14	38			
	579	0·667	401	79	480	1496	299	1796
14.	583	0·215	487	117	605			
	582	0·350	556	117	674			
	581	0·645	16	5	22			
	579	0·807	609	130	739	1668	369	2040
15.	583	0·172	566	167	736			
	582	0·183	633	134	767			
	579	0·937	498	177	677	1697	478	2180
16.	583	0·296	566	129	695			
	582	0·118	377	89	467			
	579	0·990	979	244	1224	1922	462	2386
19.	583	0·850	353	96	450			
	582	0·775	276	67	344	629	163	794
20.	583	0·937	320	107	427			
	582	0·888	224	56	281	544	163	708
21.	583	0·990	336	...	336			
	582	0·979	328	61	389			
	584	0·942	49	12	62	713	73	787
23.	585	0·405	23	18	41			
	584	0·677	23	11	34	46	29	75
25.	No spot.							

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. July 26.	585	0·366	45	18	63			
	584	0·350	72	13	85	117	31	148
27.	585	0·581	93	36	130			
	584	0·446	135	18	151	228	54	281
29.	585	0·877	95	26	122			
	584	0·715	55	12	67	150	38	189
Aug. 1.	586	0·334	81	22	103			
	587	0·915	282	94	376	363	116	479
2.	586	0·366	41	9	49			
	587	0·785	241	41	282	282	50	331
4.	587	0·452	152	28	181			
	588	0·715	164	36	201	316	64	382
5.	589	0·979	143	41	184			
	587	0·269	88	17	105			
	588	0·829	98	15	114	329	73	403
6.	590	0·979	352	123	475			
	589	0·888	75	37	111			
	587	0·038	76	21	97			
	588	0·936	83	...	83	586	181	766
8.	590	0·781	697	135	832			
	591	0·856	115	41	156			
	589	0·615	183	64	248			
	592	0·471	81	19	100			
	587	0·455	70	14	85	1146	273	1421
10.	590	0·411	661	130	791			
	591	0·588	378	120	500			
	589	0·321	427	157	586			
	593	0·203	34	13	47			
	592	0·776	365	135	500			
	589	0·802	35	21	56	1900	576	2480
11.	590	0·203	675	117	792			
	591	0·428	488	98	587			
	589	0·288	704	129	833			
	592	0·883	408	90	499			
	593	0·411	23	...	23	2298	434	2734
12.	590	0·107	440	175	616			
	591	0·294	677	191	869			
	589	0·428	1076	235	1311			
	592	0·974	378	151	529	2571	752	3325
13.	590	0·294	632	96	729			
	591	0·358	1226	195	1422			
	589	0·588	1406	332	1739			
	593	0·856	33	8	41	3297	631	3931
15.	590	0·653	541	113	654			
	591	0·642	594	122	717			
	589	0·856	1041	189	1231			
	593	0·893	56	9	65	2232	433	2667
19.	595	0·107	71	17	89	71	17	89
20.	595	0·374	133	9	142	133	9	142
21.								
25.	No spots.							
26.								
30.	597	0·776	114	54	168			
	596	0·139	94	51	146	208	105	314
Sept. 1.	597	0·471	139	28	168			
	596	0·513	99	34	133	238	62	301

Table II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. Sept. 3.	599	0·925	227	68	295	414	118	535
	597	0·385	119	27	147			
	596	0·829	68	23	93			
5.	599	0·638	199	44	244	288	77	365
	597	0·659	89	33	121			
15.	601	0·386	115	23	138	320	125	445
	600	0·978	205	102	307			
17.	602	0·920	152	21	174	177	21	199
	601	0·159	25	0	25			
19.	602	0·439	42	14	56	56	33	89
	601	0·460	14	19	33			
21.	602	0·064	25	8	34	25	8	34
22.	602	0·281	13	...	13	13	0	13
23.	No spot							
24.	604	0·963	61	30	90	61	30	90
27.	604	0·558	15	10	25	46	36	82
	603	0·316	31	26	57			
28.	603	0·529	35	10	45	35	10	45
29.	605	0·968	49	49	98	59	49	108
	603	0·631	10	...	10			
30.	605	0·868	51	25	76	51	25	76
Oct. 1.	606	0·968	148	49	196	172	73	245
	605	0·721	24	24	49			
3.	606	0·762	105	46	151	128	55	183
	605	0·378	23	9	32			
4.	606	0·579	124	20	145	167	28	196
	605	0·147	43	8	51			
5.	606	0·394	92	23	115	138	59	197
	605	0·378	46	36	82			
6.	606	0·242	104	17	122	144	42	187
	605	0·536	40	25	65			
7.	606	0·295	57	17	75	63	36	101
	605	0·757	6	19	26			
8.	607	0·684	23	11	34	120	40	160
	606	0·405	97	23	120			
	605	0·762	...	6	6			
10.	607	0·247	4	8	13	96	27	125
	606	0·762	92	19	112			
12.	606	0·875	95	26	122	95	26	122
15.	609	0·365	118	54	172	118	54	172
18.	609	0·902	19	9	29	19	9	29
19.	610	0·771	19	13	33	19	13	33
21.	610	0·625	92	43	136	92	43	136
22.	613	0·849	24	8	32	467	160	630
	612	0·912	135	52	188			
	611	0·849	224	56	281			
	610	0·286	84	44	129			
24.	613	0·495	83	9	93	320	112	436
	612	0·641	99	27	127			
	611	0·516	133	44	178			
	610	0·635	5	32	38			
28.	614	0·843	101	39	140	441	138	579
	612	0·391	203	55	258			
	611	0·505	137	44	181			
31.	616	0·985	73	49	122			
	615	0·964	277	77	355			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1864. Oct. 31.	614	0·312	290	89	380			
	612	0·886	99	18	117	739	233	974
Nov. 4.	616	0·989	1143	190	1334			
	615	0·589	521	99	621			
	614	0·443	122	28	151	1786	317	2106
14.	618	0·964	16	16	32			
	617	0·208	17	13	30			
	616	0·985	……	196	196	33	225	255
18.	620	0·713	18	12	30			
	619	0·281	48	8	57			
	618	0·208	164	26	191	230	46	278
22.	620	0·291	22	13	35			
	621	0·381	124	36	161			
	619	0·755	39	6	45			
	618	0·812	137	50	188	322	105	429
25.	624	0·849	273	120	393			
	623	0·771	284	79	364			
	622	0·511	368	118	486			
	621	0·812	181	50	231	1106	367	1473
29.	626	0·938	37	…	37			
	625	0·922	196	65	261			
	624	0·182	393	64	458			
	623	0·218	178	56	235			
	622	0·495	1023	230	1254	1827	415	2245
Dec. 1.	626	0·713	6	12	18			
	625	0·682	121	30	150			
	624	0·427	361	61	423			
	623	0·521	188	53	243			
	622	0·823	743	162	905	1419	318	1739
2.	626	0·469	14	4	19			
	625	0·485	106	34	140			
	624	0·615	210	81	291			
	623	0·703	204	42	247			
	622	0·938	809	148	959	1343	309	1656
5.	627	0·755	13	6	19			
	625	0·270	101	35	135			
	624	0·985	612	343	956			
	623	0·995	366	163	529	1092	547	1639
9.	628	0·912	94	31	124	94	31	124
19.	629	0·835	852	210	1062	852	210	1062
20.	631	0·990	……	703	703			
1865.	629	0·954	1061	207	1268	1061	910	1971
Jan. 4.	633	0·511	118	44	163			
	632	0·886	27	9	36	145	53	199
7.	634	0·407	59	9	69			
	633	0·745	75	12	88	134	21	157
9.	635	0·739	19	6	25			
	634	0·729	56	18	74			
	633	0·417	4	14	18	79	38	117
13.	636	0·609	101	42	143			
	635	0·286	240	84	325	341	126	468
23.	643	0·954	124	41	164			
	641	0·615	59	27	86			
	640	0·172	68	43	111			
	642	0·703	150	102	253			
	639	0·140	81	43	124			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1865.								
Jan. 23.	638	0·260	145	92	237			
	637	0·843	1055	296	1352	1682	644	2327
Feb. 3.	646	0·391	32	4	37			
	645	0·469	453	110	564			
	644	0·120	102	25	128			
	643	0·713	36	6	42			
	641	0·990	244	214	459	867	359	1230
9.	649	0·239	298	73	372			
	645	0·348	18	13	31			
	647	0·276	66	17	83			
	644	0·912	261	145	408	643	248	894
15.	649	0·391	203	64	268			
	647	0·417	75	28	103	278	92	371
17.	649	0·859	156	82	239			
	647	0·912	62	10	73	218	92	312
25.	650	0·920	34	79	113			
	651	0·252	26	.....	26	60	79	139
28.	651	0·826	38	15	53			
	652	0·568	317	83	400			
	653	0·973	56	18	75	411	116	528
Mar. 1.	653	0·868	85	25	110			
	652	0·726	237	87	324			
	651	0·947	......	13	13	322	125	447
3.	653	0·595	294	47	342			
	652	0·957	87	72	159	381	119	501
7.	653	0·499	220	98	319	220	98	319
8.	654	0·389	18	4	23			
	653	0·631	229	54	284	247	58	307
9.	654	0·499	39	9	49			
	653	0·778	209	40	250	248	49	299
13.	655	0·236	95	21	117			
	657	0·195	51	26	77			
	656	0·332	22	13	36	168	60	230
17.	656	0·931	225	......	225			
	655	0·736	199	62	262	424	62	487
20.	656	0·894	102	46	149	102	46	149
21.	656	0·968	32	65	98	32	65	98
22.	660	0·778	20	27	47			
	659	0·705	241	120	361			
	658	0·384	124	87	212			
	661	0·563	41	......	41	426	234	661
23.	662	0·894	75	......	75	75	......	75
24.	662	0·842	7	23	31	7	23	31
27.	663	0·789	553	227	781	553	227	781
28.	663	0·657	615	169	784	615	169	784
30.	663	0·378	473	170	644	473	170	644
31.	663	0·342	498	203	701	498	203	701
April 1.	663	0·431	408	108	517	408	108	517
3.	663	0·730	275	62	337	275	62	337
4.	663	0·893	243	93	337	243	93	337
6.	663	0·990	244	91	336	244	91	336
8.	664	0·476	72	28	100	72	28	100
10.	665	0·973	18	56	75			
	664	0·831	76	22	98	94	78	173
11.	665	0·873	26	17	43			
	664	0·936	23	35	59	49	52	102

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1865. Apr. 12.	665	0·730	6	12	18	6	12	18
13.	665	0·651	……	11	11			
	666	0·915	……	20	20	……	31	31
20.	667	0·577	109	36	145			
	668	0·265	70	35	105	179	71	250
21.	667	0·714	245	49	294			
	668	0·265	114	……	114			
	669	0·895	46	9	56	405	58	464
24.	670	0·688	5	17	23			
	668	0·529	25	……	25			
	667	0·990	91	30	121	121	47	169
25.	670	0·559	41	20	61	41	20	61
26.	670	0·372	46	4	50	46	4	50
27.	670	0·266	39	8	48			
	671	0·181	34	8	43	73	16	91
May 2.	673	0·622	43	16	59			
	672	0·670	91	40	131			
	674	0·559	……	5	5			
	671	0·814	……	7	7	134	68	202
3.	673	0·442	127	42	170			
	672	0·665	57	11	68	184	53	238
5.	676	0·990	91	……	91			
	675	0·883	81	27	108			
	673	0·287	93	13	106			
	672	0·234	8	……	8	273	40	313
6.	676	0·851	32	32	64			
	675	0·735	131	43	175			
	673	0·426	51	9	61			
	672	0·521	18	……	18	232	84	318
8.	676	0·469	14	4	19			
	675	0·266	39	……	39	53	4	58
9.	677	0·719	55	6	61			
	676	0·261	26	……	26			
	675	0·181	25	12	38	106	18	125
12.	678	0·453	95	56	152			
	679	0·293	26	……	26	121	56	178
13.	678	0·654	202	56	259			
	679	0·426	23	4	28	225	60	287
18.	680	0·835	312	54	366	312	54	366
19.	680	0·947	431	52	483			
	681	0·638	71	16	88	502	68	571
22.	681	0·985	612	147	760	612	147	760
23. 24.	No spots.							
25.	682	0·990	183	428	612			
	683	0·990	61	91	153	244	519	765
26.	682	0·931	534	83	617			
	683	0·941	161	62	223	695	145	840
27.	682	0·825	364	60	426			
	683	0·830	145	30	174	509	90	600
29.	682	0·495	363	112	476			
	683	0·479	62	29	91	425	141	567
30.	682	0·307	308	66	375			
	683	0·301	93	8	102	401	74	477
31.	682	0·984	98	49	147			
	683	0·161	60	12	73			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius = 1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1865. May 31.	682	0·177	458	77	536	616	138	756
June 5.	685	0·958	29	......	29	330	90	420
	684	0·123	30	17	46			
	682	0·915	271	73	345			
6.	686	0·985	......	710	710	392	759	1151
	682	0·985	392	49	441			
7.	686	0·453	309	90	401	415	128	546
	687	0·899	106	38	145			
8.	686	0·780	636	135	771	652	140	793
	687	0·645	16	5	22			
9.	686	0·592	626	131	758	655	131	787
	687	0·807	29	......	29			
12.	686	0·160	431	129	560	435	137	572
	687	0·107	4	8	12			
13.	686	0·321	400	81	482	400	81	482
14.	686	0·588	252	88	342	252	88	342
16.	No spot.							
23.	689	0·963	16	49	65	16	49	65
July 3.	No spot.							
4.	690	0·990	......	91	91	......	91	91
5.	690	0·942	24	24	49	24	24	49
7.	690	0·661	33	16	50	52	20	74
	691	0·516	19	4	24			
10.	690	0·092	21	8	29	149	48	198
	691	0·081	21	......	21			
	692	0·780	107	40	148			
11.	690	0·269	13	8	22	162	34	199
	691	0·280	17	......	17			
	692	0·603	132	26	160			
12.	690	0·495	14	4	19	145	27	174
	691	0·645	16	5	22			
	692	0·393	115	18	133			
14.	690	0·968	16	16	32	119	37	156
	692	0·172	103	21	124			
15.	692	0·350	94	18	112	94	18	112
20.	693	0·350	194	40	235	194	40	235
27.	694	0·231	43	8	51	43	8	51
28.	694	0·102	46	16	63	46	16	63
29.	694	0·215	17	8	26	69	29	100
	695	0·242	21	17	39			
	696	0·296	31	4	35			
Aug. 3.	698	0·850	16	8	24	878	296	1175
	697	0·813	754	275	1029			
	695	0·792	48	13	62			
	696	0·834	60	......	60			
7.	700	0·990	214	122	336	266	142	408
	699	0·615	5	16	21			
	698	0·134	47	4	51			
9.	700	0·856	115	57	173	147	75	222
	699	0·267	18	9	26			
	698	0·331	14	9	23			
10.	700	0·695	153	53	206	186	73	259
	699	0·802	7	7	14			
	698	0·946	26	13	39			
12.	700	0·342	126	27	153	126	27	153
14.	700	0·213	26	8	34	26	8	34

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1865.								
Aug. 16.	701	0·277	17	8	26	17	8	26
17.	702	0·936	47	......	47	80	16	97
	701	0·661	33	16	50			
18.	702	0·794	......	7	7	9	16	26
	701	0·508	9	9	19			
21.	703	0·873	535	95	632	535	95	632
22.	703	0·767	497	72	569	497	72	569
24.	703	0·407	466	97	563	466	97	563
26.	703	0·555	467	97	565	467	97	565
29.	703	0·699	224	41	265	309	73	381
	704	0·603	85	32	116			
30.	704	0·423	169	42	211	257	74	331
	703	0·847	88	32	120			
Sept. 1.	704	0·026	315	59	374	413	59	472
	703	0·984	98	......	98			
2.	704	0·265	303	57	360	303	57	360
4.	704	0·661	367	124	491	367	124	491
5.	704	0·831	525	83	609	525	83	609
6.	704	0·925	467	79	547	467	79	547
7.	704	0·990	428	122	550	428	122	550
8. 9.	} No spots.							
13.	705	0·952	124	......	124	124	......	124
14.	705	0·873	35	......	35	35	......	35
15.	705	0·725	49	42	91	49	42	91
16.	705	0·579	83	36	119	83	36	119
18.	705	0·684	46	17	63	81	34	115
	706	0·284	35	17	52			
19.	705	0·752	32	6	39	100	28	130
	706	0·826	68	22	91			
20.	705	0·894	28	9	37	169	41	211
	706	0·920	141	32	174			
22.	707	0·984	1398	269	1668	1398	269	1668
23.	707	0·926	1444	479	1924	1444	479	1924
25.	707	0·611	881	178	1059	881	178	1059
26.	707	0·421	709	141	850	709	141	850
27.	707	0·184	503	134	637	503	134	637
28.	707	0·037	400	123	523	400	123	523
30.	707	0·458	353	114	468	399	123	523
	711	0·405	46	9	55			
Oct. 2.	707	0·815	423	111	534	484	137	621
	712	0·874	61	26	87			
3.	707	0·936	379	71	451	416	99	516
	712	0·894	37	28	65			
4.	707	0·990	336	214	550	398	239	637
	712	0·736	62	25	87			
5.	712	0·568	52	10	62	52	10	62
6.	712	0·384	41	13	55	41	13	55
7.	712	0·158	34	12	47	401	12	414
	713	0·990	367	......	367			
10.	713	0·729	993	275	1268	993	275	1268
11.	713	0·563	1074	354	1428	1074	354	1428
12.	713	0·412	1122	246	1369	1205	276	1483
	714	0·833	83	30	114			
13.	713	0·328	1118	288	1407	1141	293	1436
	714	0·677	23	5	29			

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1865. Oct. 17.	713	0·833	1104	273	1378	1112	290	1404
	714	0·296	8	17	26			
20.	713	0·985	......	98	98	......	98	98
24.	*							
27.	No spots.							
28.								
Nov. 2.								
3.	715	0·990	......	33	33	......	33	33
4.	715	0·928	261	102	363	261	102	363
6.	715	0·317	147	44	192	160	48	210
	716	0·370	13	4	18			
13.	715	0·812	217	50	267	496	77	573
	717	0·490	4	......	4			
	718	0·954	275	27	302			
15.	715	0·990	336	91	428	564	133	699
	718	0·703	228	42	271			
22.	719	0·974	452	......	452	452	......	452
23.	719	0·869	391	93	485	391	93	485
24.	719	0·719	593	79	674	593	79	674
Dec. 2.	719	0·922	337	76	414	595	157	755
	720	0·286	168	48	218			
	721	0·859	90	33	123			
13.								
14.	No spots.							
19.								
20.	722	0·781	230	195	426	269	208	478
	723	0·948	39	13	52			
30.	724	0·729	668	74	743	668	74	743
1866. Jan. 1.	724	0·365	395	63	460	625	121	749
	725	0·338	230	58	289			
3.	724	0·270	252	44	296	629	134	764
	725	0·156	206	17	223			
	726	0·985	171	73	245			
4.	724	0·453	128	23	152	343	93	437
	725	0·391	138	41	179			
	726	0·896	77	29	106			
7.	724	0·912	292	166	460	344	189	535
	725	0·912	52	10	62			
	726	0·391	......	13	13			
8.	724	0·990	734	122	856	1642	397	2039
	725	0·985	73	......	73			
	726	0·391	9	......	9			
	727	0·990	826	275	1101			
9.	727	0·938	747	173	921	747	173	921
15.	727	0·328	698	144	842	1105	318	1424
	728	0·959	407	174	582			
19.	727	0·959	553	131	683	1067	225	1291
	728	0·276	278	70	348			
	729	0·469	236	24	260			
23.	728	0·641	366	82	449	384	91	477
	729	0·438	18	9	28			
24.	728	0·781	257	121	378	257	121	378
29.	731	0·750	324	85	410	400	94	496
	730	0·505	58	9	68			
	729	0·729	18	......	18			

TABLE II. (continued).

Date.	Group.	Mean distance from centre. radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1866. Feb. 5.	731	0·609	197	48	245			
	732	0·938	447	161	610	644	209	855
6.	732	0·817	408	74	482			
	731	0·781	189	67	256	597	141	738
8.	732	0·495	329	78	407			
	731	0·985	294	73	367	623	151	774
13.	735	0·917	690	303	995			
	734	0·693	171	35	206			
	732	0·693	265	76	342	1126	414	1543
18.	735	0·260	1428	330	1758			
	734	0·521	138	24	163	1566	354	1921
19.	735	0·365	1559	405	1964			
	734	0·703	114	24	138	1673	429	2102
20.	735	0·573	1466	478	1944			
	734	0·859	139	66	206	1605	544	2150
21.	735	0·703	1567	536	2104			
	734	0·938	......	161	161	1567	697	2265
23.	735	0·954	1186	330	1516	1186	330	1516
24.	735	0·990	795	275	1071	795	275	1071
26.	736	0·631	54	21	75	54	21	75
Mar. 2.	737	0·752	71	32	104	71	32	104
6.	738	0·684	208	40	250	208	40	250
7.	738	0·552	192	55	247	192	55	247
8.	738	0·384	194	37	231	194	37	231
9.	738	0·263	145	30	176	145	30	176
12.	738	0·647	177	38	216			
	739	0·894	281	102	384	458	140	600
14.	739	0·641	127	38	166			
	738	0·920	141	65	207	268	103	373
22.	740	0·794	247	42	290			
	739	0·555	71	30	102	318	72	392
23.	740	0·608	53	21	74			
	739	0·714	85	24	110	138	45	184
24.	741	0·540	50	15	65			
	740	0·449	143	70	214			
	739	0·858	98	24	123	291	109	402
27.	741	0·132	98	43	141			
	740	0·476	177	96	274	275	139	413
29.	741	0·847	88	56	144	88	56	144
30.	741	0·925	44	22	68	44	22	68
Apr. 3.	742	0·624	119	32	152	119	32	152
5.	742	0·307	57	31	89	57	31	89
6.	743	0·677	29	11	40			
	742	0·265	35	4	39	64	15	79
12.	745	0·925	316	44	363			
	744	0·465	62	19	81	378	63	444
14.	745	0·692	182	64	247			
	744	0·026	4	......	4	186	64	251
16.	745	0·372	174	41	225	174	41	225
18.	745	0·415	106	32	139	106	32	139
19.	745	0·585	110	31	141	110	31	141
20.	745	0·719	103	18	122	103	18	122
21.	745	0·856	49	16	66	49	16	66
23.	746	0·729	112	56	168	112	56	168
24.	746	0·559	235	56	292	235	56	292
25.	746	0·319	238	45	283	238	45	283

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra,	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1866.								
Apr. 26.	746	0·160	129	25	154	129	25	154
27.	746	0·240	82	47	131	82	47	131
30.	746	0·931	83	......	83	83	......	83
May 2.	No spots.							
3.								
4.								
5.								
7.								
8.	748	0·495	37	23	61	37	23	61
9.	748	0·251	105	22	127	155	47	202
	749	0·535	50	25	75			
10.	748	0·134	17	8	25	80	57	137
	749	0·337	63	49	112			
11.	749	0·267	206	52	259	206	52	259
12.	749	0·187	364	90	456	364	90	456
15.	749	0·877	289	122	412	289	122	412
16.	749	0·963	355	92	448	355	92	448
17.	749	0·974	170	18	189	272	52	324
	750	0·925	102	34	135			
18.	750	0·802	133	35	169	133	35	169
19.	750	0·631	32	21	54	32	21	54
21.	751	0·888	149	37	187	193	37	231
	750	0·267	44	......	44			
22.	751	0·769	125	33	158	125	33	158
23.	751	0·581	130	31	161	130	31	161
25.	751	0·285	129	26	155	129	26	155
26.	751	0·107	46	12	59	46	12	59
28.	751	0·565	102	30	132	102	30	132
29.	751	0·726	112	25	137	112	25	137
30.	751	0·372	87	43	130	87	43	130
June 1.	752	0·979	594	143	738	594	143	738
2.	752	0·861	396	98	496	396	98	496
6.	752	0·107	226	59	285	226	59	285
23.	755	0·888	233	56	290	233	56	290
26.	755	0·963	293	184	478	293	184	478
27.	755	0·990	......	244	244	......	244	244
28.	756	0·764	39	19	59	137	68	207
	757	0·968	98	49	148			
30.	756	0·377	92	27	119	254	83	337
	757	0·737	162	56	218			
July 2.	756	0·134	38	12	51	192	34	228
	757	0·366	154	22	177			
4.	757	0·188	143	13	156	143	13	156
5.	757	0·388	115	13	129	115	13	129
6.	757	0·581	36	20	57	36	20	57
7.	757	0·764	26	19	46	26	19	46
9.	757	0·973	37	18	56	37	18	56
10.	757	0·990	61	......	61	61	......	61
12.	758	0·942	124	37	161	124	37	161
13.	758	0·834	159	22	182	159	22	182
16.	758	0·296	120	35	155	120	35	155
18.	758	0·667	102	28	131	102	28	131
20.	758	0·801	133	21	155	133	21	155
21.	No spots.							
30.								
Aug. 9.	759	0·348	171	45	217	171	45	217
10.	759	0·588	189	31	220	189	31	220

TABLE II. (continued).

Date.	Group.	Mean distance from centre, radius=1.	Area of Penumbra.	Area of Umbra.	Area of whole Spot.	Whole for the day.		
						Penumbra.	Umbra.	Whole Spot.
1866.								
Aug. 11.	759	0·732	137	31	168	137	31	168
16.	760	0·588	163	67	231	163	67	231
17.	760	0·401	130	55	186	130	55	186
18.	760	0·197	151	43	194	151	43	194
20.	760	0·348	85	49	135	85	49	135
30.	761	0·265	308	96	404	308	96	404
31.	761	0·449	353	80	434	353	80	434
Sept. 1.	761	0·635	323	54	377	323	54	377
3.	761	0·925	430	113	534	430	113	534
5.	No spots.							
10.								
11.								
13.								
14.								
15.								
17.								
21.	762	0·810	246	50	297	246	50	297
24.	762	0·265	242	44	286	242	44	286
25.	762	0·132	236	60	296	236	60	296
27.	762	0·264	184	39	224	184	39	224
28.	762	0·635	224	32	257	224	32	257
Oct. 10.	763	0·925	......	34	34	......	34	34
13.	No spots.							
15.	764	0·868	102	34	136	102	34	136
16.	764	0·710	84	24	108	84	24	108
17.	764	0·552	25	20	45			
	765	0·605	48	5	53			
	766	0·963	139	77	215	212	102	313
19.	764	0·205	60	13	73			
	766	0·684	127	23	150	187	36	223
24.	766	0·353	91	18	109	91	18	109
26.	766	0·736	118	25	143	118	25	143
28.	766	0·973	207	37	245			
	767	0·316	48	26	75	255	63	320
31.	767	0·433	103	9	112	103	9	112
Nov. 2.	767	0·760	6	26	32	6	26	32
4.	No spots.							
6.								
8.								
14.								
17.								
19.	768	0·938	37	74	112	37	74	112
20.	768	0·807	36	50	86			
	769	0·938	112	49	161	148	99	247
21.	768	0·661	45	39	84			
	769	0·797	42	49	91	87	88	175
25.	768	0·202	8	8	17			
	769	0·100	50	21	71			
	770	0·526	55	10	65	113	39	153
26.	769	0·208	91	21	113	91	21	113
27.	769	0·703	18	12	30			
	770	0·469	57	14	72	75	26	102
28.	769	0·641	44	22	66	44	22	66
Dec. 7.	No spots.							
14.								
19.								
28.								

TABLE III. (continued).—Heliographic Elements of Sun-spots observed at Kew Observatory during 1864, 1865, and 1866.

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1864. Jan. 24.	3624	23·494	519	0·723	256° 13′	103° 21′	154° 33′	− 3° 19′	M.
	3625		519	0·726	255 28	102 17	153 29	− 4 1	n.
	3626		520	0·631	243 10	100 38	151 50	− 7 23	a.
	3627		520	0·642	243 17	101 12	152 24	− 7 26	A.
	3628		520	0·670	244 1	99 17	150 29	− 8 19	B.
	3629		520	0·694	244 58	99 13	150 25	− 8 25	b.
	3630		520	0·693	241 12	98 24	149 36	− 6 59	c.
	3631		521	0·577	253 19	87 37	138 49	− 3 21	P.
	3632		521	0·570	254 47	88 53	140 5	− 3 37	Q.
	3633		521	0·584	251 25	87 14	138 26	− 4 19	r.
	3634		521	0·563	252 33	86 46	137 58	− 4 12	s.
	3635		522	0·692	288 54	89 31	140 43	+20 26	S.
	3636		522	0·683	289 11	92 40	143 52	+20 51	N.
	3637		523	0·101	195 17	49 58	101 10	− 4 2	X.
	3638		523	0·093	194 13	56 13	101 25	− 3 56	x.
	3639		523	0·090	194 11	54 32	102 44	− 4 17	y.
	3640		524	0·155	131 28	41 43	92 55	− 7 26	Z.
	3641		524	0·151	130 44	42 58	94 10	− 7 34	z.
	3642		525	0·828	93 17	343 49	35 1	−13 28	Y^0.
	3643		525	0·833	93 33	344 10	35 22	−13 57	Y^1.
	3644		525	0·847	94 51	342 28	33 40	−14 9	Y^2.
	3645		526	0·888	92 19	321 33	12 45	−15 2	W.
	3646		526	0·893	92 33	325 16	16 28	−16 19	w_0.
	3647		526	0·895	94 0	325 43	16 55	−16 36	w_1.
	3648		526	0·901	94 11	325 49	17 1	−17 2	F.
	3649	23·510	519	0·724	256 20	103 30	154 40	− 3 24	M.
	3650		519	0·727	255 29	102 20	153 30	− 4 0	n.
	3651		520	0·630	243 26	100 37	151 47	− 7 26	a.
	3652		520	0·641	243 11	101 18	152 28	− 7 25	A.
	3653		520	0·671	244 10	99 24	150 34	− 8 25	B.
	3654		520	0·694	245 2	99 15	150 25	− 8 30	b.
	3655		520	0·696	241 33	98 20	149 30	− 6 49	c.
	3656		521	0·579	253 24	87 31	138 41	− 3 22	P.
	3657		521	0·571	254 46	88 55	140 5	− 3 39	Q.
	3658		521	0·584	251 28	87 3	138 13	− 4 22	r.
	3659		521	0·565	252 36	86 44	137 54	− 4 19	s.
	3660		522	0·693	288 56	89 32	140 42	+20 27	S.
	3661		522	0·684	289 0	92 49	143 59	+20 49	N.
	3662		523	0·102	195 16	49 1	101 11	− 4 10	X.
	3663		523	0·094	194 18	50 16	101 26	− 3 54	x.
	3664		523	0·091	194 5	51 38	102 48	− 4 19	y.
	3665		524	0·154	130 59	41 45	92 55	− 7 22	Z.
	3666		524	0·150	130 46	43 6	94 16	− 7 39	z.
	3667		525	0·826	93 28	344 2	35 12	−13 33	Y^0.
	3668		525	0·830	93 34	344 16	35 26	−13 51	Y^1.
	3669		525	0·847	94 57	342 34	33 44	−13 58	Y^2.
	3670		526	0·886	92 21	321 39	12 49	−15 11	W.
	3671		526	0·892	92 33	325 19	16 29	−16 22	w_0.
	3672		526	0·894	94 7	325 45	16 55	−16 38	w_1.
	3673		526	0·899	94 16	326 1	17 11	−17 5	F.
26.	3674	25·539	519	0·960	254 25	132 9	154 22	− 3 30	M.
	3675		519	0·964	255 1	131 20	153 33	− 4 12	n.
	3676		520	0·943	248 27	129 29	151 42	− 7 15	a.
	3677		520	0·946	249 33	129 37	151 50	− 7 30	A.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Jan. 26.	3678	25·239	520	0·953	249° 4′	128° 17′	150° 30′	− 8° 22′	B.
	3679		520	0·966	250 0	128 16	150 29	− 8 22	b.
	3680		520	0·970	247 52	126 55	149 8	− 6 53	c.
	3681		521	0·859	254 34	116 38	138 51	− 3 22	P.
	3682		521	0·873	251 29	118 59	141 12	− 3 42	Q.
	3683		521	0·890	253 46	117 1	139 14	− 4 22	r.
	3684		521	0·905	254 51	115 20	137 33	− 4 9	s.
	3685		523	0·514	252 48	79 44	101 57	− 4 11	X.
	3686		523	0·517	253 13	79 20	101 33	− 4 0	x.
	3687		525	0·473	94 17	13 5	35 18	−13 22	Y^0.
	3688		525	0·428	94 53	13 27	35 40	−14 0	Y^1.
	3689		525	0·439	94 10	11 46	33 59	−13 50	Y^2.
	3690		525	0·495	95 2	9 58	32 11	−12 2	G.
	3691		525	0·542	92 41	8 13	30 26	−13 14	g^1.
	3692		525	0·545	92 56	9 34	31 47	−14 4	g^2.
	3693		525	0·588	97 12	9 30	31 43	−13 3	H.
	3694		525	0·590	97 12	8 58	31 11	−13 19	h.
	3695		526	0·654	89 33	350 36	12 49	−16 33	W.
	3696		526	0·659	89 47	354 0	16 13	−15 46	w.
	3697	25·552	519	0·962	254 33	132 29	154 29	− 3 32	M.
	3698		519	0·966	255 6	131 40	153 40	− 4 17	n.
	3699		520	0·944	248 30	129 45	151 45	− 7 10	a.
	3700		520	0·947	250 0	129 45	151 45	− 7 33	A.
	3701		520	0·956	249 11	128 31	150 31	− 8 12	B.
	3702		520	0·967	249 57	128 25	150 25	− 8 20	b.
	3703		520	0·978	247 48	127 18	149 18	− 6 52	c.
	3704		521	0·861	254 30	116 27	138 27	− 3 16	P.
	3705		521	0·875	251 33	119 19	141 19	− 3 42	Q.
	3706		521	0·892	253 49	117 14	139 14	− 4 20	r.
	3707		521	0·906	254 56	115 24	137 46	− 4 18	s.
	3708		523	0·517	252 55	79 49	101 49	− 4 16	X.
	3709		523	0·519	253 27	79 25	101 25	− 3 49	x.
	3710		525	0·472	94 28	13 15	35 15	−13 17	Y^0.
	3711		525	0·426	94 56	13 43	35 43	−14 4	Y^1.
	3712		525	0·436	94 2	11 48	33 48	−13 30	Y^2.
	3713		525	0·492	95 17	10 10	32 10	−12 10	G.
	3714		525	0·541	92 44	8 36	30 36	−13 15	g^1.
	3715		525	0·540	92 57	9 50	31 50	−14 19	g^2.
	3716		525	0·586	97 13	10 0	32 0	−13 26	H.
	3717		525	0·588	97 1	9 15	31 15	−13 20	h.
	3718		526	0·652	89 36	350 58	12 58	−16 30	W.
	3719		526	0·656	89 42	354 15	16 15	−15 56	w.
28.	3720	27·526	523	0·843	251 17	107 49	101 47	− 4 43	X.
	3721		523	0·847	251 33	107 21	101 19	− 4 2	x.
	3722		525	0·145	172 6	42 14	36 12	−12 55	Y^0.
	3723		525	0·153	165 3	42 1	35 59	−13 43	Y^1.
	3724		525	0·159	166 19	39 17	33 15	−12 29	Y^2.
	3725		525	0·162	158 23	39 13	33 11	−12 2	G.
	3726		525	0·188	110 12	37 10	31 8	−13 10	g^1.
	3727		525	0·294	118 57	38 49	32 47	−13 55	g^2.
	3728		525	0·238	120 11	37 51	31 49	−12 48	H.
	3729		525	0·293	114 33	38 2	32 0	−13 15	h.
	3730		526	0·299	96 0	19 19	13 17	−15 24	W.
	3731		526	0·314	96 12	23 24	17 22	−15 55	w.
	3732	27·540	523	0·845	252 23	108 7	101 53	− 4 40	X.
	3733		523	0·849	251 47	107 43	101 29	− 3 57	x.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Jan. 28.	3734	27·540	525	0·143	173° 21′	42° 41′	36° 27′	−12° 53′	Y^0.
	3735		525	0·151	166 14	41 58	35 44	−13 30	Y^1.
	3736		525	0·157	167 35	39 16	33 2	−12 14	Y^2.
	3737		525	0·161	158 29	39 23	33 9	−12 21	G.
	3738		525	0·186	111 9	37 54	31 40	−13 8	g^1.
	3739		525	0·294	119 13	38 49	32 35	−13 44	g^2.
	3740		525	0·238	120 17	37 42	31 28	−12 41	H.
	3741		525	0·290	114 38	38 10	31 56	−13 11	h.
	3742		526	0·298	96 42	19 26	13 12	−15 15	W.
	3743		526	0·312	96 31	23 54	17 40	−15 57	w.
30.	3744	29·622	523	0·987	247 40	137 48	102 2	− 4 54	X.
	3745		523	0·988	247 21	137 27	101 41	− 3 29	x.
	3746		525	0·478	240 3	72 54	37 8	−12 44	Y.
	3747		525	0·465	241 15	71 12	35 26	−11 29	Y^0.
	3748		525	0·469	242 29	67 29	31 43	−10 59	G^0.
	3749		525	0·375	228 11	67 53	32 7	−10 58	G^1.
	3750		525	0·323	230 57	66 45	30 59	−12 49	g.
	3751		525	0·299	240 4	67 26	31 40	−11 38	H.
	3752		525	0·314	227 6	66 7	30 21	−10 4	h^1.
	3753		526	0·244	239 36	49 28	13 42	−15 12	W.
	3754		526	0·257	239 20	53 4	17 18	−15 50	w.
	3755		527	0·514	94 3	17 54	342 8	− 9 17	A_1.
	3756		527	0·522	93 19	18 19	342 33	− 9 11	A_2.
	3757		527	0·602	95 11	21 29	345 43	−10 22	a_1.
	3758		527	0·608	96 28	22 44	346 58	−10 37	a_2.
	3759		527	0·577	90 29	25 6	349 20	− 7 57	B.
	3760		527	0·573	89 54	26 17	350 31	− 5 13	b.
	3761		527	0·585	86 22	22 33	346 47	− 4 48	c.
	3762	29·636	523	0·988	247 52	138 3	102 5	− 4 50	X.
	3763		523	0·990	247 35	137 35	101 37	− 3 33	x.
	3764		525	0·479	239 54	73 20	37 22	−12 38	Y.
	3765		525	0·468	241 24	71 27	35 29	−11 34	Y^0.
	3766		525	0·472	242 37	67 42	31 44	−11 2	G^0.
	3767		525	0·378	228 5	68 10	32 12	−11 17	G^1.
	3768		525	0·327	230 59	66 46	30 48	−12 40	g.
	3769		525	0·302	240 13	67 25	31 27	−11 27	H.
	3770		525	0·316	228 10	66 27	30 29	−10 11	h^1.
	3771		526	0·248	239 33	49 48	13 50	−15 12	W.
	3772		526	0·260	239 51	53 24	17 26	−16 1	w.
	3773		527	0·512	94 17	18 11	342 13	− 9 22	A_1.
	3774		527	0·520	92 58	18 34	342 36	− 9 10	A_2.
	3775		527	0·600	95 17	21 56	345 58	−10 20	a_1.
	3776		527	0·606	96 33	23 8	347 10	−10 30	a_2.
	3777		527	0·577	90 12	25 9	349 11	− 7 50	B.
	3778		527	0·570	89 59	26 14	350 16	− 5 10	b.
	3779		527	0·582	86 16	22 40	346 42	− 4 43	c.
Feb. 4.	3780	34·464	527	0·512	252 17	88 49	344 22	− 8 11	A.
	3781		527	0·564	250 29	87 46	343 19	− 7 5	B.
	3782		527	0·577	251 38	87 29	343 2	− 7 42	C.
	3783		527	0·508	244 57	84 44	340 17	− 4 14	a.
	3784		527	0·612	239 1	85 8	340 41	− 3 50	b.
	3785		527	0·623	239 24	90 2	345 35	− 6 4	c.
	3786		528	0·470	60 57	32 43	288 16	+ 5 31	D.
	3787		528	0·472	61 13	33 57	289 30	+ 5 48	E.
	3788		528	0·481	62 2	33 19	288 52	+ 4 32	F.
	3789		528	0·512	64 11	30 22	285 55	+ 3 21	G.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Feb. 4.	3790	34·464	528	0·553	64° 47′	30° 44′	286° 17′	+3° 52′	H.
	3791		528	0·555	65 33	28 19	283 52	+3 49	d.
	3792		528	0·604	65 31	26 47	282 20	+2 1	e.
	3793		528	0·618	64 12	26 33	282 6	+7 45	f.
	3794		529	0·622	64 17	20 11	275 44	+7 38	g.
	3795		529	0·630	66 43	22 40	278 13	+6 19	h.
	3796	34·482	527	0·516	251 59	89 27	344 46	−8 8	A.
	3797		527	0·566	250 38	88 14	343 33	−7 12	B.
	3798		527	0·580	250 54	87 59	343 18	−7 44	C.
	3799		527	0·512	244 16	84 58	340 17	−4 18	a.
	3800		527	0·615	239 43	85 43	341 2	−4 1	b.
	3801		527	0·626	239 52	90 44	346 3	−5 55	c.
	3802		528	0·467	60 12	33 8	288 28	+5 30	D.
	3803		528	0·470	61 29	34 29	289 48	+5 50	E.
	3804		528	0·478	62 13	33 51	289 10	+4 21	F.
	3805		528	0·509	64 43	31 0	286 19	+3 30	G.
	3806		528	0·550	64 59	31 26	294 45	+3 59	H.
	3807		528	0·551	65 32	29 1	284 20	+3 40	d.
	3808		528	0·601	65 21	27 15	282 34	+2 24	e.
	3809		528	0·614	64 0	27 12	282 31	+7 50	f.
	3810		529	0·620	64 28	20 47	276 6	+7 38	g.
	3811		529	0·628	66 45	22 44	278 3	+6 24	h.
5.	3812	35·460	527	0·653	243 22	103 32	344 57	−8 14	A.
	3813		527	0·659	244 19	102 17	343 42	−7 2	B.
	3814		527	0·693	240 3	101 44	343 9	−7 55	C.
	3815		527	0·648	238 37	99 13	340 38	−4 32	a.
	3816		527	0·695	239 11	99 40	341 5	−4 12	b.
	3817		527	0·712	246 42	105 36	347 1	−5 53	c.
	3818		528	0·293	47 19	47 10	288 35	+5 14	D.
	3819		528	0·298	47 53	48 23	289 48	+6 11	E.
	3820		528	0·303	48 15	48 0	289 25	+4 16	F.
	3821		528	0·370	51 53	45 46	287 11	+3 21	G.
	3822		528	0·374	52 26	45 54	287 19	+4 17	H.
	3823		528	0·391	54 59	43 2	284 27	+3 44	d.
	3824		528	0·372	53 14	41 19	282 44	+2 28	e.
	3825		528	0·420	54 25	41 20	282 45	+7 40	f.
	3826		529	0·454	58 29	34 48	276 13	+7 27	g.
	3827		529	0·471	57 36	36 37	278 2	+6 37	h.
	3828	35·489	527	0·657	244 23	104 11	345 11	−8 12	A.
	3829		527	0·663	242 58	102 48	343 48	−7 8	B.
	3830		527	0·696	240 41	102 23	343 23	−7 59	C.
	3831		527	0·652	238 36	99 44	340 44	−4 22	a.
	3832		527	0·700	239 24	109 18	341 18	−4 16	b.
	3833		527	0·717	246 51	106 32	347 32	−5 55	c.
	3834		528	0·289	47 24	48 5	289 5	+5 19	D.
	3835		528	0·292	48 2	48 22	289 22	+6 7	E.
	3836		528	0·299	48 29	48 14	289 14	+4 1	F.
	3837		528	0·365	52 6	46 7	287 7	+3 31	G.
	3838		528	0·370	52 49	46 25	287 25	+4 26	H.
	3839		528	0·388	54 44	43 37	284 37	+3 40	d.
	3840		528	0·368	53 12	41 47	282 47	+2 25	e.
	3841		528	0·417	55 0	41 56	282 56	+7 41	f.
	3842		529	0·451	57 56	34 52	275 52	+7 34	g.
	3843		529	0·466	57 39	36 39	277 39	+6 33	h.
6.	3844	36·494	527	0·823	244 29	121 38	348 23	−4 23	a.
	3845		527	0·805	245 3	120 19	347 4	−3 38	b.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Feb. 6.	3846	36·494	527	0·817	245° 38′	119° 6′	345° 51′	− 5° 14′	c.
	3847		527	0·844	246 12	118 54	345 39	− 2 36	d.
	3848		527	0·859	247 52	117 25	344 10	− 1 28	e.
	3849		528	0·172	338 50	62 54	289 39	+ 7 16	A^0.
	3850		528	0·173	338 47	62 1	288 46	+ 6 52	B^0.
	3851		528	0·185	339 21	62 36	289 21	+ 7 19	C^0.
	3852		528	0·196	340 0	60 4	286 49	+ 8 12	D^0.
	3853		528	0·201	352 34	56 17	283 2	+ 5 13	E^0.
	3854		528	0·211	1 16	56 28	283 13	+ 9 18	a_1.
	3855		528	0·217	1 9	54 9	280 54	+ 2 23	b_1.
	3856		528	0·166	2 28	55 13	281 58	+ 3 26	c_1.
	3857		528	0·168	3 47	55 29	282 14	+ 6 25	d_1.
	3858		528	0·202	8 18	54 39	281 24	+ 5 8	e_1.
	3859		528	0·219	9 54	54 27	281 12	+ 5 12	M.
	3860		528	0·225	10 45	53 18	280 3	+ 5 57	N.
	3861		529	0·260	29 12	50 14	276 59	+ 8 7	P.
	3862		529	0·284	30 47	50 13	276 58	+ 7 16	p.
	3863		529	0·245	31 4	49 28	276 13	+ 9 4	Q.
	3864		529	0·257	40 12	48 31	275 16	+ 9 6	q.
	3865		529	0·290	40 7	47 54	274 39	+ 10 16	q^0.
	3866		530	0·484	88 24	34 22	261 7	− 6 23	R^1.
	3867		530	0·486	88 37	34 3	260 48	− 7 29	R^2.
	3868		530	0·495	92 3	32 28	259 13	− 7 54	r^1.
	3869		530	0·507	87 51	31 51	258 36	− 8 11	r^2.
	3870		530	0·523	86 28	29 14	255 59	− 9 54	S^1.
	3871		530	0·540	94 22	29 37	256 22	− 9 12	S^2.
	3872		530	0·558	89 57	29 58	256 43	−10 2	s^1.
	3873		530	0·571	89 14	30 12	256 57	−10 19	s^2.
	3874	36·523	527	0·829	244 38	122 19	348 40	− 4 22	a.
	3875		527	0·808	245 27	120 37	346 58	− 3 39	b.
	3876		527	0·820	244 57	119 54	346 15	− 5 16	c.
	3877		527	0·848	246 34	119 33	345 54	− 2 38	d.
	3878		527	0·864	247 43	118 1	344 22	− 1 29	e.
	3879		528	0·170	339 7	63 24	289 45	+ 7 22	A^0.
	3880		528	0·171	338 59	63 36	289 57	+ 6 57	B^0.
	3881		528	0·183	339 46	63 0	289 21	+ 7 28	C^0.
	3882		528	0·194	339 51	60 14	286 35	+ 8 0	D^0.
	3883		528	0·199	352 33	57 31	283 52	+ 5 14	E^0.
	3884		528	0·209	1 24	57 2	283 23	+ 9 31	a_1.
	3885		528	0·217	0 58	54 38	280 59	+ 2 22	b_1.
	3886		528	0·163	2 39	55 44	282 5	+ 3 28	c_1.
	3887		528	0·164	3 43	55 57	282 18	+ 6 20	d_1.
	3888		528	0·200	8 17	55 36	281 57	+ 5 19	e_1.
	3889		528	0·215	10 1	55 2	281 23	+ 5 14	M.
	3890		528	0·221	10 44	53 27	279 48	+ 5 43	N.
	3891		529	0·260	30 14	50 29	276 50	+ 8 9	P.
	3892		529	0·281	30 41	50 16	276 37	+ 7 16	p.
	3893		529	0·241	31 21	49 49	276 10	+ 9 6	Q.
	3894		529	0·257	40 19	49 6	275 27	+ 9 16	q.
	3895		529	0·288	40 50	48 18	274 39	+ 10 24	q^0.
	3896		530	0·482	88 21	34 57	261 18	− 6 28	R^1.
	3897		530	0·484	88 27	34 56	261 17	− 7 22	R^2.
	3898		530	0·491	92 28	32 20	258 41	− 7 50	r^1.
	3899		530	0·507	87 52	31 47	258 8	− 8 31	r^2.
	3900		530	0·523	86 30	29 29	255 50	− 9 50	S^1.
	3901		530	0·538	94 59	30 12	256 33	− 9 10	S^2.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Feb. 6.	3902	36·523	530	0·557	89° 28′	30° 33′	256° 54′	−10° 4′	s^1.
	3903		530	0·570	89 16	30 30	256 51	−10 24	s^2.
10.	3904	40·624	528	0·876	259 13	123 17	291 27	+ 8 19	A.
	3905		528	0·864	258 54	122 54	291 4	+ 7 26	B.
	3906		528	0·851	260 2	120 33	288 43	+ 2 29	a.
	3907		528	0·855	259 44	119 14	287 24	+ 6 45	b.
	3908		529	0·802	262 27	109 29	278 9	+ 7 48	C.
	3909		529	0·797	262 51	108 20	276 30	+ 7 49	D.
	3910		529	0·784	266 14	108 15	276 25	+ 8 13	E.
	3911		529	0·755	267 29	108 2	276 12	+ 8 40	F.
	3912		529	0·763	265 23	108 14	276 24	+ 7 13	c.
	3913		529	0·710	264 11	109 27	277 37	+ 9 57	d.
	3914		529	0·712	266 28	105 13	273 23	+10 22	e.
	3915		529	0·721	265 9	106 5	274 15	+10 14	f.
	3916		530	0·455	246 25	96 41	264 51	− 6 24	G.
	3917		530	0·463	245 39	95 21	263 31	− 7 19	H.
	3918		530	0·470	247 28	95 17	263 27	− 6 13	K.
	3919		530	0·420	247 32	93 18	261 28	− 8 23	M.
	3920		530	0·355	248 50	92 27	260 37	− 8 12	g.
	3921		530	0·370	248 56	94 38	262 48	−10 19	h.
	3922		530	0·341	249 6	88 21	256 31	−10 4	k.
	3923		530	0·312	249 38	87 25	255 35	− 9 54	m.
	3924	40·638	528	0·879	259 28	123 15	291 13	+ 8 24	A.
	3925		528	0·868	258 50	123 10	291 8	+ 7 25	B.
	3926		528	0·855	260 33	121 2	289 0	+ 2 31	a.
	3927		528	0·860	260 4	119 38	287 36	+ 6 40	b.
	3928		529	0·802	262 55	109 25	277 23	+ 7 53	C.
	3929		529	0·800	262 41	108 13	276 11	+ 7 49	D.
	3930		529	0·787	265 58	108 54	276 52	+ 8 13	E.
	3931		529	0·758	267 30	108 19	276 17	+ 8 35	F.
	3932		529	0·765	265 39	108 34	276 32	+ 7 15	c.
	3933		529	0·715	264 24	109 33	277 31	+ 9 51	d.
	3934		529	0·715	266 35	105 40	273 38	+10 20	e.
	3935		529	0·724	265 22	106 15	274 13	+10 19	f.
	3936		530	0·458	246 40	96 12	264 10	− 6 22	G.
	3937		530	0·468	245 51	95 37	263 35	− 7 17	H.
	3938		530	0·472	247 35	95 4	263 2	− 6 12	K.
	3939		530	0·422	247 40	93 28	261 26	− 8 15	M.
	3940		530	0·360	248 54	92 35	260 33	− 8 10	g.
	3941		530	0·372	249 11	94 24	262 22	−10 23	h.
	3942		530	0·345	250 0	88 20	256 18	−10 0	k.
	3943		530	0·315	249 16	87 29	255 27	− 9 55	m.
17.	3944	47·439	531	0·510	288 12	103 53	175 23	+19 12	A.
	3945		531	0·515	287 51	102 17	173 47	+21 2	B.
	3946		531	0·524	289 9	102 44	174 14	+18 57	C.
	3947		531	0·460	289 24	101 6	172 36	+17 26	D.
	3948		531	0·443	292 12	98 5	169 35	+17 24	a.
	3949		531	0·455	293 38	99 13	170 43	+20 9	b.
	3950		531	0·478	294 6	97 24	168 54	+16 13	c.
	3951		531	0·477	294 28	96 55	168 25	+16 54	d.
	3952		532	0·430	232 37	99 13	170 43	− 4 37	E.
	3953		532	0·428	230 28	94 19	165 49	− 3 38	F.
	3954		532	0·378	233 1	90 54	162 24	− 3 49	G.
	3955		533	0·054	99 6	75 12	146 42	+ 1 14	e.
	3956		533	0·059	100 4	76 33	147 3	+ 2 3	f.
*	3957		533	0·095	185 16	73 50	145 20	− 1 29	g.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Feb. 17.	3958	47·439	533	0·099	184° 45′	71° 36′	143° 6′	− 3° 36′	b.
	3959		534	0·647	80 13	33 50	105 20	− 5 44	K.
	3960		534	0·643	81 17	33 37	105 7	− 5 40	k.
	3961	47·470	531	0·516	288 19	104 12	175 16	+19 13	A.
	3962		531	0·520	287 59	102 51	173 55	+21 10	B.
	3963		531	0·526	288 58	102 53	173 57	+18 53	C.
	3964		531	0·463	289 20	101 18	172 22	+17 30	D.
	3965		531	0·448	293 2	98 55	169 59	+17 28	a.
	3966		531	0·460	293 40	99 29	170 33	+20 13	b.
	3967		531	0·480	294 16	98 2	169 6	+16 25	c.
	3968		531	0·480	294 43	97 38	168 42	+16 55	d.
	3969		532	0·432	232 41	100 0	171 4	− 4 48	E.
	3970		532	0·431	230 30	94 36	165 40	− 3 40	F.
	3971		532	0·380	233 15	91 28	162 32	− 3 55	G.
	3972		533	0·055	99 27	75 47	146 51	+ 0 52	e.
	3973		533	0·061	100 43	76 58	148 2	+ 2 28	f.
	3974		533	0·096	186 3	73 58	145 2	− 1 31	g.
	3975		533	0·100	184 50	72 30	144 34	− 3 40	h.
	3976		534	0·645	80 21	34 14	105 18	− 5 55	K.
	3977		534	0·640	81 36	35 9	106 13	− 5 39	k.
	3978	47·546	531	0·521	288 24	105 8	175 7	+19 15	A.
	3979		531	0·524	287 55	104 11	174 10	+21 8	B.
	3980		531	0·531	289 11	104 32	174 31	+18 55	C.
	3981		531	0·467	289 40	102 19	172 18	+17 31	D.
	3982		531	0·455	293 31	100 3	170 2	+17 32	a.
	3983		531	0·463	293 44	100 31	170 30	+20 20	b.
	3984		531	0·484	294 28	99 17	169 16	+16 22	c.
	3985		531	0·485	294 27	98 52	168 51	+16 50	d.
	3986		532	0·433	233 3	101 6	171 5	− 4 51	E.
	3987		532	0·437	230 39	95 35	165 34	− 3 44	F.
	3988		532	0·384	233 42	92 4	162 3	− 3 56	G.
	3989		533	0·057	99 28	76 46	146 45	+ 0 47	e.
	3990		533	0·068	101 7	78 21	148 20	+ 2 30	f.
	3991		533	0·100	185 29	74 33	144 32	− 1 36	g.
	3992		533	0·105	185 2	73 35	143 34	− 4 2	h.
	3993		534	0·639	81 19	35 28	105 27	− 6 0	K.
	3994		534	0·637	81 49	36 18	106 17	− 5 44	k.
Mar. 2.	3995	60·638	535	0·888	268 54	148 37	32 54	+14 22	A.
	3996		535	0·893	267 59	148 13	32 30	+14 51	B.
	3997		535	0·864	267 2	146 30	30 47	+15 6	C.
	3998		535	0·860	269 14	146 1	30 18	+15 39	D.
	3999		535	0·859	268 0	147 19	31 36	+16 0	a.
	4000		536	0·775	233 16	133 54	18 11	−10 17	b.
	4001		536	0·760	234 51	133 39	17 56	−11 38	c.
	4002		536	0·758	234 28	133 12	17 29	−10 19	d.
	4003		536	0·756	235 4	127 28	11 45	−12 33	E.
	4004		536	0·670	233 52	128 19	12 36	−12 37	F.
	4005		536	0·693	233 58	128 3	12 20	−12 12	G.
	4006		536	0·666	236 39	128 29	12 46	−11 4	H.
	4007		537	0·360	328 42	87 31	331 48	+19 27	h.
	4008		537	0·363	329 4	86 44	331 1	+20 44	e.
	4009		537	0·365	330 21	85 29	329 46	+20 33	f.
	4010		537	0·357	340 40	83 14	327 31	+20 47	g.
	4011		537	0·377	346 9	83 12	327 29	+19 18	K.
	4012		537	0·365	345 37	80 33	324 50	+19 39	k.
	4013		537	0·382	349 58	81 46	326 3	+21 42	K^1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 2.	4014	60·638	537	0·380	352° 14′	82° 29′	326° 46′	+21° 27′	k^1.
	4015		537	0·382	353 59	81 38	325 55	+18 14	k^2.
	4016		538	0·394	41 12	64 9	308 26	+10 1	S.
	4017		538	0·401	41 38	64 11	308 28	+10 7	s.
	4018		539	0·798	76 33	35 30	279 47	− 6 50	P.
	4019		539	0·792	76 12	35 47	279 4	− 6 33	p.
	4020		540	0·870	53 19	25 17	269 34	+18 11	Q.
	4021		540	0·875	54 37	26 32	270 49	+16 32	q^1.
	4022		540	0·866	52 26	26 39	270 56	+17 41	q^2.
	4023	60·658	535	0·890	269 3	148 53	32 53	+14 19	A.
	4024		535	0·895	267 34	148 36	32 36	+14 54	B.
	4025		535	0·867	266 54	147 1	31 1	+15 2	C.
	4026		535	0·861	269 23	146 12	30 12	+15 30	D.
	4027		535	0·859	268 13	147 28	31 28	+16 3	a.
	4028		536	0·777	233 41	133 55	17 55	−10 10	b.
	4029		536	0·763	235 0	134 2	18 2	−11 36	c.
	4030		536	0·760	234 29	133 39	17 39	−10 26	d.
	4031		536	0·759	235 6	127 50	11 50	−12 40	E.
	4032		536	0·672	233 57	128 36	12 36	−12 39	F.
	4033		536	0·695	234 13	128 34	12 34	−12 7	G.
	4034		536	0·668	236 40	129 4	13 4	−11 11	H.
	4035		537	0·363	328 49	88 12	332 12	+19 20	h.
	4036		537	0·365	329 19	87 59	331 59	+20 42	e.
	4037		537	0·367	330 22	86 0	330 0	+20 33	f.
	4038		537	0·360	340 45	83 42	327 42	+20 40	g.
	4039		537	0·380	346 19	83 40	327 40	+19 16	K.
	4040		537	0·366	346 43	81 0	325 0	+19 51	k.
	4041		537	0·384	350 8	82 13	326 13	+21 45	K^1.
	4042		537	0·383	352 24	82 46	326 46	+21 20	k^1.
	4043		537	0 384	354 16	81 26	325 26	+18 12	k^2.
	4044		538	0·392	41 1	64 38	308 38	+10 10	S.
	4045		538	0·399	41 32	64 36	308 36	+10 12	s.
	4046		539	0·796	76 34	35 33	279 33	− 6 54	P.
	4047		539	0·790	76 52	35 49	279 49	− 6 37	p.
	4048		540	0·868	53 16	25 27	269 27	+18 19	Q.
	4049		540	0·871	54 13	26 40	270 40	+16 30	q_1.
	4050		540	0·864	52 21	26 38	270 38	+17 44	q_2.
4.	4051	62·678	536	0·970	235 16	155 13	10 34	−12 31	M.
	4052		536	0·963	236 23	156 49	12 10	−12 17	N.
	4053		536	0·956	234 48	156 1	11 22	−13 38	O.
	4054		536	0·944	233 59	159 35	14 56	−13 3	m.
	4055		536	0·931	235 50	162 44	18 5	−14 2	n.
	4056		536	0·924	237 2	165 28	20 49	−14 19	o.
	4057		537	0·558	282 42	121 53	337 14	+19 39	A.
	4058		537	0·550	288 19	112 34	327 55	+19 50	B.
	4059		537	0·563	283 24	114 9	329 30	+19 36	C.
	4060		537	0·520	287 36	114 17	329 38	+20 14	D.
	4061		537	0·533	285 5	115 56	331 17	+19 15	a.
	4062		537	0·547	287 49	116 44	332 5	+20 33	b.
	4063		537	0·519	286 43	117 21	332 42	+20 13	c.
	4064		538	0·221	300 11	98 29	313 50	+11 38	P.
	4065		538	0·239	302 21	98 54	314 15	+10 41	p.
	4066		538	0·216	301 4	99 8	314 29	+10 17	q.
	4067		539*a*	0·442	86 29	65 11	280 32	− 4 10	F.
	4068		539*a*	0·446	86 45	65 27	280 48	− 6 46	f.
	4069		540*a*	0·555	41 12	60 49	275 10	+18 0	G^0.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 4.	4070	62·678	540*a*	0·559	42° 29′	60° 52′	276° 13′	+18° 13′	G^1.
	4071		540*a*	0·531	43 38	59 14	274 35	+17 28	G^2.
	4072	62·695	536	0·973	236 14	155 28	10 34	−12 35	M.
	4073		536	0·965	236 57	157 9	12 15	−12 19	N.
	4074		536	0·958	235 2	156 28	11 34	−13 30	O.
	4075		536	09·50	233 18	160 4	15 10	−13 10	m.
	4076		536	0·936	236 9	163 19	18 25	−14 9	n.
	4077		536	0·925	236 51	165 55	21 1	−14 22	o.
	4078		537	0·560	283 13	122 28	337 34	+19 50	A.
	4079		537	0·553	288 11	112 40	327 46	+19 59	B.
	4080		537	0·565	283 44	114 45	329 51	+19 40	C.
	4081		537	0·522	288 0	114 21	329 27	+20 17	D.
	4082		537	0·536	285 41	116 20	331 26	+19 16	a.
	4083		537	0·549	287 18	117 12	332 18	+20 30	b.
	4084		537	0·521	286 40	117 54	333 0	+20 16	c.
	4085		538	0·222	300 29	98 42	313 48	+11 35	P.
	4086		538	0·241	302 44	99 13	314 19	+10 47	p.
	4087		538	0·218	300 55	99 47	314 53	+10 19	q.
	4088		539	0·440	86 39	65 31	280 37	− 4 11	F.
	4089		539	0·444	87 3	65 35	280 41	− 6 50	f.
	4090		540	0·553	41 24	60 51	275 57	+17 52	G^0.
	4091		540	0·556	42 32	61 14	276 20	+18 1	G^1
	4092		540	0·527	43 37	59 38	274 44	+17 36	G^2
10.	4093	68·702	540	0·822	265 23	153 2	282 56	+18 42	g_1.
	4094		540	0·826	265 51	154 51	284 45	+17 54	g_2.
	4095		541	0·562	31 19	66 30	196 24	+22 27	A.
	4096		541	0·571	31 28	65 19	195 13	+22 6	B.
	4097		541	0·578	33 4	65 24	195 18	+23 19	C.
	4098		541	0·590	32 41	63 53	193 47	+22 33	D.
	4099		541	0·601	33 35	63 47	193 41	+24 54	a.
	4100		541	0·613	34 59	64 40	194 34	+23 21	b.
	4101		541	0·628	35 6	64 17	194 11	+22 19	c.
	4102		541	0·644	35 7	60 0	189 54	+24 15	d.
	4103		542	0·540	74 29	63 31	193 25	− 4 54	X.
	4104		542	0·549	75 34	62 46	192 40	− 4 45	x.
	4105		542	0·552	75 56	62 19	192 13	− 3 9	Y.
	4106		542	0·694	76 28	50 11	180 5	− 3 41	y.
	4107		543	0·930	59 38	28 50	158 44	+ 7 18	M.
	4108		543	0·932	60 11	27 32	157 26	+ 7 26	n.
	4109	68·712	540	0·825	265 28	153 19	283 5	+18 47	g_1.
	4110		540	0·830	266 2	155 4	284 50	+17 49	g_2.
	4111		541	0·560	31 23	66 41	286 27	+22 28	A.
	4112		541	0·569	31 46	65 22	195 8	+22 14	B.
	4113		541	0·577	33 9	65 50	195 36	+23 37	C.
	4114		541	0·588	33 10	64 1	193 47	+22 34	D.
	4115		541	0·597	34 2	63 58	193 44	+24 50	a.
	4116		541	0·609	35 50	64 47	194 33	+23 17	b.
	4117		541	0·624	34 49	65 0	194 46	+22 16	c.
	4118		541	0·640	35 12	60 17	190 3	+24 8	d.
	4119		542	0·536	76 13	63 52	193 38	− 4 33	X.
	4120		542	0·545	75 55	62 57	192 43	− 4 46	x.
	4121		542	0·550	76 38	62 25	192 11	− 3 9	Y.
	4122		542	0·692	76 13	50 37	180 23	− 3 48	y.
	4123		543	0·928	60 47	29 8	158 54	+ 7 20	M.
	4124		543	0·930	60 47	28 29	158 15	+ 7 28	n.
11.	4125	69·652	540	0·920	258 52	165 45	282 11	+17 17	g^0.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 11.	4126	69·652	540	0·918	259° 27′	168° 13′	284° 39′	+18° 54′	g^1.
	4127		541	0·471	13 8	78 53	195 19	+21 19	A^0.
	4128		541	0·483	14 34	78 51	195 17	+22 3	B^0.
	4129		541	0·490	13 28	79 18	195 44	+24 36	C^0.
	4130		541	0·492	16 25	80 48	197 14	+22 47	D^0.
	4131		541	0·506	18 24	75 57	192 23	+24 11	a^1.
	4132		541	0·511	20 21	77 12	193 38	+22 50	b^1.
	4133		541	0·528	21 37	74 15	190 41	+23 38	c^1.
	4134		541	0·532	22 50	74 3	190 29	+22 9	d^1.
	4135		542	0·388	77 9	77 36	194 2	− 4 9	X.
	4136		542	0·390	77 32	76 18	192 44	− 4 26	x.
	4137		542	0·397	78 14	75 19	191 45	− 3 52	Y.
	4138		542	0·408	78 32	65 51	182 17	− 3 37	y.
	4139		543	0·855	56 56	43 1	159 27	+ 7 49	M.
	4140		543	0·861	56 4	42 42	159 8	+ 7 18	n.
	4141	69·667	540	0·923	258 40	165 53	282 5	+17 15	g^0.
	4142		540	0·921	260 11	168 40	284 52	+19 0	g^1.
	4143		541	0·468	13 7	79 7	195 19	+21 14	A^0.
	4144		541	0·480	14 58	79 0	195 12	+22 16	B^0.
	4145		541	0·486	13 26	79 46	195 58	+24 25	C^0.
	4146		541	0·490	16 17	80 57	197 9	+22 32	D^0.
	4147		541	0·503	18 33	75 52	192 4	+24 10	a^1.
	4148		541	0·509	20 29	77 44	193 56	+22 56	b^1.
	4149		541	0·527	21 14	74 29	190 41	+23 49	c^1.
	4150		541	0·530	23 8	74 30	190 42	+22 8	d^1.
	4151		542	0·386	77 6	77 54	194 6	− 4 17	X.
	4152		542	0·388	77 51	76 33	192 45	− 4 30	x.
	4153		542	0·393	78 23	75 31	191 43	− 3 57	Y.
	4154		542	0·406	78 41	66 9	182 21	− 3 31	y.
	4155		543	0·851	56 41	43 12	159 24	+ 7 49	M.
	4156		543	0·857	56 0	43 1	159 13	+ 7 23	n.
12.	4157	70·518	540	0·973	258 13	178 59	283 7	+17 12	g^0.
	4158		540	0·976	259 56	179 31	283 39	+18 38	g^1.
	4159		541	0·381	357 14	93 17	197 25	+20 17	A.
	4160		541	0·391	359 50	94 1	198 9	+22 13	B.
	4161		541	0·386	358 7	91 28	195 36	+21 45	C.
	4162		541	0·402	4 41	88 55	193 3	+22 53	a.
	4163		541	0·417	3 26	89 34	193 42	+23 19	b.
	4164		541	0·430	3 20	87 16	191 24	+23 36	c.
	4165		542	0·182	94 25	90 12	194 20	− 5 6	D.
	4166		542	0·187	93 17	90 2	194 10	− 4 41	E.
	4167		542	0·191	95 21	89 10	193 18	− 3 52	d.
	4168		542	0·206	94 38	88 46	192 54	− 3 38	e.
	4169		543	0·722	55 13	56 37	160 45	+ 7 16	f.
	4170		543	0·725	55 47	57 58	162 6	+ 8 4	g.
16.	4171	74·507	541	0·783	273 6	150 11	197 45	+18 22	M.
	4172		541	0·784	272 19	149 37	197 11	+18 47	m^0.
	4173		541	0·775	272 53	150 4	197 38	+19 30	m^1.
	4174		541	0·762	273 21	146 27	194 1	+19 12	m^2.
	4175		541	0·754	272 16	145 21	192 55	+20 44	N.
	4176		541	0·755	274 40	147 9	194 43	+21 29	N^0.
	4177		541	0·746	273 11	143 16	190 50	+19 30	n.
	4178		541	0·749	270 2	144 47	192 21	+22 21	n^0.
	4179		541	0·700	275 33	144 11	191 45	+23 56	n^1.
	4180		542	0·740	235 44	147 34	195 8	− 6 14	G.
	4181		542	0·743	233 10	148 39	196 13	− 5 13	H.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 16.	4182	74·507	542	0·736	236° 58′	148° 11′	195′ 45°	− 4° 45′	g.
	4183		543	0·254	288 27	112 15	159 49	+ 8 25	S.
	4184		543	0·261	289 8	109 2	156 36	+ 7 0	s.
	4185		544	0·976	71 0	22 57	70 31	− 4 4	P.
	4186		544	0·962	72 57	23 42	71 16	− 4 56	p.
	4187		544	0·958	71 16	21 32	69 6	− 5 39	Q.
	4188	74·536	541	0·786	273 38	151 0	198 14	+18 27	M.
	4189		541	0·788	272 14	149 46	197 0	+18 59	m^0.
	4190		541	0·778	272 55	149 57	197 11	+19 33	m^1.
	4191		541	0·764	273 16	146 16	193 30	+19 17	m^2.
	4192		541	0·758	272 18	145 58	193 12	+20 38	N.
	4193		541	0·759	274 59	147 23	194 37	+21 32	N^0.
	4194		541	0·750	273 44	143 19	190 33	+19 47	n.
	4195		541	0·753	269 51	144 37	191 51	+22 24	n^0.
	4196		541	0·703	275 43	144 18	191 32	+24 0	n^1.
	4197		542	0·742	235 40	147 30	194 44	− 6 16	G.
	4198		542	0·745	233 18	148 45	195 59	− 5 9	H.
	4199		542	0·739	237 2	148 22	195 36	− 4 37	g.
	4200		543	0·258	289 3	112 14	159 28	+ 8 22	S.
	4201		543	0·264	289 40	109 47	157 1	+ 7 11	s.
	4202		544	0·975	71 12	23 50	71 4	− 4 3	P.
	4203		544	0·960	73 14	23 57	71 11	− 5 2	p.
	4204		544	0·956	71 15	21 35	68 49	− 5 43	Q.
17.	4205	75·610	541	0·902	264 46	166 16	198 11	+18 37	A.
	4206		541	0·897	262 21	164 38	196 33	+21 30	B.
	4207		541	0·860	262 34	161 41	193 36	+19 33	C.
	4208		541	0·857	263 19	163 7	195 2	+18 4	a.
	4209		541	0·862	264 48	160 21	192 16	+17 48	b.
	4210		542	0·884	232 9	164 21	196 16	− 6 47	D.
	4211		542	0·887	232 41	165 54	197 49	− 5 56	d.
	4212		543	0·443	262 11	127 22	159 17	+ 8 39	E.
	4213		543	0·446	262 27	125 47	157 42	+ 9 14	e.
	4214		544	0·881	67 20	38 42	70 37	− 6 56	F.
	4215		544	0·883	68 59	38 13	70 8	− 4 21	f.
	4216		544	0·897	69 2	38 14	70 9	− 5 29	g.
	4217		545	0·940	69 50	26 2	57 57	− 8 28	M.
	4218		545	0·951	68 57	25 52	57 47	− 8 43	N.
	4219		545	0·952	70 7	25 19	57 14	− 9 56	O.
	4220		546	0·975	50 23	12 42	44 37	+11 5	P.
	4221		546	0·979	50 37	11 41	43 36	+10 19	p.
	4222	75·648	541	0·907	265 3	167 0	198 23	+18 40	A.
	4223		541	0·899	262 29	164 52	196 15	+21 32	B.
	4224		541	0·865	262 35	162 18	193 41	+19 30	C.
	4225		541	0·861	263 11	163 50	195 13	+18 6	a.
	4226		541	0·866	264 52	160 51	192 14	+17 52	b.
	4227		542	0·889	232 21	164 38	196 1	− 6 43	D.
	4228		542	0·891	232 37	166 27	197 50	− 6 2	d.
	4229		543	0·449	262 32	127 49	159 12	+ 8 43	E.
	4230		543	0·454	262 4	126 18	157 41	+ 9 17	e.
	4231		544	0·879	67 19	39 4	70 27	− 6 51	F.
	4232		544	0·876	69 2	38 46	70 9	− 4 19	f.
	4233		544	0·892	69 55	38 39	70 2	− 5 38	g.
	4234		545	0·938	69 42	26 31	57 54	− 8 26	M.
	4235		545	0·948	69 13	26 17	57 40	− 8 40	N.
	4236		545	0·950	70 8	25 41	57 4	−10 1	O.
	4237		546	0·973	50 30	12 45	44 8	+11 4	P.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864.									
ar, 17.	4238	75·648	546	0·978	50° 44′	12° 14′	43° 37′	+10° 22′	p.
18.	4239	76·486	541	0·965	261 52	180 26	199 55	+19 4	M.
	4240		541	0·962	262 19	179 33	199 2	+19 31	N.
	4241		541	0·960	261 27	181 37	200 1	+21 32	m.
	4242		541	0·953	262 36	180 41	199 10	+20 33	n.
	4243		542	0·962	233 45	179 50	199 19	− 7 6	O.
	4244		542	0·967	234 1	180 39	200 8	− 6 11	o.
	4245		543	0·570	255 22	140 22	159 51	+ 9 54	P.
	4246		543	0·572	256 16	141 18	160 47	+ 8 34	p.
	4247		544	0·762	69 16	54 17	73 46	− 3 9	Q.
	4248		544	0·773	70 28	56 7	75 36	− 3 57	q.
	4249		544	0·791	69 57	56 24	75 53	− 5 18	r.
	4250		545	0·858	71 13	42 21	61 50	− 8 26	R.
	4251		545	0·861	71 26	42 56	62 25	− 8 13	S.
	4252		545	0·863	72 44	40 36	60 5	− 7 52	s.
	4253		546	0·894	49 59	27 19	46 48	+11 19	X.
	4254		546	0·892	50 55	26 16	45 45	+12 49	x.
	4255		546	0·903	50 14	28 42	48 11	+13 4	Y.
	4256		546	0·950	51 6	24 36	44 2	+11 27	y.
	4257	76·500	541	0·967	262 7	180 34	199 51	+19 8	M.
	4258		541	0·964	262 39	179 56	199 13	+19 30	N.
	4259		541	0·961	261 18	182 0	201 17	+21 25	m.
	4260		541	0·956	262 54	180 55	200 12	+20 39	n.
	4261		542	0·964	233 37	180 13	199 30	− 7 12	O.
	4262		542	0·969	233 59	180 53	200 10	− 6 13	o.
	4263		543	0·572	255 28	140 39	159 46	+ 9 58	P.
	4264		543	0·575	256 56	141 52	161 9	+ 8 30	p.
	4265		544	0·760	70 2	54 39	73 56	− 3 10	Q.
	4266		544	0·771	70 34	56 35	75 52	− 3 57	q.
	4267		544	0·790	69 50	56 56	76 13	− 5 25	r.
	4268		545	0·856	71 12	42 25	61 42	− 8 30	R.
	4269		545	0·859	71 48	43 9	62 26	− 8 11	S.
	4270		545	0·860	72 46	40 50	60 7	− 7 50	s.
	4271		546	0·892	49 42	27 25	46 42	+11 23	X.
	4272		546	0·890	50 51	26 33	45 50	+12 54	x.
	4273		546	0·901	50 29	28 58	48 15	+13 7	Y.
	4274		546	0·948	51 22	25 9	44 26	+11 27	y.
19.	4275	77·512	543	0·763	249 6	153 16	158 12	+10 2	A.
	4276		543	0·770	250 14	154 28	159 24	+ 8 37	B.
	4277		543	0·771	251 33	156 8	161 4	+ 9 42	C.
	4278		544	0·572	64 22	67 32	72 28	− 3 14	D.
	4279		544	0·583	65 19	70 23	75 19	− 4 33	a.
	4280		544	0·575	68 18	71 26	76 22	− 3 45	b.
	4281		544	0·591	67 51	71 59	76 55	− 5 52	c.
	4282		544	0·584	68 46	69 37	74 33	− 3 19	d.
	4283		544	0·595	67 19	69 40	74 36	− 4 0	e.
	4284		545	0·689	69 24	56 17	61 13	− 8 36	E.
	4285		545	0·712	68 37	57 23	62 19	− 7 31	F.
	4286		545	0·694	68 45	57 49	62 45	− 8 28	G.
	4287		545	0·701	69 2	55 8	60 4	− 9 23	f.
	4288		545	0·722	70 5	55 35	60 31	− 7 7	g.
	4289		546	0·780	44 19	40 54	45 50	+11 18	K.
	4290		546	0·782	43 52	43 42	48 38	+12 36	k.
	4291		546	0·790	45 0	39 26	44 22	+12 39	M.
	4292		546	0·796	44 36	42 57	47 53	+13 25	m.
	4293	77·534	543	0·768	249 18	153 40	158 17	+10 4	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 19.	4294	77·534	543	0·772	250° 13′	154° 44′	159° 21′	+ 8° 41′	B.
	4295		543	0·774	252 27	156 19	160 56	+ 9 40	C.
	4296		544	0·569	64 26	67 53	72 30	− 3 15	D.
	4297		544	0·578	65 0	70 38	75 15	− 4 37	a.
	4298		544	0·572	68 13	71 54	76 31	− 3 39	b.
	4299		544	0·589	67 40	72 22	76 59	− 5 51	c.
	4300		544	0·582	69 4	69 50	74 27	− 3 17	d.
	4301		544	0·592	67 11	70 2	74 39	− 4 1	e.
	4302		545	0·687	69 22	57 8	61 45	− 8 34	E.
	4303		545	0·710	68 54	57 59	62 36	− 7 25	F.
	4304		545	0·693	69 6	58 10	62 47	− 8 26	G.
	4305		545	0·695	69 14	55 35	60 12	− 9 19	f.
	4306		545	0·719	70 42	56 38	61 5	− 7 11	g.
	4307		546	0·776	44 23	41 15	45 52	+11 19	K.
	4308		546	0·780	44 16	43 56	48 33	+12 44	k.
	4309		546	0·786	45 28	39 52	44 29	+12 41	M.
	4310		546	0·794	44 31	43 9	47 46	+13 16	m.
23.	4311	81·530	544	0·304	239 13	126 18	74 15	− 3 17	A.
	4312		544	0·319	240 28	124 46	72 43	− 2 37	a.
	4313		545	0·170	237 6	119 51	67 48	− 3 21	B.
	4314		545	0·182	237 54	116 52	64 49	− 2 35	b.
	4315		546*a*	0·322	317 34	115 36	63 33	+14 48	C.
	4316		546*a*	0·327	316 25	115 1	63 58	+15 2	c.
	4317		547	0·570	37 12	76 33	24 30	+17 50	D.
	4318		547	0·572	38 48	75 29	23 26	+18 22	E.
	4319		547	0·581	37 24	76 14	24 11	+17 23	d.
	4320		547	0·584	36 3	77 47	25 44	+18 47	e.
	4321		548	0·740	43 17	63 9	11 6	+20 34	F.
	4322		548	0·742	42 46	64 6	12 3	+19 0	f.
	4323		548	0·755	43 49	62 27	10 24	+19 8	g.
29.	4324	87·476	547	0·781	264 3	167 27	31 3	+17 16	A.
	4325		547	0·784	265 19	166 37	30 13	+17 58	B.
	4326		547	0·796	265 37	167 42	31 18	+18 40	a.
	4327		547	0·799	265 23	164 9	27 45	+17 18	b.
	4328		548	0·619	272 45	155 36	19 12	+20 22	C.
	4329		548	0·640	272 44	154 31	18 7	+19 36	D.
	4330		548	0·622	274 26	153 15	16 51	+21 4	E.
	4331		548	0·637	273 50	155 44	19 20	+20 45	c.
	4332		548	0·643	274 55	152 41	16 17	+19 58	d.
	4333		549	0·355	355 9	108 26	332 2	+21 27	F.
	4334		549	0·360	358 42	106 21	329 57	+21 9	G.
	4335		549	0·364	0 36	108 47	332 23	+23 50	H.
	4336		549	0·390	357 4	104 34	328 10	+23 19	e.
	4337		549	0·401	359 47	103 50	327 26	+22 18	f.
	4338		549	0·372	0 34	105 52	329 28	+33 32	g.
	4339		549	0·422	2 32	104 28	328 4	+21 37	h.
	4340		549	0·420	3 28	104 6	327 42	+22 1	m.
	4341		550	0·870	47 0	55 34	279 10	+17 50	X.
	4342		550	0·873	48 16	55 12	278 48	+16 24	x.
	4343	87·561	547	0·786	264 26	168 24	30 48	+17 22	A.
	4344		547	0·788	265 33	167 32	29 56	+18 3	B.
	4345		547	0·799	265 21	168 40	31 4	+18 47	a.
	4346		547	0·802	265 17	165 2	27 26	+17 25	b.
	4347		548	0·622	272 44	156 27	18 51	+20 19	C.
	4348		548	0·641	272 59	155 19	17 43	+19 42	D.
	4349		548	0·625	274 35	154 2	16 26	+21 15	E.

TABLE III. (continued).

Date	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Mar. 29.	4350	87·561	548	0·640	273° 49′	156° 50′	19° 14′	+21° 0′	e.
	4351		548	0·645	274 51	153 37	16 1	+19 52	d.
	4352		549	0·353	356 13	109 20	331 44	+21 27	F.
	4353		549	0·361	358 49	107 11	329 35	+21 12	G.
	4354		549	0·362	359 59	109 31	331 55	+23 54	H.
	4355		549	0·389	357 25	105 26	327 50	+23 22	e.
	4356		549	0·402	359 30	104 28	326 52	+22 20	f.
	4357		549	0·370	1 2	106 43	329 27	+23 40	g.
	4358		549	0·420	2 46	105 16	327 40	+21 38	h.
	4359		549	0·421	3 27	104 55	327 19	+22 9	m.
	4360		550	0·868	47 19	56 24	278 48	+17 56	X.
	4361		550	0·870	48 31	56 9	278 33	+16 33	x.
30.	4362	88·659	547	0·902	257 27	181 0	27 53	+17 39	S.
	4363		547	0·899	256 34	183 38	30 31	+17 36	R.
	4364		547	0·889	257 31	181 10	28 3	+18 27	s.
	4365		548	0·795	261 26	169 56	16 49	+19 3	P.
	4366		548	0·799	263 4	171 23	18 16	+21 6	Q.
	4367		548	0·803	261 9	169 49	16 42	+20 14	M.
	4368		548	0·814	262 18	170 54	17 47	+19 8	p.
	4369		548	0·843	263 15	171 37	18 30	+20 12	q.
	4370		548	0·831	263 35	170 37	17 30	+21 39	m.
	4371		549	0·440	302 56	122 52	329 45	+23 41	A.
	4372		549	0·436	301 5	118 26	325 19	+22 22	B.
	4373		549	0·442	301 22	124 15	331 8	+22 37	C.
	4374		549	0·376	306 29	121 24	328 17	+21 7	D.
	4375		549	0·382	308 33	120 27	327 20	+23 28	a.
	4376		549	0·379	307 32	123 20	330 13	+22 22	b.
	4377		549	0·378	309 54	119 2	325 55	+20 18	c.
	4378		549	0·370	321 47	120 3	326 56	+21 3	d.
	4379		549	0·369	321 11	119 30	326 23	+23 28	e.
	4380		549	0·366	322 16	118 32	325 25	+22 55	f.
	4381		550	0·730	40 26	70 59	277 52	+16 29	Y.
	4382		550	0·728	41 45	72 11	279 4	+17 14	y.
	4383	88·672	547	0·905	257 38	181 12	27 50	+17 42	S.
	4384		547	0·902	256 22	183 48	30 26	+17 30	R.
	4385		547	0·892	257 19	181 17	27 55	+18 28	s.
	4386		548	0·796	261 38	170 2	16 40	+19 12	P.
	4387		548	0·802	263 19	171 48	18 26	+21 12	Q.
	4388		548	0·806	261 2	170 13	16 51	+20 12	M.
	4389		548	0·816	262 37	171 13	17 51	+19 37	p.
	4390		548	0·845	263 41	171 46	18 24	+20 22	q.
	4391		548	0·835	263 18	170 33	17 11	+21 41	m.
	4392		549	0·442	303 12	123 17	329 55	+23 39	A.
	4393		549	0·437	300 59	118 39	325 17	+22 31	B.
	4394		549	0·444	301 28	124 18	330 56	+22 40	C.
	4395		549	0·377	306 34	121 50	328 28	+21 9	D.
	4396		549	0·384	308 35	120 44	327 22	+23 29	a.
	4397		549	0·381	307 31	123 38	330 16	+22 20	b.
	4398		549	0·379	309 50	119 14	325 52	+20 19	c.
	4399		549	0·374	321 36	119 52	326 30	+21 14	d.
	4400		549	0·372	321 0	119 53	326 31	+23 30	e.
	4401		549	0·369	322 28	119 2	325 40	+23 4	f.
	4402		550	0·726	40 14	71 16	277 54	+16 30	Y.
	4403		550	0·725	41 36	72 35	279 13	+17 20	y.
31.	4404	89·438	547	0·962	263 11	192 54	28 40	+17 12	A.
	4405		547	0·963	264 37	194 47	30 33	+18 19	B.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864 Mar. 31.	4406	89·438	547	0·968	264° 41′	194° 31′	30° 17′	+18° 6′	C.
	4407		548	0·892	264 14	179 30	15 16	+18 57	A′.
	4408		548	0·902	263 9	183 30	19 16	+19 13	B′.
	4409		548	0·895	262 26	182 42	18 28	+21 40	C′.
	4410		548	0·897	265 57	182 55	18 41	+21 12	a.
	4411		548	0·903	266 56	181 6	16 52	+20 43	b.
	4412		548	0·906	265 25	181 26	17 12	+21 9	c.
	4413		549	0·518	290 40	134 20	330 6	+24 2	D.
	4414		549	0·521	291 18	135 57	331 43	+23 50	E.
	4415		549	0·464	295 57	130 51	326 37	+20 56	F.
	4416		549	0·462	297 6	129 41	325 27	+21 40	d.
	4417		549	0·431	303 13	134 3	329 49	+22 37	e.
	4418		549	0·427	302 12	137 8	332 54	+23 28	f.
	4419		550	0·608	39 44	85 9	280 55	+16 28	G.
	4420		550	0·609	38 33	82 23	278 9	+17 19	g.
	4421	89·500	547	0·963	263 19	193 39	28 33	+17 11	A.
	4422		547	0·965	264 53	195 52	30 46	+18 25	B.
	4423		547	0·970	265 7	195 21	30 15	+18 19	C.
	4424		548	0·894	264 33	180 25	15 19	+19 2	A′.
	4425		548	0·904	263 51	186 56	19 50	+19 11	B′.
	4426		548	0·897	262 34	183 42	18 36	+21 44	C′.
	4427		548	0·899	265 12	184 3	18 57	+21 8	a.
	4428		548	0·906	267 28	181 46	16 40	+20 40	b.
	4429		548	0·910	266 0	182 25	17 19	+21 15	c.
	4430		549	0·520	291 7	135 20	330 14	+24 0	D.
	4431		549	0·523	291 52	136 21	331 15	+23 55	E.
	4432		549	0·468	296 22	131 50	326 44	+20 59	F.
	4433		549	0·465	297 31	130 27	325 21	+21 48	d.
	4434		549	0·431	303 16	134 42	329 36	+22 33	e.
	4435		549	0·430	302 41	138 1	332 55	+23 37	f.
	4436		550	0·605	40 12	85 48	280 42	+16 19	G.
	4437		550	0·607	38 50	83 24	278 18	+17 24	g.
Apr. 1.	4438	90·432	548	0·972	257 38	194 34	16 15	+20 41	M.
	4439		548	0·980	259 19	197 21	19 2	+19 13	M^{0}.
	4440		548	0·970	258 13	198 48	20 29	+21 27	m.
	4441		548	0·983	259 27	196 49	18 30	+20 56	m^{0}.
	4442		549	0·676	274 23	147 57	329 38	+23 38	A.
	4443		549	0·672	274 24	149 52	331 33	+24 24	B.
	4444		549	0·680	275 54	144 30	326 11	+22 57	C.
	4445		549	0·692	276 13	146 33	328 14	+21 52	D.
	4446		549	0·590	274 11	151 27	333 8	+20 25	a.
	4447		549	0·586	275 0	151 28	333 9	+19 29	b.
	4448		549	0·582	282 50	144 1	325 42	+24 31	c.
	4449		549	0·550	282 6	145 2	326 43	+20 30	d.
	4450		549	0·553	283 7	147 16	328 57	+23 52	e.
	4451		550	0·464	26 23	99 36	281 17	+16 11	G.
	4452		550	0·471	26 29	96 45	278 26	+17 7	g.
	4453	90·455	548	0·974	257 33	195 18	16 39	+20 45	M.
	4454		548	0·981	258 56	197 59	19 20	+19 17	M^{0}.
	4455		548	0·973	258 35	198 53	20 14	+21 23	m.
	4456		548	0·986	259 35	197 15	18 36	+21 0	m^{0}.
	4457		549	0·680	274 28	148 16	329 37	+23 34	A.
	4458		549	0·673	274 26	150 9	331 30	+24 27	B.
	4459		549	0·682	276 1	144 54	326 15	+23 2	C.
	4460		549	0·694	276 18	146 58	328 19	+21 48	D.
	4461		549	0·591	274 10	151 39	333 0	+20 25	a.

TABLE III. (continued).

Date.	No	Mean Time of Sun-picture	No of Group in the Kew Catalogue	Distance from Centre	Angle of Position	Longitude from Node	Helio-graphical Longitude	Helio-graphical Latitude.	Spot
1864 Apr. 1.	4462	90·455	549	0·588	275° 15′	151° 33′	332° 54′	+19° 36′	b.
	4463		549	0·584	283 3	144 12	325 33	+24 30	c.
	4464		549	0·555	282 17	145 20	326 41	+20 25	d.
	4465		549	0·557	283 16	147 38	328 59	+23 46	e.
	4466		550	0·462	26 54	100 7	281 28	+16 16	G.
	4467		550	0·468	27 13	97 11	278 32	+17 20	g.
2.	4468	91·496	549	0·836	265 21	165 32	332 7	+22 26	A⁰.
	4469		549	0·821	266 38	163 16	329 51	+21 4	B⁰.
	4470		549	0·804	267 7	159 37	326 12	+20 54	C⁰.
	4471		549	0·762	264 54	161 55	328 30	+22 30	a⁰.
	4472		549	0·760	265 37	157 26	324 1	+22 28	b⁰.
	4473		550	0·344	355 16	112 54	279 29	+16 58	G.
	4474	91·506	549	0·839	265 34	165 49	332 15	+22 30	A⁰.
	4475		549	0·823	265 0	163 32	229 58	+21 8	B⁰.
	4476		549	0·806	266 59	159 45	326 11	+21 2	C⁰.
	4477		549	0·765	264 52	162 13	328 39	+22 23	a⁰.
	4478		549	0·761	265 33	157 45	324 11	+22 40	b⁰.
	4479		550	0·345	355 19	113 10	279 36	+17 1	G.
8.	4480	97·474	551	0·968	256 16	204 1	285 48	+13 45	A.
	4481		551	0·954	257 50	206 35	288 22	+13 12	B.
	4482		551	0·955	259 21	205 38	287 25	+14 52	a.
	4483		551	0·946	260 37	206 58	288 45	+15 47	b.
	4484		552	0·895	261 48	188 56	270 43	+18 35	C.
	4485		552	0·884	263 44	189 1	270 48	+17 26	D.
	4486		552	0·888	263 32	187 1	268 48	+18 47	E.
	4487		552	0·861	262 31	188 49	270 36	+17 44	F.
	4488		552	0·857	262 30	185 37	267 24	+17 34	c.
	4489		552	0·865	263 15	185 30	267 17	+18 29	d.
	4490		552	0·848	262 12	184 56	266 43	+19 20	e.
	4491		552	0·840	263 29	183 37	265 24	+18 54	f.
	4492		553	0·372	71 30	104 54	186 41	− 4 20	G.
	4493		553	0·375	72 56	103 7	184 54	− 3 30	H.
	4494		553	0·422	73 14	101 29	183 16	− 5 49	g.
	4495		553	0·431	74 31	99 20	181 7	− 6 15	h.
	4496		533	0·475	91 44	98 35	180 22	−12 5	m.
	4497		554	0·982	45 32	42 44	124 31	+16 42	M.
	4498		554	0·984	45 22	43 28	125 15	+17 47	N.
	4499		554	0·975	44 19	40 36	122 23	+17 31	n.
	4500		554	0·975	46 4	44 19	126 6	+18 36	o.
	4501	97·506	551	0·970	256 37	204 12	285 32	+13 50	A.
	4502		551	0·956	257 48	207 0	288 20	+13 9	B.
	4503		551	0·956	259 17	205 54	287 14	+14 55	a.
	4504		551	0·947	260 30	207 13	288 33	+15 42	b.
	4505		552	0·897	262 2	189 11	270 31	+18 42	C.
	4506		552	0·886	263 57	189 25	270 45	+17 25	D.
	4507		552	0·890	263 25	187 17	268 37	+18 48	E.
	4508		552	0·863	262 25	189 3	270 23	+17 48	F.
	4509		552	0·859	262 16	185 48	267 8	+17 31	c.
	4510		552	0·867	263 7	186 1	267 21	+18 35	d.
	4511		552	0·850	262 10	185 13	266 23	+19 15	e.
	4512		552	0·842	263 38	184 2	265 22	+18 50	f.
	4513		553	0·370	71 34	105 9	186 29	− 4 22	G.
	4514		553	0·374	73 5	103 32	184 52	− 3 38	H.
	4515		553	0·421	73 36	101 51	183 11	− 5 50	g.
	4516		553	0·428	74 27	99 51	181 11	− 6 11	h.
	4517		553	0·472	91 12	98 59	180 19	−12 10	m.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Apr. 8.	4518	97·506	554	0·980	45° 38′	43° 6′	124° 26′	+16° 44′	M.
	4519		554	0·982	45 19	43 46	125 6	+17 41	N.
	4520		554	0·974	44 27	40 55	122 15	+17 33	n.
	4521		554	0·973	46 25	44 35	125 55	+18 39	o.
9.	4522	98·682	552	0·978	257 16	202 50	267 29	+17 31	A^1.
	4523		552	0·972	257 44	200 35	265 14	+18 29	A^2.
	4524		552	0·971	259 18	205 46	270 25	+19 16	A^3.
	4525		552	0·971	258 39	204 47	269 26	+17 59	a.
	4526		552	0·965	257 37	201 9	265 48	+18 31	b.
	4527		552	0·968	258 17	201 49	266 28	+18 0	c.
	4528		552	0·959	260 22	200 31	265 10	+17 22	d.
	4529		552	0·953	260 7	201 26	266 5	+19 14	e.
	4530		553	0·117	104 1	119 6	183 45	− 2 4	B.
	4531		553	0·123	105 56	115 58	180 37	− 4 21	B^1.
	4532		553	0·127	103 23	120 37	185 16	− 5 28	B^2.
	4533		553	0·192	94 33	116 14	180 53	− 6 35	f.
	4534		553	0·195	95 36	114 29	179 8	− 3 10	g.
	4535		553	0·199	93 53	119 17	183 56	− 2 10	h.
	4536		553	0·295	115 58	116 58	181 37	−13 58	i.
	4537		553	0·283	116 59	115 30	180 9	−12 31	k.
	4538		554	0·883	44 57	58 47	123 26	+15 27	D.
	4539		554	0·884	44 51	58 6	122 45	+17 27	E.
	4540		554	0·896	45 48	60 14	124 53	+16 58	F.
	4541		554	0·884	47 13	60 34	125 13	+17 11	G.
	4542		554	0·902	46 37	60 51	125 30	+17 31	x^1.
	4543		554	0·911	45 9	60 11	124 50	+18 5	x^2.
	4544		554	0·921	47 28	62 17	126 56	+18 36	x^3.
	4545	98·692	552	0·979	257 28	203 4	267 35	+17 28	A^1.
	4546		552	0·972	257 53	200 39	265 10	+18 28	A^2.
	4547		552	0·972	259 14	206 7	270 38	+19 21	A^3.
	4548		552	0·973	258 44	204 59	269 30	+17 58	a.
	4549		552	0·966	257 25	201 26	265 57	+18 30	b.
	4550		552	0·968	258 26	202 9	266 40	+17 55	e.
	4551		552	0·959	260 52	200 34	265 5	+17 25	d.
	4552		552	0·955	260 18	201 40	266 11	+19 15	e.
	4553		553	0·115	104 8	119 24	183 55	− 2 10	B.
	4554		553	0·120	106 4	116 10	180 41	− 4 19	B^1.
	4555		553	0·128	103 37	121 5	185 36	− 5 34	B^2.
	4556		553	0·190	94 27	116 38	181 9	− 6 31	f.
	4557		553	0·193	95 50	114 24	178 55	− 3 14	g.
	4558		553	0·200	93 59	119 26	183 57	− 2 11	h.
	4559		553	0·292	116 7	117 13	181 44	−13 50	i.
	4560		553	0·281	117 2	115 46	180 17	−12 36	k.
	4561		554	0·881	44 47	59 3	123 34	+15 30	D.
	4562		554	0·880	44 45	58 10	122 41	+17 29	E.
	4563		554	0·895	45 58	60 30	125 1	+17 14	F.
	4564		554	0·882	47 6	60 41	125 12	+17 22	G.
	4565		554	0·901	46 22	61 5	125 36	+17 35	x^1.
	4566		554	0·911	44 57	60 23	124 54	+18 9	x^2.
	4567		554	0·919	47 26	62 32	127 3	+18 43	x^3.
11.	4568	100·428	553	0·344	230 31	145 3	184 46	− 3 16	A.
	4569		553	0·342	231 28	144 5	183 58	− 2 15	B.
	4570		553	0·287	222 27	141 16	181 9	− 4 21	C.
	4571		553	0·289	222 54	141 11	181 4	− 5 4	a.
	4572		553	0·308	198 46	140 0	179 53	−12 44	b.
	4573		554	0·628	32 17	78 35	118 28	+16 36	D.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Apr. 11.	4574	100·428	554	0·634	33° 25′	78° 28′	118° 21′	+18° 29′	E.
	4575		554	0·644	35 14	77 31	117 24	+17 31	F.
	4576		554	0·669	38 14	83 38	123 31	+18 29	G.
	4577		554	0·702	40 49	82 41	122 34	+16 2	H.
	4578		554	0·695	36 51	75 32	115 25	+18 58	c.
	4579		554	0·721	37 32	76 19	116 12	+18 33	d.
	4580		554	0·724	40 44	77 7	117 0	+17 41	e.
	4581		554	0·711	41 16	77 56	117 49	+17 36	f.
	4582		554	0·732	41 23	74 34	114 27	+17 45	g.
	4583	100·495	553	0·347	230 24	145 43	184 39	− 3 15	A.
	4584		553	0·346	231 49	144 37	183 33	− 2 21	B.
	4585		553	0·290	222 46	141 55	180 51	− 4 25	C.
	4586		553	0·292	223 11	141 28	180 24	− 5 0	a.
	4587		553	0·310	198 40	140 28	179 24	−12 41	b.
	4588		554	0·626	32 19	78 58	117 54	+16 41	D.
	4589		554	0·632	33 22	78 46	117 42	+18 25	E.
	4590		554	0·641	35 40	78 5	117 1	+17 30	F.
	4591		554	0·665	38 31	83 56	122 52	+18 31	G.
	4592		554	0·700	40 22	83 4	122 0	+15 54	H.
	4593		554	0·692	37 0	75 49	114 45	+18 57	c.
	4594		554	0·718	37 49	76 39	115 35	+18 30	d.
	4595		554	0·720	40 35	77 24	116 20	+17 36	e.
	4596		554	0·708	41 19	78 19	117 15	+17 40	f.
	4597		554	0·729	41 28	74 50	113 46	+17 49	g.
12.	4598	101·464	553	0·533	236 22	154 6	179 18	− 2 37	A^{0}.
	4599		553	0·530	237 38	157 19	182 31	− 3 44	B^{0}.
	4600		553	0·481	232 39	158 12	183 24	− 6 18	C^{0}.
	4601		553	0·477	233 6	156 42	181 54	− 5 7	a^{0}.
	4602		553	0·465	218 24	153 55	179 7	−12 59	b^{0}.
	4603		554	0·495	24 7	91 4	116 16	+17 23	M.
	4604		554	0·492	24 58	97 16	122 28	+18 9	N.
	4605		554	0·501	33 36	97 59	123 11	+18 55	O.
	4606		554	0·533	31 27	89 7	114 19	+15 44	m.
	4607		554	0·528	29 9	89 53	115 5	+17 13	n.
	4608		554	0·540	33 12	87 47	112 59	+17 47	o.
13.	4609	102·499	553	0·722	234 19	170 46	181 17	− 2 14	a.
	4610		553	0·719	235 41	169 1	179 32	− 4 9	b.
	4611		553	0·666	232 46	171 33	182 4	− 5 58	c.
	4612		553	0·662	232 49	172 43	183 14	− 3 32	d.
	4613		554	0·406	355 32	111 57	122 28	+17 25	A.
	4614		554	0·402	357 51	103 36	114 7	+17 44	B.
	4615		554	0·396	359 14	102 8	112 39	+18 19	C.
	4616		554	0·420	0 23	113 17	123 48	+15 31	D.
	4617		554	0·417	1 30	102 27	112 58	+18 32	e.
	4618		554	0·435	8 57	103 40	114 11	+17 6	f.
	4619		554	0·431	10 41	106 34	117 5	+18 15	g.
	4620		554	0·446	10 5	107 34	118 5	+18 22	h.
14.	4621	103·543	553	0·864	234 11	184 25	180 7	− 4 7	M.
	4622		553	0·831	235 28	188 54	184 36	− 2 51	N.
	4623		554	0·376	317 9	128 7	123 49	+18 37	O.
15.	4624	104 493	553	0·950	234 28	198 20	180 34	− 4 19	A.
	4625		553	0·926	232 39	203 3	185 17	− 3 2	B.
	4626		554	0·454	291 27	141 40	123 54	+18 35	C.
	4627		555	0·642	68 12	92 29	74 43	− 6 24	M.
	4628		555	0·654	69 44	92 37	74 51	− 6 38	N.
	4629		555	0·655	71 12	93 56	76 10	− 8 4	O.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Apr. 15.	4630	104·493	555	0·661	72° 51′	93° 59′	76° 13′	− 8° 21′	m.
	4631		556	0·922	49 47	67 14	49 28	+ 9 34	n.
	4632		556	0·917	50 8	69 3	51 17	+10 14	o.
18.	4633	107·482	554*a*	0·976	253 21	220 39	160 29	+ 8 27	G.
	4634		556*a*	0·805	66 24	83 32	23 22	+ 4 39	A.
	4635		556*a*	0·817	67 39	83 7	22 57	+ 4 41	B.
	4636		556*a*	0·844	68 11	79 21	19 11	− 1 10	C.
	4637		556*a*	0·852	69 56	78 43	18 33	− 0 11	D.
	4638		556*b*	0·840	44 30	77 17	17 7	+21 38	a.
	4639		556*b*	0·855	44 55	76 19	16 9	+21 21	b.
19.	4640	108·451	556*a*	0·642	64 35	96 54	22 59	+ 1 4	M.
	4641		556*a*	0·700	66 58	92 13	18 18	− 0 2	N.
	4642	108·484	556*a*	0·640	64 28	97 18	22 55	+ 1 7	M.
	4643		556*a*	0·697	67 10	92 29	18 6	+ 0 5	N.
20.	4644	109·444	556*a*	0·464	66 37	110 31	22 31	+ 2 16	A.
	4645		556*a*	0·471	67 44	109 54	21 54	+ 1 53	B.
	4646		556*a*	0·503	69 9	105 25	17 25	− 0 2	a.
	4647		556*a*	0·511	69 27	105 23	17 23	+ 0 14	b.
	4648		556*c*	0·862	61 25	77 54	349 54	+ 8 19	C.
	4649		556*d*	0·950	44 38	56 12	328 12	+26 33	D.
21.	4650	110·610	556*a*	0·243	62 26	128 0	23 28	+ 2 28	A^0.
	4651		556*a*	0·245	62 45	126 41	22 9	+ 2 0	B^0.
	4652		556*a*	0·306	66 34	122 36	18 4	+ 0 6	a^0.
	4653		556*a*	0·308	66 12	122 8	17 36	+ 0 13	b^0.
	4654		556*c*	0·724	57 54	94 40	350 8	+ 7 28	C.
	4655		556*d*	0·814	39 4	73 58	329 24	+25 35	D.
22.	4656	111·465	556*a*	0·056	70 49	136 41	20 1	+ 1 34	M.
	4657		556*a*	0·062	70 58	136 27	19 47	+ 2 7	N.
	4658	111·515	556*a*	0·055	71 14	137 35	20 12	+ 1 39	M.
	4659		556*a*	0·060	71 37	137 3	19 40	+ 2 6	N.
23.	4660	112·446	556*a*	0·208	245 13	151 19	20 44	+ 0 14	A.
	4661	112·469	556*a*	0·211	245 52	151 48	20 53	+ 0 17	A.
25.	4662	114·540	556*a*	0·594	238 47	179 32	19 15	− 0 12	A^0.
26.	4663	115·639	No spots.						
	4664	115·654							
29.	4665	118·521							
	4666	118·549							
May 3.	4667	122·545	557	0·340	240 31	169 20	255 30	− 3 28	M.
	4668		557	0·339	239 54	168 21	254 31	− 2 38	M^1.
	4669		557	0·328	241 3	167 43	253 53	− 3 54	m.
	4670		557	0·314	234 38	164 36	250 46	− 4 22	n.
	4671		557	0·297	232 16	164 11	250 21	− 4 16	o.
	4672		557	0·285	230 27	163 16	249 26	− 5 44	p.
	4673		557*a*	0·696	73 22	105 4	191 14	− 6 37	A.
	4674		558	0·982	54 27	69 3	155 13	+13 13	B.
	4675		558	0·980	55 2	67 12	153 22	+12 54	B^1.
	4676	122·622	557	0·344	241 7	170 11	255 16	− 3 32	M.
	4677		557	0·341	240 13	168 47	253 52	− 2 40	M^1.
	4678		557	0·333	240 58	168 22	253 27	− 3 49	m.
	4679		557	0·316	235 0	166 10	251 15	− 4 30	n.
	4680		557	0·301	232 42	165 3	250 8	− 4 19	o.
	4681		557	0·287	230 29	163 47	248 52	− 5 40	p.
	4682		557*a*	0·696	73 12	105 36	190 41	− 6 38	A.
	4683		558	0·979	55 1	70 11	155 16	+13 10	B.
	4684		558	0·977	55 34	67 33	152 38	+13 1	B^1.
5.	4685	124·659	557	0·720	241 42	198 22	254 33	− 4 12	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 5.	4686	124·659	557	0·718	240° 59′	199° 8′	255° 19′	− 4° 2′	B.
	4687		557	0·715	239 16	194 17	250 28	− 3 28	C.
	4688		557	0·670	240 12	195 26	251 37	− 2 19	a.
	4689		557	0·654	239 24	194 29	250 40	− 2 57	b.
	4690		557	0·645	238 53	192 54	249 5	− 3 11	c.
	4691		558	0·448	42 22	100 20	156 31	+12 44	D.
	4692		558	0·452	42 37	99 11	155 22	+12 42	E.
	4693		558	0·470	43 19	97 40	153 51	+13 19	d.
	4694		558	0·492	44 24	93 31	149 42	+14 2	e.
	4695		558	0·495	44 46	96 6	152 17	+11 57	f.
	4696		559	0·734	52 5	74 17	130 28	+14 22	G.
	4697		559	0·741	52 33	73 43	129 54	+14 17	F.
6.	4698	125·484	557	0·870	241 54	211 25	255 54	− 4 27	P.
	4699		557	0·864	242 13	209 48	254 17	− 3 51	p.
	4700		558	0·317	26 42	112 37	157 6	+12 42	Q.
	4701		558	0·324	28 19	110 51	155 20	+11 16	R.
	4702		558	0·339	31 18	108 53	153 22	+12 19	q.
	4703		558	0·362	34 28	110 7	154 36	+13 36	r.
	4704		559	0·599	47 2	85 11	129 40	+14 59	S.
	4705		559	0·610	48 11	85 40	130 9	+15 2	s.
	4706	125·530	557	0·873	242 3	211 58	255 48	− 4 30	P.
	4707		557	0·866	242 54	210 34	254 24	− 3 46	p.
	4708		558	0·315	27 19	113 5	156 55	+12 44	Q.
	4709		558	0·321	28 11	111 44	155 34	+11 22	R.
	4710		558	0·336	30 47	109 41	153 31	+12 29	q.
	4711		558	0·360	34 29	111 10	155 0	+13 37	r.
	4712		559	0·596	47 20	85 52	129 42	+15 2	S.
	4713		559	0·608	48 10	86 35	130 25	+15 0	s.
7.	4714	126·480	557	0·957	241 24	226 6	256 28	− 3 27	A.
	4715		558	0·207	330 29	123 39	154 1	+12 22	B.
	4716		558	0·211	342 38	122 15	152 37	+13 4	C.
	4717		558	0·219	354 7	123 20	153 42	+10 58	D.
	4718		558	0·235	329 52	118 54	149 16	+12 24	a.
	4719		558	0·223	338 40	120 33	150 55	+12 29	b.
	4720		558	0·241	0 14	118 7	148 29	+11 33	c.
	4721		559	0·880	46 46	98 35	128 57	+14 43	M.
	4722		559	0·894	45 3	99 35	129 57	+15 26	N.
	4723		559	0·911	43 18	100 59	131 21	+15 55	n.
10.	4724	129·638	558	0·672	277 22	195 18	180 52	+12 7	A.
	4725		558	0·685	276 31	196 37	182 11	+12 19	B.
	4726		558	0·698	275 13	193 49	179 23	+11 12	a.
	4727		558	0·712	277 54	194 21	179 55	+13 54	b.
	4728		560	0·422	20 7	139 47	125 21	+21 31	C.
12.	4729	131·530	560	0·364	308 40	170 29	129 12	+20 24	M.
	4730		560	0·375	307 38	169 11	127 54	+21 31	N.
	4731		560	0·382	308 12	170 47	129 30	+19 26	O.
	4732		561	0·870	82 19	101 26	60 9	− 7 48	A.
	4733		561	0·869	81 15	100 54	59 37	− 6 20	B.
	4734		561	0·883	82 24	99 32	58 15	− 8 3	C.
	4735		561	0·877	83 33	97 42	56 25	− 8 8	a.
	4736		561	0·892	80 30	95 4	53 47	− 6 27	b.
	4737		561	0·897	79 26	99 37	58 20	− 7 53	c.
13.	4738	132·511	560	0·505	287 28	184 32	129 21	+17 28	A.
	4739		561	0·701	81 9	116 2	60 51	− 7 14	B.
	4740		561	0·717	81 26	115 46	60 35	− 6 33	b.
	4741		561	0·719	82 8	111 31	56 20	− 7 51	C.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 13.	4742	132·511	561	0·744	82° 34′	112° 55′	57° 44′	− 7° 23′	D.
	4743		561	0·763	81 40	113 14	58 3	− 8 38	c.
	4744		561	0·751	83 57	111 19	56 8	− 6 39	d.
	4745		561	0·784	83 34	110 23	55 12	− 6 1	e.
14.	4746	133·536	560	0·709	282 31	200 32	130 48	+17 24	M.
	4747		560	0·692	281 22	198 51	129 7	+17 23	N.
	4748		560	0·733	280 53	201 17	131 33	+16 39	m.
	4749		560	0·745	278 57	198 34	128 50	+15 13	n.
	4750		561	0·497	91 12	131 15	61 31	− 6 26	A.
	4751		561	0·502	91 19	125 1	55 17	− 6 30	B
	4752		561	0·513	90 54	125 46	56 2	− 7 38	C.
	4753		561	0·550	91 33	130 41	60 57	− 6 12	D.
	4754		561	0·541	89 5	124 37	54 53	− 7 40	E.
	4755		561	0·533	89 10	132 3	62 19	− 7 41	a.
	4756		561	0·570	88 12	129 56	60 12	− 8 9	b.
	4757		561	0·571	87 29	127 22	57 38	− 7 57	c.
	4758		561	0·585	86 44	128 10	58 26	− 7 38	d.
	4759		561	0·587	87 17	128 14	58 30	− 8 14	e.
15.	4760	134·482	560	0·800	271 15	212 13	129 4	+17 28	A.
	4761		560	0·802	272 12	214 35	131 26	+17 20	B.
	4762		560	0·763	286 48	213 28	130 19	+18 53	C.
	4763		560	0·767	287 26	210 20	127 11	+16 37	a.
	4764		560	0·782	286 31	215 46	132 37	+15 48	b.
	4765		561	0·314	94 11	143 44	60 35	− 6 22	D.
	4766		561	0·321	94 54	145 2	61 53	− 6 46	E.
	4767		561	0·316	93 29	146 8	62 59	− 7 45	F.
	4768		561	0·320	95 59	140 38	57 29	− 7 58	d.
	4769		561	0·333	96 17	137 20	54 11	− 8 35	e.
	4770		561	0·339	92 31	138 32	55 23	− 6 18	f.
	4771		561	0·401	91 10	142 41	59 32	− 6 14	G.
	4772		561	0·387	93 0	141 52	58 43	− 7 33	g.
	4773		561	0·410	94 46	143 18	60 9	− 7 50	H.
	4774		561	0·412	93 18	139 1	55 52	− 7 44	h.
	4775		561	0·417	92 54	141 15	58 6	− 6 30	K.
	4776		561	0·405	94 26	137 33	54 24	− 6 22	k.
	4777	134·490	560	0·801	271 28	212 18	129 2	+17 30	A.
	4778		560	0·804	272 24	214 36	131 20	+17 25	B.
	4779		560	0·765	287 13	213 34	130 18	+19 4	C.
	4780		560	0·769	287 54	210 16	127 0	+16 44	a.
	4781		560	0·783	286 50	216 0	132 44	+15 43	h.
	4782		561	0·312	94 42	143 55	60 39	− 6 29	D.
	4783		561	0·319	94 51	144 58	61 42	− 6 47	E.
	4784		561	0·315	93 26	146 24	63 8	− 7 48	F.
	4785		561	0·319	96 4	140 42	57 26	− 8 3	d.
	4786		561	0·332	96 28	137 26	54 10	− 8 31	e.
	4787		561	0·336	92 15	138 45	55 29	− 6 17	f.
	4788		561	0·400	91 1	142 52	59 36	− 6 10	G.
	4789		561	0·385	93 24	141 56	58 40	− 7 25	g.
	4790		561	0·408	95 14	143 22	60 6	− 7 54	H.
	4791		561	0·410	93 2	139 16	56 0	− 7 42	h.
	4792		561	0·416	92 55	140 45	57 59	− 6 26	K.
	4793		561	0·404	94 28	137 48	54 32	− 6 17	k.
16.	4794	135·487	560	0·942	274 26	227 39	130 15	+17 24	M.
	4795		560	0·938	273 8	223 47	126 23	+16 30	N.
	4796		560	0·910	276 59	228 28	131 4	+18 21	m.
	4797		560	0·915	277 41	226 3	128 39	+17 36	n.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 16.	4798	135·487	561	0·199	153° 17′	159° 32′	62° 8′	− 6° 15′	P.
	4799		561	0·202	154 6	156 49	59 25	− 6 46	Q.
	4800		561	0·204	155 52	155 10	57 46	− 8 34	R.
	4801		561	0·211	140 53	153 14	55 50	− 8 53	S.
	4802		561	0·223	138 39	155 35	58 11	− 8 19	p.
	4803		561	0·240	129 46	152 58	55 34	− 7 47	q.
	4804		561	0·244	126 16	151 46	54 22	− 7 13	r.
	4805		561	0·251	122 24	159 22	61 58	− 6 48	s.
	4806		561	0·239	120 26	158 23	60 59	− 7 0	T.
	4807		561	0·265	118 29	157 23	59 59	− 8 25	T¹.
	4808		561	0·258	120 50	155 54	58 30	− 6 27	t.
	4809		561	0·264	117 33	155 30	58 6	− 6 14	t¹.
	4810		562	0·382	37 46	144 29	47 5	+15 23	A.
	4811		562	0·389	39 24	141 36	44 12	+15 16	B.
	4812		562	0·412	40 25	140 31	43 7	+17 48	a.
	4813		562*	0·434	41 0	139 46	42 24	+16 54	b.
	4814	135·523	560	0·944	273 54	228 28	130 33	+17 22	M.
	4815		560	0·939	273 41	224 10	126 15	+16 36	N.
	4816		560	0·908	277 4	228 47	130 52	+18 18	m.
	4817		560	0·916	276 38	226 35	128 40	+17 35	n.
	4818		561	0·196	154 19	160 2	62 7	− 6 14	P.
	4819		561	0·199	154 42	157 55	60 0	− 6 43	Q.
	4820		561	0·200	157 3	155 35	57 40	− 8 32	R.
	4821		561	0·208	141 12	153 39	55 44	− 8 50	S.
	4822		561	0·219	139 6	156 12	58 17	− 8 17	p.
	4823		561	0·236	128 44	153 36	55 41	− 7 45	q.
	4824		561	0·241	125 28	152 15	54 20	− 7 12	r.
	4825		561	0·247	123 1	159 57	62 2	− 6 50	s.
	4826		561	0·238	119 38	158 50	60 55	− 7 2	T.
	4827		561	0·262	117 25	157 45	59 50	− 8 28	T¹.
	4828		561	0·255	121 44	156 22	58 27	− 6 25	t.
	4829		561	0·261	117 20	156 5	58 10	− 6 16	t¹.
	4830		562	0·380	36 44	145 5	47 10	+15 25	A.
	4831		562	0·385	39 29	142 15	44 20	+15 17	B.
	4832		562	0·410	40 27	141 9	43 14	+17 52	a.
	4833		562	0·431	41 58	140 20	42 25	+16 55	b.
17.	4834	136·507	560	0·995	267 47	240 14	128 21	+17 36	N.
	4835		561	0·172	174 33	172 8	60 15	− 7 28	A.
	4836		561	0·176	175 18	175 23	63 30	− 6 46	B.
	4837		561	0·181	183 46	169 44	57 51	− 6 28	C.
	4838		561	0·194	177 12	171 52	59 59	− 6 58	D.
	4839		561	0·195	179 19	168 50	56 57	− 7 11	a.
	4840		561	0·202	184 52	170 8	58 15	− 8 23	b.
	4841		561	0·224	185 45	169 57	58 4	− 7 33	c.
	4842		561	0·219	191 25	173 18	61 25	− 7 36	d.
	4843		561	0·238	188 13	173 11	61 18	− 8 14	F.
	4844		561	0·224	195 9	168 41	56 48	− 8 57	f.
	4845		561	0·241	195 6	169 14	57 21	− 7 7	G.
	4846		561	0·237	196 58	170 49	58 56	− 7 16	g.
	4847		561	0·245	194 54	168 38	56 45	− 6 46	H.
	4848		561	0·243	197 39	167 49	55 56	− 8 4	h.
	4849		562	0·273	358 2	158 9	46 16	+15 14	K.
	4850		562	0·278	359 27	159 12	47 19	+16 1	k.
	4851		562	0·301	7 11	157 33	45 40	+16 50	O.
	4852		562	0·312	7 48	155 46	43 53	+17 27	o.
	4853		562	0·324	8 39	154 49	42 56	+17 51	P.

* Group No. 562 was visible on the 15th, but partially hidden by the wire of the telescope, hence positions not determinable.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 17.	4854	136·507	562	0·336	11° 16′	154° 51′	42° 58′	+17° 21′	Q.
	4855		562	0·325	10 14	156 15	44 22	+16 56	p.
	4856		562	0·326	12 20	157 4	45 11	+15 23	q.
	4857	136·520	560	0·996	267 28	240 31	128 29	+17 29	N.
	4858		561	0·173	175 2	172 14	60 12	− 7 34	A.
	4859		561	0·176	175 47	175 27	63 25	− 6 50	B.
	4860		561	0·180	184 19	169 52	57 50	− 6 29	C.
	4861		561	0·196	177 0	171 10	59 8	− 6 55	D.
	4862		561	0·197	178 53	168 48	56 46	− 7 10	a.
	4863		561	0·203	184 40	170 19	58 17	− 8 28	b.
	4864		561	0·225	185 36	170 19	58 17	− 7 39	c.
	4865		561	0·221	191 12	173 43	61 41	− 7 28	d.
	4866		561	0·240	188 31	173 50	61 48	− 8 10	F.
	4867		561	0·223	195 42	168 8	56 6	− 8 59	f.
	4868		561	0·240	195 50	169 33	57 31	− 7 8	G.
	4869		561	0·236	197 4	170 39	58 37	− 7 17	g.
	4870		561	0·246	194 59	168 42	56 40	− 6 44	H.
	4871		561	0·244	198 14	167 9	55 7	− 8 11	h.
	4872		562	0·271	358 31	158 29	46 27	+15 17	K.
	4873		562	0·275	359 46	159 26	47 24	+15 54	k.
	4874		562	0·302	6 53	157 21	45 19	+16 44	O.
	4875		562	0·310	6 49	155 17	43 15	+17 25	o.
	4876		562	0·322	8 31	154 18	42 16	+17 55	P.
	4877		562	0·335	11 45	154 32	42 30	+17 20	Q.
	4878		562	0·326	10 54	156 43	44 41	+16 51	p.
	4879		562	0·325	12 23	157 38	45 36	+15 25	q.
18.	4880	137·479	561	0·320	304 47	186 14	60 35	− 7 55	A.
	4881		561	0·322	305 36	184 1	58 22	− 7 22	B.
	4882		561	0·334	307 20	188 1	62 22	− 6 21	C.
	4883		561	0·333	308 15	187 5	61 26	− 8 39	D.
	4884		561	0·298	309 2	185 27	59 48	− 7 28	E.
	4885		561	0·297	310 11	180 43	55 4	− 9 18	a.
	4886		561	0·282	312 20	186 33	60 54	− 7 33	b.
	4887		561	0·291	314 23	180 32	54 53	− 6 25	c.
	4888		561	0·294	315 25	181 5	55 26	− 6 8	d.
	4889		561	0·305	317 51	182 25	56 46	− 6 33	e.
	4890		561	0·303	324 37	183 43	58 4	− 5 14	f.
	4891		561	0·277	326 56	185 5	59 26	− 7 28	F.
	4892		561	0·279	324 5	181 28	55 49	− 8 46	G.
	4893		561	0·265	325 28	185 59	60 20	− 7 25	H.
	4894		561	0·266	328 39	186 49	61 10	− 7 34	g.
	4895		562	0·324	229 33	173 35	47 56	+14 49	M.
	4896		562	0·328	225 12	171 41	46 2	+15 51	N.
	4897		562	0·327	223 10	170 47	45 8	+15 12	O.
	4898		562	0·349	222 49	171 8	45 29	+16 28	m.
	4899		562	0·356	221 11	167 40	42 1	+14 34	n.
	4900		562	0·374	220 51	168 53	43 14	+18 33	o.
	4901		562	0·381	221 43	169 51	44 12	+17 48	M^{0}.
	4902		562	0·394	219 37	171 58	46 19	+17 3	N^{0}.
	4903		562	0·364	218 19	173 22	47 43	+16 57	O^{0}.
	4904		562	0·402	218 29	169 17	43 38	+15 46	m^{1}.
	4905		562	0·398	217 39	167 48	42 9	+14 22	n^{1}.
	4906		562	0·417	217 16	168 15	42 36	+15 33	o^{1}.
	4907		562	0·424	216 48	168 30	42 51	+15 35	S.
	4908		562	0·433	215 19	170 0	44 21	+15 19	s.
	4909		562	0·431	215 30	172 48	47 9	+14 58	R.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 18.	4910	137·494	561	0·321	304° 10′	186° 34′	60° 41′	− 7° 59′	A.
	4911		561	0·324	305 53	184 14	58 21	− 7 16	B.
	4912		561	0·336	307 52	188 39	62 46	− 6 19	C.
	4913		561	0·334	308 15	187 49	61 56	− 8 30	D.
	4914		561	0·301	309 49	185 48	59 55	− 7 25	E.
	4915		561	0·300	310 24	181 15	55 22	− 9 32	a.
	4916		561	0·285	312 25	186 14	60 21	− 7 34	b.
	4917		561	0·294	314 50	180 32	54 39	− 6 28	c.
	4918		561	0·297	315 28	181 21	55 28	− 6 17	d.
	4919		561	0·307	318 11	182 11	56 18	− 6 32	e.
	4920		561	0·305	324 34	184 26	58 33	− 5 16	f.
	4921		561	0·280	326 3	185 7	59 14	− 7 24	F.
	4922		561	0·282	324 20	181 19	55 26	− 8 41	G.
	4923		561	0·266	325 37	186 20	60 27	− 7 32	H.
	4924		561	0·268	328 27	187 39	61 46	− 7 30	g.
	4925		562	0·326	229 15	173 49	47 56	+14 40	M.
	4926		562	0·330	225 44	172 27	46 34	+15 56	N.
	4927		562	0·331	223 36	171 23	45 30	+15 7	O.
	4928		562	0·353	223 25	170 53	45 0	+16 22	m.
	4929		562	0·360	221 12	168 31	42 38	+14 47	n.
	4930		562	0·375	220 54	169 15	43 22	+18 28	o.
	4931		562	0·382	221 14	170 0	44 7	+17 47	M^{0}.
	4932		562	0·397	219 15	172 21	46 28	+17 10	N^{0}.
	4933		562	0·368	218 27	173 18	47 25	+16 59	O^{0}.
	4934		562	0·403	218 57	169 25	43 32	+15 50	m^{1}.
	4935		562	0·405	217 48	168 49	42 56	+14 16	n^{1}.
	4936		562	0·419	217 18	168 29	42 36	+15 32	o^{1}.
	4937		562	0·427	216 51	167 57	42 4	+15 33	S.
	4938		562	0·435	215 53	169 53	44 0	+15 26	s.
	4939		562	0·436	215 13	172 54	47 1	+14 52	R.
19.	4940	138·494	561	0·472	276 16	200 58	60 55	− 7 54	S.
	4941		561	0·464	278 57	202 13	62 10	− 8 56	P.
	4942		561	0·456	277 3	199 59	59 56	− 5 32	Q.
	4943		561	0·450	291 19	195 7	55 4	− 5 8	R.
	4944		561	0·417	288 31	203 34	63 31	− 8 6	s.
	4945		561	0·423	285 24	194 40	54 37	− 7 53	p.
	4946		561	0·395	289 22	200 47	60 44	− 6 24	q.
	4947		561	0·385	294 12	201 52	61 49	− 6 7	r.
	4948		562	0·551	231 47	188 53	48 50	+15 27	A.
	4949		562	0·536	231 44	186 31	46 28	+17 56	B.
	4950		562	0·530	230 33	184 34	44 31	+14 16	C.
	4951		562	0·519	228 6	185 35	45 32	+18 46	D.
	4952		562	0·511	229 18	183 18	43 15	+16 11	a.
	4953		562	0·499	227 7	186 38	46 35	+14 20	b.
	4954		562	0·494	228 39	182 49	42 46	+13 32	c.
	4955		562	0·495	227 40	182 14	42 11	+14 1	d.
	4956		562	0·486	227 16	184 40	44 37	+15 21	E.
	4957		562	0·482	227 55	185 28	45 25	+13 26	e.
	4958		563	0·961	58 28	85 57	305 57	+ 6 33	X.
	4959		563	0·965	60 30	88 12	308 12	+ 7 52	Y.
	4960		563	0·973	62 20	90 6	310 6	+ 8 28	x.
	4961		563	0·980	64 4	94 20	314 20	+ 9 39	y.
	4962	138·509	561	0·475	276 17	200 31	60 15	− 7 53	S.
	4963		561	0·466	278 41	202 43	62 27	− 9 2	P.
	4964		561	0·460	277 21	199 26	59 10	− 5 40	Q
	4965		561	0·453	291 14	195 28	55 12	− 5 5	R.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 19.	4966	138·509	561	0·419	288° 59′	203° 22′	63° 6′	− 8° 7′	s.
	4967		561	0·427	285 30	194 45	54 29	− 7 33	p.
	4968		561	0·400	289 57	200 26	60 10	− 6 29	q.
	4969		561	0·387	294 33	201 38	61 22	− 6 16	r.
	4970		562	0·552	231 57	189 28	49 12	+15 22	A.
	4971		562	0·537	231 11	187 10	46 54	+18 2	B.
	4972		562	0·536	230 4	184 40	44 24	+14 20	C.
	4973		562	0·524	228 6	185 27	45 11	+18 41	D.
	4974		562	0·513	229 54	183 45	43 29	+16 16	a.
	4975		562	0·504	227 33	186 51	46 35	+14 16	b.
	4976		562	0·497	228 25	183 2	42 46	+13 35	c.
	4977		562	0·499	227 20	183 3	42 47	+13 56	d.
	4978		562	0·488	227 5	184 24	44 8	+15 20	E.
	4979		562	0·486	228 12	185 56	45 40	+13 18	e.
	4980		563	0·959	58 39	85 48	305 32	+ 6 32	X.
	4981		563	0·964	60 32	88 12	307 56	+ 8 7	Y.
	4982		563	0·972	62 50	90 1	309 45	+ 8 34	x.
	4983		563	0·980	64 25	94 44	314 28	+ 9 26	y.
20.	4984	139·480	561	0·712	237 37	213 54	59 52	− 7 3	A.
	4985		561	0·718	238 26	214 41	60 39	− 8 42	B.
	4986		561	0·754	238 29	213 1	58 59	− 5 23	C.
	4987		561	0·733	236 19	214 34	60 32	− 6 39	D.
	4988		561	0·788	236 18	213 16	59 14	− 8 18	a.
	4989		561	0·762	237 6	210 48	56 46	− 7 26	b.
	4990		561	0·769	235 45	208 13	54 11	− 6 11	c.
	4991		561	0·793	235 54	209 36	55 34	− 5 51	d.
	4992		562	0·564	274 19	200 5	46 3	+14 5	M.
	4993		562	0·570	277 49	203 56	49 54	+15 19	N.
	4994		562	0·624	280 33	202 20	48 18	+14 35	O.
	4995		562	0·655	282 23	199 54	45 52	+16 22	m.
	4996		562	0·682	282 29	202 59	48 57	+16 9	n.
	4997		563	0·830	58 45	104 5	310 3	+15 6	P.
	4998		563	0·862	59 40	102 36	308 34	+13 5	Q.
	4999		563	0·889	63 57	101 20	307 18	+14 50	R.
	5000		563	0·902	67 46	100 45	306 43	+13 49	p.
	5001		564	0·422	305 4	178 10	24 8	+18 25	S.
	5002		564	0·416	306 28	179 28	25 26	+19 14	s.
	5003		564	0·391	308 13	180 16	26 14	+18 36	s¹.
	5004	139·493	561	0·715	237 57	213 46	59 32	− 7 9	A.
	5005		561	0·721	238 40	214 37	60 23	− 8 40	B.
	5006		561	0·756	238 26	212 50	58 36	− 5 26	C.
	5007		561	0·736	236 43	214 34	60 20	− 6 40	D.
	5008		561	0·791	236 12	214 11	59 57	− 8 22	a.
	5009		561	0·765	237 27	210 30	56 16	− 7 25	b.
	5010		561	0·772	235 28	208 15	54 1	− 6 16	c.
	5011		561	0·795	235 29	209 33	55 19	− 5 54	d.
	5012		562	0·566	274 29	200 26	46 12	+14 2	M.
	5013		562	0·571	277 2	203 40	49 26	+15 22	N.
	5014		562	0·625	280 39	203 12	48 58	+14 26	O.
	5015		562	0·658	282 47	200 9	45 55	+16 24	m.
	5016		562	0·685	282 43	203 22	49 8	+16 10	n.
	5017		563	0·828	58 39	104 51	310 37	+15 5	P.
	5018		563	0·859	59 57	102 33	308 19	+13 5	Q.
	5019		563	0·887	63 51	101 20	307 6	+14 55	R.
	5020		563	0·900	67 57	101 2	306 48	+13 46	p.
	5021		564	0·425	305 18	178 47	24 33	+18 20	s.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 20.	5022	139·493	564	0·417	306° 11′	179° 34′	25° 20′	+19° 15′	s.
	5023		564	0·395	308 24	180 26	26 12	+18 46	s^1.
24.	5024	143·602	564	0·888	269 59	238 3	25 32	+19 26	m.
	5025		564	0·889	271 20	237 20	24 49	+17 32	n.
	5026		564	0·902	270 25	239 39	27 8	+18 50	o.
	5027	143·661	564	0·892	269 57	239 1	25 40	+19 33	m.
	5028		564	0·895	271 2	238 34	25 13	+17 42	n.
	5029		564	0·908	270 49	240 16	26 55	+18 54	o.
27.	5030	146·462	565	0·420	262 25	199 17	306 12	+ 7 9	A.
	5031		565	0·444	260 33	198 29	305 24	+ 6 26	B.
	5032		565	0·454	259 40	195 36	302 31	+ 8 40	C.
	5033		565	0·468	259 17	193 40	300 35	+ 8 12	a.
	5034		565	0·473	257 4	192 1	298 56	+ 9 31	b.
	5035		566	0·964	47 19	91 19	198 14	+21 25	S.
	5036		566	0·966	48 31	90 7	197 2	+22 38	s.
	5037	146·509	565	0·424	263 3	199 43	305 58	+ 7 12	A.
	5038		565	0·446	260 17	199 24	305 39	+ 6 31	B.
	5039		565	0·457	259 58	196 15	302 30	+ 8 34	C.
	5040		565	0·470	258 32	194 34	300 49	+ 8 11	a.
	5041		565	0·475	257 42	192 59	299 14	+ 9 42	b.
	5042		566	0·962	47 26	91 51	198 16	+21 20	S.
	5043		566	0·965	48 21	90 56	197 11	+22 27	s.
28.	5044	147·455	565	0·622	268 46	211 10	304 0	+ 8 36	P.
	5045		565	0·582	270 42	212 44	305 34	+ 7 42	Q.
	5046		565	0·571	266 52	210 32	303 22	+ 8 11	R.
	5047		566	0·872	53 45	106 19	199 9	+21 19	S.
	5048		566	0·884	54 34	105 57	198 47	+20 39	s.
	5049		567	0·512	281 52	203 48	296 38	+18 26	M.
	5050		567	0·521	283 6	206 14	299 4	+17 13	N.
	5051		567	0·565	283 32	208 2	300 52	+16 57	O.
	5052	147·496	565	0·625	267 13	211 57	304 12	+ 8 30	P.
	5053		565	0·584	270 9	213 13	305 28	+ 7 45	Q.
	5054		565	0·574	266 34	211 7	303 22	+ 8 12	R.
	5055		566	0·870	53 9	107 3	199 18	+21 20	S.
	5056		566	0·882	54 40	106 30	198 45	+20 44	s.
	5057		567	0·515	282 6	204 25	296 40	+18 37	M.
	5058		567	0·525	283 57	207 0	299 15	+17 16	N.
	5059		567	0·568	282 10	208 42	300 57	+17 4	O.
30.	5060	149·664	567	0·877	267 40	237 57	299 27	+18 22	B.
	5061		567	0·894	269 10	239 5	300 35	+17 16	b.
	5062		566	0·609	33 26	135 32	197 2	+21 54	D.
	5063		566	0·611	34 1	137 34	199 4	+20 39	d.
	5064		568	0·972	58 11	95 43	157 13	+ 8 53	A.
	5065		568	0·983	61 36	94 11	155 41	+ 7 39	a.
	5066		568	0·986	64 58	96 21	157 51	+ 5 27	a^1.
	5067		569	0·954	87 43	102 51	164 21	−18 20	C.
	5068		569	0·959	88 19	100 12	161 42	−19 4	c.
	5069		570	0·992	86 12	89 19	150 49	−16 46	E.
	5070		570	0·994	85 13	90 38	152 8	−17 31	e.
	5071	149·672	567	0·880	268 10	238 30	299 53	+18 25	B.
	5072		567	0·897	269 21	239 21	300 44	+17 22	b.
	5073		566	0 608	32 28	135 50	197 13	+22 6	D.
	5074		566	0·608	34 12	137 55	199 18	+20 44	d.
	5075		568	0·970	58 41	95 46	157 9	+ 8 46	A.
	5076		568	0·981	60 59	94 30	155 53	+ 7 35	a.
	5077		568	0·984	65 10	96 29	157 52	+ 5 22	a^1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. May 30.	5078	149·672	569	0·952	87° 33′	103° 11′	164° 34′	−18° 15′	C.
	5079		569	0·955	88 15	100 40	162 3	−18 58	c.
	5080		570	0·990	86 10	89 31	150 54	−16 40	E.
	5081		570	0·989	85 9	91 2	152 25	−17 31	e.
June 4.	5082	154·464	566	0·641	302 55	213 50	207 15	+22 6	A.
	5083		566	0·645	302 31	214 56	208 21	+20 14	B.
	5084		568	0·287	32 26	170 50	164 15	+10 25	M.
	5085		568	0·291	32 16	169 19	162 44	+ 8 39	N.
	5086		568	0·314	35 36	170 20	163 45	+ 8 18	O.
	5087		568	0·308	34 20	155 43	149 8	+ 7 19	P.
	5088		568	0·312	45 29	156 44	150 9	+ 7 27	m.
	5089		568	0·315	45 47	158 47	152 12	+ 9 8	n.
	5090		568	0·324	35 54	164 28	157 53	+10 13	o.
	5091		568	0·338	53 33	166 48	160 13	+13 18	p.
	5092		568	0·310	50 33	163 8	156 33	+11 26	a.
	5093		568	0·398	49 1	154 32	147 57	+10 5	b.
	5094		568	0·384	56 58	158 17	151 42	+10 42	Q.
	5095		568	0·365	57 41	167 43	161 8	+11 6	Q¹.
	5096		568	0·366	36 31	165 47	159 12	+12 45	R.
	5097		568	0·402	36 17	159 22	152 47	+10 2	S.
	5098		568	0·385	59 56	159 42	153 7	+12 12	T.
	5099		568	0·424	58 27	154 26	147 51	+11 35	r.
	5100		568	0·436	54 22	153 58	147 23	+ 9 47	s.
	5101		568	0·421	52 56	153 34	146 59	+ 8 19	t.
	5102		568	0·419	57 9	154 18	147 43	+12 25	u.
	5103		568	0·433	58 15	155 19	148 44	+12 53	v.
	5104		569	0·224	168 48	178 20	171 45	−11 5	C.
	5105		569	0·236	164 46	176 29	169 54	−11 50	D.
	5106		569	0·237	157 58	169 22	162 47	−12 37	E.
	5107		569	0·254	159 3	172 4	165 29	−11 27	F.
	5108		569	0·258	155 18	174 43	168 8	−13 35	G.
	5109		569	0·263	153 42	168 21	161 46	−13 53	c.
	5110		570	0·333	128 12	165 30	158 55	−12 55	d.
	5111		570	0·336	126 27	164 31	157 56	−15 55	e.
	5112		570	0·371	119 21	158 19	151 44	−16 28	f.
	5113		570	0·385	119 16	157 28	150 53	−14 43	g.
	5114		570	0·360	122 54	155 13	148 38	−13 34	H.
	5115		570	0·370	125 43	156 54	150 19	−14 29	h.
	5116		570	0·394	116 45	157 39	151 4	−15 42	h¹.
	5117		570	0·398	115 53	158 37	152 2	−12 40	K.
	5118		570	0·408	116 11	156 21	149 46	−10 13	k¹.
	5119		570	0·410	117 15	155 36	149 1	−10 35	l.
	5120		571	0·594	51 2	142 51	136 16	+16 12	S.
	5121		571	0·596	52 8	137 27	130 52	+15 56	s
	5122		571	0·635	52 28	140 28	133 53	+17 26	X.
	5123		571	0·642	54 9	142 5	135 30	+17 45	x.
	5124	154·520	566	0·644	303 31	214 17	206 54	+22 13	A.
	5125		566	0·649	302 44	215 28	208 5	+20 13	B.
	5126		568	0·285	32 10	171 23	164 0	+10 31	M.
	5127		568	0·288	31 54	169 54	162 31	+ 8 34	N.
	5128		568	0·312	35 55	170 41	163 18	+ 8 16	O.
	5129		568	0·305	34 8	156 24	149 1	+ 7 14	P.
	5130		568	0·308	45 50	157 29	150 6	+ 7 21	m.
	5131		568	0·310	45 41	159 33	152 10	+ 9 7	n.
	5132		568	0·321	36 3	164 37	157 14	+10 12	o.
	5133		568	0·335	53 25	166 59	149 36	+13 13	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 4.	5134	154·520	568	0·307	50° 17′	163° 29′	156° 6′	+11° 34′	a.
	5135		568	0·388	49 15	155 10	147 47	+10 1	b.
	5136		568	0·380	57 13	158 28	151 5	+10 50	Q.
	5137		568	0·362	57 35	168 21	160 58	+11 11	Q^1.
	5138		568	0·362	36 44	165 59	158 36	+12 44	R.
	5139		568	0·401	35 58	159 50	152 27	+10 6	S.
	5140		568	0·380	60 5	159 57	152 34	+12 6	T.
	5141		568	0·420	58 16	154 37	147 14	+11 40	r.
	5142		568	0·430	54 43	154 18	146 55	+ 9 51	s.
	5143		568	0·417	53 12	153 46	146 23	+ 8 28	t.
	5144		568	0·415	56 50	154 56	147 33	+12 34	u.
	5145		568	0·430	58 17	155 27	148 4	+13 1	v.
	5146		569	0·221	169 7	178 39	171 16	−11 16	C.
	5147		569	0·235	164 50	176 45	169 22	−11 54	D.
	5148		569	0·232	158 21	169 40	162 17	−12 28	E.
	5149		569	0·250	158 47	172 18	164 55	−11 22	F.
	5150		569	0·255	155 19	175 12	167 49	−13 31	G.
	5151		570	0·331	127 56	165 52	168 29	−13 3	d.
	5152		570	0·330	126 51	164 39	157 16	−16 7	e.
	5153		570	0·367	119 30	158 29	151 6	−16 24	f.
	5154		570	0·379	119 14	157 59	150 36	−14 41	g.
	5155		570	0·354	123 6	155 41	148 18	−13 36	H.
	5156		570	0·365	125 50	157 28	150 5	−14 34	h.
	5157		570	0·388	116 52	157 57	150 34	−15 40	h^1.
	5158		570	0·396	116 9	158 46	151 23	−12 43	K.
	5159		570	0·399	116 18	156 40	149 17	−10 19	k^1.
	5160		570	0·406	117 24	156 43	149 20	−10 41	l.
	5161		571	0·591	51 8	142 53	135 30	+16 18	S.
	5162		571	0·592	52 15	137 48	130 25	+16 4	s.
	5163		571	0·630	52 34	140 46	133 23	+17 25	X.
	5164		571	0·640	54 31	142 27	135 4	+17 41	x.
6.	5165	156·545	566	0·880	281 47	243 25	207 19	+21 47	M.
	5166		566	0·884	282 2	245 0	208 54	+20 31	N.
	5167		568	0·374	285 19	194 39	158 33	+ 9 4	A.
	5168		568	0·365	286 26	198 29	162 23	+10 46	B.
	5169		568	0·357	286 37	191 35	155 29	+11 13	C.
	5170		568	0·328	287 18	186 51	150 45	+11 43	D.
	5171		568	0·310	290 6	192 46	156 40	+10 26	a.
	5172		568	0·348	289 45	195 3	158 57	+ 9 28	b.
	5173		568	0·302	291 49	185 34	149 28	+12 27	c.
	5174		568	0·301	292 43	188 33	152 27	+12 4	d.
	5175		568	0·297	292 47	188 24	152 18	+11 46	E.
	5176		568	0·282	298 39	189 6	153 0	+11 10	F.
	5177		568	0·275	295 39	185 31	149 25	+ 9 10	G.
	5178		568	0·251	299 57	191 59	155 53	+11 33	H.
	5179		568	0·248	300 51	194 38	158 32	+10 2	e.
	5180		568	0·265	300 19	194 8	158 2	+11 54	f.
	5181		568	0·262	301 52	189 49	153 43	+11 52	g.
	5182		568	0·245	302 26	188 57	152 51	+10 18	h.
	5183		569	0·471	226 10	205 43	169 37	−13 57	X.
	5184		569	0·454	224 30	204 27	168 21	−12 3	x.
	5185		569	0·462	223 2	203 59	167 53	−11 18	Y.
	5186		569	0·450	221 43	205 53	169 47	−13 33	y.
	5187		570	0·310	200 39	192 6	156 0	−15 43	W.
	5188		570	0·272	194 38	190 9	154 3	−16 51	w.
	5189		570	0·220	189 26	184 48	148 42	−13 14	v.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 6.	5190	156·545	570	0·231	185° 53′	186° 29′	150′ 23°	−14° 35′	v.
	5191		571	0·320	6 41	172 45	136 39	+17 20	P.
	5192		571	0·325	11 43	169 21	133 15	+17 2	Q.
	5193		541	0·331	7 4	170 48	134 42	+16 21	R.
	5194		571	0·351	8 58	166 32	130 26	+15 11	S.
	5195		571	0·361	8 19	165 34	129 28	+16 19	p.
	5196		571	0·345	17 39	164 11	128 5	+17 31	q.
	5197		571	0·347	19 55	165 29	129 23	+15 29	r.
	5198		571	0·363	22 58	164 12	128 6	+18 32	s.
	5199		572	0·750	94 31	137 11	101 5	−17 24	T.
	5200		572	0·746	95 6	138 54	102 48	−17 9	t^1.
	5201		572	0·822	93 21	132 19	96 13	−18 20	t''.
	5202		572	0·829	92 13	131 52	95 46	−18 29	u.
7.	5203	157·521	566	0·963	281 49	258 47	208 50	+20 27	A.
	5204		566	0·965	280 14	258 46	208 49	+22 8	a.
	5205		568	0·522	275 0	207 11	157 14	+ 9 50	B.
	5206		568	0·517	277 17	196 17	146 20	+10 15	C.
	5207		568	0·519	276 48	194 4	144 7	+12 25	D.
	5208		568	0·505	281 54	199 31	149 34	+11 55	b.
	5209		568	0·487	279 11	198 45	148 48	+12 51	c.
	5210		568	0·491	282 13	206 1	156 4	+ 9 27	d.
	5211		568	0·413	278 28	195 53	145 56	+ 9 18	E.
	5212		568	0·426	279 12	205 6	155 9	+10 16	E^1.
	5213		568	0·438	283 43	208 48	158 51	+11 44	F
	5214		568	0·457	281 37	207 20	157 23	+11 34	F^1.
	5215		568	0·501	277 48	202 50	152 53	+11 42	e.
	5216		568	0·412	283 30	210 25	160 28	+11 44	f.
	5217		569	0·644	236 57	220 9	170 12	−13 51	G.
	5218		569	0·642	232 18	216 14	166 17	−10 20	G^1.
	5219		569	0·636	234 11	215 48	165 51	−12 18	g.
	5220		569	0·631	231 27	216 36	166 39	−12 4	g^1.
	5221		570	0·384	222 14	206 56	156 59	−16 3	H.
	5222		570	0·371	223 50	201 29	151 32	−15 21	h.
	5223		570	0·361	224 5	200 10	150 13	−15 39	h^1.
	5224		571	0·274	328 9	179 33	129 36	+15 36	S.
	5225		571	0·281	331 46	182 4	132 7	+17 38	s.
	5226		571	0·282	336 3	184 56	134 59	+17 59	s^1.
	5227		571	0·295	332 31	186 35	136 38	+16 35	T.
	5228		571	0·301	334 35	180 0	130 3	+15 7	t^1.
	5229		571	0·305	345 45	185 22	135 25	+17 3	t^1.
	5230		571	0·317	346 58	185 27	135 30	+16 45	t''.
	5231		541	0·328	349 34	184 41	134 44	+16 58	u.
	5232		572	0·584	104 30	150 42	100 55	−17 25	W.
	5233		572	0·579	103 54	152 24	102 27	−17 30	w.
	5234		572	0·632	101 58	147 15	97 18	−18 34	V.
	5235		572	0·628	99 58	145 17	95 20	−19 44	v.
	5236	157·540	566	0·966	282 56	258 21	208 8	+20 25	A.
	5237		566	0·968	280 26	259 5	208 52	+22 15	a.
	5238		568	0·525	274 0	207 48	157 35	+ 9 48	B.
	5239		568	0·520	277 38	196 46	146 33	+10 17	C.
	5240		568	0·524	276 9	195 8	144 55	+12 16	D.
	5241		568	0·508	282 48	199 52	149 39	+12 4	b.
	5242		568	0·489	278 44	199 56	148 43	+13 4	c.
	5243		568	0·493	282 39	206 14	156 1	+ 9 30	d.
	5244		568	0·416	279 49	195 27	145 14	+ 9 21	E.
	5245		568	0·429	278 39	205 27	155 14	+10 15	E^1.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 7.	5246	157·540	568	0·441	284° 44′	208° 57′	158° 44′	+11° 50′	F.
	5247		568	0·460	281 40	207 39	157 26	+11 42	F^1.
	5248		568	0·505	277 36	203 56	153 43	+11 44	e.
	5249		568	0·417	283 49	210 42	160 29	+11 45	f.
	5250		569	0·650	237 58	220 48	170 35	−13 56	G.
	5251		569	0·645	232 33	216 36	166 23	−10 14	G^1.
	5252		569	0·640	234 55	216 52	166 39	−12 23	g.
	5253		569	0·633	231 9	217 1	166 48	−11 54	g^1.
	5254		570	0·385	222 33	206 22	156 9	−16 8	H.
	5255		570	0·373	224 41	201 54	151 41	−15 17	h.
	5256		570	0·362	223 6	200 48	150 35	−15 44	h^1.
	5257		571	0·275	328 28	180 9	129 56	+15 41	S.
	5258		571	0·282	331 25	182 40	132 27	+17 33	s.
	5259		571	0·281	337 56	184 20	134 7	+18 10	s^1.
	5260		571	0·294	331 48	186 26	136 13	+16 36	T.
	5261		571	0·299	333 13	180 55	130 42	+14 59	t.
	5262		571	0·306	344 13	185 35	135 18	+17 5	t^1.
	5263		571	0·318	345 15	186 8	135 55	+16 49	t''.
	5264		571	0·330	349 18	185 4	134 51	+17 2	u.
	5265		572	0·580	105 43	150 19	100 6	−17 33	W.
	5266		572	0·575	103 17	152 46	102 33	−17 32	w.
	5267		572	0·627	102 47	147 24	97 11	−18 39	V.
	5268		572	0·622	99 7	146 1	95 48	−19 41	v.
8.	5269	158·548	568	0·691	268 4	221 18	156 47	+10 44	A.
	5270		568	0·683	269 38	212 48	148 17	+ 9 18	B.
	5271		568	0·675	272 49	214 15	149 44	+10 18	C.
	5272		568	0·688	271 43	214 6	149 35	+10 46	D.
	5273		568	0·674	273 9	218 48	154 17	+11 47	E.
	5274		568	0·688	274 48	211 39	147 8	+11 4	F.
	5275		568	0·672	275 26	218 40	154 9	+12 32	a.
	5276		568	0·610	275 54	223 59	159 28	+10 27	b.
	5277		568	0·625	277 35	224 47	160 16	+12 57	c.
	5278		568	0·636	277 35	220 2	155 31	+12 34	d.
	5279		568	0·651	277 57	212 59	148 28	+11 34	e.
	5280		568	0·605	278 11	213 35	149 4	+10 50	f.
	5281		569	0·812	237 21	232 26	167 55	−13 25	G.
	5282		569	0·810	237 34	233 9	168 38	−11 45	G^1.
	5283		569	0·807	236 9	234 47	170 16	−12 17	g.
	5284		569	0·804	235 57	233 50	169 19	−12 59	g^1.
	5285		570	0·550	232 10	217 10	152 39	−16 17	H.
	5286		570	0·544	231 36	220 16	155 45	−15 49	h.
	5287		571	0·415	297 4	198 30	133 59	+17 55	M.
	5288		571	0·406	298 50	194 27	129 56	+16 36	N.
	5289		571	0·392	300 44	198 50	134 19	+18 29	O.
	5290		571	0·378	302 6	199 44	135 13	+14 39	m.
	5291		571	0·388	304 28	200 43	136 12	+15 42	n.
	5292		571	0·361	307 42	199 37	135 6	+16 21	o.
	5293		571	0·355	306 51	199 47	135 16	+17 51	P.
	5294		571	0·341	308 6	195 9	130 38	+17 45	p.
	5295		572	0·415	116 35	165 43	101 12	−17 35	Q.
	5296		572	0·417	117 53	166 45	102 14	−17 18	Q^1.
	5297		572	0·488	109 47	160 49	96 18	−19 54	q.
	5298		572	0·483	109 17	162 21	97 50	−18 42	q^1.
	5299	158·610	568	0·688	268 5	221 39	156 16	+10 40	A.
	5300		568	0·679	269 32	213 48	148 25	+ 9 23	B.
	5301		568	0·670	272 24	215 11	149 48	+10 15	C.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 8.	5302	158·610	568	0·685	272° 44′	225° 17′	149° 54′	+10° 50′	D.
	5303		568	0·672	273 12	219 35	154 12	+11 49	E.
	5304		568	0·684	274 40	212 49	147 26	+11 8	F.
	5305		568	0·668	275 44	220 22	154 59	+12 35	a.
	5306		568	0·605	276 32	224 23	159 0	+10 29	b.
	5307		568	0·622	276 47	226 20	160 57	+12 52	c.
	5308		568	0·631	277 52	220 40	155 17	+12 31	d.
	5309		568	0·645	277 12	213 34	148 11	+11 37	e.
	5310		568	0·601	278 24	214 53	149 30	+10 54	f.
	5311		569	0·807	237 0	233 9	167 46	−13 26	G.
	5312		569	0·805	237 37	233 34	168 11	−11 40	G^1.
	5313		569	0·806	236 56	236 18	170 55	−12 17	g.
	5314		569	0·800	235 39	234 32	169 9	−13 4	g^1.
	5315		570	0·545	232 48	217 56	152 33	−16 16	H.
	5316		570	0·541	231 9	221 4	155 41	−15 41	h.
	5317		571	0·411	298 41	198 29	133 6	+17 50	M.
	5318		571	0·402	297 35	194 51	129 28	+16 31	N.
	5319		571	0·387	300 56	199 48	134 25	+18 22	O.
	5320		571	0·375	301 25	201 19	135 56	+14 33	m.
	5321		571	0·383	304 7	202 11	136 48	+15 44	n.
	5322		571	0·356	307 13	200 36	135 13	+16 20	o.
	5323		571	0·350	306 42	200 56	135 33	+17 46	P.
	5324		571	0·346	308 22	195 45	130 22	+17 46	p.
	5325		572	0·411	116 49	166 50	101 27	−17 42	Q.
	5326		572	0·415	118 22	167 49	102 26	−17 13	Q^1.
	5327		572	0·483	109 58	161 33	96 10	−18 0	q.
	5328		572	0·479	108 52	163 26	98 3	−18 37	q^1.
10.	5329	160·492	568	0·942	266 18	251 46	159 41	+11 33	M.
	5330		568	0·930	267 14	246 26	154 21	+12 57	N.
	5331		568	0·927	267 51	252 42	160 37	+12 40	O.
	5332		568	0·925	269 43	250 14	158 9	+10 28	P.
	5333		568	0·906	268 0	250 45	158 40	+11 40	m.
	5334		568	0·901	268 25	248 9	156 4	+12 53	n.
	5335		568	0·909	269 16	252 39	160 34	+12 23	o.
	5336		568	0·898	270 6	242 28	150 23	+11 5	p.
	5337		569	0·971	243 42	261 4	168 59	−13 4	X.
	5338		571	0·680	278 6	224 34	132 29	+16 42	G.
	5339		571	0·665	282 2	221 23	129 18	+15 5	G^1.
	5340		571	0·614	286 12	225 59	133 54	+14 47	g.
	5341		571	0·592	287 39	226 36	134 31	+18 17	g^1.
	5342		572	0·294	184 32	194 30	102 25	−17 11	Y.
	5343		572	0·299	185 44	193 37	101 32	−18 6	Y''.
	5344		572	0·308	186 50	189 32	97 27	−18 54	y^1
	5345		572	0·299	170 38	190 33	98 28	−19 12	y''.
	5346		572	0·307	172 53	188 55	96 50	−18 52	z.
	5347		573	0·946	58 54	115 33	23 28	+16 43	W.
	5348	160·503	568	0·945	266 51	252 6	159 51	+11 36	M.
	5349		568	0·933	267 42	246 26	154 11	+12 50	N.
	5350		568	0·929	267 17	253 4	160 49	+12 46	O.
	5351		568	0·926	269 41	251 9	158 54	+10 27	P.
	5352		568	0·908	268 48	250 48	158 33	+11 42	m.
	5353		568	0·905	268 17	248 55	156 40	+12 50	n.
	5354		568	0·911	269 4	252 34	160 19	+12 18	o.
	5355		568	0·902	270 14	242 46	150 31	+11 10	p.
	5356		569	0·975	243 16	260 20	168 5	−13 11	X.
	5357		571	0·683	278 51	224·55	132 40	+16 40	G.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 10.	5358	160·503	571	0·670	282° 1′	221° 29′	129° 14′	+15° 7′	G^1.
	5359		571	0·617	286 26	226 9	133 54	+14 48	g.
	5360		571	0·595	287 4	227 5	134 50	+18 12	g^1.
	5361		572	0·297	184 10	195 14	102 59	−17 21	Y.
	5362		572	0·303	185 40	193 44	101 29	−18 9	Y^0.
	5363		572	0·311	186 3	189 21	97 6	−18 55	y^1.
	5364		572	0·302	170 11	191 4	98 49	−19 15	y''.
	5365		572	0·308	172 16	188 38	96 23	−18 50	z.
	5366		573	0·944	58 40	115 27	23 12	+16 48	W.
11.	5367	161·468	568	0·984	269 9	265 36	159 40	+11 56	A.
	5368		568	0·992	270 1	266 43	160 47	+12 19	B.
	5369		568	0·990	270 50	262 37	156 41	+10 7	a.
	5370		568	0·962	269 59	258 22	152 26	+11 21	b.
	5371		572	0·402	219 47	207 33	101 37	−17 8	C.
	5372		572	0·408	220 10	204 43	98 47	−17 33	c.
	5373		573	0·884	56 44	131 44	25 48	+16 16	D.
	5374		573	0·880	57 37	129 22	23 26	+15 20	E.
	5375		573	0·875	57 47	126 2	20 6	+16 52	d.
	5376		573	0·862	58 24	127 45	21 49	+14 40	e.
	5377	161·491	568	0·988	269 1	265 44	159 29	+12 5	A.
	5378		568	0·993	270 32	265 53	159 38	+12 17	B.
	5379		568	0·991	271 6	262 40	156 25	+10 9	a.
	5380		572	0·406	219 20	208 13	101 58	−17 25	C.
	5381		572	0·413	220 30	205 19	99 4	−17 40	c.
	5382		573	0·880	56 29	131 23	25 8	+16 12	D.
	5383		573	0·876	57 52	130 4	23 49	+15 31	E.
	5384		573	0·871	56 17	126 21	20 6	+17 0	d.
	5385		573	0·857	58 16	127 52	21 37	+14 46	e.
13.	5386	163·644	572	0·781	237 25	235 36	98 49	−18 28	A.
	5387		572	0·786	238 18	236 28	99 41	−17 34	a.
	5388		573	0·472	45 38	163 31	26 44	+14 9	B.
	5389		573	0·486	46 23	160 14	23 27	+16 29	C.
	5390		573	0·493	46 28	162 59	26 12	+17 19	D.
	5391		573	0·502	49 24	158 4	21 17	+17 50	b.
	5392		573	0·517	50 25	157 37	20 50	+14 37	c.
	5393		573	0·510	48 29	159 41	22 54	+15 16	d.
	5394		573	0·534	51 15	156 26	19 39	+16 11	E.
	5395		573	0·541	52 38	156 13	19 26	+18 40	e.
	5396		574	0·222	331 37	193 18	56 31	+12 14	E.
	5397		574	0·226	337 17	192 9	55 22	+12 21	G.
	5398		574	0·231	345 33	190 20	53 33	+12 40	H.
	5399		574	0·237	346 47	191 54	55 7	+12 46	f.
	5400		574	0·242	348 40	188 51	52 4	+13 19	g.
	5401		574	0·225	355 12	189 40	52 53	+14 11	h.
	5402		574	0·240	1 21	187 28	50 41	+13 14	K.
	5403		574	0·245	359 14	186 48	50 1	+14 25	k.
14.	5404	164·589	572	0·892	241 18	249 32	99 20	−17 19	M.
	5405		572	0·895	242 10	249 23	99 11	−18 46	m.
	5406		573	0·308	25 17	175 39	25 27	+14 18	A.
	5407		573	0·353	27 29	170 29	20 17	+17 31	B.
	5408		573	0·377	33 3	174 27	24 15	+16 57	C.
	5409		573	0·392	36 56	174 11	23 59	+15 19	a.
	5410		573	0·402	38 13	171 50	21 38	+15 59	b.
	5411		574	0·341	295 19	205 45	55 33	+12 7	D.
	5412		574	0·333	296 25	202 32	52 20	+14 28	d.
	5413		574	0·328	299 18	204 32	54 20	+13 32	E.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 14.	5414	164·589	574	0·299	301° 37′	204° 53′	54° 41′	+14° 19′	e.
15.	5415	165·711	572	0·965	244 17	265 36	99 29	−17 14	X.
	5416		572	0·970	243 25	264 24	98 17	−18 55	x.
	5417		573	0·296	331 49	190 7	24 0	+17 35	A.
	5418		573	0·298	337 50	192 5	25 58	+16 41	B.
	5419		573	0·283	340 35	188 12	22 5	+16 19	C.
	5420		574	0·523	279 12	220 44	54 37	+12 31	D.
	5421		574	0·517	280 2	222 26	56 19	+14 5	d.
	5422		574	0·504	282 15	217 28	51 21	+13 28	E.
	5423		574	0·502	283 34	218 52	52 45	+12 34	e.
	5424		574	0·465	286 47	218 55	52 48	+14 17	F.
	5425		574	0·468	285 57	219 16	52 9	+14 0	f.
	5426		575	0·965	65 0	114 44	308 37	+14 29	M.
	5427		575	0·966	66 53	113 17	307 11	+12 10	m.
	5428		575	0·988	64 49	108 14	302 7	+12 0	m¹.
16.	5429	166·501	573	0·366	308 12	203 9	25 50	+16 43	P.
	5430		573	0·371	309 33	200 3	22 44	+15 15	p.
	5431		573	0·382	311 58	201 0	23 41	+17 33	p¹.
	5432		574	0·670	277 58	232 51	55 32	+13 14	A.
	5433		574	0·662	278 20	228 50	51 31	+14 5	B.
	5434		574	0·624	278 21	232 17	54 58	+14 17	C.
	5435		574	0·638	279 21	233 22	56 3	+13 57	a.
	5436		574	0·611	282 19	233 12	55 53	+13 20	b.
	5437		574	0·583	283 9	229 34	52 15	+12 41	c.
	5438		575	0·912	67 24	125 4	307 45	+14 58	M.
	5439		575	0·917	66 45	123 29	306 10	+13 40	M⁰.
	5440		575	0·918	66 0	126 12	308 53	+12 3	m.
	5441		575	0·954	65 54	118 35	301 16	+12 24	n.
	5442		575	0·959	65 6	117 58	300 39	+12 19	o.
	5443	166·534	573	0·370	308 57	203 46	25 58	+16 44	P.
	5444		573	0·374	309 27	199 55	22 7	+15 19	p.
	5445		573	0·385	310 44	200 58	23 10	+17 37	p¹.
	5446		574	0·672	277 19	232 57	55 9	+13 11	A.
	5447		574	0·666	279 32	228 49	51 1	+14 9	B.
	5448		574	0·630	278 25	232 31	54 43	+14 29	C.
	5449		574	0·641	279 42	234 9	56 21	+14 2	a.
	5450		574	0·615	282 39	233 20	55 32	+13 23	b.
	5451		574	0·588	282 50	230 10	52 22	+12 45	c.
	5452		575	0·910	67 44	125 3	307 15	+14 50	M.
	5453		575	0·913	67 12	123 53	306 5	+13 41	M⁰.
	5454		575	0·912	66 16	126 3	308 15	+12 14	m.
	5455		575	0·950	65 47	119 46	301 58	+12 26	n.
	5456		575	0·955	64 30	118 3	300 15	+12 21	o.
17.	5457	167·580	573	0·588	281 49	217 8	24 31	+17 37	A.
	5458		573	0·602	281 48	214 59	22 22	+15 38	B.
	5459		573	0·624	283 15	218 15	25 38	+17 34	C.
	5460		573	0·520	293 20	213 51	21 14	+15 36	a.
	5461		573	0·509	294 10	213 14	20 37	+16 44	b.
	5462		573	0·497	295 10	212 53	20 16	+16 49	c.
	5463		574	0·855	274 33	248 57	56 20	+13 45	D.
	5464		574	0·843	272 15	243 12	50 35	+14 18	E.
	5465		574	0·849	274 28	244 18	51 37	+13 29	F.
	5466		574	0·812	273 51	248 36	55 59	+14 41	d.
	5467		574	0·800	276 14	247 34	54 57	+14 19	e.
	5468		574	0·792	278 13	246 8	53 31	+14 23	f.
	5469		575	0·790	63 35	137 53	305 16	+13 35	G.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 17.	5470	167·580	575	0·795	64° 46′	139° 42′	307° 5′	+14° 56′	G^0.
	5471		575	0·814	63 59	140 55	308 18	+12 51	g^1.
	5472		575	0·845	62 37	134 43	302 6	+13 59	g^2.
	5473		575	0·862	62 15	133 45	301 8	+12 21	g^3.
18.	5474	168·477	573	0·746	278 0	230 38	25 17	+17 33	A.
	5475		573	0·742	278 28	228 10	22 49	+15 20	a.
	5476		574	0·934	273 37	260 5	54 44	+13 1	M.
	5477		574	0·930	274 50	256 2	50 41	+13 24	M^1.
	5478		574	0·886	275 31	256 55	51 34	+13 51	m.
	5479		574	0·890	275 44	260 37	55 16	+14 29	n.
	5480		574	0·892	276 48	258 11	52 50	+13 45	o.
	5481		575	0·633	62 39	148 50	303 29	+12 36	P.
	5482		575	0·648	62 17	147 58	302 37	+14 38	P^0.
	5483		575	0·650	61 33	155 0	309 39	+14 3	p.
	5484		575	0·710	60 12	152 49	307 28	+12 12	p^1.
	5485		575	0·666	60 28	146 44	301 23	+12 39	Q.
	5486		575	0·689	61 12	146 30	300 59	+13 20	q.
	5487		575	0·724	63 30	146 32	301 11	+14 34	q^1.*
20.	5488	170·499	575	0·310	38 35	183 53	309 51	+11 40	M.
	5489		575	0·322	39 20	182 2	308 0	+12 12	N.
	5490		575	0·324	39 17	180 11	306 9	+12 52	O.
	5491		575	0·348	40 20	179 8	305 6	+12 27	m.
	5492		575	0·371	40 44	179 11	305 9	+13 14	n.
	5493		575	0·390	42 33	178 40	304 38	+14 5	o.
	5494		575	0·384	41 26	177 11	303 9	+13 6	P.
	5495		575	0·394	43 42	176 4	302 2	+14 51	p.
21.	5496	171·467	575	0·210	355 38	188 13	300 28	+12 43	M.
	5497		575	0·247	356 51	195 0	307 15	+12 38	M^0.
	5498		575	0·252	1 6	194 22	306 37	+13 21	N.
	5499		575	0·281	3 27	196 1	308 16	+13 4	a.
	5500		575	0·279	6 9	190 40	302 55	+14 27	b.
	5501	171·511	575	0·208	355 26	188 54	300 31	+12 38	M.
	5502		575	0·244	356 9	195 17	306 54	+12 44	M^0.
	5503		575	0·248	0 52	194 53	306 30	+13 15	N.
	5504		575	0·276	3 18	197 4	308 41	+13 10	a.
	5505		575	0·271	5 37	191 32	303 9	+14 34	b.
23.	5506	173·607	575	0·354	276 38	216 4	297 57	+ 9 58	A.
	5507		575	0·362	277 54	218 0	299 53	+ 8 8	B.
	5508		575	0·384	281 28	217 22	299 15	+ 7 37	C.
	5509		575	0·385	280 20	224 23	306 16	+ 6 41	a.
	5510		575	0·402	285 35	225 29	307 22	+10 29	b.
	5511		575	0·474	286 17	227 41	309 34	+11 8	C.
	5512		575	0·490	288 48	226 36	308 29	+12 50	D.
	5513		575	0·455	284 16	228 43	310 36	+12 24	d.
	5514		575	0·493	289 36	227 24	309 17	+12 17	d^1.
24.	5515	174·539	575	0·515	278 12	233 1	301 41	+10 59	M.
	5516		575	0·537	289 3	230 24	299 4	+ 6 26	N.
	5517		575	0·526	286 39	237 39	306 19	+ 6 40	O.
	5518		575	0·588	282 15	239 43	308 23	+ 8 45	m.
	5519		575	0·602	279 38	242 9	310 49	+11 30	n.
	5520		575	0·594	283 28	238 45	307 25	+12 25	o.
	5521		575	0·641	280 44	234 33	303 13	+ 9 44	m^0.
	5522		575	0·636	284 9	238 58	307 38	+12 37	n^0.
	5523		575	0·658	286 56	238 26	307 6	+11 49	o^0.
	5524	174·549	575	0·520	277 54	233 40	302 12	+11 4	M.
	5525		575	0·539	288 43	230 32	299 4	+ 6 27	N.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. July 9.	5638	189·492	579	0·171	51° 23′	201° 15′	57° 49′	+ 8° 57′	b.
	5639		579	0·194	54 16	199 56	56 30	+ 8 19	C.
	5640		579	0·221	56 34	198 48	55 22	+10 13	c.
	5641		581	0·398	77 42	191 39	48 13	+ 6 32	D.
	5642		581	0·405	77 0	190 59	47 33	+ 6 13	E.
	5643		581	0·464	78 59	188 42	45 16	+ 5 1	F.
	5644		581	0·481	78 0	188 23	44 57	+ 6 39	d.
	5645		581	0·495	76 5	185 7	41 41	+ 7 20	e.
	5646		581	0·502	76 42	185 58	42 32	+ 8 33	f.
	5647		582	0·971	73 21	138 54	355 28	+ 9 34	G.
	5648		582	0·976	74 41	132 10	348 44	+10 2	g.
	5649		582	0·988	75 8	135 48	352 22	+ 8 20	H.
	5650		582	0·992	76 38	130 3	346 37	+ 8 39	h.
	5651	189·518	578	0·520	233 30	239 17	95 29	−16 4	A.
	5652		579	0·152	50 7	203 34	59 46	+ 9 29	B.
	5653		579	0·169	51 47	201 48	58 0	+ 8 53	b.
	5654		579	0·190	54 27	200 10	56 22	+ 8 16	C.
	5655		579	0·217	56 11	198 59	55 11	+10 11	c.
	5656		581	0·395	77 16	191 44	47 56	+ 6 28	D.
	5657		581	0·402	77 58	191 22	47 34	+ 6 18	E.
	5658		581	0·461	78 22	188 49	45 1	+ 5 9	F.
	5659		581	0·479	78 26	188 17	45 29	+ 6 44	d.
	5660		581	0·492	76 7	185 31	41 43	+ 7 29	e.
	5661		581	0·499	76 20	186 11	42 23	+ 8 28	f.
	5662		582	0·966	73 34	139 2	355 14	+ 9 37	G.
	5663		582	0·975	74 2	132 24	349 36	+ 9 54	g.
	5664		582	0·984	74 34	136 0	353 12	+ 8 22	H.
	5665		582	0·989	75 31	130 14	346 26	+ 8 43	h.
11.	5666	191·499	578	0·830	254 39	266 25	94 31	−15 32	M.
	5667		579	0·299	301 46	229 40	57 46	+ 8 39	A.
	5668		579	0·304	302 31	228 2	56 8	+ 8 46	B.
	5669		579	0·317	305 20	231 35	59 41	+10 47	C.
	5670		579	0·311	306 55	229 57	58 3	+ 9 35	a.
	5671		579	0·325	304 6	229 28	57 34	+10 7	b.
	5672		579	0·328	308 42	229 16	57 22	+ 8 58	c.
	5673		581	0·194	300 8	213 29	43 35	+ 5 19	D.
	5674		581	0·190	307 10	219 26	47 32	+ 7 58	E.
	5675		581	0·146	326 39	218 26	46 32	+ 6 41	F.
	5676		581	0·123	331 4	215 20	43 26	+ 6 1	d.
	5677		581	0·106	354 29	214 49	42 55	+ 7 51	e.
	5678		581	0·090	2 2	217 26	45 32	+ 5 48	f.
	5679		582	0·799	75 13	164 44	352 50	+ 9 50	P.
	5680		582	0·821	76 51	163 0	351 6	+10 20	Q.
	5681		582	0·852	77 29	161 21	349 27	+ 8 55	R.
	5682		582	0·855	76 58	159 29	347 35	+ 8 9	p.
	5683		582	0·869	77 47	167 13	355 19	+ 9 16	q.
	5684		582	0·876	79 18	165 26	353 32	+ 9 57	r.
	5685	191·516	578	0·832	253 26	267 8	95 0	−15 28	M.
	5686		579	0·302	300 54	229 49	57 41	+ 8 43	A.
	5687		579	0·308	301 28	228 56	56 48	+ 8 41	B.
	5688		579	0·320	305 24	231 52	59 44	+10 49	C.
	5689		579	0·316	306 46	230 45	58 37	+ 9 40	a.
	5690		579	0·330	304 26	229 25	57 17	+ 9 59	b.
	5691		579	0·333	308 14	229 28	57 20	+ 8 47	c.
	5692		581	0·197	300 47	215 20	43 12	+ 5 22	D.
	5693		581	0·193	307 10	219 40	47 32	+ 7 50	E.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. July 11.	5694	191·516	581	0·148	331° 38′	219° 5′	46° 57′	+ 6° 39′	F.
	5695		581	0·126	335 59	215 22	43 14	+ 5 54	d.
	5696		581	0·111	356 32	215 7	42 59	+ 7 52	e.
	5697		581	0·096	2 46	217 38	45 30	+ 5 44	f.
	5698		582	0·796	75 31	164 14	352 6	+ 9 52	P.
	5699		582	0·816	76 51	163 52	351 44	+10 7	Q.
	5700		582	0·848	77 53	161 49	349 41	+ 8 50	R.
	5701		582	0·853	76 21	159 58	347 50	+ 8 8	p.
	5702		582	0·865	76 12	167 32	355 24	+ 9 11	q.
	5703		582	0·873	79 2	165 22	353 14	+10 3	r.
13.	5704	193·627	579	0·671	283 12	259 9	57 3	+ 8 6	M.
	5705		579	0·689	285 38	257 29	55 23	+ 9 12	m.
	5706		579	0 698	286 28	260 40	58 34	+10 15	N.
	5707		581	0·622	278 19	248 32	46 26	+ 6 42	A.
	5708		581	0·617	277 24	249 22	47 16	+ 7 56	a.
	5709		581	0·493	281 1	249 49	47 43	+ 6 4	a⁰.
	5710		582	0·393	61 48	196 33	354 28	+ 9 22	B.
	5711		582	0·427	63 27	197 57	355 52	+ 8 8	C.
	5712		582	0·439	65 50	195 44	353 39	+ 8 10	D.
	5713		582	0·474	62 8	188 57	346 52	+ 7 7	E.
	5714		582	0·502	69 34	189 26	347 21	+11 33	b.
	5715		582	0·491	67 5	187 24	345 19	+10 6	c.
	5716		582	0·546	68 58	190 5	348 0	+ 8 28	d.
	5717		582	0·552	71 21	186 57	344 52	+ 8 22	e.
14.	5718	194·521	579	0·825	285 59	270 29	55 43	+ 8 55	A.
	5719		579	0·817	285 38	272 30	57 44	+ 9 34	B.
	5720		579	0·803	287 30	271 57	57 11	+ 9 13	C.
	5721		581	0·784	281 4	262 24	47 38	+ 6 3	a.
	5722		581	0·709	280 57	260 30	45 44	+ 5 9	b.
	5723		581	0·637	280 6	261 58	47 12	+ 7 15	c.
	5724		582	0·223	37 32	210 58	356 12	+11 47	D.
	5725		582	0·256	39 15	210 10	355 24	+10 51	E.
	5726		582	0·241	44 13	207 14	352 28	+10 29	F.
	5727		582	0·273	47 35	208 15	353 29	+ 8 35	d.
	5728		582	0·265	49 21	208 42	353 56	+ 8 53	e.
	5729		582	0·284	51 38	207 7	352 21	+ 7 42	f.
	5730		583	0·311	58 7	202 21	347 35	+ 9 27	G.
	5731		583	0·357	58 56	202 18	347 32	+ 9 35	g.
	5732		583	0·389	61 47	199 44	344 58	+10 8	H.
	5733		583	0·394	63 23	198 56	344 10	+10 29	h.
	5734		583	0·404	65 13	197 41	342 55	+11 16	n.
15.	5735	195·506	579	0·932	281 46	283 52	55 8	+ 8 20	S.
	5736		579	0·945	283 24	284 54	56 10	+ 9 58	s.
	5737		579	0·956	284 32	285 43	56 59	+ 8 8	s¹.
	5738		581	0·899	278 22	274 18	45 34	+ 6 19	A.
	5739		581	0·771	277 56	276 8	47 24	+ 5 30	a.
	5740		582	0·229	332 47	223 59	355 15	+10 22	B.
	5741		582	0·215	337 11	224 56	356 12	+10 2	C.
	5742		582	0·196	345 20	220 27	351 43	+11 20	b.
	5743		582	0·174	351 34	220 51	352 7	+ 9 46	c.
	5744		583	0·196	14 22	216 11	347 27	+ 7 9	D.
	5745		583	0·209	18 11	215 37	346 53	+ 8 41	E.
	5746		583	0·238	25 28	213 1	344 17	+ 9 40	F.
	5747		583	0·242	27 16	210 58	342 14	+ 9 54	d.
	5748		583	0·247	32 55	213 44	345 0	+10 9	e
	5749		583	0·251	35 48	213 37	344 53	+ 8 48	f.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. July 16.	5750	196·499	579	0·982	285° 49′	299° 35′	56° 46′	+ 8° 18′	A.
	5751		579	0·994	286 55	298 45	55 56	+ 8 5	a.
	5752		581	0·970	281 5	289 9	46 20	+ 6 56	M.
	5753		582	0·311	297 17	237 35	354 46	+10 30	B.
	5754		582	0·348	299 37	239 9	356 20	+11 52	C.
	5755		582	0·375	304 13	238 3	355 14	+ 9 32	D.
	5756		582	0·392	305 21	235 9	352 20	+ 9 15	b.
	5757		582	0·407	308 7	234 28	351 39	+ 9 1	c.
	5758		583	0·286	311 38	227 27	344 38	+ 8 40	S.
	5759		583	0·251	317 36	229 21	346 32	+ 9 35	O.
	5760		583	0·269	324 21	228 0	345 11	+ 8 5	P.
	5761		583	0·222	314 34	224 12	341 23	+ 9 41	s.
	5762		583	0·251	325 25	224 59	342 10	+ 9 13	o.
	5763		583	0·219	328 42	223 53	341 4	+10 55	p.
	5764		583	0·216	329 22	227 36	344 47	+ 8 20	q.
19.	5765	199·623	582	0·807	287 15	276 31	349 43	+ 8 57	A.
	5766		582	0·841	288 47	279 40	352 32	+ 9 55	B.
	5767		582	0·852	288 57	280 30	353 22	+ 9 58	a.
	5768		582	0·866	289 14	283 59	356 51	+10 31	b.
	5769		583	0·705	288 9	269 51	342 43	+11 11	C.
	5770		583	0·724	289 26	272 19	345 11	+ 8 45	D.
	5771		583	0·754	290 43	272 2	344 4	+ 9 52	E.
	5772		583	0·769	288 42	272 20	345 12	+ 8 55	c.
	5773		583	0·771	290 19	268 15	341 7	+10 46	d.
20.	5774	200·503	582	0·902	289 11	293 29	353 53	+ 9 33	S.
	5775		582	0·946	290 53	295 3	355 27	+ 8 2	s.
	5776		582	0·956	291 43	295 27	356 21	+10 13	s^1.
	5777		583	0·894	288 55	283 46	344 10	+11 52	A.
	5778		583	0·887	290 42	282 50	343 14	+ 9 55	a.
	5779		583	0·881	290 42	283 49	344 13	+10 9	a^1.
21.	5780	201·524	583	0·991	287 54	299 33	345 27	+ 8 2	A.
	5781		583	0·985	289 10	298 42	344 36	+10 54	a.
	5782		585	0·394	130 0	206 33	252 27	−11 27	B.
	5783		584	0·922	111 16	158 3	203 57	−19 45	C.
23.	5784	203·603	581	0·411	236 18	240 20	256 45	−11 46	A.
	5785		585	0·387	229 50	235 37	252 2	− 9 27	a.
	5786		584	0·694	126 20	188 5	204 30	−20 43	B.
25.	5787	205·606	No spot visible.						
26.	5788	206·497	585*a*	0·372	301 21	247 28	222 50	+11 9	A.
	5789		584*a*	0·384	227 14	237 39	213 1	−17 4	B.
	5790		584*a*	0·376	198 37	235 27	210 59	−19 39	C.
	5791		584*a*	0·389	191 46	233 19	208 41	−18 52	b.
	5792	206·591	585*a*	0·376	301 52	248 42	222 44	+11 13	A.
	5793		584*a*	0·379	227 38	239 24	213 26	−17 10	B.
	5794		584*a*	0·370	197 50	237 6	211 8	−19 36	C.
	5795		584*a*	0·382	191 2	234 40	208 42	−18 57	b.
27.	5796	207·641	585*a*	0·591	293 33	263 16	222 24	+11 37	a.
	5797		585*a*	0·572	294 4	265 30	224 38	+10 8	b.
	5798		585*a*	0·554	295 28	262 6	221 14	+11 5	c.
	5799		584*a*	0·423	225 47	251 39	210 47	−17 56	a^0.
	5800		584*a*	0·454	227 16	252 33	211 41	−17 9	b^0.
	5801		584*a*	0·467	229 21	254 43	213 51	−19 25	c^0.
	5802	207·651	585*a*	0·594	293 46	263 3	222 3	+11 46	a.
	5803		585*a*	0·576	294 29	265 23	224 23	+10 7	b.
	5804		585*a*	0·560	295 7	262 41	221 41	+11 12	c.
	5805		584*a*	0·427	225 8	251 10	210 10	−18 0	a^0.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Heliographical Longitude.	Heliographical Latitude.	Spot.
1864. July 27.	5806	207·651	584*a*	0·459	227° 42′	252° 30′	211° 30′	−17° 6′	b°.
	5807		584*a*	0·470	229 32	254 33	213 33	−19 33	c°.
29.	5808	209·509	585*a*	0·888	291 30	291 13	223 51	+11 9	A.
	5809		584*a*	0·711	250 28	279 0	211 38	−18 32	B.
Aug. 1.	5810	212·505	586	0·376	171 21	236 53	127 1	−19 34	A.
	5811		586	0·361	179 0	231 33	121 41	−15 35	B.
	5812		586	0·347	182 59	232 0	122 8	−17 33	a.
	5813		586	0·321	185 18	229 32	119 40	−14 9	b.
	5814		587	0·884	85 57	167 34	57 42	+ 8 34	C.
	5815		587	0·886	86 57	168 26	58 34	+ 7 46	D.
	5816		587	0·912	92 55	165 56	56 4	+ 3 30	E.
	5817		587	0·929	98 36	167 0	57 8	+ 2 57	c.
	5818		587	0·943	94 56	169 31	59 39	+ 0 17	d.
2.	5819	213·678	586	0·394	230 38	252 2	125 32	−15 50	a.
	5820		586	0·370	228 11	251 15	124 45	−18 4	b.
	5821		586	0·357	224 13	249 19	122 49	−19 43	c.
	5822		587	0·741	88 8	183 49	57 19	+ 6 27	A.
	5823		587	0·792	86 9	185 59	59 29	+ 5 14	B.
	5824		587	0·810	93 34	185 1	58 31	+ 6 52	C.
	5825		587	0·817	94 58	183 11	56 41	+ 1 26	D.
4.	5826	215·616	586	0·699	258 11	276 15	122 16	−12 37	M.
	5827		586	0·682	256 38	275 17	121 18	−11 20	N.
	5828		586	0·644	253 13	273 48	119 49	−11 14	m.
	5829		586	0·624	251 41	270 51	116 52	−10 11	n.
	5830		587	0·432	73 38	215 40	61 41	+ 8 44	S.
	5831		587	0·450	81 15	213 12	59 13	+ 7 11	T.
	5832		587	0·471	84 28	211 54	57 55	+ 1 36	s.
	5833		587	0·489	93 48	209 3	55 4	− 0 35	t.
5.	5834	216·505	586	0·847	263 23	289 15	122 39	−12 18	A.
	5835		586	0·823	262 29	286 30	119 54	−10 2	a.
	5836		586	0·820	259 11	287 7	120 31	−11 31	B.
	5837		586	0·809	259 16	288 22	121 46	−11 26	b.
	5838		587	0·304	94 25	225 8	58 32	+ 7 31	C.
	5839		587	0·307	94 39	223 49	57 13	+ 8 4	c.
	5840		589	0·972	113 29	223 12	56 36	−18 39	S.
6.	5841	217·530	586	0·902	267 30	302 35	121 27	−11 47	M.
	5842		586	0·884	266 22	300 58	119 50	−11 2	m.
	5843		587	0·060	58 40	239 22	58 14	+ 6 43	O.
	5844		589	0·892	117 20	185 37	4 29	−19 31	P.
	5845		589	0·917	117 35	182 51	1 43	−18 38	p.
	5846		590	0·966	94 29	164 5	342 57	+ 5 13	A.
	5847		590	0·964	95 41	160 37	339 29	+ 4 28	B.
8.	5848	219·605	587	0·473	286 27	268 22	57 48	+ 8 58	A.
	5849		587	0·439	287 6	266 41	56 7	+ 8 28	a.
	5850		592	0·494	239 26	263 28	52 54	−12 3	M.
	5851		592	0·487	237 51	263 48	53 14	−12 44	N.
	5852		592	0·483	237 22	261 12	50 38	−13 15	O.
	5853		589	0·611	132 35	214 36	4 2	−16 53	B.
	5854		589	0·599	130 16	215 50	5 16	−21 46	C.
	5855		589	0·575	128 42	212 8	1 34	−19 44	b.
	5856		589	0·564	127 12	210 24	359 50	−19 7	c.
	5857		588	0·674	126 44	205 56	355 22	−17 54	S.
	5858		588	0·678	125 48	208 22	357 48	−16 12	s.
	5859		591	0·855	122 58	186 34	336 0	−18 7	T.
	5860		591	0·859	115 30	185 24	334 50	−19 38	t.
	5861		590	0·764	96 2	196 32	345 58	+ 4 31	P.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Aug. 8.	5862	219·605	590	0·777	95° 5′	192° 12′	341° 38′	+ 6° 7′	Q.
	5863		590	0·795	90 1	190 59	340 25	+ 4 33	R.
	5864		590	0·782	89 9	189 24	338 50	+ 4 58	p.
	5865		590	0·819	92 49	188 15	337 41	+ 1 31	q.
	5866		590	0·821	89 44	189 32	338 58	+ 1 11	r.
10.	5867	221·482	587	0·795	287 54	296 32	59 21	+ 9 0	A^0.
	5868		592	0·782	262 2	290 6	52 55	−12 26	A.
	5869		592	0·765	261 54	289 40	52 29	−13 19	B.
	5870		592	0·754	259 39	288 41	51 30	−13 55	C.
	5871		592	0·748	258 58	285 29	48 18	−12 5	a.
	5872		592	0·711	256 35	286 10	48 59	−14 51	b.
	5873		592	0·708	256 22	284 0	46 59	−11 19	c.
	5874		593	0·208	305 29	253 43	16 32	+ 7 58	M.
	5875		589	0·302	182 20	242 38	5 27	−16 42	D.
	5876		589	0·327	178 18	241 43	4 32	−18 29	E.
	5877		589	0·354	175 28	236 1	358 50	−19 26	d.
	5878		589	0·395	170 50	237 16	0 5	−17 21	e.
	5879		589	0·383	171 17	238 21	1 10	−20 11	T.
	5880		589	0·408	165 41	235 5	357 54	−21 50	t.
	5881		588	0·375	158 41	233 29	356 18	−17 46	F.
	5882		588	0·381	157 2	233 28	356 17	−18 21	f.
	5883		591	0·552	135 45	218 31	341 20	−16 57	Q.
	5884		591	0·558	136 5	215 21	338 10	−18 54	R.
	5885		591	0·567	138 19	216 2	338 51	−19 48	S.
	5886		591	0·561	139 29	213 51	336 40	−20 40	q.
	5887		591	0·588	136 47	212 56	335 45	−22 5	r.
	5888		591	0·596	131 8	211 42	334 31	−22 47	s.
	5889		591	0·633	129 17	211 35	334 24	−22 27	s^1.
	5890		591	0·647	126 40	209 58	332 47	−23 27	s$^{\prime\prime}$.
	5891		590	0·412	89 50	221 5	343 54	+ 1 40	K.
	5892		590	0·433	90 39	220 35	343 24	+ 3 28	L.
	5893		590	0·431	92 0	219 42	342 31	+ 4 57	k.
	5894		590	0·482	94 37	218 44	341 33	+ 6 27	l.
	5895		590	0·495	93 35	217 21	340 10	+ 4 25	w.
	5896		590	0·529	91 35	215 57	338 46	+ 7 5	w^1.
	5897		590	0·554	88 12	214 56	337 45	+ 1 26	u.
	5898		590	0·567	86 35	212 11	335 0	+ 2 44	u^1.
11.	5899	222·466	592	0·874	262 36	298 55	47 46	−12 54	A.
	5900		592	0·902	265 8	303 44	52 25	−13 40	B.
	5901		592	0·923	266 15	300 48	49 39	−12 22	a.
	5902		592	0·925	266 26	302 19	51 10	−12 39	b.
	5903		588	0·302	191 13	246 24	355 15	−17 58	C.
	5904		588	0·323	192 57	247 56	356 47	−18 37	c.
	5905		588	0·338	195 7	245 17	354 8	−16 58	D.
	5906		588	0·357	196 37	247 34	356 25	−17 44	d.
	5907		589	0·315	208 58	252 59	1 50	−21 6	E.
	5908		589	0·339	210 20	248 47	357 38	−18 12	e^0.
	5909		589	0·342	217 5	255 32	4 23	−17 55	e^1.
	5910		589	0·387	223 5	250 29	359 20	−20 22	e$^{\prime\prime}$.
	5911		589	0·366	214 20	254 47	3 38	−16 28	F.
	5912		589	0·387	225 34	256 56	5 47	−18 24	f.
	5913		589	0·392	226 52	257 22	6 13	−18 17	G.
	5914		589	0·399	225 0	255 12	4 3	−18 17	g.
	5915		590	0·215	73 33	233 7	341 58	+ 2 34	H.
	5916		590	0·264	76 57	235 52	344 43	+ 4 41	L.
	5917		590	0·242	77 43	230 38	339 29	+ 4 50	h.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Aug. 11.	5918	222·466	590	0·341	79° 31′	229° 52′	338° 43′	+ 4° 44′	l.
	5919		590	0·306	84 1	235 5	343 56	+ 2 27	M.
	5920		590	0·361	87 6	233 49	342 40	+ 5 31	N.
	5921		590	0·299	79 31	232 31	341 22	+ 1 42	m.
	5922		590	0·376	88 36	233 23	342 14	+ 4 16	n.
	5923		591	0·337	149 59	226 1	334 52	−21 53	O.
	5924		591	0·366	141 43	229 28	338 19	−22 8	P.
	5925		591	0·412	143 15	234 17	343 8	−23 33	Q.
	5926		591	0·389	149 10	235 49	344 40	−21 20	R.
	5927		591	0·477	145 28	232 48	341 39	−19 59	S.
	5928		591	0·451	146 13	230 10	339 1	−20 36	o.
	5929		591	0·502	151 5	234 5	342 56	−20 41	p.
	5930		591	0·489	148 30	233 54	342 45	−21 27	q.
	5931		591	0·495	153 27	232 35	341 26	−21 14	r.
	5932		591	0·508	154 33	234 25	343 16	−21 30	s.
	5933		593	0·446	297 26	268 12	17 3	+ 8 28	T.
12.	5934	223·586	592	0·991	271 2	314 22	47 20	−12 10	A.
	5935		592	0·964	268 42	316 38	49 36	−12 13	a.
	5936		588	0·384	241 45	262 43	355 41	−16 21	B.
	5937		588	0·417	239 9	261 1	353 59	−17 50	b.
	5938		588	0·442	237 4	262 27	355 25	−17 7	b⁰.
	5939		589	0·465	244 52	269 12	2 10	−20 37	C.
	5940		589	0·476	246 24	266 44	359 42	−18 56	D.
	5941		589	0·497	251 50	270 5	3 3	−21 15	E.
	5942		589	0·508	248 41	271 14	4 12	−17 2	c.
	5943		589	0·481	255 19	271 26	4 24	−18 22	d.
	5944		589	0·469	253 37	265 28	358 26	−18 9	e.
	5945		589	0·512	257 58	268 10	1 8	−19 17	x.
	5946		590	0·102	335 27	251 39	344 37	+ 5 45	M.
	5947		590	0·129	341 29	248 33	341 31	+ 4 3	N.
	5948		590	0·157	26 25	252 29	345 27	+ 4 2	m.
	5949		590	0·166	37 48	249 41	342 39	+ 2 13	n.
	5950		591	0·276	221 37	245 53	338 51	−19 4	O.
	5951		591	0·294	219 42	247 55	340 53	−19 54	P.
	5952		591	0·312	202 21	246 44	339 42	−20 12	Q.
	5953		591	0·387	189 56	248 50	341 48	−21 17	R.
	5954		591	0·353	184 19	247 11	340 9	−21 6	S.
	5955		591	0·360	195 42	244 28	337 26	−18 41	o.
	5956		591	0·394	186 53	245 16	338 14	−18 40	p.
	5957		591	0·401	177 26	249 19	342 17	−20 14	q.
	5958		593	0·623	295 33	283 19	16 17	+ 8 17	T.
13.	5959	224·629	588	0·531	257 14	278 42	356 52	−16 35	M.
	5960		588	0·572	255 37	273 37	351 47	−16 56	N.
	5961		588	0·591	251 6	275 6	353 16	−17 24	m.
	5962		588	0·608	249 12	277 4	355 14	−17 41	n.
	5963		589	0·627	255 3	280 55	359 5	−18 17	S.
	5964		589	0·634	261 3	282 44	0 54	−20 2	s.
	5965		589	0·688	259 26	283 30	1 40	−19 53	T.
	5966		589	0·676	257 31	280 13	358 23	−20 24	t.
	5967		589	0·652	260 27	285 38	3 48	−20 38	U.
	5968		589	0·679	262 22	287 16	5 26	−20 55	u.
	5969		589	0·693	264 8	286 48	4 58	−20 55	v.
	5970		591	0·384	198 31	260 5	338 15	−18 30	A.
	5971		591	0·395	199 42	261 4	339 14	−19 40	a.
	5972		591	0·401	207 26	262 7	340 17	−18 14	B.
	5973		591	0·402	213 36	261 10	339 20	−18 22	b.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Aug. 13.	5974	224·629	591	0·411	223° 36′	259° 4′	337° 14′	−21° 17′	C.
	5975		591	0·418	229 26	260 30	338 40	−21 14	c.
	5976		591	0·433	226 42	264 20	342 30	−21 15	D.
	5977		591	0·422	231 31	262 45	340 55	−20 57	E.
	5978		591	0·429	228 8	262 45	340 55	−20 25	d.
	5979		591	0·439	234 26	263 12	341 22	−21 38	e.
	5980		590	0·305	292 45	259 14	337 24	+ 4 49	F.
	5981		590	0·331	297 46	259 5	337 15	+ 5 20	G.
	5982		590	0·267	335 35	261 26	339 36	+ 4 5	H.
	5983		590	0·347	298 38	263 9	341 19	+ 3 2	f.
	5984		590	0·357	306 35	262 44	340 54	+ 4 22	g.
	5985		593	0·762	309 25	298 25	16 35	+ 8 5	T.
15.	5986	226·476	590	0·653	295 32	290 19	342 17	+ 4 48	A.
	5987		590	0·661	296 8	290 29	342 27	+ 5 32	B.
	5988		590	0·679	297 29	289 33	341 31	+ 5 31	C.
	5989		590	0·694	298 50	292 25	344 23	+ 4 24	D.
	5990		590	0·576	302 12	291 36	343 34	+ 3 36	a.
	5991		590	0·602	308 41	291 22	343 20	+ 4 9	b.
	5992		590	0·609	306 16	288 47	340 45	+ 4 14	c.
	5993		591	0·604	252 11	285 31	337 29	−18 37	E.
	5994		591	0·612	262 26	288 21	340 19	−21 18	F.
	5995		591	0·637	260 41	289 16	341 14	−21 9	G.
	5996		591	0·655	259 57	284 49	336 47	−20 10	e.
	5997		591	0·645	258 40	287 27	339 25	−19 32	f.
	5998		591	0·666	264 23	286 5	338 3	−20 50	g.
	5999		591	0·755	276 10	285 21	337 19	−20 19	H.
	6000		591	0·762	275 34	286 6	338 4	−20 41	h.
	6001		588	0·834	269 51	303 5	355 3	−17 42	M.
	6002		588	0·846	269 9	304 48	356 46	−18 47	N.
	6003		588	0·851	271 54	298 7	350 5	−18 57	m.
	6004		589	0·874	267 11	307 10	359 8	−19 11	O.
	6005		589	0·882	269 42	308 7	0 5	−18 58	o.
	6006		589	0·909	270 58	306 49	358 47	−20 39	P.
	6007		589	0·889	272 34	309 4	1 2	−18 28	p.
	6008		589	0·895	273 31	314 36	6 34	−19 39	Q.
	6009		589	0·911	273 47	313 4	5 2	−19 28	q.
	6010		593	0·948	307 31	325 42	17 40	+ 7 14	T.
	6011		593	0·957	308 28	324 34	16 32	+ 8 10	t.
18.	6012	229·518	590	0·988	296 53	332 14	341 4	+ 5 23	A.
	6013		590	0·991	297 47	334 35	343 25	+ 4 42	a.
	6014		591	0·962	269 28	330 9	338 59	−19 16	B.
	6015		595	0·129	79 0	243 6	251 56	+ 3 35	C.
	6016		595	0·133	81 0	241 53	250 43	+ 2 7	c.
19.	6017	230·524	595	0·205	304 15	256 3	264 53	+ 3 13	A.
	6018		595	0·131	311 38	254 55	263 45	+ 5 40	a.
	6019		595	0·217	305 13	258 18	267 8	+ 4 3	a^{0}.
20.	6020	231·496	595	0·414	294 40	273 12	282 2	+ 3 53	A.
	6021		595	0·355	290 21	272 47	281 37	+ 4 1	a.
	6022		595	0·428	291 55	269 21	278 11	+ 2 58	a^{0}.
21.	6023	232·472	No spot visible.						
25.	6024	236·476							
26.	6025	237·546							
30.	6026	241·524	596	0·193	312 8	269 48	108 20	+ 5 13	A.
	6027		596	0·185	324 59	266 22	104 54	+ 5 7	B.
	6028		596	0·167	331 40	267 22	105 54	+ 5 39	C.
	6029		596	0·156	355 24	268 46	107 18	+ 7 56	a.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Aug. 30.	6030	241·524	596	0·145	6° 26′	263° 52′	102° 24′	+ 6° 13′	b.
	6031		596	0·149	8 23	262 46	101 18	+ 7 43	c.
	6032		597	0·764	135 28	217 45	56 17	−21 3	D.
	6033		597	0·771	134 26	216 46	55 18	−22 33	d.
Sept. 1.	6034	243·465	596	0·583	304 24	296 40	107 40	+ 7 43	M.
	6035		596	0·512	306 29	294 32	105 32	+ 7 6	m.
	6036		596	0·517	307 4	292 25	103 25	+ 7 27	n.
	6037		597	0·489	160 49	248 56	59 56	−22 21	B.
	6038		597	0·483	161 14	249 12	60 12	−22 2	b.
3.	6039	245·486	596	0·892	298 37	323 51	106 11	+ 5 47	A.
	6040		596	0·834	301 25	321 28	103 48	+ 5 7	a.
	6041		597	0·407	223 52	278 12	60 32	−20 4	B.
	6042		597	0·405	224 13	277 6	59 26	−21 13	b.
	6043		599	0·898	100 46	201 35	343 55	+ 6 59	C.
	6044		599	0·900	101 17	202 17	344 37	+ 7 55	c.
5.	6045	247·490	597	0·654	259 37	305 13	59 7	−19 30	A.
	6046		597	0·662	258 3	304 29	58 23	−20 27	A^{0}.
	6047		597	0·684	260 25	303 20	57 14	−20 35	a.
	6048		599	0·622	99 6	230 26	344 20	+ 7 29	B.
	6049		599	0·626	100 6	230 49	344 43	+ 7 56	b.
15.	6050	257·493	600	0·973	283 20	5 43	337 44	+ 5 4	A.
	6051		600	0·968	282 5	4 52	336 53	+ 6 30	a.
	6052		601	0·424	135 20	259 11	231 12	− 9 39	B.
	6053		601	0·433	134 31	255 50	227 51	− 7 40	C.
	6054		601	0·436	129 30	256 6	228 7	− 8 15	D.
	6055		601	0·448	132 20	258 32	230 33	− 7 28	E.
	6056		601	0·481	132 20	254 26	226 27	− 9 21	b.
	6057		601	0·476	132 45	255 15	227 16	−11 46	c.
	6058		601	0·467	129 24	256 26	228 27	−10 4	d.
	6059		601	0·492	128 8	254 27	226 28	−10 26	e.
17.	6060	259·458	601*a*	0·157	204 42	270 43	229 3	− 7 34	A.
	6061		601*a*	0·193	211 10	269 34	227 54	− 8 48	a.
	6062		602	0·824	115 39	227 59	186 19	− 5 10	B.
	6063		602	0·837	114 49	227 3	185 23	− 5 32	b.
19.	6064	261·528	601*a*	0·567	280 1	299 8	228 6	− 8 2	A^{0}.
	6065		601*a*	0·532	282 45	298 37	227 35	− 8 55	a^{0}.
	6066		602	0·476	121 54	258 33	187 31	− 5 20	B^{0}.
	6067		602	0·491	122 9	256 18	185 16	− 6 43	b^{0}.
20.	6068	262·458	601*a*	0·702	284 7	312 12	227 58	− 8 8	S.
	6069		601*a*	0·674	285 12	312 33	228 19	− 7 11	s.
	6070		602	0·266	132 8	271 52	187 38	− 5 18	T.
	6071		602	0·278	130 7	270 21	186 7	− 5 49	t.
21.	6072	263·416	602	0·094	192 33	285 33	187 44	− 5 52	M.
	6073		602	0·099	195 38	286 50	189 1	− 6 4	m.
22.	6074	264·482	602	0·287	281 46	299 29	186 33	− 5 45	A.
23.	6075	265·490	No spot visible.						
24.	6076	266·524	602*a*	0·867	276 21	343 11	191 17	− 8 43	A.
	6077		604	0·961	124 22	227 45	75 51	−15 55	B.
27.	6078	269·490	603	0·354	249 34	303 48	119 50	−12 24	A.
	6079		603	0·351	245 6	301 32	117 34	−12 47	a.
	6080		603	0·348	241 58	301 1	117 3	−13 14	a^{0}.
	6081		604	0·577	139 7	259 17	75 19	−16 22	B.
	6082		604	0·573	139 1	259 46	75 48	−15 59	b.
28.	6083	270·661	603	0·555	267 9	318 49	118 14	−12 11	M.
	6084		603	0·537	269 22	319 39	119 4	−12 52	m.
	6085		604	0·372	166 58	276 14	75 39	−14 23	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Sept. 28.	6086	270·661	604	0·379	165° 57′	277° 22′	76° 47′	−16° 7′	a.
29.	6087	271·454	603	0·633	275 11	330 9	118 29	−11 33	A.
	6088		605	0·964	104 47	218 52	7 2	+ 3 43	B.
30.	6089	272·619	605	0·879	106 33	235 40	7 19	+ 2 25	A.
Oct. 1.	6090	273·552	605	0·722	104 49	249 19	7 44	+ 2 7	A^0.
	6091		606	0·958	124 25	224 36	343 1	−14 50	B.
3.	6092	275·440	605	0·405	95 44	275 30	7 8	+ 3 48	A.
	6093		606	0·764	129 47	251 8	342 46	−15 51	B.
4.	6094	276·532	605	0·174	71 55	290 35	6 43	+ 3 3	a.
	6095		605*a*	0·366	211 18	301 2	17 10	−18 25	a^0.
	6096		605*a*	0·384	199 17	299 4	15 12	−17 31	a^1.
	6097		606	0·588	138 6	266 18	342 26	−16 4	A.
5.	6098	277·538	605*a*	0·423	241 19	315 52	17 45	−18 20	M.
	6099		605*a*	0·388	228 40	314 41	16 34	−17 11	m.
	6100		606	0·444	147 5	281 23	343 36	−16 15	O.
6.	6101	278·462	605*a*	0·602	259 19	328 32	17 18	−17 58	A.
	6102		605*a*	0·534	252 42	328 40	17 26	−18 41	B.
	6103		606	0·295	179 43	293 29	342 15	−16 34	a.
7.	6104	279·528	605*a*	0·671	264 18	344 16	17 55	−18 7	A.
	6105		606	0·291	239 10	309 43	343 22	−16 26	B.
	6106		607	0·849	108 58	245 52	279 31	+ 7 37	C.
8.	6107	280·480	605*a*	0·812	273 40	356 52	17 0	−17 2	M.
	6108		606	0·438	263 6	323 12	343 20	−16 22	m.
	6109		607	0·692	103 58	261 36	281 44	+ 7 47	n.
	6110		607	0·743	104 26	257 21	277 29	+ 5 3	o.
	6111		607	0·769	106 22	259 49	279 57	+ 3 47	p.
10.	6112	282·458	606	0·774	281 14	350 17	342 22	−16 33	A.
	6113		607	0·229	88 30	290 32	282 37	+ 7 30	a.
	6114		607	0·354	95 37	285 45	277 50	+ 4 33	a^0.
11.	6115	283·440	606	0·962	288 53	4 29	342 38	−16 28	M.
12.	6116	284·518	608	0·908	286 14	3 29	326 21	− 3 43	N.
15.	6117	287·462	609	0·362	259 23	329 7	250 13	−10 39	A.
	6118		609	0·384	257 3	325 4	246 10	− 9 15	B.
	6119		609	0·411	249 47	326 17	247 23	− 7 23	C.
	6120		609	0·399	252 29	328 1	249 7	− 8 40	a.
	6121		609	0·442	255 5	322 38	243 44	−10 42	b.
	6122		609	0·437	253 34	323 19	244 25	−11 18	c.
	6123		609	0·454	248 24	321 11	242 17	−12 47	D.
18.	6124	290·479	609	0·887	283 14	7 50	246 9	− 8 13	d.
19.	6125	291·476	610	0·811	129 9	268 29	132 39	−12 17	A.
21.	6126	293·472	610	0·708	133 59	297 24	133 16	−14 8	A.
	6127		610	0·711	133 0	299 39	135 31	−12 44	a.
	6128		610	0·749	134 2	296 55	132 47	−14 12	B.
	6129		610	0·754	134 29	295 7	130 59	−15 19	b.
22.	6130	294·465	610	0·248	187 32	314 5	135 51	−11 6	M.
	6131		610	0·257	185 2	309 9	130 55	−12 11	N.
	6132		610	0·263	174 40	312 47	134 33	−12 14	m.
	6133		610	0·304	169 30	310 11	131 57	−14 32	n.
	6134		610	0·287	178 0	308 42	130 28	−16 51	O.
	6135		610	0·312	168 13	308 18	130 4	−15 54	o.
	6136		611	0·874	129 6	261 0	82 46	−16 44	P.
	6137		611	0·880	130 1	261 23	83 9	−16 54	p.
	6138		612	0·905	128 20	248 49	70 35	−17 38	Q.
	6139		612	0·907	129 14	249 7	70 53	−17 0	q.
	6140		613	0·871	99 7	257 14	79 0	+ 8 9	R.
	6141		613	0·873	101 40	260 27	82 13	+ 9 27	r.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Oct. 24.	6142	296·538	610	0·398	259° 10′	342° 2′	134° 25′	−10° 52′	A.
	6143		610	0·421	264 36	339 31	131 54	− 9 27	B.
	6144		610	0·457	261 7	338 52	131 15	−11 3	a.
	6145		610	0·497	265 10	335 7	127 30	−12 47	b.
	6146		611	0·505	142 20	290 43	83 6	−16 57	C.
	6147		611	0·509	143 5	288 29	80 52	−16 37	c.
	6148		612	0·622	137 25	278 27	70 50	−17 30	D.
	6149		612	0·629	136 39	277 2	69 25	−17 15	d.
	6150		613	0·484	85 48	287 30	79 53	+ 8 17	E.
	6151		613	0·495	88 39	288 49	81 12	+ 8 29	F.
	6152		613	0·505	85 23	289 49	82 12	+ 9 51	G.
	6153		613	0·517	87 34	285 11	77 34	+ 7 24	g.
	6154		613	0·519	89 22	283 43	76 6	+ 6 2	f.
28.	6155	300·493	611	0·497	264 17	346 15	82 31	−14 10	M.
	6156		611	0·502	265 47	346 58	83 14	−15 8	m.
	6157		612	0·437	258 24	337 46	74 2	−13 57	S.
	6158		612	0·440	249 29	336 5	72 21	−12 48	T.
	6159		612	0·402	253 7	333 17	69 33	−13 12	s.
	6160		612	0·395	247 29	335 59	72 15	−16 17	t.
	6161		612	0·423	250 47	335 46	72 2	−16 51	P.
	6162		612	0·389	251 57	334 45	71 1	−17 21	p.
	6163		612	0·436	249 14	333 49	70 5	−18 48	Q.
	6164		612	0·388	249 26	334 55	71 11	−17 35	q.
	6165		614	0·822	109 22	261 21	357 37	+ 4 17	R.
	6166		614	0·854	108 30	264 58	1 14	+ 3 43	r^1.
	6167		614	0·841	109 26	262 51	359 7	+ 3 38	r^2.
	6168		614	0·867	107 31	266 7	2 23	− 1 31	r^3.
31.	6169	303·549	612	0·947	281 35	17 14	70 9	−16 38	A.
	6170		614	0·254	102 37	311 12	4 7	+ 3 29	B.
	6171		614	0·267	104 23	310 14	3 9	+ 2 49	C.
	6172		614	0·289	106 42	309 16	2 11	+ 0 57	D.
	6173		614	0·302	107 43	307 26	0 21	− 1 47	E.
	6174		614	0·307	106 0	307 4	359 59	+ 1 22	b.
	6175		614	0·341	106 42	306 0	358 55	+ 0 50	c.
	6176		614	0·357	108 26	304 23	357 18	+ 1 42	d.
	6177		614	0·374	107 6	303 7	356 2	+ 3 18	e.
	6178		614	0·365	107 22	304 23	357 18	+ 3 42	f.
	6179		614	0·383	109 55	303 6	356 1	+ 4 7	g.
	6180		615	0·963	106 55	246 48	285 32	+ 3 22	T.
	6181		615	0·965	107 46	248 13	286 57	+ 3 38	t.
	6182		616	0·989	104 8	223 56	262 40	+ 6 19	H.
Nov. 3.	6183	306·465	614	0·441	301 10	6 36	3 58	+ 3 49	M.
	6184		614	0·396	299 55	4 14	1 36	+ 4 37	M^0.
	6185		614	0·354	303 23	0 24	357 46	+ 4 49	A.
	6186		614	0·337	306 0	358 38	356 0	+ 2 8	a.
	6187		615	0·571	104 33	287 37	284 59	+ 4 17	B.
	6188		615	0·579	105 9	287 57	285 19	+ 4 56	b.
	6189		615	0·587	105 53	288 22	285 44	+ 3 55	C.
	6190		615	0·602	104 4	286 24	283 46	+ 4 33	c.
	6191		616	0·846	103 20	267 12	264 34	+ 5 15	D.
	6192		616	0·857	104 50	268 48	266 10	+ 6 1	E.
	6193		616	0·855	104 43	265 35	262 57	+ 6 11	d.
	6194		616	0·888	105 22	263 50	261 12	+ 4 9	e.
	6195		616	0·895	106 23	267 35	264 57	+ 5 58	e^0.
4.	6196	307·500	614	0·643	299 23	21 13	3 35	+ 3 36	A.
	6197		614	0·659	300 37	20 13	2 55	+ 2 29	B.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864 Nov. 4.	6198	307·500	614	0·667	299° 45′	19° 56′	2° 38′	+ 4° 17′	a.
	6199		614	0·695	302 6	16 20	359 2	+ 2 29	b.
	6200		615	0·387	101 7	304 44	287 26	+ 3 46	F.
	6201		615	0·389	101 17	303 46	286 28	+ 4 42	f.
	6202		615	0·405	98 41	304 44	287 26	+ 3 5	G.
	6203		615	0·419	98 42	301 1	283 43	+ 4 5	g.
	6204		615	0·424	97 28	301 43	284 25	+ 4 20	g^1.
	6205		616	0·682	102 36	284 48	267 30	+ 7 55	M.
	6206		616	0·661	102 0	283 53	266 35	+ 6 22	m.
	6207		616	0·699	104 50	281 20	264 2	+ 5 46	N.
	6208		616	0·719	104 22	279 23	262 5	+ 6 37	n.
	6209		616	0·744	103 59	278 55	261 37	+ 3 4	O.
	6210		616	0·763	104 23	277 48	260 30	+ 5 33	o.
14.	6211	317·476	616	0·994	287 46	61 53	263 5	+ 6 9	A.
	6212		617	0·224	222 7	343 35	184 47	− 9 9	a.
	6213		618	0·897	123 8	284 7	125 19	−10 33	a^0.
18.	6214	321·496	618	0·221	201 51	343 27	127 37	−11 12	A.
	6215		618	0·239	199 23	342 46	126 56	−12 31	B.
	6216		618	0·228	194 32	340 18	124 28	−10 22	C.
	6217		618	0·244	193 18	340 55	125 5	−10 23	D.
	6218		619	0·292	172 1	334 26	118 36	−13 6	a.
	6219		619	0·298	171 42	335 8	119 18	−14 13	b.
	6220		620	0·677	86 46	300 46	84 56	+12 20	c.
	6221		620	0·713	91 1	296 22	80 32	+ 9 32	d.
22.	6222	325·462	618	0·824	274 44	38 41	125 11	−10 12	B.
	6223		618	0·801	273 6	40 33	127 3	−11 17	b.
	6224		618	0·799	271 3	40 6	126 36	−11 37	C.
	6225		619	0·304	338 43	31 55	118 25	−14 38	M.
	6226		620	0·337	339 1	350 48	78 43	+16 0	N.
	6227		621	0·417	247 31	6 57	94 52	−14 56	D.
	6228		621	0·395	246 30	4 21	92 16	−12 36	E.
	6229		621	0·359	239 28	4 12	92 7	−11 31	d.
	6230		621	0·328	242 19	6 38	94 33	−11 49	e.
	6231		621	0·312	238 16	358 32	86 27	−12 21	f.
25.	6232	328·514	621	0·872	269 35	43 0	87 37	−10 36	A.
	6233		621	0·865	267 33	46 28	91 5	−11 45	B.
	6234		621	0·849	268 20	48 30	93 7	−11 58	C.
	6235		621	0·826	269 50	48 13	92 50	−11 6	D.
	6236		622	0·482	81 33	317 47	2 24	+14 9	a.
	6237		622	0·495	84 24	321 40	6 17	+12 22	b.
	6238		622	0·505	86 22	323 46	8 23	+16 51	c.
	6239		622	0·571	87 32	318 37	3 14	+11 58	d.
	6240		622	0·536	79 42	317 54	2 31	+12 14	e.
	6241		622	0·584	80 46	317 13	1 50	+12 34	f.
	6242		623	0·789	89 1	300 12	344 49	+13 4	E.
	6243		623	0·805	90 4	299 30	344 7	+11 43	F.
	6244		623	0·806	91 4	298 48	343 25	+12 38	G.
	6245		623	0·794	89 39	298 57	343 34	+12 57	H.
	6246		624	0·853	94 7	292 3	336 40	+ 9 51	S.
	6247		624	0·861	95 19	288 42	333 19	+11 41	s.
29.	6248	332·528	622	0·502	305 13	23 39	11 20	+ 9 33	A.
	6249		622	0·487	311 21	22 54	10 35	+ 9 39	B.
	6250		622	0·489	309 24	18 2	5 43	+10 45	C.
	6251		622	0·453	318 27	19 52	7 33	+12 43	a.
	6252		622	0·414	324 47	21 41	9 22	+14 15	b.
	6253		622	0·396	322 2	15 54	3 35	+12 38	c.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Nov. 29.	6254	332·528	622	0·409	320° 36′	15° 45′	3° 26′	+12° 22′	D.
	6255		622	0·428	319 13	14 38	2 19	+16 46	d.
	6256		622	0·387	319 10	12 32	0 13	+14 8	d^{1}.
	6257		623	0·247	2 18	357 10	344 51	+11 5	M.
	6258		623	0·244	5 48	356 57	343 38	+12 12	N.
	6259		623	0·259	6 28	356 19	344 0	+13 48	n.
	6260		624	0·192	21 9	349 58	337 39	+ 9 49	S.
	6261		624	0·216	25 43	348 51	336 32	+11 24	s.
	6262		624	0·215	26 28	346 13	333 54	+11 3	T.
	6263		624	0·229	35 58	346 32	334 13	+10 42	t.
	6264		624	0·238	37 40	347 58	335 39	+12 35	q.
	6265		625	0·933	89 29	285 22	273 3	+ 7 22	Q.
	6266		625	0·928	93 20	286 45	274 26	+ 8 38	R.
	6267		625	0·944	94 4	285 6	272 47	+ 7 17	r.
	6268		626	0·953	106 55	284 30	272 11	− 4 49	P.
Dec. 1.	6269	334·454	622	0·802	301 16	47 5	7 28	+11 58	S.
	6270		622	0·811	300 26	43 48	4 11	+14 41	s.
	6271		622	0·826	298 55	47 53	8 16	+12 51	T.
	6272		622	0·833	299 44	45 1	5 24	+10 24	t.
	6273		622	0·845	298 16	43 22	3 45	+16 57	q.
	6274		622	0·714	301 2	41 54	2 17	+11 10	m.
	6275		622	0·756	302 15	39 54	0 17	+11 25	n.
	6276		622	0·744	302 42	45 55	6 18	+11 26	o.
	6277		622	0·779	304 6	44 41	5 4	+12 38	p.
	6278		622	0·783	305 50	44 12	4 35	+11 18	p^{1}.
	6279		623	0·506	308 11	24 5	344 28	+12 20	A.
	6280		623	0·517	309 44	23 44	344 7	+11 13	A^{0}.
	6281		624	0·421	313 13	15 10	335 33	+10 27	B.
	6282		624	0·426	310 7	16 6	336 29	+11 22	b.
	6283		624	0·444	311 20	12 57	333 20	+12 13	C.
	6284		624	0·439	308 1	14 57	335 20	+12 59	c.
	6285		624	0·449	306 5	15 16	335 39	+10 2	d.
	6286		625	0·683	89 49	314 10	274 33	+ 7 56	M.
	6287		625	0·681	89 3	313 29	273 52	+ 8 42	N.
	6288		626	0·711	108 26	312 49	273 12	− 5 28	O.
2.	6289	335·504	622	0·867	303 15	60 34	6 3	+14 49	A.
	6290		622	0·911	299 18	57 45	3 14	+15 15	B.
	6291		622	0·888	301 52	59 6	4 35	+12 26	C.
	6292		622	0·922	298 0	54 35	0 4	+12 27	a.
	6293		622	0·936	302 14	57 41	3 10	+11 1	b.
	6294		622	0·940	297 53	55 57	1 26	+11 18	c.
	6295		622	0·948	297 59	59 9	4 38	+12 1	d.
	6296		623	0·704	304 29	37 50	343 19	+11 52	E.
	6297		623	0·698	305 44	39 13	344 42	+13 14	e.
	6298		624	0·633	304 32	26 55	332 24	+ 9 36	S.
	6299		624	0·616	305 55	30 45	336 14	+10 19	s.
	6300		624	0·595	304 25	29 57	335 26	+11 20	T.
	6301		624	0·588	301 55	31 10	336 39	+11 51	t.
	6302		625	0·487	84 47	328 43	274 12	+ 8 6	X.
	6303		625	0·491	85 13	327 16	272 45	+ 8 57	x.
	6304		626	0·502	109 0	327 53	273 22	− 5 3	y.
5.	6305	338·468	623	0·984	296 12	81 14	344 40	+12 19	A.
	6306		624	0·979	294 59	71 41	335 7	+10 18	B.
	6307		624	0·972	295 37	73 28	336 54	+12 57	C.
	6308		624	0·966	294 6	72 9	335 35	+11 16	b.
	6309		625	0·287	321 31	11 17	274 43	+ 8 38	D.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. Dec. 5.	6310	338·468	627	0·712	114° 42′	311° 41′	215° 7′	−11° 45′	E.
	6311		627	0·723	113 8	310 36	213 58	−12 7	e.
8.	6312	341·584	627	0·246	161 32	355 38	214 52	−11 30	A.
	6313		627	0·249	161 12	354 13	213 27	−11 32	a.
9.	6314	342·608	627*a*	0·417	279 22	23 30	268 13	− 4 47	A.
	6315		627*a*	0·412	278 34	22 53	227 36	− 3 59	a.
	6316		628	0·921	113 13	300 48	145 31	− 2 6	b.
19.	6317	352·504	629	0·879	273 48	66 51	131 11	− 7 38	A.
	6318		629	0·854	267 59	68 10	132 30	− 5 34	B.
	6319		629	0·860	268 9	66 12	130 32	− 5 42	C.
	6320		629	0·799	269 18	69 1	133 21	− 4 3	D.
	6321		629	0·814	268 29	69 25	133 45	− 6 43	a.
	6322		629	0·833	267 54	70 12	134 32	− 6 5	b.
	6323		629	0·798	269 8	70 55	135 15	− 6 31	c.
20.	6324	353·500	629	0·954	265 16	81 15	131 28	− 5 40	A.
	6325		629	0·955	269 3	83 45	133 58	− 5 50	B.
	6326		629	0·936	270 32	82 50	133 3	− 4 38	a.
	6327		629	0·928	267 14	82 13	132 26	− 6 28	b.
	6328		629	0·922	268 23	79 51	130 4	− 4 56	c.
	6329		629	0·926	271 10	83 59	134 12	− 4 28	d.
	6330		630	0·846	76 11	283 14	333 27	+18 15	M.
	6331		631	0·987	83 24	295 53	347 6	+16 1	N.
1865. Jan. 4.	6332	3·535	632	0·934	266 49	101 15	298 12	− 2 21	A.
	6333		633	0·412	60 47	8 57	205 54	+11 34	a.
	6334		633	0·476	61 36	5 29	202 26	+12 55	b.
	6335		633	0·483	61 0	3 35	200 32	+12 14	c.
	6336		633	0·508	62 25	0 9	197 6	+13 1	d.
7.	6337	6·602	633	0·424	283 21	50 38	204 10	+10 44	M.
	6338		634	0·759	99 20	346 28	140 0	−12 24	N.
9.	6339	8·479	633	0·744	276 53	82 55	209 45	+10 4	M^0.
	6340		634	0·429	115 24	12 49	139 39	−12 43	N^0.
23.	6341	22·514	637	0·864	239 50	104 58	32 43	−21 27	P.
	6342		637	0·853	238 17	105 43	33 28	−22 49	Q.
	6343		637	0·769	228 56	95 6	22 51	−18 16	p.
	6344		637	0·778	237 11	96 13	23 58	−19 30	q.
	6345		638	0·308	249 27	65 1	352 46	− 3 34	A.
	6346		638	0·292	244 43	58 8	345 53	− 4 36	B.
	6347		638	0·281	240 16	62 48	350 33	− 4 23	C.
	6348		638	0·239	242 1	63 24	351 9	− 5 1	a.
	6349		638	0·224	243 53	57 1	344 46	− 5 12	a^0.
	6350		639	0·149	182 16	49 54	337 39	− 6 35	M.
	6351		639	0·202	165 54	45 2	332 47	− 7 57	m.
	6352		639	0·187	164 26	46 11	333 56	− 7 38	n.
	6353		640	0·185	351 30	48 6	335 51	+10 48	S.
	6354		640	0·180	2 2	46 5	333 50	+ 9 25	s.
	6355		640	0·172	4 9	46 55	334 40	+11 5	s^0.
	6356		641	0·607	82 44	11 53	299 38	+ 2 51	D.
	6357		641	0·611	83 52	11 21	299 6	+ 2 42	d.
	6358		642	0·695	103 8	6 46	294 31	−11 18	E.
	6359		642	0·784	104 26	1 13	288 58	−13 4	e.
	6360		642	0·806	105 15	0 1	287 46	−14 23	f.
	6361		643	0·946	78 0	339 41	267 26	+ 6 17	F.
Feb. 2.	6362	32·532	643	0·988	274 9	124 22	270 1	+ 6 12	A.
	6363		643*a*	0·743	259 45	107 53	253 32	+ 2 27	a.
	6364		645	0·566	214 54	84 14	229 53	−25 10	B.
	6365		645	0·561	210 34	82 59	228 38	−24 55	C.

Table III. (continued).

Date.	No.	Mean Tim of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Feb. 2.	6366	32·532	645	0·498	211° 36′	79° 14′	224° 53′	−23° 50′	D.
	6367		645	0·536	205 24	80 12	225 52	−23 44	E.
	6368		645	0·540	203 21	74 45	220 24	−24 32	b.
	6369		645	0·511	202 46	75 28	221 7	−25 30	c.
	6370		645	0·482	207 18	73 11	218 50	−25 48	d.
	6371		645	0·474	202 23	74 22	220 1	−23 44	e.
	6372		644	0·164	271 1	66 58	212 37	+ 2 56	F.
	6373		644	0·162	269 5	66 16	211 55	+ 3 33	f.
	6374		646	0·402	162 48	55 16	201 55	−22 21	G.
	6375		646	0·395	159 29	53 31	199 10	−21 52	g.
9.	6376	39·504	644	0·907	271 34	165 57	212 42	+ 3 19	M.
	6377		644	0·885	273 10	165 36	212 21	+ 4 7	m.
	6378		647*a*	0·282	6 29	63 6	109 51	+16 47	P.
	6379		648	0·368	34 23	52 15	99 0	+15 28	R.
	6380		648	0·375	37 56	50 29	97 14	+17 37	r.
	6381		649	0·264	141 11	52 38	99 23	−11 57	S.
	6382		649	0·265	139 54	51 18	98 3	−12 44	T.
	6383		649	0·272	135 39	51 57	98 42	−12 41	s.
	6384		649	0·287	132 20	49 3	95 48	−10 15	t.
15.	6385	45·605	647	0·436	287 14	93 49	54 2	+14 41	A.
	6386		647	0·459	285 51	97 39	57 52	+15 24	a.
	6387		647	0·491	283 4	101 10	61 23	+16 25	B.
	6388		647	0·512	283 4	104 8	64 21	+13 17	b.
	6389		649	0·398	218 29	89 40	49 53	−13 5	C.
	6390		649	0·382	212 34	90 5	50 18	−14 41	c.
	6391		649	0·428	217 3	87 14	47 27	−12 20	d.
17.	6392	47·514	647	0·902	264 29	135 45	68 53	+13 22	M.
	6393		649	0·865	231 38	130 19	63 27	−15 3	A.
	6394		649	0·863	230 18	129 47	62 55	−15 27	a.
25.	6395	55·434	650	0·908	236 30	144 33	325 21	− 9 7	A.
	6396		651	0·264	249 59	97 11	277 59	+ 3 49	B.
28.	6397	58·507	651*a*	0·811	243 19	138 5	275 18	+ 0 28	P
	6398		651*a*	0·822	244 59	140 31	277 44	− 3 43	p.
	6399		651*a*	0·846	245 31	141 19	278 32	− 7 25	A.
	6400		651*a*	0·854	247 54	142 7	279 20	− 5 17	a.
	6401		652	0·583	277 29	120 46	257 59	+14 22	M.
	6402		652	0·575	276 46	121 16	258 29	+13 53	m.
	6403		652	0·508	283 0	115 7	252 20	+16 52	N.
	6404		652	0·514	284 50	114 13	251 26	+16 9	n.
	6405		653	0·887	52 35	26 55	164 8	+13 42	O.
	6406		653	0·906	51 58	24 28	161 51	+12 57	o.
Mar. 1.	6407	59·546	651*a*	0·900	243 32	157 19	279 47	− 3 53	M.
	6408		651*a*	0·889	244 27	155 19	277 47	− 4 56	m.
	6409		652	0·754	268 29	133 23	255 51	+13 30	N.
	6410		652	0·749	269 34	133 48	256 16	+13 54	n.
	6411		652	0·673	271 56	131 19	252 47	+13 39	O.
	6412		652	0·670	271 37	133 57	256 25	+14 24	o.
	6413		653	0·871	55 48	42 54	165 22	+12 26	P.
	6414		653	0·904	54 0	40 53	163 21	+13 11	p.
3.	6415	61·616	652	0·972	259 45	161 16	254 22	+12 3	A.
	6416		652	0·914	262 19	162 52	255 58	+13 26	a.
	6417		653	0·508	43 3	61 29	154 35	+12 46	M.
	6418		653	0·519	43 43	60 50	153 56	+12 54	m.
	6419		653	0·522	45 9	61 46	154 52	+15 13	N.
	6420		653	0·597	46 27	53 14	146 20	+16 46	n.
8.	6421	66·465	653	0·624	265 53	133 17	157 37	+12 10	S.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Mar. 8.	6422	66·465	653	0·622	266° 42′	132° 31′	156° 20′	+12° 52′	s.
	6423		654	0·417	319 34	97 9	121 29	+22 54	T.
9.	6424	67·450	653	0·788	258 21	146 3	156 0	+12 27	A.
	6425		653	0·785	257 17	147 47	157 43	+12 2	a.
	6426		654	0·517	289 12	115 58	125 55	+19 0	B.
	6427		654	0·489	287 55	116 40	126 37	+20 38	b.
	6428		654	0·492	287 59	116 41	126 38	+20 47	b^1.
	6429		655	0·467	59 25	68 5	78 2	+ 7 16	C.
	6430	67·479	653	0·791	259 3	146 18	155 38	+12 29	A.
	6431		653	0·789	256 55	148 9	157 29	+12 16	a.
	6432		654	0·519	288 42	116 48	126 8	+18 52	B.
	6433		654	0·492	288 16	117 20	126 40	+20 41	b.
	6434		654	0·495	287 30	117 19	126 39	+20 53	b^1.
	6435		655*a*	0·464	59 11	68 27	77 47	+ 7 14	C.
13.	6436	71·444	655	0·246	309 24	104 20	58 2	+13 22	A.
	6437		655	0·253	317 27	101 21	55 3	+14 56	B.
	6438		655	0·255	324 38	99 47	53 29	+13 21	C.
	6439		655	0·261	339 41	97 34	51 18	+15 1	a.
	6440		655	0·264	342 46	95 7	48 21	+16 58	b.
	6441		656	0·638	254 53	135 8	88 22	+ 7 27	D.
	6442		656	0·645	252 4	135 35	89 17	+ 6 2	c.
	6443		656	0·657	252 34	136 59	90 41	+ 7 42	d.
	6444		657	0·299	250 13	117 39	71 21	+ 1 16	P.
	6445		657	0·338	247 11	115 22	69 4	+ 2 21	p.
	6446		657	0·272	242 22	113 18	67 0	− 0 33	M.
	6447		657	0·211	245 47	110 43	64 25	− 1 13	m.
	6448		657	0·198	241 41	109 23	63 5	+ 1 11	m^0.
17.	6449	75·630	656	0·933	248 8	193 40	88 0	+ 7 44	A.
	6450		656	0·957	247 41	194 50	89 10	+ 7 4	a.
	6451		655	0·744	259 46	162 46	57 6	+14 13	B.
	6452		655	0·740	258 16	158 26	52 46	+14 3	b.
	6453		655	0·662	259 36	161 8	55 28	+13 59	C.
	6454		655	0·625	260 9	160 28	54 48	+15 26	c.
20.	6455	78·490	655	0·894	253 38	203 45	57 30	+14 49	A.
	6456		655	0·886	254 29	202 8	55 53	+13 12	a.
	6457	78·510	655	0·897	253 4	204 20	57 48	+14 45	A.
	6458		655	0·890	254 37	202 14	55 42	+13 17	a.
21.	6459	79·486	655	0·972	251 30	218 11	57 49	+14 4	M.
	6460	79·636	655	0·980	250 56	220 22	57 52	+14 0	M.
22.	6461	80·493	658	0·371	39 25	88 7	273 28	+10 22	A.
	6462		658	0·370	43 58	86 19	271 40	+11 13	B.
	6463		658	0·393	40 34	86 59	272 20	+12 30	C.
	6464		658	0·387	41 23	88 27	273 48	+11 53	a.
	6465		658	0·404	45 0	89 42	275 3	+ 9 57	a^0.
	6466		659	0·555	62 33	74 52	260 13	+ 3 48	D.
	6467		659	0·560	63 2	72 32	257 53	+ 3 38	E.
	6468		659	0·672	65 8	72 47	258 8	+ 3 42	F.
	6469		659	0·675	65 25	66 7	251 28	+ 4 28	b.
	6470		659	0·679	66 58	67 48	253 9	+ 3 19	c.
	6471		659	0·748	66 34	66 39	252 0	+ 2 8	d.
	6472		659	0·750	67 44	60 7	245 28	+ 2 21	e.
	6473		659	0·772	64 47	59 38	244 59	+ 2 5	f.
	6474		659	0·783	63 31	61 15	246 36	+ 2 58	G.
	6475		659	0·781	65 48	62 3	247 24	+ 2 13	g.
	6476		660	0·753	58 3	57 18	242 39	+10 12	M.
	6477		660	0·762	58 32	58 48	244 9	+10 29	m.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Mar. 22.	6478	80·493	660	0·794	56° 19′	55° 1′	240° 22′	+ 8° 20′	A°.
	6479		660	0·801	59 38	56 37	241 58	+ 8 5	a^{1}.
	6480		661	0·555	95 23	77 41	263 2	−13 52	P.
	6481		661	0·561	95 18	77 56	263 17	−13 13	p.
	6482		661	0·562	97 2	78 45	264 6	−14 12	Q.
	6483		661	0·574	98 47	78 27	263 48	−15 27	q.
	6484		661	0·569	107 2	78 23	263 44	−20 41	R.
	6485		661	0·573	108 53	79 8	264 29	−21 25	r.
23.	6486	81·479	662	0·891	57 25	49 24	220 46	+ 9 52	S.
	6487		662	0·897	58 7	47 42	219 4	+ 9 48	s.
24.	6488	82·476	662	0·751	54 52	63 54	221 7	+ 9 20	M.
	6489		662	0·762	53 20	61 2	218 15	+10 5	m.
	6490	82·518	662	0·746	54 29	64 53	221 30	+ 9 28	M.
	6491		662	0·757	53 10	61 34	218 11	+10 2	m.
27.	6492	85·607	663	0·783	42 55	65 35	178 24	+19 57	A.
	6493		663	0·791	43 14	62 44	175 33	+20 2	B.
	6494		663	0·795	42 39	64 21	177 10	+16 50	C.
	6495		663	0·810	45 8	62 29	175 18	+17 46	D.
	6496		663	0·807	44 23	62 10	174 59	+17 52	a.
	6497		663	0·831	44 23	60 26	173 15	+18 57	b.
	6498		663	0·836	44 46	61 3	173 52	+19 33	c.
	6499		663	0·849	45 2	59 28	172 17	+19 26	d.
	6500		663	0·844	46 47	59 17	172 6	+19 15	e.
	6501	85·624	663	0·780	42 6	66 8	178 42	+20 3	A.
	6502		663	0·788	43 3	62 49	175 23	+19 53	B.
	6503		663	0·792	42 10	64 27	177 1	+17 0	C.
	6504		663	0·808	45 25	63 4	175 38	+17 44	D.
	6505		663	0·805	44 41	61 41	174 15	+18 5	a.
	6506		663	0·824	44 27	60 59	173 33	+19 6	b.
	6507		663	0·831	44 49	61 20	173 54	+19 30	c.
	6508		663	0·844	45 34	59 26	172 0	+19 21	d.
	6509		663	0·839	46 13	59 16	171 50	+19 19	e.
28.	6510	86·482	663	0·622	35 5	71 39	172 3	+19 16	M.
	6511		663	0·643	37 36	76 53	177 17	+20 38	N.
	6512		663	0·641	36 51	77 48	178 12	+18 43	O.
	6513		663	0·657	35 14	77 12	177 36	+17 38	m.
	6514		663	0·666	36 31	76 30	176 54	+16 21	n.
	6515		663	0·692	38 14	71 21	171 45	+18 8	o.
	6516		663	0·688	35 59	71 33	171 57	+17 28	p.
	6517		663	0·711	39 59	71 36	172 0	+19 7	q.
	6518	86·588	663	0·619	35 48	72 58	171 52	+19 12	M.
	6519		663	0·639	37 0	78 36	177 30	+20 24	N.
	6520		663	0·635	36 2	79 5	177 59	+18 40	O.
	6521		663	0·650	35 26	79 10	178 4	+17 39	m.
	6522		663	0·662	36 31	78 5	176 59	+16 22	n.
	6523		663	0·690	38 9	73 8	172 2	+18 5	o.
	6524		663	0·684	36 17	73 22	172 16	+17 29	p.
	6525		663	0·707	39 37	73 21	172 15	+19 0	q.
30.	6526	88·462	663	0·384	358 13	107 31	179 50	+20 56	A.
	6527		663	0·389	3 21	106 56	179 15	+21 36	B.
	6528		663	0·382	1 7	103 11	175 30	+20 47	C.
	6529		663	0·414	359 28	100 55	173 14	+19 58	D.
	6530		663	0·427	0 26	101 6	173 25	+18 22	a.
	6531		663	0·379	7 21	102 48	175 7	+18 5	b.
	6532		663	0·395	5 5	99 47	172 6	+17 16	c.
	6533		663	0·411	8 34	98 14	170 33	+17 7	d.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Mar. 30.	6534	88·462	663	0·452	12° 42′	99° 25′	171° 44′	+16° 32′	e.
	6535		663	0·457	14 25	99 19	171 38	+17 50	f.
	6536	88·478	663	0·383	359 55	107 32	179 37	+20 55	A.
	6537		663	0·388	3 58	106 49	178 54	+20 59	B.
	6538		663	0·380	1 15	103 47	175 52	+20 45	C.
	6539		663	0·414	359 56	100 29	172 34	+19 50	D.
	6540		663	0·425	1 49	102 0	174 5	+18 20	a.
	6541		663	0·377	6 22	103 34	175 39	+18 10	b.
	6542		663	0·393	5 39	100 13	172 18	+17 24	c.
	6543		663	0·407	8 47	99 59	172 4	+17 15	d.
	6544		663	0·450	12 48	98 44	170 49	+17 0	e.
	6545		663	0·455	14 57	99 27	171 32	+18 4	f.
31.	6546	89·479	663	0·375	325 14	119 44	177 37	+18 29	M.
	6547		663	0·379	327 59	121 5	178 58	+17 47	N.
	6548		663	0·370	340 55	117 39	175 32	+16 8	O.
	6549		663	0·363	328 26	116 8	174 1	+20 43	P.
	6550		663	0·350	330 13	115 33	173 26	+21 42	m.
	6551		663	0·351	336 42	114 55	172 48	+16 5	n.
	6552		663	0·361	339 16	113 46	171 39	+17 3	o.
	6553		663	0·346	346 39	113 43	171 36	+18 25	p.
	6554	89·490	663	0·377	325 56	119 27	177 11	+18 25	M.
	6555		663	0·382	328 46	121 13	178 57	+17 49	N.
	6556		663	0·374	341 28	117 32	175 16	+16 18	O.
	6557		663	0·365	328 56	116 29	174 13	+20 44	P.
	6558		663	0·354	330 13	115 44	173 28	+21 50	m.
	6559		663	0·356	336 31	114 21	172 5	+16 0	n.
	6560		663	0·362	339 11	113 18	171 2	+17 6	o.
	6561		663	0·350	346 40	113 20	171 4	+18 40	p.
Apr. 1.	6562	90·445	663	0·452	290 57	135 36	179 47	+20 34	S.
	6563		663	0·449	294 37	132 56	177 7	+18 43	T.
	6564		663	0·399	292 22	132 16	176 27	+17 10	U.
	6565		662	0·424	301 34	133 3	177 14	+16 34	s.
	6566		663	0·430	299 51	129 25	173 36	+19 37	t.
	6567		663	0·387	305 34	127 39	171 50	+19 18	u.
	6568		663	0·381	304 34	127 45	171 56	+18 14	v.
3.	6569	92·448	663	0·738	268 26	161 31	177 17	+18 33	A.
	6570		663	0·740	269 37	162 27	178 13	+18 5	a^{0}.
	6571		663	0·692	267 2	159 58	175 44	+19 6	B.
	6572	92·469	663	0·741	269 20	161 52	177 21	+18 36	A.
	6573		663	0·745	269 20	162 40	178 9	+18 2	a^{0}.
	6574		663	0·695	267 56	160 13	175 42	+18 54	B.
4.	6575	93·524	663	0·875	262 41	177 6	177 37	+18 26	M.
	6576		663	0·873	261 35	178 6	178 37	+17 54	m.
	6577	93·535	663	0·877	262 6	177 20	177 58	+18 29	M.
	6578		663	0·876	261 57	178 8	178 46	+17 52	m.
6.	6579	95·449	663	0·988	255 21	205 52	179 5	+18 20	A.
	6580		663	0·991	254 15	206 18	179 31	+17 55	a.
	6581	95·475	663	0·990	256 24	206 43	179 33	+18 31	A.
	6582		663	0·993	254 15	206 37	179 27	+17 59	a.
8.	6583	97·446	664	0·528	281 42	152 1	96 54	+17 8	S.
	6584		664	0·520	282 35	153 24	98 17	+18 34	s.
	6585		664	0·479	289 33	150 4	94 57	+20 15	t.
10.	6586	99·487	664	0·833	261 24	182 49	98 45	+19 11	S.
	6587		664	0·835	261 5	182 7	98 3	+20 27	s.
	6588		665	0·961	46 29	58 44	334 40	+18 27	t.
11.	6589	100·495	664	0·922	258 26	196 32	98 10	+19 37	Q.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Apr. 11.	6590	100·495	665	0·875	44° 17′	73° 24′	335° 2′	+18° 21′	R.
	6591	100·510	664	0·925	258 0	196 2	97 27	+19 45	Q.
	6592		665	0·871	45 14	74 31	335 56	+18 55	R.
12.	6593	101·654	665	0·702	39 7	89 24	334 36	+19 0	M.
13.	6594	102·464	665	0·568	31 58	101 32	335 14	+18 9	A.
	6595		666	0·892	234 59	196 45	70 27	− 8 49	B.
	6596		666	0·881	233 13	194 37	68 19	− 8 4	b.
20.	6597	109·522	667	0·583	276 57	170 17	304 1	+16 45	A.
	6598		667	0·576	277 48	169 16	303 0	+16 44	B.
	6599		667	0·569	276 0	168 58	302 42	+17 24	a.
	6600		667	0·555	279 51	166 24	300 8	+18 0	b.
	6601		668	0·264	348 10	133 4	266 48	+14 13	C.
	6602		668	0·265	349 8	133 43	267 27	+14 1	D.
	6603		668	0·277	349 42	132 58	266 42	+15 41	c.
	6604		668	0·287	350 10	134 28	268 12	+16 11	d.
	6605	109·535	667	0·585	276 44	170 27	303 51	+16 40	A.
	6606		667	0·577	277 42	169 46	303 10	+16 42	B.
	6607		667	0·570	277 34	169 25	302 49	+17 28	a.
	6608		667	0·556	278 15	167 26	300 50	+17 55	b.
	6609		668	0·265	348 8	133 48	267 12	+14 12	C.
	6610		668	0·267	349 6	134 6	267 30	+14 9	D.
	6611		668	0·279	349 16	133 17	266 41	+15 47	c.
	6612		668	0·288	350 47	135 25	268 49	+16 20	d.
21.	6613	110·467	667	0·728	269 57	182 11	302 22	+17 53	S.
	6614		667	0·710	271 55	184 21	304 32	+16 4	T.
	6615		667	0·712	269 23	182 47	302 58	+16 49	s.
	6616		667	0·693	272 58	182 14	302 25	+16 0	t.
	6617		667	0·690	272 22	180 28	300 39	+17 54	u.
	6618		668	0·284	299 20	144 59	265 10	+14 5	A.
	6619		668	0·295	301 15	146 38	266 49	+14 32	B.
	6620		668	0·306	303 32	148 34	268 45	+14 18	a.
	6621		668	0·309	304 33	147 6	267 17	+15 32	b.
	6622		669	0·895	264 31	198 9	318 20	+13 31	C.
	6623		669	0·883	266 28	196 22	316 33	+14 41	c.
	6624	110·489	667	0·731	269 9	182 37	302 30	+17 56	S.
	6625		667	0·714	271 29	184 42	304 35	+16 6	T.
	6626		667	0·716	269 55	182 8	302 1	+16 53	s.
	6627		667	0·697	272 42	182 9	302 2	+15 49	t.
	6628		667	0·692	272 54	180 19	300 12	+17 55	u.
	6629		668	0·285	299 16	145 27	265 20	+14 5	A.
	6630		668	0·297	301 42	146 34	266 27	+14 27	B.
	6631		668	0·308	303 1	148 33	268 26	+14 17	a.
	6632		668	0·311	304 48	147 41	267 34	+15 32	b.
	6633		669	0·897	264 16	198 46	318 39	+13 39	C.
	6634		669	0·885	266 7	196 18	316 11	+14 40	c.
24.	6635	113·616	667	0·992	258 7	227 48	303 19	+15 3	A.
	6636		670	0·687	41 3	104 42	180 13	+13 34	a.
25.	6637	114·496	670	0·517	35 36	116 42	179 44	+15 9	M.
	6638		670	0·533	37 31	118 14	181 16	+16 26	m.
	6639		670	0·548	38 31	115 12	178 14	+16 9	n.
26.	6640	115·495	670	0·392	21 27	128 47	177 39	+13 55	A.
	6641		670	0·385	19 20	132 53	181 45	+14 29	a.
	6642		670	0·399	15 37	129 46	178 38	+16 50	B.
	6643		670	0·407	11 28	129 31	178 23	+17 36	b.
27.	6644	116·504	670	0·262	342 24	144 27	179 0	+14 56	a.
	6645		670	0·275	341 6	142 33	177 5	+14 6	b.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Apr. 27.	6646	116·504	670	0·281	339° 22′	142° 48′	177° 21′	+16° 23′	c.
	6647		671	0·223	137 46	138 54	173 27	− 8 9	A.
	6648		671	0·227	142 42	137 14	171 47	− 8 29	B.
	6649		671	0·231	139 13	135 47	170 20	− 8 14	C.
	6650		671	0·240	143 27	136 28	171 1	− 7 59	D.
May 2.	6651	121·608	674	0·800	262 51	206 35	168 44	+12 17	d.
	6652		672	0·654	77 37	111 17	73 26	− 7 31	P.
	6653		672	0·657	77 18	112 22	74 31	− 7 48	M.
	6654		672	0·663	79 16	110 17	72 26	− 8 2	m.
	6655		673	0·577	37 21	117 14	79 23	+14 8	S.
	6656		673	0·600	40 41	118 31	80 40	+15 0	s.
	6657		673	0·584	38 40	120 38	82 47	+16 51	T.
	6658		673	0·612	41 5	119 37	81 46	+14 15	t.
3.	6659	122·501	672	0·488	82 5	122 48	72 17	− 7 2	A.
	6660		673	0·431	24 25	132 8	81 37	+17 19	B.
	6661		673	0·454	37 21	129 48	79 17	+15 59	C.
	6662		673	0·472	30 22	131 51	81 20	+16 42	b.
	6663		673	0·438	29 15	126 33	76 2	+16 35	c.
	6664		673	0·487	38 35	124 31	74 0	+14 32	d.
5.	6665	124·538	672	0·217	175 36	153 11	73 47	− 8 12	M.
	6666		673	0·231	292 56	162 19	82 55	+13 29	P.
	6667		673	0·240	299 17	159 3	79 39	+13 18	Q.
	6668		673	0·307	311 57	158 1	78 37	+15 18	R.
	6669		673	0·287	309 55	160 41	81 17	+15 58	p.
	6670		673	0·343	314 37	156 43	77 19	+16 3	q.
	6671		675	0·887	71 57	93 44	14 20	− 4 50	r.
	6672		675	0·890	72 37	92 33	13 9	− 5 56	s.
	6673		675	0·892	73 0	89 0	10 0	− 5 46	t.
	6674		675	0·877	69 31	96 0	16 36	− 5 15	A.
	6675		676	0·981	71 18	83 7	3 43	+ 1 13	B.
6.	6676	125·669	673	0·407	271 19	172 33	77 6	+15 16	A.
	6677		673	0·423	272 6	176 13	80 46	+16 33	a.
	6678		675	0·658	66 10	110 30	15 3	− 4 6	B.
	6679		675	0·693	68 17	106 25	10 58	− 5 56	C.
	6680		675	0·714	70 18	106 28	11 1	− 3 59	D.
	6681		675	0·701	71 42	110 18	14 51	− 5 41	b.
	6682		675	0·688	73 43	109 16	13 49	− 5 8	c.
	6683		675	0·669	72 43	111 15	15 48	− 3 17	d.
	6684		675	0·732	69 44	112 18	16 51	− 5 35	e.
	6685		676	0·857	70 37	98 44	3 17	− 0 7	E.
8.	6686	127·491	675	0·292	74 54	135 50	14 33	− 4 42	M.
	6687		675	0·316	76 20	137 49	16 32	− 5 18	m.
	6688		675	0·348	76 21	132 52	11 35	− 5 36	m^{1}.
	6689		676	0·517	69 12	124 31	3 14	+ 0 43	N.
9.	6690	128·591	675	0·054	168 37	152 10	15 16	− 3 30	A.
	6691		675	0·089	159 23	150 55	14 1	− 5 15	a.
	6692		675	0·112	154 24	148 37	11 43	− 5 17	a^{0}.
	6693		676	0·293	77 24	140 35	3 41	− 0 24	B.
	6694		676	0·295	77 10	139 57	3 3	+ 0 20	b.
	6695		677	0·672	43 48	115 50	338 56	+17 16	C.
	6696		677	0·695	52 49	112 14	335 20	+16 48	D.
	6697		677	0·702	48 18	109 1	332 7	+15 55	c.
	6698		677	0·708	54 7	108 54	332 0	+14 29	d.
12.	6699	131·651	678	0·491	251 6	184 5	3 47	− 2 0	A.
	6700		678	0·470	250 7	183 41	3 23	− 3 18	A^{0}.
	6701		678	0·463	246 24	182 42	2 24	− 4 25	a.

Table III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. May 12.	6702	131·651	678	0·429	240° 40′	179° 36′	359° 18′	− 4° 4′	a⁰.
	6703		678	0·444	242 39	181 16	0 58	− 4 0	B.
	6704		678	0·476	242 40	178 39	358 21	− 6 40	b.
	6705		678	0·483	248 24	183 4	2 46	− 5 47	C.
	6706		678	0·487	249 39	183 14	2 56	− 7 14	c.
	6707		678	0·424	239 55	178 54	358 36	− 8 53	c¹.
	6708		679	0·273	281 56	170 14	349 56	+ 6 56	X.
	6709		679	0·259	283 48	169 19	349 1	+ 7 3	x.
13.	6710	132·660	678	0·663	244 52	198 35	3 58	− 5 45	A.
	6711		678	0·612	239 45	196 15	1 38	− 5 46	B.
	6712		678	0·604	242 37	197 51	3 14	− 5 56	C.
	6713		678	0·657	240 20	198 3	3 26	− 6 3	D.
	6714		678	0·660	241 26	194 16	359 39	− 6 12	E.
	6715		678	0·639	241 1	194 56	0 19	− 5 34	a.
	6716		678	0·623	240 53	195 22	0 45	− 7 47	b.
	6717		678	0·636	241 53	195 43	1 6	− 8 54	c.
	6718		678	0·598	239 26	193 59	359 22	− 8 35	d.
	6719		679	0·408	265 20	183 39	349 2	+ 5 33	F.
	6720		679	0·427	266 12	182 29	347 52	+ 5 26	G.
	6721		679	0·420	266 29	180 40	346 3	+ 6 22	e.
	6722		679	0·419	265 1	179 22	344 45	+ 7 27	f.
	6723		679	0·431	267 18	181 2	346 25	+ 7 13	g.
18.	6724	137·528	680	0·864	269 50	229 28	325 49	+10 16	A.
	6725		680	0·831	267 57	227 48	324 9	+11 13	B.
	6726		680	0·822	268 17	227 10	323 31	+11 9	a.
	6727		680	0·855	269 45	226 29	322 50	+10 9	b.
	6728		680	0·820	267 0	226 56	323 17	+ 8 19	c.
19.	6729	138·505	680	0·942	266 28	240 2	322 31	+10 59	M.
	6730		680	0·937	265 48	242 0	324 29	+11 2	N.
	6731		680	0·921	265 17	241 38	324 7	+11 33	O.
	6732		681	0·628	260 34	202 52	285 21	+ 9 36	A.
	6733		681	0·612	261 49	201 10	283 39	+ 8 35	a.
	6734		681	0·579	260 9	199 34	282 3	+ 8 16	B.
	6735		681	0·562	261 52	197 53	280 23	+ 7 37	b.
22.	6736	141·503	681	0·960	255 12	242 6	282 3	+ 8 6	M.
	6737		681	0·991	256 20	240 46	280 43	+ 7 50	m.
23.	6738	142·655	No spots visible.						
24.	6739	143·513							
25.	6740	144·482	682	0·982	80 56	87 35	85 17	− 9 59	P.
	6741		683	0·988	61 47	90 57	88 39	+10 24	Q.
26.	6742	145·489	682	0·941	82 21	100 51	84 16	−10 59	P.
	6743		682	0·943	81 4	100 32	83 57	−10 54	p.
	6744		682	0·957	83 25	102 29	85 54	−11 44	q.
	6745		683	0·944	62 47	105 10	88 35	+ 9 51	R.
	6746		683	0·942	62 26	103 47	87 12	+10 59	r.
27.	6747	146·462	682	0·843	81 9	118 57	88 34	− 9 54	A.
	6748		682	0·848	82 17	115 54	85 31	−11 23	a.
	6749		682	0·859	82 56	117 40	87 17	−10 13	B.
	6750		682	0·937	81 11	111 17	80 54	−11 8	b.
	6751		683	0·842	59 24	118 44	88 21	+10 13	C.
	6752		683	0·846	60 38	117 28	87 5	+ 9 35	c.
29.	6753	148·647	682	0·490	91 39	144 55	83 32	−10 3	M.
	6754		682	0·497	90 51	148 28	87 5	−11 6	m.
	6755		682	0·523	90 4	146 55	85 32	− 9 20	n.
	6756		682	0·506	91 0	144 32	83 9	−11 26	o.
	6757		682	0·519	91 18	145 52	84 29	−10 55	p.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. May 29.	6758	148·647	682	0·533	93° 30′	147° 16′	85° 47′	−10° 58′	q.
	6759		683	0·488	54 53	150 59	89 36	+10 7	R.
	6760		683	0·491	55 0	149 36	88 13	+ 8 19	r.
30.	6761	149·510	682	0·337	104 32	160 0	86 24	− 9 34	S.
	6762		682	0·347	101 37	160 46	87 10	−11 41	T.
	6763		682	0·371	104 25	159 29	85 53	−10 40	s.
	6764		682	0·359	106 51	157 14	83 38	−11 38	t.
	6765		682	0·388	104 27	157 30	83 54	−11 46	a.
	6766		683	0·323	41 35	161 54	88 18	+ 9 45	B.
	6767		683	0·330	41 10	160 42	87 6	+ 9 8	b.
31.	6768	150·683	682	0·192	169 59	174 22	84 7	− 9 23	A.
	6769		682	0·190	168 22	175 56	85 41	−10 42	a.
	6770		682	0·206	169 7	173 20	83 5	− 9 7	B.
	6771		682	0·213	167 53	172 51	82 36	−10 38	b.
	6772		683	0·187	347 5	178 25	88 10	+10 21	C.
	6773		683	0·192	346 9	177 26	87 11	+ 9 22	c.
	6774		684	0·953	77 52	107 23	17 8	− 5 14	M.
	6775		684	0·950	78 22	105 19	15 4	− 3 16	m.
June 5.	6776	155·478	682	0·895	241 36	173 12	14 56	− 4 8	P.
	6777		682	0·906	244 34	241 39	83 23	−10 40	p.
	6778		682	0·904	242 25	239 54	81 38	− 9 53	Q.
	6779		682	0·917	246 32	243 22	85 6	−11 13	R.
	6780		682	0·922	244 19	245 14	86 58	−10 5	q.
	6781		682	0·924	247 49	243 37	85 21	−11 10	r.
	6782		684	0·109	139 49	176 50	18 34	− 3 19	S.
	6783		684	0·129	131 8	173 39	15 23	− 5 37	s.
	6784		684	0·157	124 55	172 26	14 10	− 4 34	t.
6.	6785	156·533	682	0·973	243 16	259 4	85 50	−10 51	A.
	6786		682	0·974	245 21	254 7	80 53	−11 35	a.
	6787		686	0·980	68 47	108 57	295 43	+ 3 12	M.
	6788		686	0·978	68 13	114 1	300 47	+ 5 45	m.
7.	6789	157·619	686	0·907	69 46	126 9	297 31	+ 6 40	A.
	6790		686	0·900	68 33	130 22	301 44	+ 4 27	B.
	6791		686	0·862	67 53	129 14	300 36	+ 3 41	a.
	6792		686	0·844	66 34	124 37	295 59	+ 5 57	b.
	6793		687	0·444	253 41	206 19	17 41	− 7 41	C.
	6794		687	0·447	254 28	207 21	18 43	− 7 14	c.
8.	6795	158·543	686	0·705	68 36	134 48	293 3	+ 6 14	D.
	6796		686	0·771	69 57	141 3	299 18	+ 5 39	E.
	6797		686	0·753	70 53	137 36	295 51	+ 6 47	F.
	6798		686	0·722	69 26	138 20	296 35	+ 5 20	d.
	6799		686	0·794	70 51	135 49	294 4	+ 4 54	e
	6800		686	0·760	72 32	141 34	299 49	+ 7 43	f.
	6801		686	0·802	74 28	136 7	294 22	+ 8 36	g.
	6802		687	0·625	249 22	219 14	17 29	− 8 9	H.
9.	6803	159·473	686	0·521	67 51	149 5	294 9	+ 5 27	A.
	6804		686	0·536	65 46	154 41	299 45	+ 6 12	B.
	6805		686	0·628	66 15	148 14	293 18	+ 7 14	a.
	6806		686	0·579	67 30	153 52	298 56	+ 7 11	b.
	6807		686	0·637	65 32	150 39	295 43	+ 7 38	c.
	6808		687	0·801	251 51	233 37	18 41	− 7 3	D.
12.	6809	162·515	686	0·107	281 26	199 44	301 39	+ 4 53	M.
	6810		686	0·240	312 45	192 9	294 4	+ 4 7	m.
	6811		686	0·117	292 29	198 42	300 37	+ 5 36	N.
	6812		686	0·129	328 23	197 8	299 3	+ 4 54	n.
	6813		686	0·179	329 32	192 27	294 22	+ 6 22	o.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1864. June 12.	6814	162·515	686	0·223	330° 51′	191° 24′	293° 19′	+ 6° 46′	p.
	6815		686	0·189	317 29	191 21	293 16	+ 7 18	q.
	6816		686	0·242	295 11	190 40	292 35	+ 7 38	r.
	6817		688	0·167	154 25	184 59	286 54	− 5 29	S.
13.	6818	163·476	686	0·295	282 18	206 19	294 36	+ 5 21	A.
	6819		686	0·299	275 31	209 58	298 15	+ 6 4	a.
	6820		686	0·407	278 40	207 33	295 50	+ 7 37	B.
	6821		686	0·359	280 44	210 48	299 5	+ 7 10	b.
	6822		686	0·443	271 16	212 21	300 38	+ 7 33	c.
14.	6823	164·519	686	0·612	269 19	220 51	294 20	+ 7 58	M.
	6824		686	0·599	267 36	225 31	299 0	+ 6 39	A.
	6825		686	0·547	269 24	221 22	294 51	+ 5 16	a.
	6826		686	0·572	268 7	222 13	295 42	+ 7 50	B.
	6827		686	0·511	269 44	227 45	301 14	+ 7 18	b.
16.	6828	166·648	No spots visible.						
23.	6829	173·513	689	0·968	95 49	124 44	70 39	− 4 52	M.
July 3.	6830	183·515	No spots visible.						
4.	6831	184·546	690	0·982	88 53	128 54	278 19	− 5 40	S.
5.	6832	185·640	690	0·921	93 14	145 24	279 18	− 6 6	S^{0}.
7.	6833	187·521	690	0·643	97 14	171 31	278 44	− 6 45	S^{1}.
	6834		691	0·497	83 0	186 32	293 45	+ 3 8	A.
	6835		691	0·533	78 16	185 5	292 18	+ 4 50	a.
10.	6836	190·512	690	0·137	168 1	214 29	279 17	− 5 55	M.
	6837		691	0·197	281 47	228 7	292 55	+ 3 16	A.
	6838		691	0·212	288 0	227 29	292 17	+ 4 38	a.
	6839		691	0·240	292 36	224 45	289 33	+ 6 45	B.
	6840		692	0·792	101 17	165 28	230 16	− 7 16	b.
11.	6841	191·644	690	0·302	249 3	231 44	280 28	− 6 8	A.
	6842		692	0·576	104 12	180 29	229 13	− 6 24	a.
12.	6843	192·694	690	0·498	263 58	245 58	279 49	− 5 0	M.
	6844		691	0·627	282 46	258 54	292 45	+ 4 40	A.
	6845		691	0·658	282 32	259 30	293 21	+ 5 38	a.
	6846		692	0·404	117 40	197 6	230 57	− 7 48	B.
14.	6847	194·544	690	0·814	267 3	271 39	279 15	− 6 3	A.
	6848		692	0·190	207 24	221 32	229 8	− 7 22	a.
15.	6849	195·569	692	0·357	246 10	237 34	230 38	− 7 48	B.
20.	6850	200·458	693	0·346	149 56	213 24	137 7	−16 24	M.
	6851		693	0·377	145 2	208 26	132 9	−17 23	N.
	6852		693	0·406	142 30	207 35	131 18	−16 28	m.
	6853		693	0·419	139 59	205 56	129 39	−17 49	n.
26.	6854	206·522	694	0·430	85 29	201 42	39 24	+ 5 44	S.
	6855		694	0·491	86 51	203 16	40 58	+ 5 27	s.
27.	6856	207·506	694	0·238	76 32	217 13	40 58	+ 4 25	S^{0}.
	6857		694	0·243	75 45	215 27	39 12	+ 5 30	s^{0}.
28.	6858	208·545	694	0·098	358 2	232 39	41 39	+ 4 53	A.
	6859		694	0·132	0 4	229 23	38 23	+ 6 20	B.
	6860		694	0·119	3 21	231 42	40 42	+ 5 34	a.
	6861		694	0·144	4 39	228 56	37 56	+ 7 30	b.
29.	6862	209·624	694	0·386	291 57	246 53	40 25	+ 7 37	A.
	6863		694	0·395	296 59	247 52	41 34	+ 7 22	a.
	6864		694	0·402	299 2	244 0	37 42	+ 8 36	a^{0}.
	6865		695	0·238	71 43	220 23	14 5	+ 5 47	B.
	6866		695	0·243	71 48	219 37	13 19	+ 5 7	b.
	6867		695	0·280	72 3	217 32	11 14	+ 7 44	b^{0}.
	6868		696	0·283	106 57	216 32	10 14	− 3 58	C.
	6869		696	0·349	97 3	212 11	5 53	− 1 18	c.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Aug. 3.	6870	214·516	695	0·779	279° 9′	290° 12′	14° 31′	+ 5° 15′	M.
	6871		695	0·785	279 23	289 24	13 43	+ 7 3	m.
	6872		695	0·799	282 33	285 56	10 15	+ 6 34	N.
	6873		695	0·802	280 29	287 18	11 37	+ 4 6	n.
	6874		696	0·789	273 31	284 2	8 21	− 4 34	A.
	6875		696	0·749	272 19	286 22	10 41	− 3 26	a.
	6876		696	0·824	273 35	284 53	9 12	+ 0 3	b.
	6877		697	0·766	290 57	304 38	28 57	+11 18	B.
	6878		697	0·777	293 4	300 57	25 16	+11 47	C.
	6879		697	0·823	294 13	298 18	22 37	+14 29	D.
	6880		697	0·792	297 29	297 8	21 27	+15 30	E.
	6881		697	0·849	296 6	294 24	18 43	+13 51	b.
	6882		697	0·855	292 34	295 44	20 3	+14 20	c.
	6883		697	0·861	291 22	295 59	20 18	+12 21	d.
	6884		698	0·846	95 36	175 33	259 52	+ 5 17	F.
7.	6885	218·572	698	0·102	88 17	235 4	261 51	+ 5 25	M.
	6886		698	0·147	85 46	230 47	257 34	+ 3 40	N.
	6887		698	0·129	87 54	231 29	258 16	+ 3 33	O.
	6888		698	0·187	83 25	234 37	261 24	+ 4 9	m.
	6889		698	0·206	79 35	229 13	266 0	+ 6 7	n.
	6890		699	0·595	117 55	205 15	232 2	− 8 55	o.
	6891		699	0·613	115 21	205 11	231 58	− 9 40	p.
	6892		700	0·990	90 57	158 49	185 36	+ 5 4	Q.
	6893		700	0·987	91 27	156 13	183 0	+ 5 14	q.
9.	6894	220·439	698	0·358	290 22	259 6	259 24	+ 3 30	A.
	6895		698	0·347	294 43	261 11	261 29	+ 3 17	a.
	6896		699	0·282	146 19	231 28	231 46	− 9 51	B.
	6897		700	0·820	92 35	185 20	185 38	+ 6 41	C.
	6898		700	0·867	91 6	186 30	186 58	+ 7 44	c.
10.	6899	221·606	699	0·223	205 32	248 31	232 16	− 9 46	A^{0}.
	6900		700	0·614	91 4	201 55	185 40	+ 5 22	B.
	6901		700	0·682	92 7	203 27	187 12	+ 5 20	b.
	6902		700	0·679	91 5	202 1	185 46	+ 6 14	b^{0}.
12.	6903	223·429	700	0·288	85 23	229 37	187 30	+ 5 49	M.
	6904		700	0·319	83 55	227 13	185 6	+ 6 49	N.
	6905		700	0·331	81 22	226 45	184 38	+ 7 50	m.
14.	6906	225·495	700	0·196	310 39	254 42	183 16	+ 7 28	M^{0}.
	6907		700	0·240	329 21	256 34	185 8	+ 6 30	N^{0}.
	6908		701	0·972	115 26	170 49	99 23	−21 59	A.
16.	6909	227·490	701	0·812	123 52	196 30	96 47	−20 12	A^{0}.
	6910		701	0·851	121 41	197 32	97 49	−20 51	a.
17.	6911	228·485	701	0·640	129 11	212 54	99 4	−21 52	A.
	6912		701	0·708	126 25	212 18	98 28	−21 35	a.
	6913		702	0·990	117 51	173 41	59 51	−19 15	B.
	6914		702	0·981	116 40	174 22	60 32	−22 33	b.
	6915		702	0·964	115 59	174 53	61 3	−21 7	c.
18.	6916	229·472	701	0·512	139 19	227 31	99 41	−21 31	A.
	6917		701	0·569	135 11	222 17	94 27	−21 31	a.
	6918		702	0·870	121 0	193 17	65 27	−20 53	B.
	6919		702	0·902	119 53	189 14	61 24	−21 35	b.
	6920		702	0·921	117 25	186 2	58 12	−20 36	c.
21.	6921	232·667	703	0·859	99 39	192 53	19 44	+ 4 5	D.
	6922		703	0·876	94 28	190 33	17 24	+ 3 23	d.
	6923		703	0·897	96 7	193 13	20 4	+ 3 40	e.
	6924		703	0·891	95 10	191 9	18 0	+ 4 50	F.
	6925		703	0·906	96 9	193 52	20 43	+ 4 56	f.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Aug. 21.	6926	232·667	703	0·922	94° 50′	192° 50′	19° 41′	+ 3° 38′	g.
22.	6927	233·626	703	0·704	98 38	208 4	21 19	+ 5 27	M.
	6928		703	0·772	98 11	199 58	13 13	+ 6 4	N.
	6929		703	0·763	97 39	201 0	14 15	+ 4 10	O.
	6930		703	0·800	96 15	203 45	17 0	+ 7 43	P.
	6931		703	0·755	97 41	205 45	19 0	+ 6 40	m.
	6932		703	0·731	96 29	208 7	21 22	+ 8 15	n.
	6933		703	0·813	96 22	207 4	20 19	+ 3 21	o.
24.	6934	235·612	703	0·367	91 31	230 16	15 21	+ 4 23	A.
	6935		703	0·372	92 57	231 40	16 45	+ 4 6	B.
	6936		703	0·411	97 53	235 9	20 14	+ 5 10	C.
	6937		703	0·395	95 36	236 31	21 36	+ 6 16	D.
	6938		703	0·452	95 28	229 10	14 15	+ 8 58	a.
	6039		703	0·463	96 31	228 19	13 24	+ 7 30	b.
	6940		703	0·414	95 20	236 58	22 3	+ 6 5	c.
	6941		703	0·399	97 6	232 20	17 25	+ 5 11	d.
	6942		703	0·435	95 33	233 56	19 1	+ 4 10	e.
	6943		703	0·470	98 36	235 12	20 17	+ 3 35	f.
26.	6944	237·546	703	0·087	328 17	263 47	21 26	+ 4 27	M.
	6945		703	0·091	334 42	264 50	22 29	+ 3 2	N.
	6946		703	0·088	355 56	258 56	16 35	+ 3 3	O.
	6947		703	0·102	347 59	262 31	20 10	+ 4 15	P.
	6948		703	0·154	359 26	256 59	14 38	+ 3 1	Q.
	6949		703	0·133	13 4	257 7	14 46	+ 6 52	R.
	6950		703	0·141	1 54	255 3	12 42	+ 5 27	m.
	6951		703	0·150	47 20	261 36	19 15	+ 4 40	n.
	6952		703	0·160	50 40	259 57	17 36	+ 4 20	o.
	6953		703	0·123	26 19	256 35	14 14	+ 5 51	p.
	6954		703	0·129	9 48	257 0	14 39	+ 8 38	q.
	6955		703	0·162	55 23	255 33	13 12	+ 7 49	r.
29.	6956	240·426	703	0·707	299 47	297 55	14 43	+ 4 0	S.
	6957		703	0·692	295 34	299 25	16 13	+ 5 46	s.
	6958		703	0·695	294 2	304 53	21 41	+ 6 15	T.
	6959		703	0 611	293 13	303 44	20 32	+ 8 40	t.
	6960		703	0·649	293 4	301 13	18 1	+ 5 47	U.
	6961		703	0·671	296 45	300 43	17 31	+ 5 32	Z.
	6962		703	0·602	293 55	297 47	14 35	+ 5 54	u.
	6963		704	0·607	111 6	223 33	300 21	− 2 19	a.
	6964		704	0·615	112 18	220 16	297 4	− 3 33	b.
	6965		704	0·660	109 37	221 46	298 34	− 3 29	c.
30.	6966	241·462	703	0·862	298 11	314 31	16 37	+ 7 10	M.
	6967		703	0·870	297 22	317 28	19 34	+ 3 35	m.
	6968		703	0·881	296 48	318 59	21 5	+ 5 49	n.
	6969		704	0·408	114 57	237 29	299 35	− 3 19	A.
	6970		704	0·424	110 27	237 58	300 4	− 2 17	B.
	6971		704	0·472	111 38	239 30	301 36	− 1 42	C.
	6972		704	0·460	109 26	234 53	296 59	− 2 5	a.
	6973		704	0·491	109 32	233 18	295 24	− 4 18	b.
Sept. 1.	6974	243·476	703	0·970	297 15	344 47	18 19	+ 3 55	A.
	6975		703	0·972	299 21	341 5	14 37	+ 6 23	a.
	6976		704	0·008	125 31	263 33	297 5	− 1 57	B.
	6977		704	0·012	184 26	266 23	299 55	− 2 33	C.
	6978		704	0·088	237 20	265 0	298 32	− 1 15	D.
	6979		704	0·037	146 10	264 47	298 19	− 3 5	b.
	6980		704	0·059	239 14	266 19	299 51	− 2 44	c.
	6981		704	0·097	269 25	268 9	301 41	− 3 48	d.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Sept. 2.	6982	244·431	704	0·318	281° 20′	279° 1′	299° 0′	− 2° 50′	A.
	6983		704	0·307	282 10	277 51	297 50	− 1 17	B.
	6984		704	0·270	285 19	279 9	299 8	− 3 39	C.
	6985		704	0·260	287 3	276 29	296 28	+ 0 19	a.
	6986		704	0·232	289 2	278 51	298 50	− 1 16	b.
4.	6987	246·436	704	0·606	289 2	309 11	300 44	− 3 43	M.
	6988		704	0·684	288 1	306 29	298 2	− 3 36	m.
	6989		704	0·641	287 14	305 48	297 21	+ 1 35	n.
	6990		704	0·690	287 17	307 56	299 29	− 2 5	o.
5.	6991	247·650	704	0·838	291 17	325 32	299 52	− 3 32	S.
	6992		704	0·870	291 41	326 50	301 10	− 1 35	T.
	6993		704	0·864	294 36	324 17	298 37	− 3 13	s.
	6994		704	0·852	292 50	323 12	297 32	− 2 11	t.
	6995		704	0·840	293 50	325 11	299 31	− 3 38	u.
	6996		704	0·872	295 14	322 29	296 49	− 1 7	v.
6.	6997	248·636	704	0·961	294 44	340 51	301 12	− 3 17	Z.
	6998		704	0·940	295 11	339 43	300 4	+ 0 56	a.
	6999		704	0·956	295 11	340 5	300 26	− 1 4	b.
	7000		704	0·939	297 17	338 23	298 44	− 2 33	c.
7.	7001	249·656	704	0·989	294 20	353 17	299 9	− 3 7	M.
	7002		704	0·993	296 50	355 25	301 17	− 3 9	M^{0}.
8.	7003	250·476	No spots visible.						
9.	7004	251·444							
13.	7005	255·454	705	0·962	125 9	211 5	74 43	−17 13	A.
	7006		705	0·973	127 4	213 21	76 59	−18 11	a.
14.	7007	256·461	705	0·824	126 39	226 31	75 52	−16 3	A^{0}.
	7008		705	0·833	124 16	223 23	72 44	−18 44	a^{0}.
15.	7009	257·466	705	0·710	137 35	239 27	74 32	−19 36	a.
	7010		705	0·764	138 40	237 44	72 49	−18 17	b.
16.	7011	258·448	705	0·622	148 31	249 49	70 59	−17 54	M.
	7012		705	0·631	147 42	254 41	75 51	−17 46	N.
	7013		705	0·688	146 37	251 52	73 2	−19 49	m.
	7014		705	0·699	146 51	253 11	74 21	−18 40	n.
18.	7015	260·513	705	0·311	211 2	283 39	75 32	−16 5	A.
	7016		705	0·363	199 31	279 34	71 27	−19 39	a.
	7017		706	0·712	276 26	324 59	116 52	− 8 28	B.
	7018		706	0·708	274 1	323 21	115 14	− 8 49	b.
	7019		706	0·664	277 10	320 17	112 10	− 7 37	b^{0}.
19.	7020	261·518	705	0·320	251 26	298 53	76 30	−16 12	Z.
	7021		706	0·850	277 41	337 45	115 22	− 8 4	a.
	7022		706	0·860	277 58	336 24	114 1	− 7 59	b.
	7023		706	0·866	278 59	335 45	113 22	− 8 52	c.
	7024		706	0·843	277 26	334 54	112 31	− 7 5	d.
	7025		706	0·838	289 58	338 35	116 12	− 7 54	e.
20.	7026	262·474	705	0·522	259 11	310 23	74 26	−16 26	M.
	7027		706	0·902	277 36	350 49	114 52	− 7 7	A.
	7028		706	0·940	277 59	352 21	116 24	− 8 49	a.
	7029		706	0·952	279 2	348 12	112 15	− 8 15	b.
	7030		706	0·919	278 19	351 40	115 43	− 7 56	B.
	7031		706	0·948	279 38	351 2	115 5	− 9 46	C.
22.	7032	264·500	707	0·970	104 25	210 19	305 38	+ 2 38	P.
	7033		707	0·975	109 0	209 35	304 54	− 1 43	Q.
	7034		707	0·976	107 17	207 37	302 56	+ 3 2	p.
	7035		707	0·983	111 9	211 35	306 54	− 2 1	q.
23.	7036	265·442	707	0·901	107 19	222 29	304 27	+ 2 53	A.
	7037		707	0·912	108 5	219 44	301 42	− 2 49	B.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Sept. 23.	7038	265·442	707	0·914	111° 22′	223° 13′	305° 11′	− 0° 49′	C.
	7039		707	0 959	112 34	223 19	306 17	− 1 27	a.
	7040		707	0·966	112 11	224 1	305 59	+ 2 19	b.
25.	7041	267·517	707	0·569	108 57	252 54	305 26	− 2 39	P.
	7042		707	0·572	109 14	253 30	306 2	− 3 31	Q.
	7043		707	0·644	114 52	254 37	307 9	+ 1 35	R.
	7044		707	0·606	112 36	250 9	302 41	+ 0 27	p.
	7045		707	0·657	109 15	251 17	303 49	+ 0 4	q.
	7046		707	0·593	108 32	252 7	304 39	− 2 5	r.
	7047		707	0·680	115 31	252 35	305 7	− 2 47	s.
26.	7048	268·471	707	0·397	104 20	267 31	306 39	− 2 39	M.
	7049		707	0·399	112 43	266 29	305 37	− 3 17	N.
	7050		707	0·450	110 0	265 42	304 50	+ 0 23	O.
	7051		707	0·441	106 54	261 28	300 36	+ 1 56	P.
	7052		707	0·472	114 18	268 10	307 18	+ 2 41	m.
	7053		707	0·480	115 14	262 47	301 55	+ 3 34	n.
	7054		707	0·490	115 31	266 20	305 28	− 3 50	o.
	7055		707	0·457	106 4	267 36	306 44	− 2 2	p.
	7056		707	0·460	108 18	268 18	307 26	+ 2 47	Q.
	7057		707	0·427	113 9	266 29	305 37	+ 2 24	R.
	7058		707	0·406	114 21	265 32	304 40	− 2 32	q.
	7059		707	0·499	116 39	267 15	306 23	+ 3 1	r.
27.	7060	269·536	707	0·136	91 35	284 20	308 13	+ 3 44	A.
	7061		707	0·159	104 37	282 32	306 25	+ 2 20	B.
	7062		707	0·253	97 49	281 51	305 44	+ 2 12	C.
	7063		707	0·272	103 25	280 31	304 24	+ 2 18	a.
	7064		707	0·190	117 51	283 37	307 30	+ 1 39	b.
	7065		707	0·281	120 28	280 9	304 2	+ 1 59	c.
	7066		707	0·226	99 36	279 4	302 57	− 0 10	D.
	7067		707	0·202	100 40	276 34	300 27	− 1 16	d.
	7068		707	0·279	122 41	275 9	299 2	− 1 55	E.
	7069		707	0·281	115 46	277 21	301 14	− 2 54	e.
	7070		707	0·288	122 48	275 49	299 42	− 2 35	f.
	7071		708	0·881	283 0	350 32	14 25	− 4 41	S.
	7072		708	0·890	283 50	351 39	15 32	− 5 57	T.
	7073		708	0·897	284 22	349 45	13 38	− 5 20	s.
28.	7074	270·490	707	0·043	268 36	291 13	301 34	+ 1 9	A.
	7075		707	0·057	194 39	290 24	300 45	− 0 53	a.
	7076		707	0·069	165 9	289 29	299 50	+ 1 11	B.
	7077		707	0·102	121 38	287 22	297 43	− 3 24	b.
	7078		709	0·252	248 29	300 47	311 8	− 9 29	C.
	7079		709	0·254	250 43	301 33	311 54	−11 53	c.
	7080		710	0·961	104 37	219 37	229 58	+ 4 7	D.
30.	7081	272·419	707	0·468	298 39	315 25	298 25	− 1 46	a.
	7082		707	0·473	299 4	317 2	300 2	+ 0 18	b.
	7083		711	0·431	284 48	317 15	300 15	− 4 56	x.
Oct. 2.	7084	274·451	707	0·809	301 31	345 44	299 54	+ 1 10	a^0.
	7085		707	0·814	301 48	346 1	300 11	+ 1 59	b^0.
	7086		712	0·975	108 30	220 58	175 8	− 0 31	A.
3.	7087	275·469	707	0·910	303 37	0 52	300 36	+ 1 11	a.
	7088		712	0·895	111 57	235 2	174 46	+ 0 47	B.
4.	7089	276·605	707	0·988	303 6	16 4	299 37	+ 1 27	B.
	7090		712	0·728	111 15	251 41	175 14	− 1 59	c.
5.	7091	277·501	712	0·556	111 44	263 16	174 11	− 1 0	A.
6.	7092	278·478	712	0·405	110 47	278 32	175 35	− 0 42	a.
7.	7093	279·434	712	0·188	115 37	291 9	174 38	+ 0 16	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Oct. 7.	7094	279·434	713	0·994	130° 52′	215° 44′	99° 13′	−18° 12′	B.
10.	7095	282·488	713	0·715	140 52	259 53	100 3	−17 14	C.
	7096		713	0·728	135 30	257 22	97 32	−20 54	D.
	7097		713	0·751	138 11	256 0	96 10	−21 53	c.
	7098		713	0·794	134 43	258 16	98 26	−17 29	d.
	7099		713	0·813	133 52	257 33	97 43	−18 15	e.
11.	7100	283·532	713	0·553	151 47	277 32	102 54	−20 43	M.
	7101		713	0·594	149 54	274 1	99 23	−19 59	N.
	7102		713	0·638	150 57	271 55	97 17	−19 41	m.
	7103		713	0·559	145 51	275 39	101 1	−18 7	n.
	7104		713	0·641	147 49	271 32	96 54	−17 18	P.
	7105		713	0·662	149 45	270 28	95 50	−21 59	p.
	7106		713	0·581	147 6	268 29	93 51	−18 34	Q.
	7107		713	0·612	147 21	274 23	99 45	−17 17	q.
	7108		713	0·633	145 46	274 57	100 19	−17 8	S.
	7109		713	0·641	145 3	275 7	100 29	−19 22	s.
	7110		713	0·584	146 19	266 49	92 11	−20 19	T.
	7111		713	0·672	144 21	272 17	97 39	−21 54	t.
12.	7112	284·514	713	0·422	168 19	289 36	101 2	−18 6	A.
	7113		713	0·471	165 42	286 15	97 41	−19 9	B.
	7114		713	0·515	159 33	284 7	95 33	−20 49	C.
	7115		713	0·450	161 55	284 58	96 24	−21 19	D.
	7116		713	0·462	156 56	289 49	101 15	−20 30	a.
	7117		713	0·491	158 44	287 23	98 49	−17 40	b.
	7118		713	0·527	154 6	287 25	98 51	−17 1	c.
	7119		714	0·841	123 6	286 1	97 27	−18 48	d.
	7120		714	0·860	122 58	287 6	98 32	−17 33	e.
13.	7121	285·512	713	0·338	199 37	305 38	102 55	−18 49	M.
	7122		713	0·342	196 25	304 23	101 40	−19 19	m.
	7123		713	0·354	193 6	301 1	98 18	−19 45	n.
	7124		713	0·388	190 3	300 38	97 55	−17 11	A.
	7125		713	0·372	187 44	301 32	98 49	−18 20	a.
	7126		713	0·391	181 55	300 58	98 15	−20 18	B.
	7127		713	0·436	185 35	297 10	94 27	−21 21	b.
	7128		713	0·414	186 53	299 2	96 19	−20 47	C.
	7129		713	0·444	178 29	296 41	95 58	−23 26	c.
	7130		714	0·650	125 3	265 38	62 55	− 7 10	D.
	7131		714	0·691	125 58	262 13	59 30	− 7 20	d.
17.	7132	289·635	713	0·805	275 44	0 23	99 11	−18 53	S.
	7133		713	0·832	273 50	358 17	97 5	−20 27	s.
	7134		713	0·827	275 57	2 8	100 56	−19 54	T.
	7135		713	0·848	272 6	359 44	98 32	−22 48	t.
	7136		714	0·349	268 36	326 17	65 5	− 6 2	v.
24.	7137	296·459	No spots visible.						
27.	7138	299·501							
28.	7139	300·429							
Nov. 2.	7140	305·535	715	0·983	126 15	246 3	105 49	−13 4	A.
3.	7141	306·487	715	0·940	127 53	259 38	105 32	−12 9	A.
4.	7142	307·464	715	0·942	126 13	259 0	104 54	−13 7	a.
	7143		715	0·962	126 53	258 56	104 50	−12 46	b.
	7144		715	0·347	152 38	317 23	104 34	−12 52	B.
8.	7145	311·604	715	0·354	154 59	318 7	105 18	−13 59	b.
	7146		716	0·371	22 40	329 3	116 14	+23 35	C.
	7147		716	0·531	8 40	334 59	122 10	+34 5	c.
	7148		716	0·555	10 10	335 46	122 57	+36 4	D.
	7149								

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1865. Nov. 8.	7150	311·604	716	0·598	8° 50′	337° 27′	114° 38′	+38° 15′	d.
10.	7151	313·455	715	0·353	235 51	345 8	106 3	−14 50	M.
	7152		715	0·360	240 8	344 46	105 41	−14 59	m.
	7153		717	0·544	100 8	303 9	64 4	+12 34	N.
	7154		717	0·557	99 22	301 59	62 54	+13 42	n.
13.	7155	316·503	715	0·831	274 42	26 32	104 13	−12 14	O.
	7156		718	0·946	117 53	265 23	343 4	− 4 49	S.
	7157		718	0·969	116 37	267 11	344 52	− 4 28	s.
15.	7158	318·448	715	0·979	277 24	55 32	105 38	−13 44	A.
	7159		718	0·665	121 47	298 38	348 44	− 5 13	C.
	7160		718	0·731	119 17	295 54	346 0	− 4 28	c.
	7161		718	0·701	120 30	294 26	344 32	− 4 11	c^1.
	7162		718	0·754	118 28	292 20	342 26	− 4 19	c^2.
22.	7163	325·471	719	0·951	94 30	276 14	226 43	+12 49	A.
	7164		719	0·956	95 7	272 8	222 37	+13 17	a.
	7165		719	0·974	96 0	272 58	223 27	+13 43	b.
23.	7166	326·522	719	0·851	93 31	283 57	219 31	+12 54	a.
	7167		719	0·867	94 0	287 23	222 57	+14 21	b.
	7168		719	0·895	93 25	288 49	224 23	+13 58	c.
	7169		719	0·902	95 20	290 3	225 37	+12 23	d.
24.	7170	327·479	719	0·705	88 25	304 1	226 1	+14 30	M.
	7171		719	0·711	89 54	299 0	221 0	+13 49	m.
	7172		719	0·784	91 45	302 47	224 47	+14 36	N.
	7173		719	0·796	92 31	297 34	219 34	+12 34	n.
Dec. 2.	7174	335·475	719	0·892	297 5	55 58	224 33	+13 9	B.
	7175		719	0·889	298 40	56 32	225 7	+14 47	b.
	7176		720	0·290	341 1	6 51	175 26	+15 7	S.
	7177		721	0·869	119 48	297 29	106 4	−12 20	s.
13.	7178	346·462	No spots visible.						
14.	7179	347·460							
19.	7180	352·543							
30.	7181	363·609	724*	0·704	112 12	340 38	110 9	−14 28	A.
	7182		724	0·752	111 54	338 21	107 52	−13 43	B.
	7183		724	0·741	112 15	338 43	108 14	−14 9	a.
	7184		724	0·766	113 51	337 29	107 0	−15 21	b.
1866. Jan. 1.	7185	0·462	724	0·366	129 6	8 26	111 40	−13 18	M.
	7186		724	0·411	125 4	7 4	110 18	−13 53	m.
	7187		724	0·394	127 54	5 37	108 51	−14 16	N.
	7188		724	0·429	124 32	3 59	107 13	−13 56	n.
	7189		725	0·305	103 42	9 29	112 43	− 4 54	P.
	7190		725	0·309	102 29	9 15	112 29	− 4 27	Q.
	7191		725	0·398	102 52	3 7	106 21	− 3 17	p.
	7192		725	0·406	99 1	4 58	108 12	− 3 49	q.
3.	7193	2·450	724	0·288	216 33	30 22	105 24	−13 34	A.
	7194		724	0·294	213 56	32 18	107 20	−14 23	a.
	7195		724	0·305	209 33	33 28	108 30	−13 24	a^1.
	7196		725	0·184	245 29	37 14	112 16	− 4 55	B.
	7197		725	0·172	234 46	36 0	111 2	− 3 3	C.
	7198		725	0·169	240 52	31 37	106 39	− 3 24	b.
	7199		725	0·151	228 49	32 49	107 51	− 2 44	c.
	7200		726	0·973	77 25	315 42	30 44	+12 51	D.
4.	7201	3·534	724	0·455	238 24	46 41	106 21	−13 48	A.
	7202		724	0·460	239 15	48 33	108 13	−14 15	a.
	7203		725	0·402	260 25	53 16	112 56	− 4 21	B.
	7204		725	0·409	262 24	58 0	107 40	− 3 11	c.
	7205		725	0·336	259 38	59 0	108 40	− 3 33	d.

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* Groups 722 and 723 were visible on the 20th December, but the limb of the Sun-picture was so ill-defined that the determination of their heliographic position was rendered impossible.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Jan. 4.	7206	3·534	726	0·894	75° 7′	331° 9′	30° 49′	+13° 39′	b.
7.	7207	6·583	724	0·9[illegible]7	251 8	90 56	107 21	−13 27	M.
	7208		724	0·911	252 23	92 7	108 32	−14 14	m.
	7209		725	0·913	263 33	92 59	109 24	− 3 19	N.
	7210		726	0·404	51 19	12 46	29 11	+12 29	O.
8.	7211	7·501	724	0·972	251 31	103 18	106 41	−13 33	S.
	7212		724	0·976	251 50	105 30	108 53	−14 9	T.
	7213		725	0·970	262 27	107 13	110 36	− 3 47	s.
	7214		726	0·274	26 39	25 51	29 14	+13 6	t.
	7215		727	0·985	78 53	313 54	317 17	+14 58	A.
9.	7216	8·459	727	0·948	76 38	325 41	315 29	+13 4	M.
	7217		727	0·942	77 57	326 56	316 44	+12 26	m.
	7218		727	0·959	77 2	322 39	312 27	+13 27	n.
	7219		727	0·963	79 7	324 0	313 48	+13 54	o.
15.	7220	14·504	727	0·351	300 38	54 26	318 29	+15 18	M.
	7221		727	0·340	301 52	52 9	316 12	+14 19	m.
	7222		727	0·333	302 13	51 29	315 32	+13 41	n.
	7223		727	0·327	303 20	52 35	316 38	+14 21	o.
	7224		727	0·318	301 14	51 17	315 20	+14 49	a.
	7225		727	0·310	308 4	49 15	313 18	+13 33	b.
	7226		727	0·270	304 32	49 39	313 42	+13 2	c.
	7227		727	0·246	309 16	48 39	312 42	+12 37	d.
	7228		728	0·948	90 25	332 17	236 20	− 6 5	D.
	7229		728	0·950	91 40	330 29	234 32	− 5 28	E.
	7230		728	0·980	92 39	335 29	239 32	− 6 21	f.
	7231		728	0·972	91 13	332 21	236 24	− 5 58	g.
	7232		728	0·987	93 27	333 39	237 42	− 6 9	h.
19.	7233	18·496	727	0·954	271 52	109 9	316 35	+12 42	A.
	7234		727	0·958	273 41	105 37	313 3	+14 27	a.
	7235		728	0·274	94 4	30 16	237 42	− 6 55	M.
	7236		728	0·282	92 54	29 27	236 53	− 6 22	m.
	7237		728	0·295	91 50	29 41	237 7	− 5 24	n.
	7238		728	0·311	90 59	28 17	235 43	− 6 11	o.
	7239		728	0·324	91 50	27 18	234 44	− 6 54	p.
	7240		728	0·380	90 20	22 59	230 25	− 5 22	q.
	7241		729	0·444	94 15	20 15	227 41	− 9 4	S.
	7242		729	0·461	92 41	19 10	226 36	− 8 27	s.
	7243		729	0·455	93 20	19 1	226 27	− 9 48	T.
	7244		729	0·453	93 16	17 4	224 30	− 8 15	t.
	7245		729	0·469	91 22	16 57	224 23	− 7 56	v.
23.	7246	22·510	728	0·620	252 8	87 4	237 34	− 6 33	M.
	7247		728	0·611	251 34	82 58	233 28	− 6 16	m.
	7248		729	0·446	250 17	74 52	225 22	− 8 59	N.
24.	7249	23·494	728	0·783	251 36	100 8	236 40	− 5 36	B.
	7250		728	0·768	250 15	94 8	230 40	− 6 15	b.
29.	7251	28·502	730	0·508	51 13	25 21	90 51	+14 51	S.
	7252		730	0·564	54 7	23 16	88 46	+13 19	s.
	7253		730	0·572	54 11	21 8	86 38	+13 7	t.
	7254		731	0·802	86 19	1 11	66 41	− 6 27	a.
	7255		731	0·807	87 58	1 16	66 46	− 7 39	b.
	7256		731	0·839	89 15	358 2	63 32	− 7 39	c.
	7257		731	0·848	89 30	358 37	64 7	− 8 15	d.
Feb. 5.	7258	35·505	731	0·612	237 19	99 13	65 23	− 7 59	A.
	7259		731	0·600	237 19	98 24	64 34	− 7 54	a.
	7260		731	0·594	239 29	96 28	62 38	− 6 45	a^1.
	7261		732	0·943	64 48	354 37	320 47	+14 36	B.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Feb. 5.	7262	35·505	732	0·946	65° 19′	356° 46′	322° 56′	+13° 15′	b.
	7263		732	0·938	65 25	356 56	323 6	+13 43	b^1.
6.	7264	36·478	731	0·788	243 16	112 3	64 26	− 7 42	C.
	7265		732	0·937	59 6	11 33	323 56	+13 11	D.
	7266		732	0·932	59 41	12 4	324 27	+14 14	d.
8.	7267	38·485	731	0·976	243 12	141 26	65 20	− 6 54	S.
	7268		732	0·465	41 29	38 40	322 34	+14 17	a.
	7269		732	0·471	42 7	38 21	322 15	+14 13	b.
	7270		732	0·483	43 2	37 10	321 4	+13 22	c.
	7271		732	0·540	45 56	32 50	316 44	+13 23	d.
10.	7272	40·480	732	0·277	338 10	67 16	322 52	+14 12	B.
	7273		732	0·279	341 23	68 27	324 3	+14 13	b.
	7274		732	0·297	335 42	69 49	325 25	+16 10	b^1.
	7275		733	0·743	271 9	114 37	10 13	+14 7	C.
	7276		733	0·708	274 6	113 33	9 9	+15 57	c.
	7277		733	0·694	275 8	112 37	8 13	+16 10	c^1.
	7278		734	0·995	61 17	348 24	244 0	+20 26	D.
13.	7279	43·494	732	0·690	271 8	110 38	323 29	+13 46	A.
	7280		732	0·694	271 52	111 38	324 29	+14 28	a.
	7281		734	0·699	49 0	31 38	244 29	+19 57	B.
	7282		735	0·892	56 36	7 39	220 30	+15 31	C.
	7283		735	0·916	57 18	358 36	211 27	+18 32	D.
	7284		735	0·917	59 14	2 42	215 33	+19 40	E.
	7285		735	0·955	58 21	0 44	213 35	+16 59	e.
	7286		735	0·961	58 42	3 55	216 46	+16 16	d.
	7287		735	0·963	60 21	5 39	218 30	+17 8	e.
18.	7288	48·423	734	0·512	84 40	101 56	244 52	+19 19	M.
	7289		735	0·310	311 43	80 23	222 19	+16 13	A.
	7290		735	0·315	317 30	79 48	222 44	+14 5	B.
	7291		735	0·320	334 45	75 44	218 40	+14 23	C.
	7292		735	0·321	346 48	76 16	219 12	+13 50	D.
	7293		735	0·323	355 47	70 24	213 20	+13 2	E.
	7294		735	0·305	319 11	71 13	214 9	+15 22	a
	7295		735	0·299	351 46	72 49	215 45	+16 34	b.
	7296		735	0·272	346 17	72 16	215 12	+18 8	c.
	7297		735	0·281	345 52	69 13	212 9	+19 56	d.
	7298		735	0·283	358 13	68 58	211 54	+18 13	e.
	7299		735	0·260	358 46	68 6	211 2	+17 18	f.
19.	7300	49·537	734	0·694	272 50	118 3	245 11	+19 14	D.
	7301		735	0·456	283 44	95 41	222 49	+14 26	S.
	7302		735	0·432	291 12	91 52	219 0	+15 40	T.
	7303		735	0·410	295 40	91 3	218 11	+16 6	U.
	7304		735	0·419	315 33	90 21	217 29	+18 52	s.
	7305		735	0·371	312 34	85 27	212 35	+19 28	t.
	7306		735	0·384	317 4	84 30	211 38	+19 43	u.
	7307		735	0·349	288 23	86 17	213 25	+14 12	v.
	7308		735	0·361	292 17	86 49	213 57	+15 55	M.
	7309		735	0·359	317 21	87 4	214 12	+15 18	m.
	7310		735	0·344	314 37	88 11	215 19	+16 17	N.
	7311		735	0·390	316 5	84 53	212 1	+16 16	n.
	7312		735	0·345	318 45	84 37	211 45	+16 50	O.
20.	7313	50·591	734	0·853	266 3	132 31	244 42	+20 44	D^o.
	7314		735	0·648	273 6	110 20	222 31	+15 32	S.
	7315		735	0·650	281 28	104 48	216 59	+16 48	s.
	7316		735	0·539	277 31	102 56	215 7	+16 12	T.
	7317		735	0·591	275 22	105 51	217 42	+17 56	t.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Feb. 20.	7318	50·591	735	0·588	275° 39′	98° 51′	211° 2′	+16° 46′	a.
	7319		735	0·550	279 38	99 56	212 7	+15 38	b.
	7320		735	0·540	280 10	100 3	212 14	+14 36	c.
	7321		735	0·609	278 31	101 1	213 12	+14 37	d.
	7322		735	0·601	278 41	104 42	216 53	+13 24	e.
	7323		735	0·528	282 55	106 1	218 12	+14 1	f.
	7324		735	0·466	283 53	99 8	211 19	+13 28	M.
	7325		735	0·471	284 33	101 58	214 9	+14 17	m.
	7326		735	0·510	284 37	107 33	219 44	+14 27	N.
	7327		735	0·519	285 0	108 57	221 8	+15 20	n.
21.	7328	51·541	734	0·946	263 33	147 2	245 45	+19 0	A.
	7329		735	0·800	266 18	124 14	222 57	+18 59	B.
	7330		735	0·790	268 9	120 13	218 56	+13 31	C.
	7331		735	0·672	272 12	120 54	219 37	+14 14	D.
	7332		735	0·755	270 40	124 48	223 31	+15 19	b.
	7333		735	0·709	274 44	113 28	212 11	+15 55	c.
	7334		735	0·684	269 5	116 53	215 36	+14 8	d.
	7335		735	0·645	275 13	117 48	216 31	+13 57	e.
23.	7336	53·632	735	0·971	258 3	153 23	222 26	+15 32	M.
	7337		735	0·951	261 34	150 9	219 12	+18 24	m.
	7338		735	0·917	259 7	149 19	218 22	+17 54	N.
	7339		735	0·936	260 10	150 46	219 49	+16 16	n.
	7340		735	0·944	260 48	151 50	220 53	+14 28	O.
	7341		735	0·909	263 2	144 20	213 23	+14 48	P.
24.	7342	54·486	735	0·977	257 46	159 53	216 49	+17 44	A^0.
	7343		735	0·980	259 32	162 21	219 17	+14 3	a^1.
26.	7344	56·482	736	0·608	74 28	48 24	77 1	− 5 14	S.
	7345		736	0·623	74 10	44 38	73 15	− 4 38	s.
	7346		736	0·647	72 16	43 5	71 42	− 5 58	t.
Mar. 2.	7347	60·471	737	0·747	44 56	41 14	13 16	+16 55	B.
	7348		737	0·751	45 12	40 56	12 58	+16 13	b.
6.	7349	64·650	738	0·692	44 59	52 41	325 27	+19 48	M.
7.	7350	65·458	738	0·556	36 27	62 58	324 16	+18 17	M^0.
8.	7351	66·497	738	0·398	21 11	77 27	324 1	+19 3	M^1.
12.	7352	70·479	738	0·636	269 41	135 26	325 31	+19 26	A.
	7353		739	0·892	48 3	38 24	228 29	+15 38	a.
	7354		739	0·895	49 21	39 47	229 52	+15 55	b.
	7355		739	0·901	51 33	36 8	226 13	+14 13	c.
14.	7356	72·413	738	0·903	258 43	162 48	325 27	+19 36	A^0.
	7357		739	0·577	37 24	66 31	229 10	+16 51	a^1.
	7358		739	0·579	38 43	65 26	228 5	+15 44	b^1.
	7359		739	0·619	40 45	63 45	226 24	+14 18	c^1.
22.	7360	80·497	739	0·570	276 44	177 42	225 41	+14 18	M.
	7361		740	0·788	44 22	65 20	113 19	+16 57	a.
	7362		740	0·793	45 16	63 1	111 0	+15 54	b.
	7363		740	0·809	46 9	63 28	111 27	+17 43	c.
23.	7364	81·502	739	0·705	269 13	192 56	226 40	+14 41	N.
	7365		740	0·556	33 6	81 32	115 16	+17 47	A.
	7366		740	0·580	38 38	79 1	112 45	+16 35	a.
	7367		740	0·612	36 55	82 48	116 32	+18 36	B.
	7368		740	0·564	33 17	81 8	114 52	+19 24	b.
	7369		740	0·620	39 35	78 0	111 44	+19 9	c.
24.	7370	82·443	739	0·841	261 2	206 7	226 31	+15 49	D.
	7371		739	0·843	261 28	207 3	227 27	+15 21	d.
	7372		740	0·396	23 37	94 43	115 7	+17 9	E.
	7373		740	0·408	24 20	91 50	112 14	+17 43	e.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Mar. 24.	7374	82·443	740	0·421	22° 53′	93° 29′	113° 53′	+16° 8′	A^1.
	7375		740	0·483	27 20	89 39	110 3	+15 12	B^1.
	7376		740	0·459	24 12	90 41	111 5	+17 59	a^1.
	7377		740	0·510	29 32	92 27	112 51	+15 15	b^1.
	7378		741	0·558	69 23	75 5	95 29	− 3 34	C.
	7379		741	0·571	70 49	77 0	97 24	− 2 7	c.
27.	7380	85·534	740	0·493	277 49	139 52	116 24	+15 50	A.
	7381		740	0·495	279 32	137 25	113 57	+15 39	B.
	7382		740	0·498	280 4	137 8	113 40	+16 48	C.
	7383		740	0·456	287 24	134 2	110 34	+17 52	a.
	7384		740	0·459	287 23	134 56	111 28	+17 19	b.
	7385		740	0·461	289 18	133 1	109 33	+18 32	c.
	7386		741	0·187	241 11	121 23	97 55	− 2 19	D.
	7387		741	0·136	212 22	117 52	94 24	− 3 59	d.
	7388		741	0·129	213 0	116 40	93 12	− 3 15	E.
	7389		741	0·092	169 44	112 17	88 49	− 2 17	e.
29.	7390	87·418	741	0·812	259 0	144 47	94 36	− 3 23	M.
	7391		741	0·807	260 34	140 45	90 34	− 3 13	m.
	7392		741	0·792	260 33	139 46	89 35	− 2 55	N.
	7393		741	0·745	264 7	143 47	93 36	− 3 59	n.
30.	7394	88·476	741	0·919	256 13	157 25	92 14	− 3 49	M^0.
Apr. 3.	7395	92·498	742	0·609	39 2	87 27	325 12	+15 17	A.
	7396		742	0·615	39 34	86 45	324 30	+14 45	a.
	7397		742	0·626	40 29	86 42	324 27	+14 51	a^1.
5.	7398	94·499	742	0·287	1 48	115 48	325 11	+15 52	B.
	7399		742	0·292	1 31	116 0	325 23	+14 39	C.
	7400		742	0·301	0 39	114 17	323 40	+15 55	b.
	7401		742	0·305	358 29	114 53	324 16	+16 5	c.
6.	7402	95·476	742	0·285	316 10	129 54	325 25	+15 23	x.
	7403		743	0·635	25 6	91 17	286 48	+26 38	M.
	7404		743	0·650	27 3	90 44	286 15	+28 13	m.
	7405		743	0·661	27 40	88 17	283 48	+29 48	n.
12.	7406	101·487	744	0·410	62 30	97 12	207 27	− 3 47	S.
	7407		744	0·413	62 52	99 8	209 23	− 2 39	T.
	7408		744	0·488	64 30	98 32	208 47	− 3 33	s.
	7409		744	0·491	64 54	97 54	208 9	− 3 46	t.
	7410		745	0·910	43 43	65 48	176 3	+18 41	K.
	7411		745	0·915	44 19	68 19	178 34	+18 19	k.
14.	7412	103·437	744	0·099	93 16	126 22	208 58	− 2 41	A.
	7413		744	0·084	78 32	125 49	208 25	− 3 18	a^1.
	7414		745	0·661	36 5	95 10	177 46	+19 56	S.
	7415		745	0·670	37 38	93 41	176 17	+18 18	s.
	7416		745	0·675	37 9	93 24	176 0	+18 53	t.
16.	7417	105·649	745	0·392	358 48	126 57	178 10	+19 50	A.
	7418		745	0·405	0 46	125 53	177 6	+19 8	B.
	7419		745	0·411	359 12	126 40	177 53	+21 11	a.
	7420		745	0·427	1 13	125 18	176 31	+20 6	b.
18.	7421	107·451	745	0·417	294 11	150 59	176 39	+19 2	C
	7422		745	0·422	296 19	151 42	177 22	+20 10	c.
	7423		745	0·431	299 29	152 58	178 38	+19 56	D.
19.	7424	108·541	745	0·556	278 24	167 23	177 35	+18 7	S.
	7425		745	0·577	282 26	169 7	179 19	+19 23	s.
	7426		745	0·582	279 28	166 5	176 17	+19 50	T.
	7427		745	0·563	281 34	167 59	178 11	+18 28	t.
	7428		745	0·589	283 27	165 46	175 58	+20 39	m.
20.	7429	109·463	745	0·715	268 52	179 38	176 45	+19 7	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Apr. 20.	7430	109·463	745	0·726	270° 45′	180° 37′	177° 44′	+18° 52′	a.
	7431		745	0·733	269 39	181 5	178 12	+19 51	b.
21.	7432	110·468	745	0·866	264 12	195 3	177 55	+18 17	M.
	7433		745	0·871	266 56	193 51	176 43	+20 20	m.
23.	7434	112·434	746	0·714	51 16	96 27	51 26	+10 15	A.
	7435		746	0·729	53 5	95 55	50 54	+ 8 3	B.
	7436		746	0·751	53 29	93 14	48 13	+ 9 59	a.
	7437		746	0·765	53 30	92 40	47 39	+10 5	b.
24.	7438	113·460	746	0·506	49 38	109 9	49 35	+10 19	M.
	7439		746	0·554	50 37	106 46	47 12	+ 9 43	m.
	7440		746	0·560	52 32	109 59	50 25	+ 9 32	n.
	7441		746	0·519	51 28	110 50	51 16	+ 8 6	P.
	7442		746	0·537	51 41	110 48	51 14	+10 16	p.
	7443		746	0·561	52 14	107 43	48 9	+10 29	p⁰.
25.	7444	114·482	746	0·300	37 22	125 31	51 27	+ 8 34	A.
	7445		746	0·317	39 41	124 25	50 21	+ 9 20	a.
	7446		746	0·333	38 18	123 10	49 6	+ 9 43	b.
	7447		746	0·336	39 13	124 48	50 44	+ 9 4	B.
	7448		746	0·387	40 55	120 31	46 27	+10 48	b.
26.	7449	115·466	746	0·146	341 0	140 53	52 50	+10 42	S.
	7450		746	0·179	346 26	139 57	51 54	+10 29	s.
	7451		746	0·161	355 9	138 29	50 26	+11 0	T.
	7452		746	0·155	350 58	138 58	50 55	+11 32	t.
	7453		746	0·184	357 47	136 48	48 45	+ 8 21	v.
27.	7454	116·478	746	0·192	284 8	158 36	56 13	+ 9 18	A.
	7455		746	0·226	281 1	159 23	57 0	+10 55	a.
	7456		746	0·294	279 3	152 37	50 14	+11 45	b.
	7457		746	0·271	273 25	155 14	52 51	+10 32	c.
	7458		746	0·305	275 39	158 29	56 6	+10 26	B
	7459		746	0·250	281 23	151 8	48 45	+ 9 9	C.
	7460		746	0·246	277 46	150 24	48 1	+ 9 30	e.
	7461		746	0·317	272 20	150 47	48 24	+ 8 57	f.
30.	7462	119·449	746	0·814	254 4	195 21	50 50	+10 12	G.
May 2.	7463	121·660	No spots visible.						
3.	7464	122·504							
4.	7465	123·442							
5.	7466	124·441							
7.	7467	126·487							
8.	7468	127·443	748	0·492	67 49	126 28	228 33	+ 3 57	A.
	7469		748	0·507	69 57	125 9	227 14	+ 3 9	a.
9.	7470	128·504	748	0·263	69 41	142 52	229 54	+ 3 20	A⁰.
	7471		748	0·299	68 33	141 39	228 41	+ 4 6	a⁰.
	7472		748	0·301	69 34	139 4	226 6	+ 4 36	B.
	7473		749	0·466	74 0	129 9	216 11	+ 1 21	b.
	7474		749	0·528	73 14	126 8	213 10	+ 2 37	b⁰.
	7475		749	0·534	72 6	125 41	212 43	+ 2 57	c.
10.	7476	129·458	748	0·008	41 10	154 46	228 16	+ 3 11	A.
	7477		748	0·052	56 10	151 4	224 34	+ 2 40	a.
	7478		749	0·156	84 50	143 8	216 38	− 1 12	B.
	7479		749	0·174	80 40	140 58	214 28	− 2 44	C.
	7480		749	0·254	79 8	138 18	211 48	− 2 16	D.
	7481		749	0·226	77 36	141 35	215 5	+ 0 30	b.
	7482		749	0·279	79 38	137 31	211 1	+ 1 53	c.
	7483		749	0·298	72 57	136 55	210 25	+ 2 0	d.
11.	7484	130·524	749	0·079	179 37	158 47	217 10	− 2 25	A.
	7485		749	0·092	180 25	154 4	212 27	− 1 17	B.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. May 11.	7486	130·524	749	0·155	136° 49′	157° 38′	216° 1′	+ 1° 54′	C.
	7487		749	0·110	97 16	154 35	212 58	− 2 17	a.
	7488		749	0·129	155 10	152 30	210 53	− 2 4	b.
	7489		749	0·164	95 29	151 28	209 51	− 2 58	c.
	7490		749	0·175	126 48	150 40	209 3	− 4 50	S.
	7491		749	0·172	92 39	150 32	208 55	− 3 1	s.
12.	7492	131·470	749	0·305	231 55	174 18	219 16	− 3 55	M.
	7493		749	0·287	223 24	171 23	216 21	− 2 34	N.
	7494		749	0·195	218 50	169 11	214 9	− 2 15	O.
	7495		749	0·214	226 3	173 22	218 20	+ 0 42	P.
	7496		749	0·257	209 48	169 54	214 52	− 2 13	m.
	7497		749	0·176	210 8	168 30	213 28	− 3 9	n.
	7498		749	0·150	230 53	167 34	212 32	− 3 10	o.
	7499		749	0·211	225 53	170 50	215 48	− 3 51	p.
	7500		749	0·146	209 39	166 29	211 27	− 4 44	q.
15.	7501	134·602	749	0·873	244 27	213 32	214 4	− 3 8	S.
	7502		749	0·870	243 44	215 56	216 28	− 2 2	s.
	7503		749	0·810	240 54	210 36	211 8	+ 1 53	T.
	7504		749	0·812	241 34	212 42	213 14	+ 0 27	t.
16.	7505	135·618	749	0·904	240 51	227 37	213 44	− 3 16	A.
	7506		749	0·923	238 26	230 38	216 45	− 2 53	a.
17.	7507	136·470	749	0·972	241 7	241 23	215 30	− 2 56	A^0.
	7508		750	0·922	81 56	100 19	74 21	− 7 2	B.
	7509		750	0·950	81 40	99 16	73 18	− 7 50	b.
18.	7510	137·490	750	0·805	81 42	115 26	75 0	− 7 18	A.
	7511		750	0·817	82 7	114 26	74 0	− 8 44	a.
	7512		750	0·829	81 16	114 23	73 57	− 7 9	b.
19.	7513	138·480	750	0·602	85 31	128 46	74 18	− 7 2	M.
	7514		750	0·617	85 47	127 4	72 36	− 7 35	m.
21.	7515	140·474	750	0·298	121 6	157 57	75 12	− 7 57	N.
	7516		750	0·305	116 0	155 42	82 57	− 8 24	n.
	7517		751	0·901	56 25	108 16	25 31	+14 16	O.
22.	7518	141·504	751	0·756	54 16	121 35	24 13	+15 3	S.
23.	7519	142·552	751	0·584	49 21	137 40	25 26	+14 42	S^1.
25.	7520	144·461	751	0·296	12 56	164 37	25 19	+15 18	A
26.	7521	145·540	751	0·264	314 54	179 29	24 52	+14 38	A^0.
28.	7522	147·464	751	0·555	275 11	206 12	25 18	+14 34	a^0.
29.	7523	148·514	751	0·713	271 8	222 24	25 37	+14 24	a^1.
30.	7524	149·520	751	0·879	267 5	236 52	25 48	+14 47	M.
June 1.	7525	151·464	752	0·964	77 54	103 1	224 23	− 4 39	A.
	7526		752	0·970	78 43	100 57	222 19	− 3 41	a.
2.	7527	152·511	752	0·860	78 15	118 57	225 28	− 4 26	A.
	7528		752	0·875	79 56	115 48	222 19	− 3 47	a.
	7529		752	0 880	82 29	116 42	223 13	− 4 34	B.
	7530		752	0·869	80 37	114 57	221 28	− 4 30	b.
	7531		752	0·881	83 51	116 12	222 43	− 4 57	c.
6.	7532	156·497	752	0·138	116 14	175 49	225 47	− 3 55	M.
	7533		752	0·145	112 2	174 8	224 6	− 3 11	m.
	7534		752	0·187	114 21	173 30	223 28	− 4 33	N.
	7535		752	0·162	115 18	172 28	221 26	− 3 44	n.
	7536		752	0·159	113 9	174 46	224 44	− 4 4	O.
	7537		752	0 197	111 43	172 29	222 27	− 4 22	o.
24.	7538	174·529	755	0·923	254 22	258 53	53 5	− 8 12	A.
	7539		755	0·899	254 12	256 58	51 10	−10 28	B.
	7540		755	0·912	252 54	252 28	46 40	−10 25	C.
	7541		755	0·878	253 29	255 54	50 6	−10 26	a.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. June 24.	7542	174·529	755	0·889	253° 31′	257° 17′	51° 29′	−11° 57′	b.
	7543		755	0·895	254 23	254 40	48 52	− 9 34	c.
	7544		755	0·875	252 29	250 9	44 21	−12 43	d.
26.	7545	176·486	755	0·951	255 56	275 45	42 11	−10 38	A^{0}.
	7546		755	0·976	256 32	279 52	46 18	−10 11	M^{0}.
	7547		755	0·980	257 24	277 44	44 10	−11 6	N^{0}.
	7548		755	0·958	255 30	280 37	47 3	− 9 14	a.
	7549		755	0·982	257 19	281 42	48 8	−12 39	b.
27.	7550	177·504	755	0·998	255 3	289 49	41 49	−12 14	A.
	7551		755	0·997	256 54	289 20	41 20	−10 48	a.
28.	7552	178·493	756	0·716	74 22	154 44	252 42	+ 6 50	A.
	7553		756	0·733	73 12	155 3	253 1	+ 7 10	a.
	7554		756	0·749	73 20	155 29	253 27	+ 6 40	b.
	7555		757	0·958	93 58	130 16	228 14	− 6 20	C.
	7556		757	0·957	92 16	129 8	227 6	− 7 8	c.
30.	7557	180·496	756	0·348	61 50	184 57	254 31	+ 6 9	D.
	7558		756	0·387	65 39	182 56	252 30	+ 5 7	d.
	7559		756	0·365	64 12	183 2	252 36	+ 6 55	E.
	7560		756	0·392	67 25	180 15	249 49	+ 7 47	e.
	7561		757	0·702	97 51	160 35	230 9	− 6 59	G.
	7562		757	0·734	96 43	159 41	229 15	− 6 6	g.
	7563		757	0·769	95 14	157 18	226 52	− 7 17	H.
	7564		757	0·805	94 47	152 50	222 24	− 7 59	h.
July 2.	7565	182·515	756	0·157	314 26	211 19	252 14	+ 8 50	M.
	7566		757	0·375	113 23	186 48	227 43	− 7 31	a.
	7567		757	0·399	112 27	185 54	226 49	− 7 45	b.
	7568		757	0·408	112 47	183 36	224 31	− 8 39	c.
4.	7569	184·518	756	0·540	278 18	240 37	253 8	+ 8 31	N.
	7570		757	0·239	221 48	215 9	227 40	− 7 4	a.
	7571		757	0·196	216 23	214 34	227 5	− 7 57	a^{0}.
5.	7572	185·505	757	0·395	250 18	227 12	225 43	− 8 28	B.
	7573		757	0·390	249 21	224 35	223 6	− 7 44	b.
6.	7574	186·494	757	0·594	254 58	241 47	226 16	− 7 43	A.
7.	7575	187·489	757	0·754	260 1	256 5	226 27	− 7 14	a.
9.	7576	189·505	757	0·964	263 17	285 18	227 4	− 7 39	M.
10.	7577	190·476	757	0·999	264 38	299 0	227 0	− 7 54	M^{0}.
12.	7578	192·496	758	0·946	99 29	158 15	57 36	−10 22	A.
13.	7579	193·497	758	0·857	104 37	173 40	58 49	− 9 58	A^{0}.
16.	7580	196·473	758	0·354	126 14	205 17	48 13	− 9 56	a.
19.	7581	199·632	758	0·634	261 42	260 14	58 21	−10 53	a^{0}.
20.	7582	200·622	758	0·806	264 49	273 11	57 16	−10 19	a^{1}.
21.	7583	201·469	No spots visible.						
30.	7584	210·466							
Aug. 7.	7585	218·471	759	0·157	70 37	233 40	124 35	+ 4 16	A.
	7586		759	0·192	68 9	232 14	123 9	+ 4 48	B.
	7587		759	0·236	65 25	228 2	118 57	+ 6 35	a.
	7588		759	0·208	71 0	230 55	121 50	+ 5 43	b.
	7589		759	0·257	74 8	227 26	118 21	+ 3 4	C.
	7590		759	0·263	78 38	226 26	117 21	+ 3 16	c.
9.	7591	220·490	759	0·361	291 2	261 9	123 25	+ 5 0	M.
	7592		759	0·287	290 48	256 4	118 20	+ 4 11	m.
10.	7593	221·538	759	0·563	290 15	275 4	123 28	+ 5 37	M^{0}.
	7594		759	0·559	289 45	274 25	121 49	+ 4 8	m^{0}.
11.	7595	222·518	759	0·714	289 53	289 46	123 16	+ 4 20	N.
	7596		759	0·710	289 14	287 8	120 38	+ 5 51	n.
16.	7597	227·630	760	0·555	102 5	212 19	333 19	+ 3 25	A.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866.			*						
Aug. 16.	7598	227·630	760	0·592	99° 36′	209° 34′	330° 34′	+ 1° 37′	B.
	7599		760	0·588	102 10	208 55	329 55	− 1 5	C.
	7600		760	0·612	100 16	207 4	328 4	+ 0 46	a.
	7601		760	0·571	102 50	209 27	330 27	+ 0 30	b.
	7602		760	0·622	103 34	213 48	334 48	+ 1 22	c.
17.	7603	228·626	760	0·405	101 26	226 11	333 3	+ 3 32	A^{0}.
	7604		760	0·436	105 19	224 40	331 32	− 1 22	M.
	7605		760	0·425	102 32	222 19	329 11	+ 2 36	N.
	7606		760	0·426	105 47	225 29	332 21	− 0 16	m.
	7607		760	0·483	103 18	225 45	332 37	− 1 10	n.
	7608		760	0·475	103 34	223 3	329 55	+ 1 57	o.
	7609		760	0·488	104 31	224 38	331 30	+ 0 32	p.
	7610		760	0·492	106 29	221 10	328 2	+ 1 39	q.
18.	7611	229·445	760	0·164	91 59	242 34	337 49	+ 2 15	S.
	7612		760	0·173	95 24	240 58	336 13	+ 1 21	s.
	7613		760	0·205	102 52	238 33	333 48	− 0 18	T.
	7614		760	0·198	99 1	240 8	335 23	− 0 19	t.
	7615		760	0·255	104 58	236 51	332 6	+ 1 32	N^{0}.
	7616		760	0·276	107 33	235 45	331 0	+ 2 3	P^{0}.
	7617		760	0·260	106 36	237 8	332 23	− 1 39	n^{0}.
	7618		760	0·291	109 42	235 37	330 52	− 2 12	p.
20.	7619	231·667	760	0·305	285 34	271 41	335 25	+ 2 38	A.
	7620		760	0·316	288 41	270 5	333 49	− 2 2	a.
	7621		760	0·395	287 18	267 36	331 20	− 0 19	B.
	7622		760	0·371	289 18	269 15	332 59	+ 0 25	b.
	7623		760	0·417	290 30	272 20	336 4	+ 1 0	s.
	7624		760	0·425	293 51	266 14	329 58	− 2 16	S.
30.	7625	241·436	761	0·273	309 12	274 2	199 12	+ 6 47	A.
	7626		761	0·278	310 55	274 54	200 4	+ 6 33	a.
31.	7627	242·455	761	0·466	303 6	289 48	200 31	+ 6 27	B.
	7628		761	0·457	304 25	289 7	199 50	+ 7 1	b.
Sept. 1.	7629	243·464	761	0·651	301 30	304 14	200 39	+ 6 9	A^{0}.
3.	7630	245·461	761	0·940	302 57	332 44	200 48	+ 6 38	A.
	7631		761	0·942	303 17	331 21	199 25	+ 7 11	a.
5.	7632	247·591	No spots visible.						
10.	7633	252·525							
11.	7634	253·470							
13.	7635	255·528							
14.	7636	256·534							
15.	7637	257·457							
17.	7638	259·440							
21.	7639	263·602	762	0·802	99 18	231 51	202 36	+ 7 48	M.
24.	7640	266·508	762	0·306	89 54	272 35	202 7	+ 6 25	M
25.	7641	267·451	762	0·148	40 19	286 19	202 28	+ 7 45	A.
27.	7642	269·598	762	0·473	312 47	316 16	201 58	+ 7 4	A^{1}.
28.	7643	270·526	762	0·609	311 17	329 47	202 19	+ 7 16	A.
Oct. 10.	7644	282·514	No spots visible.						
13.	7645	285·515							
15.	7646	287·479	764	0·871	125 48	251 12	243 16	−14 28	S.
	7647		765	0·894	113 16	247 45	239 49	− 2 22	a.
	7648		765	0·902	113 25	248 0	240 4	− 2 9	b.
16.	7649	288·509	764	0·708	129 14	265 31	242 59	−14 5	S.
	7650		765	0·754	114 32	262 56	240 24	− 2 36	A.
17.	7651	289·465	764	0·556	136 58	279 7	243 1	−14 34	s.
	7652		766	0·958	102 51	236 19	200 13	+12 9	A.
	7653		766	0·961	103 13	235 32	199 26	+13 48	a.

TABLE III. (continued).

Date.	No.	Mean Time of Sun-picture.	No. of Group in the Kew Catalogue.	Distance from Centre.	Angle of Position.	Longitude from Node.	Helio-graphical Longitude.	Helio-graphical Latitude.	Spot.
1866. Oct. 19.	7654	291·596	764	0·208	204° 44′	310° 53′	244° 33′	−13° 2′	A.
	7655		766	0·679	96 57	267 4	200 44	+13 19	B.
24.	7656	296·508	766	0·406	325 8	335 59	199 59	+14 22	a.
26.	7657	298·435	766	0·708	311 5	3 5	199 45	+12 57	a^0.
28.	7658	300·479	766	0·964	311 18	32 24	200 4	+13 14	a^0.
	7659		767	0·309	139 33	304 56	112 36	− 6 5	M.
	7660		767	0·329	136 23	301 43	109 23	− 4 11	m.
	7661		767	0·371	137 10	302 39	110 19	− 5 50	n.
	7662		767	0·388	131 55	299 37	107 17	− 4 4	o.
31.	7663	303·434	767	0·451	279 31	350 17	116 3	− 6 42	M^0.
	7664		767	0·362	281 21	345 39	111 25	− 4 31	N.
	7665		767	0·365	283 7	345 8	110 54	− 3 26	n.
Nov. 2.	7666	305·482	767	0·802	279 8	18 53	115 36	− 5 22	A.
	7667		767	0·761	281 19	14 6	110 49	− 3 39	a.
	7668		767	0·765	284 32	14 28	111 11	− 4 54	a^0.
4.	7669	307·514	No spots visible.						
6.	7670	309·432							
8.	7671	311·529							
14.	7672	317·458							
17.	7673	320·466							
19.	7674	322·474	768	0·936	94 38	263 7	118 48	+13 6	A.
	7675		768	0·937	93 12	260 49	116 30	+13 7	b.
	7676		768	0·949	93 47	262 9	117 50	+14 47	c.
20.	7677	323·504	768	0·830	93 32	276 13	117 18	+13 12	M.
	7678		768	0·846	92 6	276 33	117 38	+13 55	m.
	7679		769	0·958	106 30	263 38	104 53	+ 2 6	O.
21.	7680	324·493	768	0·692	87 16	291 44	118 47	+14 30	M^0.
	7681		769	0·845	104 47	278 48	105 51	+ 1 57	O^0.
25.	7682	328·448	768	0·236	26 11	347 41	118 38	+13 58	A.
	7683		768	0·241	31 47	346 1	116 58	+14 21	a.
	7684		769	0·286	105 49	333 21	104 18	+ 1 54	B.
	7685		769	0·288	106 51	333 13	104 10	+ 2 33	b.
26.	7686	329·476	769	0·236	290 29	348 21	104 44	+ 1 20	S.
	7687		769	0·209	284 31	349 17	105 40	− 0 35	T.
	7688		769	0·211	293 9	348 11	104 34	+ 2 41	s.
	7689		769	0·219	297 30	348 12	104 35	+ 3 26	t.
27.	7690	330·535	769	0·306	331 38	3 25	104 46	+ 2 16	S.
	7691		770	0·494	80 39	324 35	65 56	+ 9 9	A.
	7692		770	0·550	81 51	320 37	61 58	+ 8 12	a.
	7693		770	0·536	80 19	322 57	64 18	+ 8 22	B.
	7694		770	0·556	82 38	319 23	60 44	+ 8 21	b.
28.	7695	331·501	769	0·632	288 52	16 24	104 3	+ 2 9	C.
30.	7696	333·452	769	0·820	286 38	45 41	105 39	+ 1 6	C.
	7697		769	0·814	285 18	44 10	104 8	+ 1 37	c.
Dec. 7.	7698	340·495	No spots visible.						
14.	7699	347·460							
19.	7700	352·475							
28.	7701	361·502							

XXII. *On the Mechanical Performance of Logical Inference.* *By* W. STANLEY JEVONS, *M.A.* (*Lond.*), *Professor of Logic, &c. in Owens College, Manchester. Communicated by Professor* H. E. ROSCOE, *F.R.S.*

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1. IT is an interesting subject for reflection that from the earliest times mechanical assistance has been required in mental operations. The word *calculation* at once reminds us of the employment of pebbles for marking units, and it is asserted that the word ἀριθμὸς is also derived from the like notion of a pebble or material sign*. Even in the time of ARISTOTLE the wide extension of the decimal system of numeration had been remarked and referred to the use of the fingers in reckoning; and there can be no doubt that the form of the most available arithmetical instrument, the human hand, has reacted upon the mind and moulded our numerical system into a form which we should not otherwise have selected as the best.

2. From early times, too, distinct mechanical instruments were devised to facilitate computation. The Greeks and Romans habitually employed the *abacus* or arithmetical board, consisting, in its most convenient form, of an oblong frame with a series of cross wires, each bearing ten sliding beads. The *abacus* thus supplied, as it were, an unlimited series of fingers, which furnished marks for successive higher units and allowed of the representation of any number. The Russians employ the abacus at the present day under the name of the *shtshob*, and the Chinese have from time immemorial made use of an almost exactly similar instrument called the *schwanpan*.

3. The introduction into Europe of the Arabic system of numeration caused the abacus to be generally superseded by a far more convenient system of written signs; but mathematicians are well aware that their science, however much it may advance, always requires a corresponding development of material symbols for relieving the memory and guiding the thoughts. Almost every step accomplished in the progress of the arts and sciences has produced some mechanical device for facilitating calculation or representing its result. I may mention astronomical clocks, mechanical globes, planetariums, slide rules, &c. The ingenious rods known as NAPIER'S Bones, from the name of their inventor, or the Promptuarium Multiplicationis of the same celebrated mathematician†, are curious examples of the tendency to the use of material instruments.

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* Professor DE MORGAN "On the word 'Αριθμὸς," Proceedings of the Philological Society (p. 9).

† Rabdologiæ seu numerationis per virgulas libri duo: cum appendice de expeditissimo multiplicationis Promptuario. Quibus accessit et Arithmeticæ Localis Liber Unus. Authore et Inventore JOANNE NEPERO, Barone Merchistonii &c. Lugduni, 1626.

4. As early as the 17th century we find that machinery was made to perform actual arithmetical calculation. The arithmetical machine of PASCAL was constructed in the years 1642–45, and was an invention worthy of that great genius. Into the peculiarities of the machines subsequently proposed or constructed by the Marquis of WORCESTER, Sir SAMUEL MORLAND, LEIBNITZ, GERSTEN, SCHEUTZ, DONKIN, and others we need not inquire; but it is worthy of notice that M. THOMAS, of Colmar, has recently manufactured an arithmetical machine so perfect in construction and so moderate in cost, that it is frequently employed with profit in mercantile, engineering, and other calculations.

5. It was reserved for the profound genius of Mr. BABBAGE to make the greatest advance in mechanical calculation, by embodying in a machine the principles of the calculus of differences. Automatic machinery thus became capable of computing the most complicated mathematical tables*; and in his subsequent design for an Analytical Engine Mr. BABBAGE has shown that material machinery is capable, in theory at least, of rivalling the labours of the most practised mathematicians in all branches of their science. Mind thus seems able to impress some of its highest attributes upon matter, and to create its own rival in the wheels and levers of an insensible machine.

6. It is highly remarkable that when we turn to the kindred science of logic we meet with no real mechanical aids or devices. Logical works abound, it is true, with metaphorical expressions implying a consciousness that our reasoning powers require such assistance, even in the most abstract operations of thought. In or before the 15th century the logical works of the greatest logician came to be commonly known as the *Organon* or *Instrument*, and, for several centuries, logic itself was defined as *Ars instrumentalis dirigens mentem nostram in cognitionem omnium intelligibilium.*

When FRANCIS BACON exposed the futility of the ancient deductive logic, he still held that the mind is helpless without some mechanical rule, and in the second aphorism of his 'New Instrument' he thus strikingly asserts the need:—

Nec manus nuda, nec Intellectus sibi permissus, multum valet; Instrumentis et auxiliis res perficitur; quibus opus est, non minus ad intellectum, quam ad manum. Atque ut instrumenta manus motum aut cient, aut regunt; ita et Instrumenta mentis, Intellectui aut suggerunt aut cavent.

7. In all such expressions, however, the word *Instrument* is used metaphorically to denote an invariable formula or rule of words, or system of procedure. Even when RAYMOND LULLY put forth his futile scheme of a mechanical syllogistic, the mechanical apparatus consisted of nothing but written diagrams. It is rarely indeed that any invention is made without some anticipation being sooner or later discovered; but up to the present time I am totally unaware of even a single previous attempt to devise or construct a machine which should perform the operations of logical inference†; and it is only I believe in the satirical writings of SWIFT that an allusion to an actual reasoning machine is to be found‡.

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* See Companion to the Almanack for 1866, p. 5.

† See note at the end of this paper, p. 518.

‡ In the recent Life of Sir W. HAMILTON, by Professor VEITCH, is given an account and figure of a wooden instrument employed by Sir W. HAMILTON in his logical lectures to represent the comparative extent and intent

8. The only reason which I can assign for this complete inability of logicians to devise a real logical instrument, is the great imperfection of the doctrines which they entertained. Until the present century logic has remained substantially as it was moulded by ARISTOTLE 2200 years ago. Had the science of quantity thus remained stationary since the days of PYTHAGORAS or EUCLID, it is certain that we should not have heard of the arithmetical machine of PASCAL, or the difference-engine of BABBAGE. And I venture to look upon the logical machine which I am about to describe as equally a result and indication of a profound reform and extension of logical science accomplished within the present century by a series of English writers, of whom I may specially name JEREMY BENTHAM, GEORGE BENTHAM, Professor DE MORGAN, Archbishop THOMSON, Sir W. HAMILTON, and the late distinguished Fellow of the Royal Society, Dr. BOOLE. The result of their exertions has been to effect a breach in the supremacy of the Aristotelian logic, and to furnish us, as I shall hope to show by visible proof, with a system of logical deduction almost infinitely more general and powerful than anything to be found in the old writers. The ancient syllogism was incapable of mechanical performance because of its extreme incompleteness and crudeness, and it is only when we found our system upon the fundamental laws of thought themselves that we arrive at a system of deduction which can be embodied in a machine acting by simple and uniform movements.

9. To GEORGE BOOLE, even more than to any of the logicians I have named, this great advance in logical doctrine is due. In his 'Mathematical Analysis of Logic' (1847), and in his most remarkable work 'Of the Laws of Thought' (London, 1854), he first put forth the problem of logical science in its complete generality:—*Given certain logical premises or conditions, to determine the description of any class of objects under those conditions.* Such was the general problem of which the ancient logic had solved but a few isolated cases—the nineteen moods of the syllogism, the sorites, the dilemma, the disjunctive syllogism, and a few other forms. BOOLE showed incontestably that it was possible, by the aid of a system of mathematical signs, to deduce the conclusions of all these ancient modes of reasoning, and an indefinite number of other conclusions. Any conclusion, in short, that it was possible to deduce from any set of premises or conditions, however numerous and complicated, could be calculated by his method.

10. Yet BOOLE's achievement was rather to point out the extent of the problem and the possibility of solving it, than himself to give a clear and final solution. As readers of his logical works must be well aware, he shrouded the simplest logical processes in the mysterious operations of a mathematical calculus. The intricate trains of symbolic transformations, by which many of the examples in the 'Laws of Thought' are solved, can be followed only by highly accomplished mathematical minds; and even a mathematician would fail to find any demonstrative force in a calculus which fearlessly employs unmeaning and incomprehensible symbols, and attributes a signification to

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of meaning of terms; but it was merely of an illustrative character, and does not seem to have been capable of performing any mechanical operations.

them by a subsequent process of interpretation. It is surely sufficient to condemn the peculiar mathematical form of BOOLE's method, that if it were the true form of logical deduction, only well-trained mathematicians could ever comprehend the action of those laws of thought, on the habitual use of which our existence as superior beings depends.

11. Having made BOOLE's logical works a subject of study for many years past, I endeavoured to show in my work on Pure Logic* that the mysterious mathematical forms of BOOLE's logic are altogether superfluous, and that in one point of great importance, the employment of exclusive instead of unexclusive alternatives, he was deeply mistaken. Rejecting the mathematical dress and the erroneous conditions of his symbols, we arrive at a logical method of the utmost generality and simplicity. In a later work† I have given a more mature and clear view of the principles of this Calculus of Logic, and of the processes of reasoning in general, and to these works I must refer readers who may be interested in the speculative or theoretical views of the subject. In the present paper my sole purpose is to bring forward a visible and tangible proof that a new system of logical deduction has been attained. The logical machine which I am about to describe is no mere model illustrative of the fixed forms of the syllogism. It is an analytical engine of a very simple character, which performs a complete analysis of any logical problem impressed upon it. By merely reading down the premises or data of an argument on a key board representing the terms, conjunctions, copula, and stops of a sentence, the machine is caused to make such a comparison of those premises that it becomes capable of returning any answer which may be logically deduced from them. It It is charged, as it were, with a certain amount of information which can be drawn from it again in any logical form which may be desired. The actual process of logical deduction is thus reduced to a purely mechanical form, and we arrive at a machine embodying the 'Laws of Thought,' which may almost be said to fulfil in a substantial manner the vague idea of an organon or instrumental logic which has flitted during many centuries before the minds of logicians.

12. As the ordinary views of logic and the doctrine of the syllogism would give little or no assistance in comprehending the action of the machine, I find it necessary to preface the description of the machine itself with a brief and simple explanation of the principles of the indirect method of inference which is embodied in it, avoiding any reference to points of abstract or speculative interest which could not be suitably treated in the present paper.

13. Whatever be the form in which the rules of deductive logic are presented, their validity must rest ultimately upon the Three Fundamental Laws of Thought which develope the nature of Identity and Diversity. These laws are three in number. The first appears to give a definition of Identity by asserting that *a thing is identical with*

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* Pure Logic, or the Logic of Quality apart from Quantity: with Remarks on BOOLE's System, and on the relation of Logic and Mathematics. London, 1864 (Stanford).

† The Substitution of Similars, the True Principle of Reasoning: derived from a Modification of ARISTOTLE's Dictum. London, 1869 (MACMILLAN).

itself; the second, known as the Law of Contradiction, states that *a thing cannot at the same time and place combine contradictory or opposite attributes*; whatever A and B may be it is certain that A cannot be both B and not B. This law, then, excludes from real, or even conceivable existence, any combination of opposite attributes.

The third law, commonly known as the Law of Excluded Middle, but which I prefer to call by the simpler title of the Law of Duality, asserts that *every thing must either possess any given attribute or must not possess it.* A must either be B or not B. It enables us to predict anterior to all particular experience the alternatives which may be asserted of any object. When united, these laws give us the all-sufficient means of analyzing the results of any assertion: the Law of Duality developes for us the classes of objects which may exist; the Law of Identity allows us to substitute for any name or term that which is asserted or known to be identical with it; while the Law of Contradiction directs us to exclude any class or alternative which is thus found to involve self-contradiction.

14. To illustrate this by the simplest possible instance, suppose we have given the assertion that

A metal is an element,

and it is required to arrive at the description of the class of *compound* or *not-elementary bodies* so far as affected by this assertion. The process of thought is as follows:—

By the Law of Duality I develope the class *not-element* into two possible parts, those which are *metal* and those which are *not metal,* thus—

What is not element is either metal or not-metal.

The given premise, however, enables me to assert that *what is metal is element*; so that if I allowed the first of these alternatives to stand there would be a *not-element* which is yet an *element.* The law of contradiction directs me to exclude this alternative from further consideration, and there remains the inference, commonly known as the contrapositive of the premise, that

What is not element is not metal.

Though this is a case of the utmost simplicity, the process is capable of repeated application *ad infinitum,* and logical problems of any degree of complication can thus be solved by the direct use of the most fundamental Laws of Thought.

15. To take an instance involving three instead of two terms, let the premises be—

Iron is a metal . . . . . (1)
Metal is element . . . . (2)

We can, by the Law of Duality, develope any of these terms into four possible combinations. Thus

Iron is metal, element; . . . (α)
or metal, not-element; . . (β)
or not-metal, element; . . (γ)
or not-metal, not-element . (δ)

But the first premise informs us that iron is a metal, and thus excludes the combinations (γ) and (δ), while the second premise informs us that *metal* must be *element*, and thus further excludes the combination (β). It follows that iron must be described by the first alternative (α) only, and that it is an element, thus proving the conclusion of the syllogistic mood Barbara.

16. In employing this method of inference, it is soon found to be tedious to write out at full length in words the combinations of terms to be considered. It is much better to substitute for the words single letters, A, B, C, &c., which may stand in their place and bear in each problem a different meaning, just as x, y, z in algebra signify different quantities in different problems, and are really used as brief marks to be substituted for the full descriptions of those quantities. At the same time it is convenient to substitute for the corresponding negative terms small italic letters, a, b, c, &c.; thus

if A denote	*iron,*	a	denotes	what is	*not-iron.*
B	,, *metal,*	b	,,	,,	*not-metal.*
C	,, *element,*	c	,,	,,	*not-element.*

When these general terms are combined side by side, as in A B C, a B C, they denote a term or thing combining the properties of the separate terms. Thus A B C denotes *iron which is metal and element*; a B C denotes *metal which is element but not iron.* These letter terms A, B, C, a, b, c, &c. can, in short, be joined together in the manner of adjectives and nouns.

17. I must particularly insist upon the fact, however, that there is nothing peculiar or mysterious in these letter symbols. They have no force or meaning but such as they derive from the nouns and adjectives for which they stand as mere abbreviations, intended to save the labour of writing, and the want of clearness and conciseness attaching to a long clause or series of words. In the system put forth by BOOLE various symbols of obscure or even incomprehensible meaning were introduced; and it was implied that the inference came from operations different from those of common thought and common language. I am particularly anxious to prevent the misapprehension that the method of inference embodied in the machine is at all symbolic and dark, or differs from what the unaided human mind can perform in simple cases.

18. Great clearness and brevity are, however, gained by the use of letter terms; for if we take

A = iron,
B = metal,
C = element;

then the premises of the problem considered are simply

Iron is metal . . . A is B, ... (1)
Metal is element . . B is C. . . . . (2)

The combinations in which A may manifest itself are, according to the Laws of Thought,

A B C, ... (α)
A B *c*, ... (β)
A *b* C, ... (γ)
A *b* *c*. . . . . (δ)

But of these (γ) and (δ) are contradicted by (1) and (β) by (2). Hence

A is identical with A B C,

and this term, A B C, contains the full description of A or *iron* under the conditions (1) and (2).

Similarly, we may obtain the description of the term or class of things *not-element*, denoted by *c*. For by the Law of Duality *c* may be developed into its alternatives or possible combinations.

A B *c*, ... (β)
A *b* *c*, ... (δ)
a B *c*, ... (ζ)
a *b* *c*. . . . . (θ)

Of these (β) and (ζ) are contradicted by (2) and (δ) by (1); so that, excluding these contradictory terms, *a b c* alone remains as the description or equivalent of the class *c*. Hence what is *not-element*, is always *not-metallic* and is also *not-iron*.

19. In practising this process of indirect inference upon problems of even moderate complexity, it is found to be tedious in consequence of the number of alternatives which have to be written and considered time after time. Modes of abbreviation can, however, be readily devised. In the problem already considered it is evident that the same combination sometimes occurs over again, as in the cases of (β) and (δ); and if we were desirous of deducing all the conclusions which could be drawn from the premises we should find the combination (α) occurring in all the separate classes A, A B, B, B C, A C. Similarly, the combination *a b* C occurs in the classes *a*, *b*, C, *a b*, *a* C, *b* C, and it would be an absurd loss of labour to examine again and again whether the same combination is or is not contradicted by the premises. It is certain that all the combinations of the terms A, B, C, *a*, *b*, *c*, which are possible under the universal conditions of thought and existence are but eight in number, as follows:—

(α)	A B C
(β)	A B *c*
(γ)	A *b* C
(δ)	A *b* *c*
(ε)	*a* B C
(ζ)	*a* B *c*
(η)	*a* *b* C
(θ)	*a* *b* *c*

All the classes of things which can possibly exist will be represented by an appropriate selection from this list; B will consist of (α), (β), (ϵ) and (ζ); C will consist of (α), (γ), (ϵ) and (η); B C will consist of the combinations common to these classes, as (α) and (ϵ); and so on. If we wish, then, to effect a complete solution of a logical problem, it will save much labour to make out in the first place the complete development of combinations, to examine each of these in connexion with the premises, to eliminate the inconsistent combinations, and afterwards to select from the remaining consistent combinations such as may form any class of which we desire the description. Performing these processes in the case of the premises (1) and (2), we find that of the eight conceivable combinations only four remain consistent with the premises, viz.:—

A B C	(α)
a B C	(ϵ)
a b C	(η)
a b c	(θ)

In this list of combinations the conditions (1) and (2) are, as it were, embodied and expressed, so that we at once learn that A according to those conditions consists of A B C only;

B consists of (α) or (ϵ)
b ,, ,, (η) or (θ)
c ,, ,, (θ)
a ,, ,, (ϵ), (η) or (θ).

20. It is easily seen that the solution of every problem which involves three terms A, B, C will consist in making a similar selection of consistent combinations from the same series of eight conceivable combinations. Problems involving four distinct terms would similarly require a series of sixteen conceivable combinations, and if five or six terms enter, there will be thirty-two or sixty-four of such combinations. These series of combinations appear to hold a position in logical science at least as important as that of the multiplication table in arithmetic or the coefficients of the binomial theorem in the higher parts of mathematics. I propose to call any such complete series of combinations a *Logical Abecedarium*, but the number of combinations increases so rapidly with the number of separate terms that I have not found it convenient to go beyond the sixty-four combinations of the six terms A, B, C, D, E, F and their negatives.

21. To a person who has once comprehended the extreme significance and utility of the Logical Abecedarium, the whole indirect process of inference becomes reduced to the repetition of a few uniform operations of classification, selection, and elimination of contradictories. Logical deduction becomes, in short, a matter of routine, and the amount of labour required the only impediment to the solution of any question. I have directed much attention, therefore, to reduce the labour required, and have in previous publications described devices which partially accomplish this purpose. The Logical Slate consists of the complete Abecedarium engraved upon a common writing slate, and merely

saves the labour of writing out the combinations*. The same purpose may be effected by having series of combinations printed ready upon separate sheets of paper, a series of proper length being selected for the solution of any problem, and the inconsistent combinations being struck out with the pen as they are discovered on examination with the premises.

22. A second step towards a mechanical logic was soon seen to be easy and desirable. The fixed order of the combinations in the written abecedarium renders it necessary to consider them separately, and to pick out by repeated acts of mental attention those which fall into any particular class. Considerable labour and risk of mistake thus arise. The Logical Abacus was devised to avoid these objections, and was constructed by placing the combinations of the *abecedarium* upon separate moveable slips of wood, which can then be easily classified, selected and arranged according to the conditions of the problem. The construction and use of this Abacus have, however, been sufficiently described both in the 'Proceedings of the Manchester Literary and Philosophical Society' for 3rd April, 1866, and more fully in my recently published work, called 'The Substitution of Similars,' which contains a figure of the Abacus. I will only remark, therefore, that while the logical slate or printed abecedarium is convenient for the private study of logical problems, the abacus is peculiarly adapted for the logical class-room. By its use the operations of classification and selection, on which Boole's logic, and in fact any logic must be founded, can be represented, and the clearest possible solution of any question can be shown to a class of students, each step in the solution being made distinctly apparent.

23. In proceeding to explain how the process of logical deduction by the use of the abecedarium can be reduced to a purely mechanical form, I must first point out that certain simple acts of classification are alone required for the purpose. If we take the eight conceivable combinations of the terms A, B, C, and compare them with a proposition of the form

A is B, ... (1)

we find that the combinations fall apart into three distinct groups, which may be thus indicated:—

Excluded combinations . . . . $\left\{\begin{array}{cccc} a & a & a & a \\ \mathrm{B} & \mathrm{B} & b & b \\ \mathrm{C} & c & \mathrm{C} & \mathrm{D} \end{array}\right.$

Included combinations consistent with premise (1) . . . . . $\left\{\begin{array}{cc} \mathrm{A} & \mathrm{A} \\ \mathrm{B} & \mathrm{B} \\ \mathrm{C} & c \end{array}\right.$

Included combinations inconsistent with premise (1) . . . . $\left\{\begin{array}{cc} \mathrm{A} & \mathrm{A} \\ b & b \\ \mathrm{C} & c \end{array}\right.$

The highest group contains those combinations which are all *a*'s, and on account of the absence of A are unaffected by the statement that A's are B's; they are thus *excluded* from the sphere of meaning of the premise, and their consistency with truth cannot be affected by that premise. The middle group contains A-combinations, included within the meaning of the premise, but which also are B-combinations, and therefore comply with the condition expressed in the premise. The lowest group consists of A-combinations also, but such as are distinguished by the absence of B, and which are therefore inconsistent with the premise requiring that where A is, there B shall be likewise. This analysis would evidently be effected most simply by placing the eight combinations of the abecedarium in the middle rank, raising the *a*'s into a higher rank, and then lowering such *b*'s as remain in the middle rank into a lower rank. But as we only require in the solution of a problem to eliminate the inconsistent combinations, we must unite again the two upper ranks, and we then have

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* See Pure Logic, p. 68.

$$\text{Combinations consistent with the premise (1)} \ldots \left\{\begin{array}{cccccc} A & A & a & a & a & a \\ B & B & B & B & b & b \\ C & c & C & c & C & c \end{array}\right.$$

$$\text{Combinations inconsistent with the premise (1)} \ldots \left\{\begin{array}{cc} A & A \\ b & b \\ C & c \end{array}\right.$$

24. Supposing we now introduce the second premise,

$$B \text{ is } C, \qquad \ldots \qquad (2)$$

the operations will be exactly similar, with the exception that certain combinations have already been eliminated from the abecedarium by the first premise. These contradicted combinations may or may not be consistent with the second premise, but in any case they cannot be readmitted. Whatever is inconsistent with any one condition, is to be deemed inconsistent throughout the problem. Hence the analysis effected by the second premise may be thus represented:—

$$\text{Combinations consistent with (1).} \left\{\begin{array}{ll} \text{Combinations excluded from (2)} \ldots & \left\{\begin{array}{cc} a & a \\ b & b \\ C & c \end{array}\right. \\ \text{Combinations included and consistent with (2)} \ldots & \left\{\begin{array}{cc} A & a \\ B & B \\ C & C \end{array}\right. \\ \text{Combinations inconsistent with (2)} & \left\{\begin{array}{cc} A & a \\ B & B \\ c & c \end{array}\right. \end{array}\right.$$

$$\text{Combinations inconsistent with (1)} \ldots \left\{\begin{array}{cc} A & A \\ b & b \\ C & c \end{array}\right.$$

To effect the above classification, we first move down to a lower rank the combinations inconsistent with (1); we then raise the *b*'s, and out of the remaining B's lower the *c*'s.

But as all our operations are directed only to distinguish the consistent and inconsistent combinations, we now join the highest to the second rank, and the third to the lowest, as follows:—

Combinations consistent with (1) and (2) . . . $\left\{\begin{array}{llll} A & a & a & a \\ B & B & b & b \\ C & C & C & c \end{array}\right.$

Combinations inconsistent with (1) or (2), or both . $\left\{\begin{array}{llll} A & A & A & a \\ B & b & b & B \\ c & C & c & c \end{array}\right.$

25. The problem is now solved, and it only remains to put any question we may desire. Thus if we want the description of the class A, we may raise out of the consistent combinations such as are *a*'s, and the sole remaining combination A B C gives the description required, agreeably to our former conclusion. To obtain the description of B, we unite the consistent combinations again and raise the *b*'s; there will remain two combinations A B C and *a* B C, showing that B is always C, but that, so far as the conditions of the problem go, it may or may not be A.

26. In considering such other kinds of propositions as might occur, we meet the case where two or more terms are combined together to form the subject or predicate, as in the example

A B is C,

meaning that whatever combinations contain both A and B, ought also to contain C. This case presents no difficulty; and to obtain the included combinations it is only necessary to raise out of the whole series of combinations the *a*'s *and* *b*'s, simultaneously or successively. The result, in whatever way we do it, is as follows:—

Excluded combinations . . . . . . $\left\{\begin{array}{llllll} A & A & a & a & a & a \\ b & b & B & B & b & b \\ C & c & C & c & C & c \end{array}\right.$

Included combinations . . . $\left\{\begin{array}{ll} A & A \\ B & B \\ C & c \end{array}\right.$

We may then remove such of the included combinations, *i. e.* A B *c* only, as may be inconsistent with the premise, and proceed as before.

27. Had the predicate instead of the subject contained two terms as in

A is B C,

we should have required to raise the *a*'s and then lower the *b*'s and the *c*'s, in an exactly similar manner.

28. The only further complication to be considered arises from the occurrence of the disjunctive conjunction *or* in the subject or predicate, as in the case

A is B or C (or both).

To investigate the proper mode of treating this condition, we may take the same series of eight conceivable combinations and raise those containing *a*, in order to separate the excluded combinations. But it is not now sufficient simply to lower such of the included combinations as contain *b*, and condemn these as inconsistent with the premise. For though these combinations do not contain B they may contain C, and may require to be admitted as consistent on account of the second alternative of the predicate. While the A B's are certainly to be admitted, the A *b*'s must be subjected to a new process of selection. Now the simplest mode of preparing for this new selection is to join the A B's to the *a*'s or excluded combinations, to move up the A *b*'s into the place last occupied by the A's, to lower such of the A *b*'s as do not contain C. The result will then be as follows:—

Excluded combinations and included combinations consistent with 1st alternative	A B C	A B *c*	*a* B C	*a* B *c*	*a* *b* C	*a* *b* *c*
Included combination inconsistent with 1st but consistent with 2nd alternative . . .	A *b* C					
Included combination inconsistent with both alternatives	A *b* *c*					

It is only the lowest rank of combinations, in this case containing only A *b c*, which is inconsistent with the premise as a whole, and which is therefore to be condemned as contradictory; and if we join the two higher ranks we have effected the requisite analysis.

29. It will be apparent that should the subject of the premise contain a disjunctive conjunction, as in

A or B is C,

a similar series of operations would have to be performed. We must not merely raise the *a*'s and treat them as excluded combinations, but must return them to undergo a new sifting, whereby the *a* B's will be recognized as included in the meaning of the subject, and only the *a b*'s will be treated as excluded. This analysis effected, the remaining operations are exactly as before.

30. The reader will perhaps have remarked that in the case of none of the premises considered has it been requisite to separate the combinations of the abecedarium into more than four groups or ranks, and it may be added that all problems involving simple logical relations only have been sufficiently represented by the examples used. The task of constructing a mechanical logic is thus reduced to that of classifying a series of wooden rods representing the conceivable combinations of the abecedarium into certain definite groups distinguished by their positions, and providing such mechanical arrangements, that wherever a letter term occurs in the subject or predicate of a proposition,

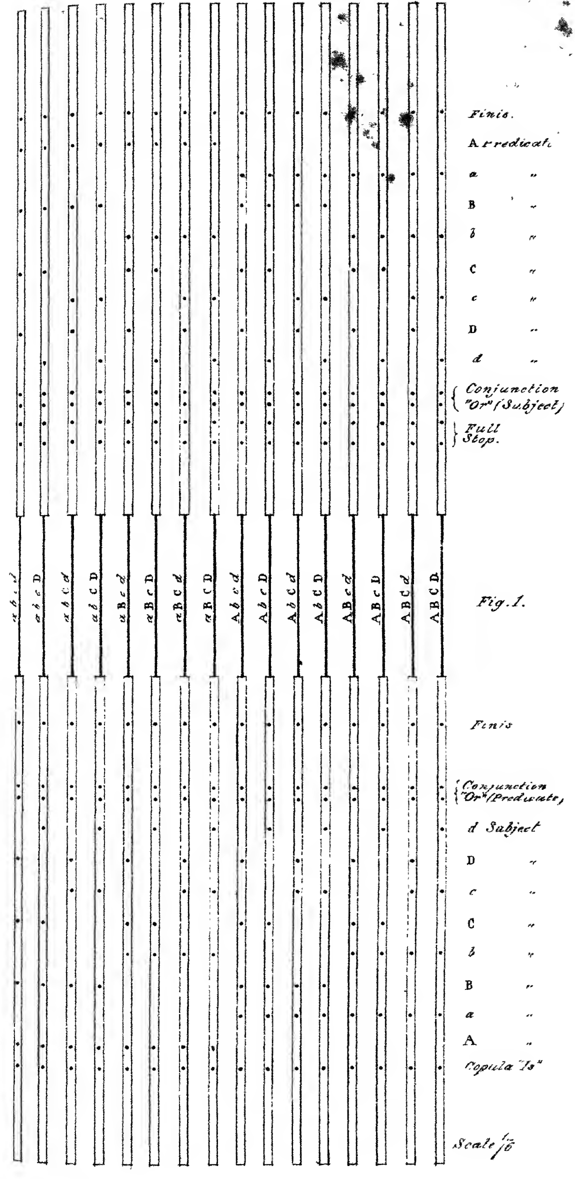
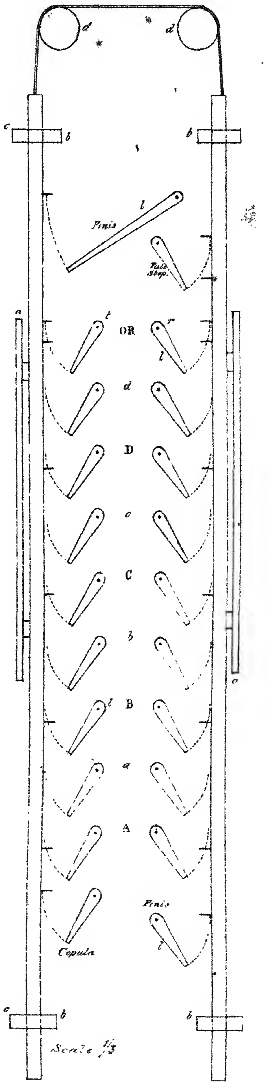

Finis.
A Predicate
a "
B "
b "
C "
c "
D "
d "
Conjunction "Or" (Subject)
Full Stop.
a b c d
a b c D
a b C d
a b C D
a B c d
a B c D
a B C d
a B C D
A b c d
A b c D
A b C d
A b C D
A B c d
A B c D
A B C d
A B C D
Finis
Conjunction "Or" (Predicate)
d Subject
D "
c "
C "
b "
B "
a "
A "
Copula "Is"
Scale 1/6

Fig. 1.

Fig. 2


d
d
c
b
b
l
Finis
Full Stop.
a
t
OR
r
l
d
D
c
C
b
l
B
a
A
Finis
Copula
l
c
b
b
Scale 1/3

or a conjunction, copula or stop intervenes, the pressure of a corresponding lever or key shall execute systematically the required movements of the combinations.

31. The principles upon which the logical machine is based will now be apparent to the reader; and as the construction of the machine involves no mechanical difficulties of any importance, it only remains for me to give as clear a description of its component parts and movements as their somewhat perplexing character admits of.

32. The Machine, which has been actually finished, is adapted to the solution of any problems not involving more than four distinct positive terms, indicated by A, B, C, D, with, of course, their corresponding negatives, *a*, *b*, *c*, *d*. The requisite combinations of the abecedarium are, therefore, sixteen in number (§ 20, p. 504), and each combination is represented by a pair of square rods of baywood (Plate XXXII. fig. 1), united by a short piece of cord and slung over two round horizontal bars of wood (*d*, *d*, figs. 2 & 3), so as to balance each other and to slide freely and perpendicularly in wooden collars (*b*, *b*, figs. 2, 3, & 4) closed by plain wooden bars (*c*, *c*). To each rod is attached a thin piece of baywood, 8½ inches long and 1 inch wide (*a*, *a*, figs. 2 & 3), bearing the letters of the combination represented. Each letter occupies a space of ½ inch in height, but is separated from the adjoining letter by a blank space of white paper 1½ inch long. Both at the front and back of the machine are pierced four horizontal slits, 1½ inch apart, extending the whole width of the case, and ½ inch in height, so placed that when the rods are in their normal position each letter shall be visible through a slit. The machine thus exhibits on its two sides, when the rods are in a certain position, the combinations of the abecedarium as shown in fig. 5; but should any of the rods be moved upwards or downwards through a certain limited distance the letters will become invisible as at *f f* (Plate XXXIII. fig. 5).

33. Externally the machine consists of a framework, seen in perpendicular section in fig. 3 (*g*, *g*), and in horizontal section in fig. 4 (*g*, *g*), which serves at once to support and contain the moving parts. It is closed at the front and back by large doors (*h*, *h*, fig. 3), in the middle panels of which are pierced the slits rendering the letters of the abecedarium visible.

34. The rods are moved upwards, and the opposite rods of each pair are thus caused to fall downwards, by a series of long flat levers seen in section at *l*, *l*, *l* (figs. 2, 3, 6 ... 13). These levers revolve on pivots inserted in the thicker part, and move in sockets attached to the inner side of the framework. Brass arms (*m*, *m*, figs. 3 & 13), connected by copper wires (*n*, *n*) with the keys of the machine (*o*, *o*), actuate the levers, which are caused to return, when the key is released, by spiral brass springs (*s*, *s*).

35. The levers communicate motion to the rods by means of brass pins fixed in the inner side of the rods (fig. 2). As it is upon the peculiar arrangement of these pins that the whole action of the machine depends, the position of each of the 272 pins is shown by a dot in fig. 1, in which are also indicated the function of each pin and the combination represented by each pair of rods. It is seen that certain pins are placed uniformly in all the adjoining rods, as in the rows opposite the words *Finis*, *Conjunction*, *Copula*, *Full Stop*. These may be called *operation pins* and must be distinguished from

the *letter pins*, representing the terms of the combination, and varied in each pair of rods to correspond with the letters of the abecedarium. On examining fig. 1, it will be apparent that the pins are distributed in a negative manner; that is to say, it is the absence of a pin in the space A, and its presence in the space *a*, which constitutes the rod a representative of the term A. The rods belonging to the combination A *b* C *d*, for instance, have pins in the spaces belonging to the letters *a*, B, *c*, D.

36. The key board of the instrument is shown in fig. 4, where are seen two sets of term or letter keys, marked A, *a*, B, *b*, C, *c*, D, *d*, separated by a key marked COPULA—IS. The letter keys on the left belong to the subject of a proposition, those on the right to the predicate, and on either side just beyond the letter keys is a *Conjunction* key, appropriated to the disjunctive conjunction OR, according as it occurs in the subject or predicate. The last key on the right hand is marked FULL STOP, and is to be pressed at the end of each proposition, where the full stop is properly placed On the extreme left, lastly, is a key marked FINIS, which is used to terminate one problem and prepare the machine for a new one.

37. In order to gain a clear comprehension of the action of these keys, we must now turn to fig. 2, where all the levers are shown in position, only three of them being inserted in fig. 3, and to figs 6–13 (Plate XXXIV.), which represent, in the full natural size, the relative positions of each kind of lever with regard to the pins in every possible position of the rods.

If the subject key A be pressed it actuates the lever A at the back of the machine; and supposing all the rods to be in their proper initial positions, it moves upwards, as in fig. 6, all the back *a* rods through exactly half an inch, the front rods connected with them of course falling through half an inch. All the *a* combinations are thus caused to disappear from the abecedarium; but as the A rods have no pins opposite to the A lever, they will remain unmoved, and continue visible. Thus the pressure of the A key effects the selection of the class A of the conceivable combinations. Each subject letter key similarly acts upon a lever at the back, and should several of them be pressed, either simultaneously or in succession, the combinations containing the corresponding letters will be selected.

38. Each predicate letter key is connected with a lever in the front of the machine, and when pressed the effect is exactly the same as that of a subject key, but in the opposite direction (fig. 11). If the B predicate key be pressed it raises through half an inch all the front rods which happen to have corresponding pins, and to be in the initial position. The back rods will at the same time fall, and the combinations containing *b* will disappear from the abecedarium, but in the opposite direction.

39. It is now necessary to explain that each rod has four possible positions fully indicated in the figs. 6–13. The first of these positions is the neutral or initial position, in which the letters are visible in the *abecedarium*, and the letter pins are opposite letter levers so as to be acted upon by them. The second position is that into which a rod is thrown by a subject key; the third position lies in the opposite direction, and is that

Fig. 6.

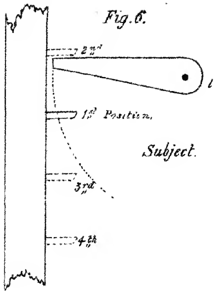

2nd
l
1st Position.
Subject.
3rd
4th

Fig. 7.

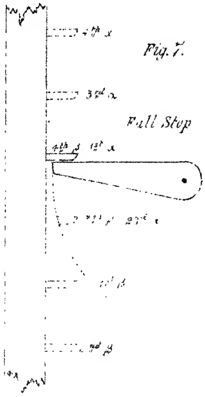

4th α
3rd α
Full Stop
4th β
1st α
3rd β
2nd α
1st β
2nd β

Fig. 8.

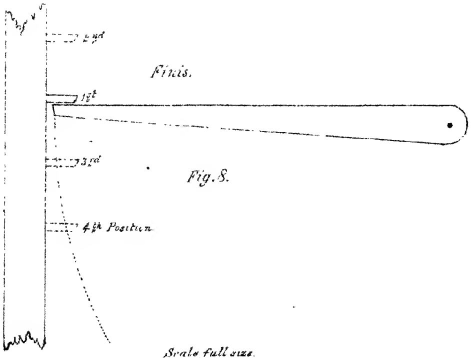

2nd
Finis.
1st
3rd
4th Position

Fig. 9.

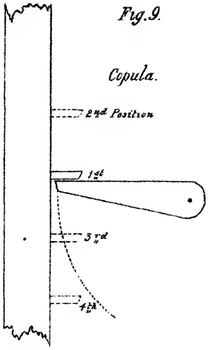

Copula.
2nd Position
1st
3rd
4th

Fig. 10.

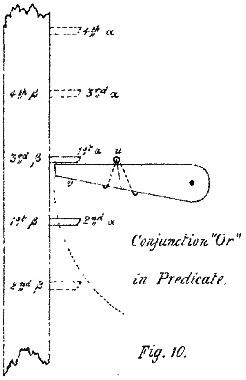

4th α
4th β
3rd α
3rd β
1st α
u
v
1st β
2nd α
2nd β
Conjunction "Or"
in Predicate.

Fig. 11.

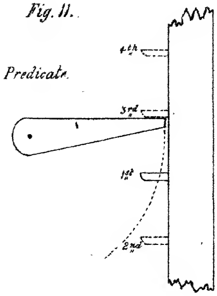

4th
Predicate.
3rd
1st
2nd

Fig. 12.

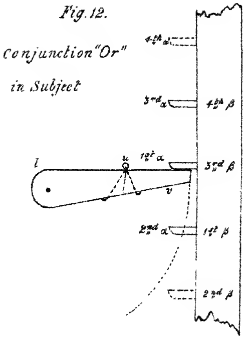
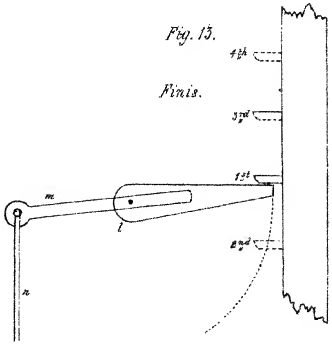

Conjunction "Or"
in Subject
4th α
3rd α
4th β
l
u
1st α
3rd β
v
2nd α
1st β
2nd β

Fig. 13.


4th
Finis.
3rd
1st
m
l
2nd
n

Scale full size.

into which a rod is thrown by a predicate key. The fourth position lies one half inch beyond the third. The four positions evidently correspond to the four classes into which combinations were classified in the previous part of the paper as follows:—

Second Position.—Combinations excluded from the sphere of the premise.

First Position.—Combinations included, but consistent with the premise.

Third Position.—Temporary position of combinations contradicted by the premise: also temporary position of combinations excluded from some of the alternatives of a disjunctive predicate.

Fourth Position.—Final position of contradictory or inconsistent combinations.

40. Let us now follow out the motions produced by impressing the simple proposition

A is B

upon the machine, all the rods being at first in the initial or first position. The keys to be pressed in succession are—

First. The subject key, A.
Second. The copula key.
Third The predicate key, B.
Fourth. The full-stop key.

The subject key A has the effect of throwing all the *a* rods from the first into the second position, the back rods rising and the front rods falling ½ inch

The copula key will in this case have no effect, for, as seen in fig. 9 (Plate XXXIV.), it acts only on rods in the third position, of which there are at present none.

The predicate key (fig. 11) does not act upon such of the rods (those marked *a*) as are in the second position, but it acts upon those in the first position, provided they have pins opposite the lever. The effect thus far will be that the *a* rods are in the second position, the A *b* rods in the third, while the A B rods remain undisturbed in the first position. An analysis has been effected exactly similar to that explained above (§ 23, p 505).

41. The full-stop key being now pressed has a double effect. It acts only on a single lever at the front of the machine (figs. 2 & 7), but the front rods all have in the space opposite to the lever two pins one inch apart (fig. 1). These pins we may distinguish as the α and β pins, the α pin being the uppermost While a rod is in the first position the lever passes between the pins and has no effect; but if the rod be lowered ½ inch into the second position, the lever will cause the rod to return to the first position by means of the α pin; but if the rod be raised into the third position, the β pin will come into gear, and the rod will be pushed ½ inch further into the fourth position. Now in the case we are examining, the AB's are in the first position and will so remain; the *a*'s are in the second and will return to the first, the A*b*'s are in the third, and will therefore proceed onwards to the fourth. The reader will now see that we have effected the classification of the combinations as required into those consistent with the premise A is B, whether they be included or not in the term A, and those contradicted by the premise which have been ejected into the fourth position. An examination of the figures 6–13 will show that only one lever (fig. 8) moved by the Finis key affects rods in

the fourth position, so that any combination rod once condemned as contradictory so remains until the close of the problem, and its letters are no more seen upon the abecedarium.

42. Any other proposition, for instance, B is C, can now be impressed on the keys, and the effects are exactly similar, except that the A*b* combinations are out of reach of the levers. The B subject key throws the *b*'s into the second, the C predicate key throws the B*c*'s into the third, and the full stop throws the latter into the fourth, where they join the A*b*'s already in that place of exclusion, while the remainder all return to the first position.

The combinations now visible in the abecedarium will be as follows:—

A	A	*a*	*a*	*a*	*a*	*a*	*a*
B	B	B	B	*b*	*b*	*b*	*b*
C	C	C	C	C	C	*c*	*c*
D	*d*	D	*d*	D	*d*	D	*d*

They correspond exactly to those previously obtained from the same premises (see § 24), except that each combination of A, B, C, *a*, *b*, *c* is repeated with D and *d*. If we now want a description of the term A, we press the subject key A, and all disappear except

A B C D, A B C *d*,

which contain the information that A is always associated with B and with C, but that it may appear with D or without D, the conditions of the problem having given us no information on this point. The series of consistent combinations is restored at any time by the full-stop key, the contradictory ones remaining excluded.

43. Any other subject key or succession of subject keys being pressed gives us the description of the corresponding terms. Thus the key *c* gives us two combinations, *a b c* D, *a b c d*, informing us that the absence of C is always accompanied by the absence of A and B. Of *b* we get the description

a	*a*	*a*	*a*
b	*b*	*b*	*b*
C	C	*c*	*c*
D	*d*	D	*d*

whence we learn that the absence of B always causes the absence of A, but that C and D are indifferently absent or present.

44. We can at any time add a new condition to the problem by pressing the full stop to bring the combinations as yet possible into the first position, and then impressing the new condition on the keys as before. Let this condition be

C is D.

The effect will obviously be to remove such C *d* combinations as yet remain into the fourth position, leaving only five:

A	*a*	*a*	*a*	*a*
B	B	*b*	*b*	*b*
C	C	C	*c*	*c*
D	D	D	D	*d*

hence we learn that A, B, and C are all D; that B, C, and D may or may not be A; that what is not D is not A, not B and not C; and so on. The conditions of this problem form what would be called a Sorites in the old logic, and we have not only obtained its conclusion A is D, but have performed a complete analysis of its conditions, and the inferences which may be drawn from those conditions.

45. The problem being supposed complete, we press the Finis key, which differs from all the others in moving two levers, one of which (fig. 13) is of the ordinary character and returns any rods which may happen to be in the second position into the first, while the other (fig. 8) has a much longer radius, is moved by a cord or flexible wire *p*, passing over a pulley *q* and through a perforation *r* in the flat board which forms the lever itself, in this case a lever of the second order. This broad lever sweeps the rods from the fourth position as well as any which may be in the third into the first, and together with the other lever (fig. 13) it reduces the whole of the rods to the neutral position, and renders the machine, as it were, a *tabula rasa*, upon which an entirely new set of conditions may be impressed independently of previous ones. Its office thus is to obliterate the effects of former problems.

46. When several of the letter keys on the subject side only or the predicate side only are pressed in succession, the effect is to select the combinations possessing all the letters marked on the keys. Thus if the keys A, B, C be pressed there will remain in the abecedarium only the combinations A B C D and A B C *d*; and if the key D be now pressed, the latter combination will disappear, and A B C D will alone remain. The effect will be exactly the same whatever the order in which the keys are pressed, and if they be pressed simultaneously there will be no difference in the result. The machine thus perfectly represents the commutative character of logical symbols which Mr. Boole has dwelt upon in pp. 29–30 of the 'Laws of Thought.' What I have called the Law of Simplicity of logical symbols, expressed by the formula AA=A *, is also perfectly fulfilled in the machine; for if the same key be pressed two or more times in succession, there will be no more effect than when it is pressed once. Thus the succession of keys A A C B B A C would have merely the effect of A B C. This applies also to the predicate keys, but not of course to an alternation of subject and predicate keys.

47. To impress upon the machine the condition

A B is C D,

or *whatever combines the properties of* A *and* B *combines the properties of* C D, we strike in succession the subject keys A and B, the Copula, the predicate keys C and D and the Full stop. The subject keys throw into the second position both the *a* combinations and the *b*'s; the predicate keys, out of the remaining A B's, throw the *c*'s and *d*'s into the third position; and the full stop completes the separation of the consistent and contradictory combinations in the usual manner.

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* Pure Logic, p. 15. Boole's 'Laws of Thought,' p. 31.

48. It yet remains for us to consider a proposition with a disjunctive term in subject, predicate, or both members. For such propositions the conjunction keys are requisite, that adjoining the subject keys (fig. 4) for the subject, and the other for the predicate. These keys act in opposition to each other, and each is opposed, again, to its corresponding letter keys. Thus while the subject keys act on levers at the back of the machine (Plate XXXIII. fig. 3), the subject conjunction key acts on the lever *r* in front, while the predicate conjunction key *t* is at the back. These levers are shown in their full size in figs. 10 & 12, and are seen to differ from all the other levers in having the edge *v* moving on small wire hinges *u* in such a way that it can exert force upwards but not downwards. The lever can thus raise the rods; but in case it should strike a pin in returning, the edge yields and passes the pin without moving the rod. In connexion with these levers each rod has two pins (figs. 1 & 2) at a distance of only ½ an inch, and the peculiar effect of these pins will be gathered from figs. 10 & 12 (Plate XXXIV.). Thus if we press in succession the predicate keys

A *or* B,

the key A will throw the *a*'s into the third position. The conjunction key will now act upon the α pins of the A's and move them into the second position, and at the same time upon the β pins of the *a*'s and return them into the first position. The key B now selects from the *a*'s those which are *b*'s, and puts them into the third position ready for exclusion by the full stop, which will also join to the *a* B's still remaining in the first position the A's which were temporarily put out of the way in the second position. Should there be, however, another alternative, as in the term

A *or* B *or* C,

the conjunction key would be again pressed, which gives the *a b*'s a new chance by returning them to the first, and the key C selects only the *a b c*'s for exclusion. The action would be exactly similar with a fourth alternative.

49. The subject conjunction key is similar but opposite in action. If the subject key A be pressed it throws the *a*'s into the second position; the conjunction key then acts upon the α pin of the *a*'s returning them to the first position, and also upon the β pin of the A's, sending them to a temporary seclusion in the third position. The key B would now select the *a b*'s for the second position; the conjunction key again pressed would return them, and add the *a* B's to those in the third, and so on. The final result would be that those combinations excluded from all the alternatives would be found in the second position, while those included in one or more alternatives would be partly in the first and partly in the third positions.

In the progress of a proposition the copula key would now have to be pressed, and when the subject is a disjunctive term its action is essential. It has the effect (fig. 9) of throwing any combinations which are in the third back into the first. It thus joins together all the combinations included in one or more alternatives of the subject, and prepares them for the due action of the predicate keys.

50. It must be carefully observed that any doubly universal proposition of the form

all A's are *all* B's,

or, in another form of expression,

A=B,

can only be impressed upon the logical machine in the form of two ordinary propositions; thus

all A's are B's,

and

all B's are A's.

The first of these excludes such A's as may be not-B's; the second excludes such B's as may be not-A's.

If we impress upon the keys of the machine the six propositions expressing the complete identity of A, B, C, and D, it is obvious that there would remain only the two combinations

A B C D,
a b c d,

the identity of the positive terms involving the identity also of their negatives.

The premise

A or B=C or D

would require to be read

A or B is C or D,
C or D is A or B.

51. To give some notion of the degree of facility with which logical problems may be solved with the machine, I will adduce the logical problem employed by Boole to illustrate the powers of his system at p. 118 of the 'Laws of Thought.'

"Suppose that an analysis of the properties of a particular class of substances has led to the following general conclusions, viz.:—

"1st. That wherever the properties A and B are combined, either the property C, or the property D, is present also; but they are not jointly present.

"2nd. That wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent.

"3rd. That wherever the properties A and B are both absent, the properties C and D are both absent also; and *vice versâ*, where the properties C and D are both absent, A and B are both absent also."

This somewhat complex problem is solved in Boole's work by a very difficult and lengthy series of eliminations, developments, and algebraic multiplications. Two or three pages are required to indicate the successive stages of the solution, and the details of the algebraic work would probably occupy many more pages. Upon the machine the problem is worked by the successive pressure of the following keys:—

1st. A, B, Copula, C, *d*, Conjunction, *c*, D, Full stop.
2nd. B, C, Copula, A, D, Conjunction, *a*, *d*, Full stop.
3rd. *a*, *b*, Copula, *c*, *d*, Full stop.
c, *d*, Copula, *a*, *b*, Full stop.

There will then be found to remain in the abecedarium the following combinations:

A B *c* D
A *b* C D
A *b* C *d*
A *b* *c* D

a B C *d*
a B *c* D
a *b* *c* *d*

On pressing the subject key A, the A combinations printed above in the left-hand column will alone remain, and on examining them they yield the same conclusion as Boole's equation (p. 120), namely, "Wherever the property A is present, there either C is present and B absent, or C is absent."

Pressing the full-stop key to restore the *a* combinations, and then the keys *b*, C, we have the two combinations

A *b* C D,
A *b* C *d*,

from which we read Boole's conclusion, p. 120, "Wherever the property C is present, and the property B absent, there the property A is present." In a similar manner the other conclusions given by Boole in p. 129 can be drawn from the abecedarium.

52. It is to be allowed that a certain mental process of interpreting and reducing to simple terms the indications of the combinations is required, for which no mechanical provision is made in the machine as at present constructed, but an exactly similar mental process is required in the Indirect Process of Inference, as stated in my 'Pure Logic,' pp. 44, 45; and equivalent processes are necessary in Boole's mathematical system. The machine does not therefore supersede the use of mental agency altogether, but it nevertheless supersedes it in most important steps of the process.

53. This mechanical process of inference proceeds by the continual selection and classification of the conceivable combinations into three or four groups. It should be noticed that in Boole's system the same groups are indicated by certain quasi-mathematical symbols as follows:—

the coefficient $\frac{0}{0}$ indicates an excluded combination
" $\frac{1}{1}$ " included "
" $\frac{0}{1}$ " inconsistent "
" $\frac{1}{0}$ " inconsistent "

It is exceedingly questionable whether there is any analogy at all between the significations of these symbols in mathematics and those which Boole imposed upon them in logic. In reality the symbol 1 denotes in Boole's logic inclusion of a combination under a term, and 0 exclusion. Accordingly $\frac{1}{0}$ indicates that the combination is included in the subject and not in the predicate, and is therefore inconsistent with the proposition, and $\frac{0}{1}$ indicates inclusion in the predicate and exclusion from the subject of an equational proposition or identity, from which also results inconsistency. Inclusion in both terms is indicated by $\frac{1}{1}$, and exclusion from both $\frac{0}{0}$, in which case the combination is consistent with the proposition.

54. To the reader of the preceding paper it will be evident that mechanism is capable of replacing for the most part the action of thought required in the performance of logical deduction. Having once written down the conditions or premises of an argument in a clear and logical form, we have but to press a succession of keys in the order corresponding to the terms, conjunctions, and other parts of the propositions, in order to effect a complete analysis of the argument. Mental agency is required only in interpreting correctly the grammatical structure of the premises, and in gathering from the letters of the abecedarium the purport of the reply. The intermediate process of deduction is effected in a material form. The parts of the machine embody the conditions of correct thinking; the rods are just as numerous as the Law of Duality requires in order that every conceivable union of qualities may have its representative; no rod breaks the Law of Contradiction by representing at the same time terms that are necessarily inconsistent; and it has been pointed out that the peculiar characters of logical symbols expressed in the Laws of Simplicity and Commutativeness are also observed in the action of the keys and levers. The machine is thus the embodiment of a true symbolic method or Calculus. The representative rods must be classified, selected, or rejected by the reading of a proposition in a manner exactly answering to that in which a reasoning mind should treat its ideas. At every step in the progress of a problem, therefore, the abecedarium necessarily indicates the proper condition of a mind exempt from mistake.

55. I may add a few words to deprecate the notion that I attribute much practical utility to this mechanical device. I believe, indeed, that it may be used with much advantage in the logical class-room, for which purpose it is more convenient than the logical abacus which I have already employed in this manner. The logical machine may become a powerful means of instruction at some future time by presenting to a body of students a clear and visible analysis of logical problems of any degree of complexity, and rendering each step of its solution plain. Its employment, however, in this way must for the present be restricted, or almost entirely prevented, by the predominance of the ancient Aristotelian logic, and the almost puerile character of the current logical examples.

56. The chief importance of the machine is of a purely theoretical kind. It demonstrates in a convincing manner the existence of an all-embracing system of Indirect Inference, the very existence of which was hardly suspected before the appearance of BOOLE'S logical works. I have often deplored the fact that though these works were published in the years 1847 and 1854, the current handbooks, and even the most extensive treatises on logic, have remained wholly unaffected thereby*. It would be possible

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* Professor BAIN'S treatise on 'Logic,' which has been published since this paper was written, is an exception. In the first Part, which treats of Deductive Logic, pp. 190–207, he gives a description and review of BOOLE'S

to search the works of two very different but leading thinkers, Mr. J. S. MILL and Sir W. HAMILTON, without meeting the name of Dr. BOOLE, or the slightest hint of his great logical discoveries; and other eminent logicians, such as Professor DE MORGAN or Archbishop THOMSON, barely refer to his works in a few appreciative sentences. This unfortunate neglect is partly due to the great novelty of BOOLE's views, which prevents them from fitting readily into the current logical doctrines. It is partly due also to the obscure, difficult, and, in many important points, the mistaken form in which BOOLE put forth his system; and my object will be fully accomplished should this machine be considered to demonstrate the existence and illustrate the nature of a very simple and obvious method of Indirect Inference of which Dr. BOOLE was substantially the discoverer.

Mathematical System; but it is significant that he omits the process of mathematical deduction where it is in the least complex, and merely quotes BOOLE's conclusions. Thus we have the anomalous result that in a treatise on Logical Deduction, the reader has to look elsewhere for processes which, according to BOOLE, must form the very basis of Deduction.

NOTE to § 7.

It has been pointed out to me by Mr. WHITE, and has also been noticed in 'Nature' (March 10th, 1870, vol. i. p. 487), that in the year 1851, Mr. ALFRED SMEE, F.R.S., the Surgeon of the Bank of England, published a work called 'The Process of Thought adapted to words and language, together with a description of the Relational and Differential Machines' (Longmans), which alludes to the mechanical performance of thought.

After perusing this work, which was unknown to me when writing the paper, it cannot be doubted that Mr. SMEE contemplated the representation by mechanism of *certain* mental processes. His ideas on this subject are characterized by much of the ingenuity which he is well known to have displayed in other branches of science. But it will be found on examination that his designs have no connexion with mine. His represent the mental states or operations of memory and judgment, whereas my machine performs logical inference. So far as I can ascertain from the obscure descriptions and imperfect drawings given by Mr. SMEE, his Relational Machine is a kind of Mechanical Dictionary, so constructed that if one word be proposed its relations to all other words will be mechanically exhibited. The Differential Machine was to be employed for comparing ideas and ascertaining their agreement and difference. It might be roughly likened to a patent lock, the opening of which proves the agreement of the tumblers and the key.

It does not appear, again, that the machines were ever constructed, although Mr. SMEE made some attempts to reduce his designs to practice. Indeed he almost allows that the Relational Machine is a purely visionary existence when he mentions that it would, if constructed, occupy an area as large as London.—October 10, 1870.

XXIII. *On the Fossil Mammals of Australia.*—Part III. Diprotodon australis, OWEN.

By Professor OWEN, *F.R.S. &c.*

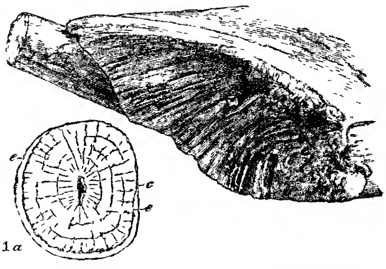
Received December 10, 1869,—Read February 3, 1870.

§ 1. *Introduction.*—IN a letter dated May 8th, 1838, addressed to Sir THOMAS MITCHELL, F.G.S., Surveyor-General of Australia, giving results of an examination of a series of Fossil Remains from caves in 'Wellington Valley,' and published in his 'Three Expeditions into the Interior of Eastern Australia,' vol. ii. 8vo. 1838, one of the specimens was described as follows.—

"Genus DIPROTODON. I apply this name to the genus of *Mammalia*, represented by the anterior extremity of the right ramus, lower jaw, with a single large procumbent incisor (IX.), fig. 1, pl. 31. This is the specimen conjectured to have belonged to the Dugong, but the incisor resembles the corresponding tooth of the Wombat in its enamelled structure and position (see fig. 2, pl. 31, and a section of the Wombat's teeth in fig. 7, pl. 30). It differs, however, in the quadrilateral figure of its transverse section, in which it corresponds with the inferior incisors of the *Hippopotamus*. To this *Diprotodon*, or to some distinct species of equal size, have belonged the fragments of bones of extremities marked X, X *a*, X *b*" (p. 362).

I reproduce the original figures (Woodcut, figs. 1 & 1 *a*) representing the specimen of half the natural size, and the section of the incisor of the full size, on which the genus was founded, but which specimen I now know to be that of a young individual.

Fig. 1



Extraordinary as seemed the magnitude of the beast which this tooth indicated, at a period when the largest known mammal of Australia was a Kangaroo, it gave only half the size of the full-grown *Diprotodon australis*.

In ignorance of this fact I was led astray by the first evidences (femur and molar teeth) of the mature animal which were transmitted to me from freshwater deposits in another and remote locality of Australia; and, for a short time, I believed them to belong to a Proboscidian, referring them, in 1843, on the authority of a drawing of part of a jaw and teeth transmitted to me by Sir THOMAS MITCHELL, to the Dinotherian section of that order*.

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* Annals and Magazine of Natural History, No. 71, for May 1843.

But I was not long under this delusion, and in 1844 realized grounds for the following rectification:—"Having since received specimens of portions of lower jaws with teeth identical in structure with the fragment figured in my first communication to the 'Annals of Natural History' (p. 9, figs. 2 & 3), I find that the reference of that portion of tooth to the genus *Dinotherium* was premature and erroneous. The extinct species to which it belonged does indeed combine molar teeth like those of *Dinotherium* with two large incisive tusks in the lower jaw; but those tusks incline upwards instead of downwards, and are identical in form and structure with the tusk from one of the bone-caves of Wellington Valley, described by me in Sir THOMAS MITCHELL'S 'Three Expeditions into the Interior of Australia,' vol. ii. 1838, p. 362, pl. 31, figs. 1 & 2, as indicative of a new genus and species of gigantic mammalian animal, to which I gave the name of *Diprotodon australis*"[*].

Of no extinct animal of which a passing glimpse, as it were, had thus been caught, did I ever feel more eager to acquire fuller knowledge than of this huge Marsupial. No chase can equal the excitement of that in which, bit by bit, and year after year, one captures the elements for reconstructing the entire creature of which a single tooth or fragment of bone may have initiated the quest; in the course of which one finally realizes, with more or less exactitude, the picture which the laws of correlation had led one to frame of an animal which may have passed out of existence long ages ago[†].

Appeals to friendly correspondents in Australia had met, in 1845, with so much success as enabled me to give the entire dental formula of the lower jaw, viz. $i, 1, c, 0, m, 5; =6$: and also to indicate a second genus of large herbivorous Marsupial (*Nototherium*) only inferior in size to *Diprotodon*[‡].

Further evidences fell in at longer intervals, and I was occasionally flattered with the hope of obtaining an entire skeleton, as by the subjoined extract from my old ally in researches of Australian Zoology, Dr. GEORGE BENNETT, F.L.S., of Sydney, New South Wales.

Copy of part of a letter from GEORGE BENNETT, *Esq., F.L.S., dated Sydney, September* 18*th*, 1863, *referring to a skeleton of* Diprotodon.

". . . . I have some expectation of getting you more of the bones of the *Diprotodon*; my son George, who is now on a station in Queensland, writes me as follows:—'I have been along the bank of a creek called 'King's Creek,' and searched it very minutely: I have found several bones, and also the place where the whole of a skeleton is imbedded in the ground. The bones are immense, some of the vertebræ are about 12 or 14 inches in length. I have now one of the smaller vertebræ, and it measures 6 inches in diameter.

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[*] Annals and Magazine of Natural History for October 1844.

[†] I hazarded the expression, in 1843, of such an ideal picture, as "of a heavy terrestrial quadruped, like the Mastodon, with thick and stout extremities adapted to the support and progression of a massive frame."—Annals and Magazine of Natural History, vol. ix. p. 332.

[‡] "Report on the Extinct Mammals of Australia, and on the Geographical Distribution of Pliocene and Post-pliocene Mammals in general," Reports of the British Association, 8vo, 1845.

The head of the same skeleton was sent some years ago to Sydney. The shepherd who discovered the head is here still, and it was he who showed me the place. When I can get time and men in that direction I will dig it out and then forward it by degrees to you.'

"Since I received this account," proceeds Dr. BENNETT, "I have seen Mr. W. B. TOOTH, the owner of the station, and he informed me that he has a large blade-bone, and that when he visits the station and has the men at leisure, he will gradually dig out the skeleton as perfect as possible and forward it to me. Mr. T. left on the 15th of this month for the station. I suggested to him to preserve every bone however small, which he has promised to do. On my receiving only a few at a time I will immediately transmit them to you, as I expect it will take some time to excavate the whole skeleton, as men cannot be spared at all times from a large sheep station."

I have long (perhaps too long) deferred entering upon the work of the present communication, hoping to complete the materials for the entire reconstruction of the *Diprotodon*. But the quick lapse of time, its inevitable effect on mind and body, and the venial impatience of the possessors of nondescript bones of the great Marsupial, combine to put an end to delay, and I proceed, therefore, to the description of the parts of this extinct animal at present at my command.

§ 2. *Skull.*—It is probable that the specimen in the British Museum (Plate XXXV. figs. 1, 2, 3), purchased at the sale of a series of Australian Fossils sent to London from Sydney by a Mr. BOYD, and stated to have been obtained from the bed of a creek at Gowrie, near Drayton, Darling Downs, Queensland*, may be the "head" referred to in the letter above cited.

The chief dimensions of this skull are given in the 'Table of Admeasurements' of parts of the skeleton of *Diprotodon*, p. 573.

The skull shows the general marsupial character of that part in its degree of depression or flattening from above downward, in the small proportion devoted to receive the brain, and in the large proportion given to the olfactory chamber and precranial air-sinuses.

The occipital region (Plate XXXV. fig. 1, $_3$, fig. 3), instead of being vertical, as in *Macropus* (ib. fig. 5) and most existing Marsupials, slopes forward from the terminal condyles at an angle of 45° with the basicranial axis.

The basioccipital (ib. fig. 3, $_1$) forms by a thick border convex vertically, slightly concave transversely, the lower part of the rim of the foramen magnum (ib. *o*) an inch in extent, separating in the same degree the lower ends of the occipital condyles (ib. 2, $_2$). These ends may be contributed by the basioccipital element, but the sutures between it and the exoccipitals are obliterated.

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* "All the above fossil remains are from King's Creek, Darling Downs, being the same locality whence the entire skull of the *Diprotodon* was obtained some years ago."—W. S. MACLEAY, in 'Report on Donations to the Australian Museum during August, 1857.' See also "OWEN, On *Nototherium*," in Proceedings of the Geological Society of London, March 1858, p. 158.

The basioccipital, as it advances, expands and gains a thickness of nearly an inch of compact and minutely cellular bony tissue.

The occipital condyles (Plate XXXV. fig. 3, $_{2}$), an inch apart below and 2½ inches apart above, have their lower extremity obtuse, about an inch broad, curved inward and forward; they expand as they ascend, diverging to a breadth of 2 inches at their upper ends. The vertical convexity of each condyle describes a semicircle, the extent of the articular surface following this curve being 6 inches. The outer border is longer than the inner one, so that the upper margin of the condyle rises obliquely from within outward. The narrow lower ends of the condyles rise or project more abruptly from the intervening basioccipital border of the foramen magnum than in *Macropus*. Each condyle is here impressed by a rough surface or shallow notch at its inner surface, indicative of tendinal insertion. A low narrow ridge extends from the outer part of the lower end of the condyle, forward, and may indicate the lateral extent of the basioccipital at this part. The transverse convexity of the condyle is greater, less angular, especially at the lower half, than in *Macropus*. The oblique base or upper end of the condyle projects proportionally more from the non-articular part of the exoccipital. A similar better definition or greater prominence characterizes the outer margin of the condyle; the inner margin forming the sides of the foramen magnum is sharper and better defined than in *Macropus*: these borders are also less divergent as they rise. The inner, non-articular side of the condyle is slightly concave, rough, subtuberculate. The outer border of the articular part is sharp, and projects over the inner non-articular side of the condyle. In their posterior terminal position and degree of prominence the occipital condyles of *Diprotodon* resemble those in *Dinotherium*.

The foramen magnum (ib. fig. 3, *o*) is bounded above by the arched obtuse border of the exoccipitals, which bones rapidly gain thickness as they extend from the foramen. I assume that the exoccipitals met above the foramen, as in many Marsupials; or, if not meeting, had their interspace filled by the superoccipital, as in *Phascolarctos*: they left here no notch, such as one sees in *Macropus Bennettii*. The upper border of the foramen magnum is non-emarginate in *Macropus major*; but it is relatively of greater extent in that Kangaroo, through the greater divergence of the condyles, and it is less arched or concave transversely than in *Diprotodon*. Two precondyloid foramina open upon each exoccipital, opposite the junction of the lower and middle thirds of the condyle, from which the hindmost foramen is distant 9 lines, the next 1 inch 2 lines; each foramen is about 4 lines in diameter. An irregular or tubercular ridge curves over the fossa of the precondyloid foramina, expanding to be lost on the paroccipital. This process (ib. figs. 1 & 3, $_{4}$) is tuberous, thick, and short; it is not so much produced as in *Macropus*.

The foramen magnum is more *evasé* than in *Macropus*, i. e. it expands funnel-wise to its outlet, backward, and especially above and below; it has more the character of a short (neural) canal than a foramen, through the antero-posterior extent of its wall or rim. It is transversely elliptical, 2 inches 5 lines in long, 1 inch 3 lines in short, diameters. It opens directly backward, the plane of its outlet being vertical. On each side of the inlet of the foramen magnum a wide and deep impression of a sinus curves downward to the jugular foramen at the inner side of the base of the paroccipital. The inner openings of the precondyloid canal are in a slight depression on each side the foramen magnum, a little nearer together than the outlets.

In the almost entire skull the upper border of the foramen magnum and contiguous part of the superoccipital surface are wanting. In the part of the occipital of another skull that surface is preserved to an extent of 2½ inches in advance of the upper border of the foramen, and for a breadth of 6 inches. This surface slopes forward from the foramen and condyles as in the entire skull; it is externally smooth and transversely undulating, showing a shallow medial concavity between two broad gentle convexities, which fall outwardly into concavities bounded by the oblique bases of the condyles. Nearly the lower half of the superoccipital surface is preserved in the fragment: the upper half, present in the skull, shows a strong medial vertical ridge (Plate XXXV. fig. 3, $_{3}$), and is bounded above by the ridge between the superocciptal ($_{3}$) and parietals ($_{7}$), continued outwardly upon the mastoids ($_{8}$). The cranial air-sinuses are continued backward into the super- and ex-occipitals, not into the basioccipital. On each side the mid-ridge ($_{3}$) is a shorter vertical ridge. The mastoid ($_{8}$) makes a much less projection than the paroccipital ($_{4}$); it is confluent above with the petrosal, as in other Marsupials; not preserving its primitive distinction from that sense-capsule as in the Babyroussa*.

The brain being small in Marsupials, and the disproportionate smallness of its case to the rest of the skull increasing, as in other natural groups of mammals, with the general bulk of the species in such group, this character is a striking one in the skull of *Diprotodon*. Like its carnivorous contemporary the *Thylacoleo*†, the brain-case makes no convexity or out-swelling into the temporal fossæ; the inner as well as the outer and hind walls of these long and large lateral vacuities are concave, and form parts of a general though not uniform excavation.

The broad and low triangular superoccipital surface, strongly sloping forward as it rises from the condyles, contracts above to its apex, and is continuous there with a (sagittal) ridge of the coalesced parietals (ib. fig. 1, $_{7}$) extending forward to the interorbital region. There the upper surface of the cranium begins to expand, and to swell into a pair of low convexities (ib. fig. 1, $_{11}$) which roof over the frontal sinuses. The outer wall of this pneumatic part of the cranium has been crushed down by posthumous pressure or injury in the entire skull.

The nasals (Plate XXXV. figs. 1, 2, $_{15}$), in continuing forward from the frontals the upper line of the skull, rise gently toward their terminations, which again curve downward, giving a sigmoid contour to that part of the cranial profile in a degree peculiar to the present species. The vertical diameter of the facial part of the skull at the termination of the nasals rather exceeds that of the cranial part at the parietal region; it is also greater than the beginning of the facial part of the skull in advance of the orbits and molar alveoli, whence there is a gain both in depth and breadth as it approaches the anterior terminations; but the uniformity of this diameter of the skull along the medial line, from the superoccipital forward to the premaxillo-nasal part, viewed sideways or in direct profile, is a remarkable characteristic of *Diprotodon.*

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* Owen, 'Anatomy of Vertebrates,' 8vo, vol. ii. p. 469; and 'Catalogue of the Osteology, in the Museum of the Royal College of Surgeons,' 4to, no. 3345, p. 557.

† Philosophical Transactions, 1866, Plate I.

The part of the maxillary (Plate XXXV. fig. 1, $_{21}$) lodging the molar series of teeth (*d* 3–*m* 3) breaks the lower line of the profile, descending below it along the middle third of the length of the skull. The zygomatic arch is deep, long, but proportionally less convex outwardly, or less expanded, than in *Thylacoleo.* Its base (ib. figs. 1 & 3, $_{27}$) seems as if continued from the whole side of the occipital plane, contracting rapidly at the upper border as the arch sweeps outward and forward; the superoccipital crest being continued into the upper border of the arch, and this apparently without break or abrupt rise in any part of that border*. The frame of the orifice of the "meatus auditorius" (ib. fig. 1, $_{28}$) projects downward from the hind part of the lower border of the base of the zygoma, indicative of the tympanic. Immediately in front of this descends the postglenoid process (*a*) of the squamosal, and in advance of this is a second downward projection or convexity due to the "eminentia articularis" (*b*), which is here, as in Marsupials, a process of the malar ($_{26}$). From this part the lower border of the zygoma runs forward nearly parallel to the upper one, but with a slight concavity, as far as the maxillary element of the zygoma, which sends down a strong, moderately long, obtuse, subcompressed masseteric process (ib. $_{21'}$)—a cranial feature which is peculiar to herbivorous Marsupials†.

The orbit (ib. fig. 1, *r*) is a relatively small vertically oval cavity, communicating widely behind with the temporal fossa (ib. $_{7}$). The external nostril (ib. figs. 1 & 2, *n*) is terminal, subvertical, rather expanded, and divided in great part by an upward extension of the medial nasal plate of each premaxillary (ib. figs. 1 & 2,$_{22'}$), which plates, being in close contact, form the lower part of a long "septum narium" at the outlet of the nasal cavity, recalling its condition in the extinct *Rhinoceros tichorhinus.* There is a narrow and short descending ridge at the coadapted medial borders of the nasals, which seems to have been continued into the septum by cartilage rather than by bone. I have alluded to the analogy which the structure of the external nostrils in *Diprotodon* suggest to those of an extinct Pachyderm, but the truer and closer resemblance is found in the Marsupial group. The cavity of the nose is divided by a complete bony septum to within one-fourth of the outer opening in *Macropus* and *Phascolomys*‡, advancing, in one species of Wombat, as in *Nototherium,* nearer to that outlet.

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* Some mutilation of the hind part of this upper border in both zygomata begets reserve in definitely pronouncing as to its normal outline.

† The descending masseteric process in Glyptodonts, Sloths, and Megatherioids is formed by the malar bone exclusively. OWEN, '*Anatomy of Vertebrates,*' vol. ii. p. 405, figs. 273, 274, 26, *a.*

‡ "*On the Osteology of the Marsupialia,*" *Zoological Transactions,* vol. ii. p. 391. Anatomy of Vertebrates, vol. ii. p. 348.

The base of the masseteric process of the maxillary (Plate XXXV. fig. 1, $_{21'}$) is a vertical outstanding ridge, beginning below about an inch above the fore part of the last molar alveolus, or above the interspace between the last and penultimate sockets, according to the age of the individual. It becomes thinned as it rises and projects, and then suddenly expands to form the fore part of the zygoma and to send down the process. This is slightly twisted upon itself outward and backward, concave on the hinder and inner surface, convex at the opposite surface; the fore part of the vertical base of the zygomatic process of the maxillary is smooth and concave.

The alveolar border of the maxillary contracts and terminates obtusely behind the last molar (*m* 3, Plate XXXVIII. fig. 2). It articulates with the palatine, leaving a hinder angular interval into which the lower and fore part of the pterygoid is wedged. The outer part of the concave hind border of the bony palate curves from the pterygoid inward and forward to opposite the mid-division of the last molar: the palatines appear to complete, with the maxillaries, the hind part of the roof of the mouth, without leaving a vacuity.

Portions of the maxillary and palatine of two other individuals are equally without indications of any wide postpalatal vacuity opposite the interspace between the last and penultimate molars. In advance of this interval the bony palate, due here to the maxillaries exclusively, extends so as to give a breadth of the palate between the penultimate molars of 4½ inches.

Anterior to the masseteric process the outer alveolar wall of the maxillary is undulated by the vertical prominences, indicative of the large and thick roots of the molar teeth. The alveolar border contracts as the teeth advance in position and decrease in size, and becomes a ridge anterior to the first molar in place (usually the second of the series (d_4) in full-sized *Diprotodons*). This antalveolar or diastemal ridge (Plate XXXV. fig. 1, x) curves upward and inward, approaching its fellow, then arches downward and terminates at the back part of the socket of the third incisor, where the maxillo-premaxillary suture begins. At the wider hind part of the interval, between the antalveolar ridges, there seems to have opened an anterior or prepalatal canal leading to the fore part of the nasal cavity, the orifice being elongate. In advance of this the deep and narrow channel between the fore part of the diastemata is entire. Above these ridges, the outer plates of the maxillaries swell outward as they ascend to form the lateral walls of the antorbital part of the nasal chamber, arching inward again above to join the nasal bones ($_{15}$).

The maxillo-nasal suture seems to have a relatively greater extent than in *Phascolomys*; but owing to the short facial or antorbital part of the skull, as compared with *Macropus*, it is of much less relative extent than in that genus. The antorbital foramen (ib. figs. 1 & 2, $_{21}$) is longest vertically.

Each premaxillary (Plate XXXV. figs. 1 & 2, $_{22}$) is deeply excavated by three alveoli, the foremost the longest, largest, and most curved. The inner walls of these alveoli rise as a strong vertical crest ($_{22'}$) dividing the lower part of the nasal outlet. The incisive alveoli succeed each other from before backward; and, owing to its superior size, the outer border of the first projects further from the midline than does that of the last. Viewed from the palatal aspect the two series of incisive alveoli converge backward, instead of forward as in the Kangaroo and most other quadrupeds. The malar bone (Plate XXXV. fig. 1, $_{26}$) ascends from its junction with the maxillary to join the lacrymal ($_{73}$) at the fore part of the orbit, by a very narrow curved strip or process; its main body is suspended in the zygomatic arch, of which it constitutes the anterior half, and the lower part, as far as, and including, the "eminentia articularis." The suture between the squamosal and malar elements of the zygoma is almost straight, extending from behind the orbit obliquely backward and downward to the glenoid cavity, of which articular surface the malar "eminentia," here more flattened than usual, contributes the fore part. This articulation (ib. fig. 4) is most extended transversely to the skull's axis; its hinder half (ib. ib. $_{27}$) is concave from before backward, its fore part (ib. ib. $_{26}$) convex, but becomes flattened or a little hollowed on the "eminentia."

The lacrymal (ib. fig. 1, $_{73}$) is perforated by the canal, marsupial-wise, in advance of and external to the orbital cavity.

§ 3. *Mandible.*—A transversely extended subconvex condyle (Plate XLII. figs. 3 & 4) adapts itself to the cavity offered by the base of the zygoma. The condyle is 5½ inches in transverse extent, 1 inch 9 lines from before backward; it is, in that direction, most convex. The condyloid process is supported by a three-sided neck quickly contracting to 1 inch 9 lines in transverse diameter (ib. fig. 3, *n*); it is broadest and flattened behind, contracted in front to the ridge-like beginning of the "coronoid" plate (ib. figs. 2 & 4, *r*), which extends forward near the outer side of the neck. The condyle is more extended inward (ib. fig. 4, *c*) than outward (ib. ib. *c'*) of this advancing vertical coronoid plate. The flat surface at the back part of the neck is continued into a suddenly expanded hinder facet of the ascending ramus, formed by the outward production of the hind wall or boundary of the outer depression for the insertion of the temporal muscle, and by the inward production (Plate XXXV. fig. 3, *e*) of the hind wall or boundary of the deep inner concavity of the ascending ramus, where opens the large entry (Plate XLII. fig. 2, *o*) of the dental canal. Below this orifice the concavity extends downward through the concomitant extension of the inner plate or hind wall to the lower border of the horizontal ramus, where it gradually subsides. The hind wall of the outer depression of the ascending ramus (Plate XXXV. fig. 1, *e*) follows the contour of that of the inner depression, but sooner subsides; the interspace is a continuation of the broad hind flattened facet which, as it descends, gets a more outward aspect. Beyond the subsidence of the outer plate it gives the appearance of a bending inward of the angle of the jaw (ib. fig. 2, *e'*), and that to a degree which is characteristic of Marsupials. The outer or crotaphyte depression of the ascending ramus (Plate XXXV. fig. 1, *f*) gradually gains the ordinary level of the outer surface of the horizontal ramus, and does not undermine the ascending branch to communicate with the inner concavity as in *Macropus*. In the shape of the condyle *Diprotodon* resembles *Phascolomys*, in which the intercommunicating canal is much reduced.

The anterior border of the ascending ramus is straight and subvertical; it is thickened at its lower part to be continued into the convex outswelling of the horizontal ramus outside the last molar (Plate XXXV. fig. 1, *m* 3), a distance of an inch intervening between the alveolus of this molar and the convexity rising and thinning into the fore part of the coronoid plate (*f*). The alveolar border is continued into an obtuse ridge or prominence, 2 inches behind the last alveolus; from which prominence the ridge subsides and expands, retrograding to form the internal border of the entry of the dental canal (ib. *d*).

The horizontal ramus gains slightly in depth as it advances from the last to the first molar socket (*d* 3). Two and a half inches below this socket, and a little in advance, is the vertically elliptic outlet of the dental canal (ib. 32). Below this orifice the ramus bulges out into a rather rough tumefaction, then slopes and contracts upward and forward to form the socket of the huge procumbent lower incisor (*i*). From the socket of *d* 3 the alveolar border sinks and expands into the upper part of the socket of the incisor. The under border of the horizontal ramus is smoothly and broadly convex transversely. The inner surface sinks sheer from the openings of the molar alveoli, and curves inward below the anterior ones to the symphysis (Plate XLI. fig. 2, *s*, *s*). The fore part of the mandible below the incisive alveoli, expanding to the tuberous outswellings abovementioned, has a broad, subquadrate form, recalling the shape of that part in the *Hippopotamus* (Plate XXXV. fig. 2, *t*, *t*).

The symphysis (Plates XLI., XLII. fig. 2, *s*, *s*)begins behind, at a line dropped vertically from the front lobe of the third molar (*m* 1); it is 6 inches in length, 4 inches in depth in the full-grown animal. It gains in vertical direction more than in length during the growth of the mandible, with reference apparently to the provision of a sufficient lodgment of the progressively increasing incisive tusk. (Compare Plate XLI. fig. 2, *s*, *s* with Plate XLII. fig. 2, *s*, *s*.)

The large size of the dental canal exposed by the posterior fracture of the ramus of another mutilated mandible indicates the ample supply of vessels and nerves which minister to the growth and nutrition of the incisive tusk; the depth of the symphysis of the jaw corresponds with the tusks, which it helps to support; contributing to the required strength for the operations of those eroding implements, with space for the deep implantation and for the lodgment of the large persistent matrix of each tusk (Plate XLII. fig. 5). The direction of the symphysis is oblique, from below upward and forward; its upper margin is nearly straight, its lower one convex; the rough articular surface stands out a very little way from the vertical plane of the inner surface of the ramus.

In comparing the symphysial part of the jaw of *Diprotodon* with that of any other large quadruped carrying a single incisor in each ramus there are well-marked differences. The symphysis in the Sumatran *Rhinoceros* and in *Acerotherium* is less deep and is proportionately broader; the great length of that part in the *Mastodon longirostris*, and its deflection in *Dinotherium* more conspicuously differentiate them. In the remarkably large proportion of the symphysis in *Diprotodon* to the size of its molar teeth there are no quadrupeds which so nearly resemble it as the Nototheré and the Wombat; but in this existing Marsupial the symphysial part of the jaw is broader in proportion to its depth. The long and narrow symphysial junction in the Kangaroo is peculiar for the yielding movements allowed to the rami upon each other, which is betrayed by those of the long procumbent depressed incisors in the living animal*.

§ 4. *Dentition.*—The dental formula of *Diprotodon* is:—$i\frac{3-3}{1-1}$, $c\frac{0-0}{0-0}$, $m\frac{5-5}{5-5}=28$. Of the upper incisors the first or anterior pair (Plate XXXV. figs. 1 & 2, *i* 1; Plate XXXVI. figs. 1–6) are large curved scalpriform teeth, of which I have not found indications of cessation of growth in any specimen. The skull above described and figured (Plate XXXV.) has been that of an aged male, judging from the size and degree of attrition of the teeth which are retained; but the anterior incisors above, like the pair below, are continued to the bottom of their deep alveoli without contraction, and with the retention of a widely open pulp-cavity (Plate XXXVI. fig. 6). It is obvious that these strong anterior incisors (ib. figs. 1–4) worked with the evergrowing power of the "dentes scalprarii" of the Wombat, the Aye-aye, and the Rodents.

In the above skull the length of *i* 1, following the convex curve of the tooth, is 11 inches; its circumference is 4 inches 9 lines; the breadth of the oblique abraded working surface is 1 inch 9 lines; the longitudinal extent of that surface is 2 inches; but this varies in other specimens. An extent of the tooth of 8½ inches (following the outer curve) is lodged in the socket of the premaxillary.

I made a transverse section of a fragment of the skull of a *Diprotodon*, including the fore part of the premaxillaries and their scalpriform teeth (Plate XXXVI. fig. 5). Such section of the tooth (*i* 1, *c*, *d*, *e*) is irregularly three-sided, with the angles broadly rounded off. The inner side, or that next the fellow tooth, is the narrowest; the front or enamelled side is the broadest: this side is traversed lengthwise by a wide and shallow mid channel; the opposite side is grooved by a narrower and rather deeper channel, running along its outer half; and the inner more prominent half of this side (the concave one lengthwise) also shows a narrow and feeble impression near the mid-line of the tooth, and a broader more shallow impression nearer the angle, dividing the hinder from the inner surface. This surface, 1 inch 3 lines across (ib. fig. 2), is generally somewhat convex, but wavy through two or three low obtuse longitudinal ridges, with intervening shallow channels. A fossil fragment of a similarly sized tooth yielding such transverse section as that shown by this remarkable scalpriform incisor would, according to present experience, determine the genus of Mammal to which it had belonged.

The enamel coating the anterior convex curve of the tooth is continued over the major part of the outer rounded surface, terminating abruptly along a line (ib. fig. 5, *e'*) external to the outer longitudinal ridge (*c*) of the posterior surface. In like manner the enamel is continued over the rounded angle between the anterior and inner or medial sides of the incisor, and terminates abruptly at *e*, fig. 5, after covering about one-third of the inner surface. The hinder and two-thirds of the inner sides of *i* 1 are thus uncoated by enamel, the dentine (*d*) showing there only a thin coating of cement (*c*).

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* First noticed by MASON GOOD, 'Book of Nature,' vol. i. p. 283.

The surface of the enamel is longitudinally striate and punctate (Plate XXXVI. figs. 3 & 4); the fine pits being chiefly but not wholly between the striæ; so that in some parts the surface seems to be minutely reticulate or reticulo-punctate. The surface of the dentine to which the enamel was applied shows a similar but less marked character. The cement (*c*) is thickest where it overlaps the terminal edges of the enamel.

The second upper incisor (Plate XXXV. fig. 1, *i* 2; Plate XXXIX. fig. 7) is slightly curved, but in an opposite direction to the first, the anterior longitudinal outline being concave: the degree of this bend seems greater through the oblique attrition of the tooth from behind downward and forward. The transverse section of this incisor is subcircular. The length of the exposed part of the tooth is 3 inches; the circumference is 3 inches 6 lines; but this slightly diminishes to the margin of the socket, and more so to the inserted end. The fore-and-aft extent of the abraded working-surface of the tooth (ib. fig. 8) is 1 inch 6 lines. The length of the entire tooth does not exceed 4 inches.

The third upper incisor (Plate XXXV. fig. 1, *i* 3), of similar form, is smaller. The length of the exposed part is 2 inches 10 lines; its circumference is the same; the fore-and-aft extent of the worn surface is 1 inch. This surface runs upon the same level as that of the second incisor. The crown of the large lower incisor, besides applying its trenchant edge against that of the broader front incisor, scraped upon both the smaller incisive teeth. Probably, by reason of the age of the individual and the extent of tooth worn away, the original enamelled crown has gone, and both *i* 2 and *i* 3 are here represented only by their cylindrical cement-covered portion.

A specimen of a detached second upper incisor is in the same condition: the enamelled crown is worn away, the root contracts to its implanted end, which shows a small remnant of a conical pulp-cavity 8 lines in depth and the same in width, as in fig. 7, Plate XXXIX.

The second and third incisors of *Diprotodon* were teeth of limited growth, and with the enamel confined to and thus defining a crown as in the Kangaroos; whilst the front incisor was a scalpriform tooth as in the Wombats, in which the second and third incisors are not developed. The extinct *Diprotodon* thus exemplifies an interesting intermediate or transitional condition of the upper "dentes primores" unknown in any existing form of *Marsupialia*.

In the upper jaw of the skull above described (Plate XXXV.) the molar series is in place, with the exception of the first small tooth (*d* 3). The other four teeth occupy, on each side the jaw, a longitudinal alveolar extent of 7 inches 4 lines. The homologies of these teeth with those in *Macropus* are indicated by the symbols used in my 'Anatomy of Vertebrates,' vol. iii. fig. 296, where the grounds for such use are given, and in fig. 1, Plate XXXV. of the present Memoir. Scarcely a trace of the socket of the first small molar (*d* 3) remains in the skull; the other molars progressively increase in size to the last (*m* 3), which has a minor breadth of the hind lobe than in *m* 2. The line of the working surfaces of the four molars describes a slight convexity downwards (Plate XXXVIII. fig. 1); the exterior line is also slightly convex (ib. fig. 2); the interior line is concave in a less degree; the right and left series are moderately convergent anteriorly. The interspace between the hind lobes of the last molars (*m* 3) is 4 inches 1 line; that between the front lobes of the first molars (*d* 4) is 3 inches 1 line; these dimensions give the breadth of the palate between the right and left teeth above symbolized.

All the molars in place have an enamelled crown divided into two transverse lobes (Plates XXXVII., XXXVIII. *a*, *b*), with accessory ridges (*f*, *g*), and are inserted by cement-clad contracting roots as in *Macropus*. The summits of the transverse lobes are abraded in all the molars of the specimens figured; but least so in the hind lobe of the subject of *m* 3, fig. 2, Plate XXXVIII.

The socket of the first small molar (fig. 1, *d* 3) is partially preserved in the entire skull; it consists of two cavities, the hindmost the largest, the tooth having only two roots. In the subject of Plate XXXVII. figs. 1 & 2, the crown of *d* 4 is ground down nearly to the bottom of the cleft (*h*); the fore-and-aft extent of the grinding-surface is 1 inch 2 lines; its transverse extent across the hind lobe is the same. Across the fore part of the base of the tooth is a low ridge (*f*), to the level of which the anterior lobe (*a*) is almost worn. The corresponding ridge at the back part of the tooth is continued along both the outer and inner borders (*g*, *g*) of so much of the back part of the hind lobe as is not ground down; the whole of the surface projects beyond the level of the worn surface of the following molar (fig. 1, *m*). The transverse cleft is deepest at its outer and inner ends; a ridge of enamel descends from each of these ends of the anterior lobe, and, meeting a corresponding projection of the oppposite lobe, it partially closes the entry of the valley.

The anterior basal ridge is strongly developed in *m* 1, especially at its inner end; the interspace between it and the anterior lobe widens toward the inner side of the tooth (*f*). The anterior lobe is worn down nearly to the level of the ridge; the surface describes a transverse irregular ellipse; that of the posterior lobe is narrower: in both a mid linear tract of osteo-dentine (*o*, *o*) is exposed. The narrow hind basal ridge (*g*) is continued upon the hind lobe as in *p* 4, and that lobe projects clear beyond the level of the grinding-surface of *m* 2. The antero-posterior and transverse diameters of the working-surface of *m* 1 are each 1 inch 6 lines.

The anterior basal ridge (*f*) is strongly developed in *m* 2, and the antero-posterior diameter of the tooth (1 inch 9 lines) rather exceeds the transverse diameter. The front lobe (*a*) is worn down to within 5 lines of the basal ridge. The minor degree of abrasion of the hind lobe shows the curve of the grinding-surface, concave backward, which is lost as the thicker part of the lobe is reached. The hind basal ridge (*g*) is feebly developed.

In the last molar (*m* 3) the hind lobe is markedly less than the front one, by its more rapid loss of transverse dimension: it is rather narrower in this line at its base, as it is in fore-and-aft extent. The last upper molar of *Diprotodon* may be readily determined by its posterior contraction. In some individuals the hind surface of the hind lobe is less evenly concave transversely; I have seen it almost canaliculate. The loss of breadth of this lobe is chiefly from the outer side, and the lobe is lower than the front one, the level of the grinding-surface reaching halfway toward that of the front lobe. The fore-and-aft extent of the base of the tooth is 2 inches; the transverse extent of the worn surface of the front lobe is 1 inch 6 lines; that of the hind lobe is 1 inch 3 lines. The anterior ridge (fig. 4, *f*) is continuous with a feeble rising of the enamel at the outer and the inner borders of the front surface of the anterior lobe. The posterior basal ridge (Plate XXXVIII. fig. 3, *g*) is more directly and conspicuously continued into the ridge along the inner border of the posterior surface of *m* 3.

Wherever sufficient of the lobes remains, their profile, especially the outer one, describes a curve concave forward (Plates XXXVII., XXXVIII. fig. 1). The inner and anterior angle of each tooth, due to the more prominent part of the front basal ridge, projects inward, a few lines beyond the inner surface of the tooth in advance (ib. ib. fig. 2). Thus there is not only a zigzag disposition in the vertical but in the transverse arrangement of the upper molars, though in the latter it be but slightly marked. The enamel is about a line in thickness, and shows strongly the reticulo-punctate or rugous surface at the less exposed parts of the crown.

The upper molars are implanted by fangs which acquire twice the length of the enamelled crown: they are at least three in number, save in the first small and early deciduous tooth (*d* 3). The base of the anterior division of the tooth bifurcates as it descends, slightly contracting in the socket, and thus forming two fangs in the same transverse line. The base of the posterior division, if it bifurcates in any molar, is divided later and to a less extent. It gradually contracts, and is longitudinally excavated at the side next the other fangs.

Figure 5, in Plate XXXVIII., gives a view of the two anterior fangs (*m*, *n*) of the last molar; fig. 6, ib., shows the single posterior fang (*l*) of the same tooth. Plate XXXIX. fig. 3 shows the sockets and implanted ends of the fangs of the antepenultimate and last molar teeth. The outstanding antero-posteriorly compressed zygomatic process of the maxillary (*n'*) is here opposite the hind lobe of *m* 2.

In the series of upper molars of *Diprotodon* there are varieties as to size, and as to order or degree of wear, the former variety being more constant. Both are exemplified in the specimens figured in Plate XXXVII. figs. 2 & 3. In fig. 3, a portion of the left upper jaw with the last three grinders, *m* 1 shows both lobes and the anterior ridge worn down to a common field of dentine (*d*) and osteo-dentine (*o*, *o*): the summits of *m* 3 are partially abraded. In fig. 2, in which the last molar (*m* 3) shows an equal degree of abrasion, the antepenultimate molar (*m* 1) is not worn to the same degree as in fig. 3; the anterior lobe is ground down near to the basal ridge (*f*), but this remains untouched; the valley between the two main lobes is not obliterated. What is still more unusual, where the last molar has come into use, the second molar (*p* 4, fig. 2, Plate XXXVII.) preserves its lobes hardly worn down to the bottom of the valley, and the two fangs of the first molar (*d* 3) remain in their alveolus.

A cursory comparison of the two foregoing specimens suggests that *m* 1 (fig. 3, Plate XXXVII.) may have been destined to be pushed out by a vertical successor, which, in place in the larger specimen (*m* 1, fig. 2, Plate XXXVIII.), shows of course a less degree of abrasion. But this is not the case. I have in vain sought for evidence of any premolar, in either upper or lower dental series of *Diprotodon*: it differs from *Macropus* and resembles *Phascolomys* in this particular. All the teeth, like the last three grinders in the type diphyodont dentition, belong to the first set. The variety as to degree of attrition in molars of the same series is due to some modified habit of mastication: the difference in respect of size I ascribe to sex, the smaller grinders belonging to the female, concomitantly with a general inferiority of bulk, as is seen in *Macropus*. The following admeasurements exemplify the difference of size in molar teeth, which is probably sexual:—

		Diprotodon.			
		Male.		Female.	
		in.	lines.	in.	lines.
m 3.	Antero-posterior diameter	1	10	1	7
	Transverse posterior diameter (base of front lobe)	1	11	1	6
m 4.	Antero-posterior diameter	2	3	2	0
	Transverse posterior diameter	2	0	1	7
m 5.	Antero-posterior diameter	2	4	2	1
	Transverse posterior diameter	2	0	1	7½
	Antero-posterior extent of *m* 3, *m* 4, *m* 5	6	0	5	10

The forms and proportions in which the four constituents of the molar teeth of *Diprotodon* are combined, are exemplified, in the vertical longitudinal section of the last three upper grinders, in Plate XLII. fig. 1. The enamel (*e*) gains thickness as it recedes to a certain extent from the summits of the lobes, giving more resistance or grinding-power as the tooth wears down; but the enamel thins again at the base of the lobe; it gains a little more thickness as it is reflected, so to speak, over the basal ridges, beyond which it extends from three to four lines before thinning off, and ceases upon the body of the tooth before its division into fangs. The usual general direction of the dentinal tubules is well displayed, as in most fossil teeth. As the dentine becomes exposed and abraded, the pulp-cavity is defended by the coarser calcification of the remaining matrix near the field of abrasion, and from 2 to 3 lines of osteo-dentine is interposed between that field and the pulp-cavity. In each lobe of the tooth most worn (*m* 1) the cavity is reduced to a linear trace. In the anterior lobe of *m* 2 it is more expanded; and it retains width in both lobes of *m* 3. In each tooth the pulp-cavity has received a lining of dark-coloured spar in the course of fossilization. The cement is thickest upon the back part of the hind root (*c*), whence it extends upon the posterior basal ridge: this partial excess of cemental development assumes a characteristically definite figure in such sections as the one described.

The lower incisors (Plates XXXV., XLI. & XLIII. *i*; Plate XXXIX. figs. 4, 5, 6) are nearly straight; the very slight degree in which they deviate from that line tends to an upward curve (Plate XXXIX. fig. 4).

The length is 10 inches, the circumference 5 inches 6 lines. The longitudinal extent of the worn surface in those of the skull (Plate XXXV.) is 3 inches; its transverse breadth is 1 inch 4 lines. The transverse section of the entire tooth (Plate XXXIX. fig. 6) is oblong; in some it presents an irregular oval with the small end upward. The outer side at its lower two-thirds is usually prominent; the inner side is more even or flat, in some instances feebly convex; in one specimen very slightly concave along its middle third. The outer side is more constantly traversed by a narrower shallow longitudinal channel, rather above the middle of that side. The enamel (Plate XXXIX. figs. 5, 6, *e*) is continued from the border of this channel round the lower part of the incisor, to about one-fifth of the extent of the inner side (ib. *e'*): its terminal borders are abrupt on both sides, with the rather thick cemental covering of the unenamelled part of the circumference extending over the enamel borders. The surface of the enamel is finely ridged lengthwise and reticulo-granulate; the minute studs of enamel being, however, more conspicuous than the holes; although these are not absent.

About two-thirds of the tooth is lodged in the socket, which extends backward a little beyond the symphysis, but without causing, as in Rodents, a prominence of the inner wall of the ramus (Plate XLI. fig. 2); in this respect *Diprotodon* resembles *Macropus* and *Phascolomys*. The line of the socket forms an angle of 147° with the basal line of the mandibular ramus. The pulp-cavity (Plate XLII. fig. 5, *p*) is a long cone widely open at the base. The pair of tusks run almost parallel, slightly approximating so as to come into contact at their working ends.

The form of the lower incisor, described as it is shown in the most perfect specimen of the lower jaw of a full-grown example, is subject to some variety. Being a tooth of unlimited growth, it increases with the size of the jaw. In young specimens the outswelling of the outer side, or the contraction of the upper third of that side, is either not apparent or not so conspicuous, and the transverse section of the incisor yields a full oval, as in that of the young *Diprotodon* from the Wellington Valley Cave* (Cut, fig. 1 *a*), and, slightly modified, in the one of similar age from Darling Downs, Queensland (Plate XLI. fig. 1, *a*).

But under all these slight varieties, which I cannot regard as specific, there prevail the same essential characters of structure, disposition of enamel, &c., pointed out in my original Memoir as differentiating *Diprotodon* from *Halichore*, *Hippopotamus*, and other Mammals with tusks of similar size.

A diastema, between three and four inches in extent, rises gently as it recedes from the incisor (Plate XLI. *i*), to the first molar (*d* 3), and more so, as the molar series becomes completed and pushed out for use as in Plate XXXV. fig. 1, and Plate XLII. fig. 2.

Of the first molar tooth (*d* 3) I have no specimen. Its existence was indicated by traces of its socket in the portion of mandible obtained by Dr. E. C. Hobson, from a gravel-bed in the "Melbourne district," described in my 'Catalogue of Fossils in the Museum of the Royal College of Surgeons' (4to, 1845), p. 308, no. 1491; and such trace of socket showed the tooth to have been implanted by two fangs. The corresponding divisions of the socket of *d* 3, with the fangs *in situ*, are better preserved in the specimen figured in Plate XLII. fig. 5, and Plate XLIII. figs. 1 & 2, *d* 3. Dr. Hobson, shortly before his death in 1848, transmitted to me a sketch of this tooth *in situ*, in a fragment of the lower jaw of a young *Diprotodon* (Cut, fig. 2), according to which the anterior as well as the posterior lobe of *d* 3 is in the form of a transverse wedge; there is a basal ridge along both the fore and hind parts of the crown, the latter being the broadest; in short, *d* 3 presents, in miniature, the bilophodont type of the succeeding molars. From the attrition of the two lobes it may be inferred that the opposing molar above was also transversely two-ridged. That the tooth (fig. 2) answers to the one which occupied the socket (*d* 3) in Plates XLI. & XLII. fig. 5, is shown by correspondence of size. The fore-and-aft extent of the socket in both is 9 lines, the breadth of the division for the anterior fang is 4 lines, of that for the posterior fang 4½ lines; the alveolar wall extending transversely between the two divisions exceeds a line in thickness; each fang is subcircular at its fractured end, with an indent at the side turned toward the other fang, indicative of a longitudinal groove into which the walls of the socket enters, giving a firmer implantation to the tooth.

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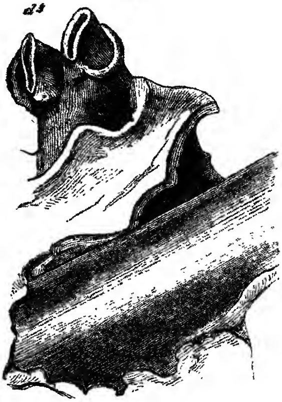
* Mitchell's 'Three Expeditions into the Interior of Australia,' 8vo. 1838, vol. ii. p. 362, pl. 31. figs. 1 & 2.

Fig. 2.



First lower molar, *d* 3, young *Diprotodon*, nat. size.

In the portion of mandible (Plate XLI. & XLII. fig. 5) the penultimate molar (*m* 2) had not risen completely into place, and the posterior lobe was barely touched by masticatory work. In the mandibular ramus (Plate XLII. fig. 2), with the last molar (*m* 3) in place and both ridges showing wear, the two divisions of the socket of *d* 3 are retained, without trace of tooth. The fore-and-aft extent of the socket is 9 lines, that of the hind fossa or division is 3½ lines, that of the front one 2½ lines, and that of the intervening bar is 2½ lines at its prominent part.

In the younger jaw the second molar (Plate XLI. figs. 1 & 2, *d* 4) has both lobes of the crown about half worn down; the fore-and-aft extent of the crown, including the anterior and posterior basal ridges, is 1 inch 6 lines. The anterior basal ridge is thickest at its outer part, and here the enamel has been worn off in mastication. The flat fore side of the front lobe rises 5 lines above the ridge. The abraded surface (Plate XL. fig. 3, *a*) of this lobe is 8 lines in transverse and 4 lines in antero-posterior extent, the mid part being increased in this direction by an outswelling of the hind surface there of the lobe. The outswelling of the front slope or surface of the hind lobe is situated more outwardly: the abraded surface (Plate XL. fig. 3, *b*) of this lobe is narrower from before backward, broader transversely, than that of the front lobe. A low and short ridge of enamel (*h*) closes both outer and inner ends of the intervening transverse valley. The length or vertical extent of crown between the end of the valley and the division into fangs is 5 lines on the inner side and 4 lines on the outer side of the tooth. The middle of the hind surface of the hind lobe swells out; and as both outer and inner ends of the hind basal ridge (*g*) bend up the corresponding parts of the hind lobe, its hind surface shows two shallow depressions divided by the above-named rising: in these depressions the reticulo-punctate character of the enamel is most strongly marked. The hind basal ridge is thicker than the front one (*f*), and thickest at its middle; its enamelled margin is irregular, it rises higher than, and seems to overlap, the front basal ridge of the following tooth. The cement upon the exposed part of the crown of *d* 4, between its enamelled lobes and implanted fangs, is thick. The fangs are two in number, broadest transversely, slightly divergent, canaliculate on the contiguous sides.

In the jaw of an older *Diprotodon* the second molar (Plate XL. fig. 4, *d* 4) shows both lobes abraded to their common base, exposing the osteo-dentine (*o*) obliterating the cavity of the fang. A small part of the enamel of the front basal ridge (*f*) shows its position as blended with the front lobe. The line of enamel of the worn hind surface of the hind lobe (*b*) forms an open angle, of which the apex shows the end of the prominence joining the middle of the hind basal ridge (*g*), and dividing the remnants of the pair of depressions between that ridge and the hind lobe. The fore-and-aft extent of the worn surface of this molar is 1 inch 6½ lines, that of the base of the crown is 1 inch 8 lines; the breadth of the hind part of the worn surface is 1 inch. The alveolar border rises into an angle between the origins of the fangs.

In Plate XL. fig. 2 shows the working-surface of the crown of *d* 4, of rather smaller size than those above described, and probably from a young female *Diprotodon*. The summit of the anterior lobe is so far worn as to expose a transverse curved line of dentine, concave forward, beginning to expand where attrition has reached the prominent part of the hind surface of the lobe. The summit of the posterior lobe (*b*) has just been touched The proportions of the basal ridges (*f*, *g*) are well shown. The reticulo-punctate character of the enamel is well marked. This tooth was from the freshwater deposits of the Province of Victoria, near Melbourne The outer side view of this tooth is given in figure 1.

From the same locality I received the third molar (*m* 1) of the same *Diprotodon* (Plate XL. figs. 5 & 6): its almost untouched lobes are more compressed than in the Tapir and Dinothere, and their lamelliform summits rise higher beyond their basal connexions than in the Kangaroo; the median connecting ridge which extends between the two transverse eminences longitudinally, or in the axis of the jaw in the molars of the Kangaroo (ib. fig. 14), is very feebly indicated by the outswelling, shown in figs. 3 & 7, at the back of lobe *a*, in the *Diprotodon*. The anteriorly concave curve of the summits of the transverse lobes, in fig. 6, is more regular, equable, and greater than in the Tapir (fig. 15), the Dinothere, or the Kangaroo. The two fangs, the contiguous surfaces of which present the deep and wide longitudinal groove, as in the Tapir, Dinothere, and Kangaroo, are connected together at their base by a ridge coated thickly with cement, and extending longitudinally between the beginnings of the opposite grooves in *Diprotodon.*'

The third molar in the young specimen (Plate XLI. figs. 1 & 2, *m* 1) has both lobes partially abraded; the fore-and-aft extent of the tooth is 1 inch 10 lines, the basal breadth of both lobes is the same, viz. 1 inch 1½ line. The reticulo-punctate or "worm-eaten" character is strongly marked on the enamel of the fore part of the front lobe; this is slightly concave transversely at its upper part, the outer and inner borders inclining forward to receive the upward continuations from those ends of the anterior transverse ridge (*f*). The middle of the hind surface of the front lobe (Plate XL. fig. 7, *a*) is prominent, making the masticatory surface widest at that part. The prominence (*b*) from the opposite surface of the hind lobe looks more like an infolding of the outer border of that lobe, a character exaggerated in most Kangaroos; the inner border of the hind lobe is slightly produced backward as well as forward The hind surface of the hind lobe does not show the mid prominence. The hind transverse basal ridge (*g*) is highest and thickest at its middle; the ends of this ridge are less distinctly continued upon the corresponding borders of the hind lobe than in *m* 2. The slight backward curve of the lobes appears in the profile view of *m* 1, fig 5

In the older jaw the lobes of *m* 1 (Plate XL fig 9) are worn down nearly to their bases. The front transverse ridge rises a little above the hind one of the antecedent tooth; about 5 lines extent of the fore part of the front lobe rises above the ridge. The anterior enamel-line of the worn surface is nearly straight, the posterior one forms a low angle answering to the prominence of that surface of the lobe. The valley between the two lobes is most shallow and narrow at its middle. The abraded surface of the hind lobe is transversely elliptical, 1 inch 4½ lines in transverse diameter, and 8 lines in the opposite diameter, its hind border is worn down within 3 lines of the posterior basal ridge (*g*), which abuts against the next tooth above its anterior ridge.

The fourth molar (*m* 2) in the younger specimen (Plate XL. figs. 9 & 10) has a line of dentine exposed on the summit of the front lobe (*a*), but the enamel is not worn off that of the hind lobe (*b*) The transverse concavity of the fore part of the front lobe is well marked at the present early stage of attrition· the convexity of the back part increases towards the base of the mid prominence. The ends of the front basal ridge (*f*) rise a little way upon the outer and inner borders of the front lobe. The transverse concavity of the fore part of the hind lobe is narrowed, as it descends, by the reciprocal and progressive inbending of the outer and inner borders of the lobe upon the front surface, as this approaches the base of the lobe. The height of the hind lobe from the middle of the valley is 1 inch 3 lines; the antero-posterior extent of the middle of the base of the lobe is 10 lines. The posterior basal ridge (*g*) resembles that of *m* 1, bearing the same proportion to the front ridge.

In the older specimen (ib. fig. 11) the two lobes of *m* 2 are half worn down; the abraded surface of each is gently bent with the concavity forward; the transverse extent of such surface is 1 inch 7 lines; the fore-and-aft extent of the tooth is 2 inches 4 lines. The abraded surfaces slope from before downward and backward.

In the last molar of the same lower jaw the summit of the hind lobe, on which a narrow tract of dentine is exposed, measures 1 inch 4 lines in transverse extent, that of the more worn front lobe being 1 inch 6 lines. The transverse extent of the base of each lobe is the same, viz. 1 inch 7 lines. The summit of the hind transverse ridge (*g*) is continuous with a short low rising upon the back part of the hind lobe. The antero-posterior extent of the tooth is 2 inches 5 lines.

There is less difference between *m* 2 and *m* 3 of the lower jaw than in the upper one. Owing to the direction of the plane of attrition, the front surface of each worn lobe is higher than the back surface; the front lobe, when unworn, rises a little higher than the back one. The fore part of each tooth rises more abruptly, and in a greater degree above the back part of the tooth in advance; thus the line of attrition of the entire molar series is zigzag. The general curve of the grinding-surface of the four molars is slightly concave from before backward, as, above, it is convex. The contour of the outer sides of the lower series of molars is slightly convex; that of their inner sides is almost straight.

In the mandible belonging to the skull (Plate XXXV. fig. 1) the outer part of *m* 2 is worn to its base, and a larger proportion of *m* 3 alone remains in the left ramus*. The fore-and-aft extent of *m* 3 is 2 inches 5 lines: the same extent of the abraded surface of the front lobe is 9 lines, its transverse extent being 1 inch 7 lines. The enamel at this part of the tooth is fully a line in thickness.

In a fragment of the left mandibular ramus of an old *Diprotodon* are the last two grinders (Plate XL. figs. 17, 18), similarly worn down but better preserved. In *m* 2 (fig. 17) a portion of the enamel at the inner end of the valley (*e*), and the enamel of the hind part of the base of the hind lobe with the contiguous basal ridge alone remain; the rest of the surface is polished dentine and osteo-dentine with the external cement. In *m* 3 (fig. 18) the enamel is worn away from the fore and outer part of the front lobe; the front basal ridge is rounded off; the outer boundary of the valley connecting there the front and hind lobes is smoothed down, and the middle of the hind transverse ridge is touched. Both lobes are worn down nearly to the bottom of the valley. At the middle of each of the smooth concave plates of dentine, a central tract of osteo-dentine (*o*) is defined. The antero-posterior extent of *m* 2 is 2 inches 4 lines; that of *m* 3 is 2 inches 5 lines; the greatest transverse diameter in each is 1 inch 7 lines. This *Diprotodon* had probably died of old age. The outer alveolar border has grown upward with the rise of the fangs and base of the teeth to bring them into grinding contact with those of the upper jaw.

In the crown of the last molar not wholly emerged from the formative alveolus of the young *Diprotodon* (Plate XL. figs. 12, 16), the unworn summit of the hind lobe is irregularly and minutely wrinkled, not divided into small mammilloid tubercles as in the Dinothere. In the largest existing species of Kangaroo (*Macropus major* and *M. laniger*, e. g.) the lower molars have no posterior basal ridge. It is interesting to find that this is present in a still larger extinct species (*Macropus atlas*, Ow., fig. 14, *g*), but it is narrower than the anterior basal ridge. In the lower molars of *Diprotodon* the posterior basal ridge is not only constant, but is broader than the anterior one.

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* The form and position of these teeth are given, in outline, from better preserved and less worn specimens.

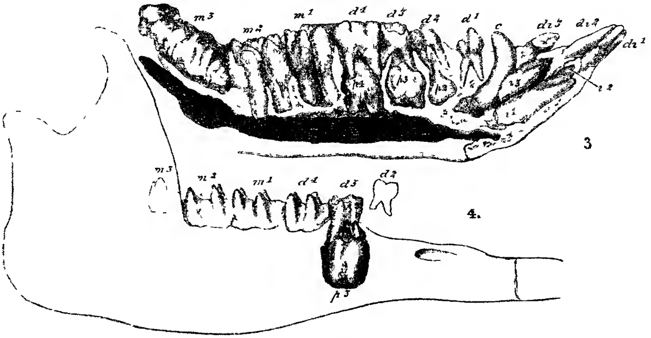
The sum of the characters of the teeth of *Diprotodon*, and the observed varieties and modifications due to sex, age, and other conditions, have been given in detail and fully illustrated. The most common evidences of extinct Mammals are detached teeth; and it seemed desirable to afford sufficient and satisfactory means of determining those of the genus *Diprotodon*, as thereby the knowledge of its geographical distribution in the Australian Continent at the period of its existence may be the more speedily acquired.

A retrospect of the dentition exhibited in the series of specimens above described and illustrated brings to view a combination of characters now shown apart in the marsupial herbivorous genera *Macropus* and *Phascolomys*; but the Macropode characters prevail in number and importance. The small upper incisors (*i* 2 and *i* 3) with definable crown and fang and concomitant limitation of growth, the same genetic character of the molars with the bilophodont type of their crown, testify to the closer affinity of *Diprotodon* to *Macropus*. The large, scalpriform, ever-growing first pair of incisors of the upper jaw, with the shape, structure, and corresponding genetic character of the lower pair of incisors, are resemblances to the Wombat's dentition; and the same affinity is exemplified in the number of the molar teeth.

In the Macropode group, although not more than five grinders are ever in place in one alveolar series of either jaw, seven may be developed. Of these teeth two have no homologues calcified in either *Phascolomys* or *Diprotodon*; these are the small anterior teeth symbolized in my 'Anatomy of Vertebrates' (vol. iii. p. 380, fig. 296) as *d* 2 and *p* 3 (Cut, fig. 4). It may be objected that, for certainty on this point, one ought to have specimens of jaws of *Diprotodon* of an earlier age than that represented in Plates XLI. & XLII. My experience in marsupial dentition begets confidence, however, that, had a true "replacing tooth" been developed in *Diprotodon* as in *Macropus*, its crown-germ would have been detected beneath the tooth marked *d* 3, in the subject of the above-cited Plates. I also believe that, had a *d* 2 ever been calcified and in use, as in the Kangaroos and Potoroons, some trace of its alveolus would have remained, in this young jaw, instead of the continuous, even subtrenchant margin which the diastema of the subject of Plate XLI. presents between *d* 3 and *i*.

Since the Wombats in their molar dentition offer precisely the same differences as to number and succession of grinders which *Diprotodon* presents, we may have the less reserve in accepting the evidences of the further resemblance which the molar series adds to the incisive one. The extension of the genetic character of the scalpriform incisors to the molars is the marked distinction of *Phascolomys* in the Marsupial series; for, with[*] continuous growth, go length of tooth without loss of breadth, depth of implantation with, commonly, curvature of socket, and continuation of enamel to the widely open base of the tooth. I have no evidence that the first and smallest of the series of five grinders in *Phascolomys* is a premolar or replacing tooth, and view it, therefore, as one of the first developed calcified series. It is analogous, in function, in retention, and long-continued use, to a premolar of the placental type-dentition. The succeeding four grinders in both *Phascolomys* and *Diprotodon* are equally members of the first set of teeth; and the last three are homologous with those that are not displaced by vertical successors in diphyodont *Placentalia.* The symbols, therefore, *d* 3, *d* 4, *m* 1, *m* 2, *m* 3, express, in my opinion, the homologies of the functional molar teeth of *Diprotodon* with those, *e. g.*, so marked in *Hyrax*, *Hippopotamus*, and *Sus**. For convenience of comprehension of the teeth symbolized in Plates XXXV.–XLII. I subjoin woodcuts of an instructive phase of dentition in the Hog (fig. 3) and Kangaroo (fig. 4).

Figs. 3 & 4.



§ 5 *Spinal Column.*—Of the atlas there is a portion of the left moiety (Plate XLIII. fig. 2) showing the deep articular cavity for the occipital condyle of the same side, between which and the diapophysis is the outlet of a canal (*a*) about 3 lines in diameter, which traverses the neural arch from within outward behind the upper part of the cavity for the condyle. The surface (*z'*) for the articular process of the axis is slightly concave; between its upper part and the ridge leading from the hind margin of the neural arch to that of the diapophysis there is a deep and wide groove for the passage of the vertebral artery into the neural canal. The above-described fragment yields evidence that, as in *Macropus*, *Phascolomys*, *Phascolarctos*, and some other Marsupials, the ring of the atlas (if indeed it were completed below by bone in *Diprotodon*) presented only the perforation

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[*] In my 'Anatomy of Vertebrates,' vol. ii. p. 465, fig. 312; vol. iii. p. 346, fig. 276; p. 357, fig. 287; p. 377, fig. 294.

(for the anterior spinal nerve or a division of it) on each side of the base of the neural arch, the vertebrarterial canal being, as above described, an open groove. In the atlas of proboscidian and of other large placental Mammals, which the present fossil resembles in size, the diapophyses are widely bored by the vertebral artery, which usually perforates also the fore part of the neurapophysis*.

I recognize, therefore, in the portion of the atlas vertebra, here referred to *Diprotodon*, marsupial characteristics; compared with that of the Kangaroo, its diapophyses are relatively shorter, thicker, terminally more obtuse, not so much expanded or depressed at that part, upon the whole more resembling those in the Wombat and Koala†.

The axis or vertebra dentata (Plates XLIII. fig. 1, & XLIV. figs. 1, 2, 3) is entire save the ends of the diapophyses, which have been broken away. The length of the body, with the odontoid process, is 6 inches 3 lines, the height of the vertebra is 8 inches 4 lines, the breadth across the anterior articular surfaces is 5 inches 9 lines. The size of this vertebra thus equals that in the largest Rhinoceros or Hippopotamus, and in length that of a full-sized Elephant. The hind surface of the centrum (Plate XLIV. fig. 3, *c*) is flat, rather rough, transversely elliptic, with a tendency to an angular or octagonal outline. The under surface (ib. fig. 2) expands as it advances to develope the bases of the parapophyses (ib. *p p*); contracting in advance of these it again expands into the anterior articular processes (ib. and fig. 1, *z*, *z*). A low hypapophysis (ib. fig. 2, *hy*) of a subtriangular form projects from the middle of the under surface towards the fore part. The anterior articular surfaces (Plates XLIII. & XLIV. fig. 1, *z*) converge to the base of the coalesced body of the atlas, called "odontoid" process. This element (Plate XLIII. fig. 1, *ca*, and Plate XLIV. fig. 2, *ca*), 2 inches in length, 1 inch 6 lines in breadth, and of similar depth, is convex transversely and longitudinally below, it has a pair of slightly concave roughened surfaces, meeting above, along the anterior sloping half (Plate XLIII. fig. 1, *m*), behind which the upper surface rises into a low broad tuberosity (ib. *t*), bounding anteriorly a smoothish elliptical surface (ib. *r*) occupying the upper part of the rest of the odontoid. A broad deep irregular depression (ib. *o*) divides this surface of the odontoid from the anterior articular surfaces of the axis. These surfaces (ib. and Plate XLIV. fig. 1, *z*, *z*), of a full oval shape, 3 inches in diameter, are moderately convex. The neurapophyses (Plate XLIII. *n*), after developing the diapophysis (ib. *d*), contract to a fore-and-aft extent of 1 inch 9 lines, then expand backward to develope the postzygapophyses (*z*′), in advance of and between which the neurapophyses converge and coalesce to form the base of the neural spine (ib. *ns*). This expands both forward and backward, as it ascends, to an obliquely truncate summit nearly 5 inches in fore-and-aft extent; narrow and ridge-like at the mid part, expanding and obtuse at the fore and hind angles, the latter being the thickest; from each side of this angle a low ridge (Plate XLIII. fig. 1, *g*) descends obliquely forward, subsiding upon the lateral surface of the spine. The neural canal is 2 inches 3 lines in width, and rather less in height, especially behind, where the vertical diameter is 1 inch 6 lines. A wide groove leads outward and downward from the canal between the postzygapophyses and the back part of the centrum. The upper (neural) surface of the centrum is impressed at its middle with a deep pit, to which a groove leads on each side; the smooth surface has been broken away before and behind the pit, indicative of its having been crossed lengthwise by a bony bar, which would have converted the lateral groove into a pair of foramina.

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* Osteological Catalogue of the Museum of the Royal College of Surgeons, 4to, 1853, p. 475, no. 2678, 'Atlas of Elephant' (by misprint the vertebral artery is called "medullary"); p. 509, no. 2945, 'Atlas of *Rhinoceros bicornis*,' "the vertebral artery perforates the diapophysis and then also the neural arch;" p. 566, no. 3404, *Hippopotamus amphibius*, "the transverse processes are perforated by the vertebral arteries."

† I am led to believe, after fresh study of Diprotodont fossils, that the one ascribed to a calcaneum in my 'Catalogue of Fossil Mammalia in the Museum of the Royal College of Surgeons' may be a fragment somewhat rolled and worn of the atlas vertebra.

Of the quadrupeds resembling in size the *Diprotodon*, the Proboscidians have the axis most like that of the Australian giant, but the following differences present themselves. In *Elephas* the odontoid is absolutely, as well as relatively shorter; the anterior articular surfaces are less uniformly convex and less convex in any direction; the neural spine is relatively lower, much thicker transversely, with a subquadrate termination or upper surface, canaliculate along the mid line, and deepening to produce a posterior bifurcation The centrum has no hypapophysis. In *Macropus*, on the other hand, we find the hypapophysis is repeated both as to size and position, the odontoid process also offers a like development, with resemblance in such details as the disposition and proportions of the pair of upper terminal surfaces for ligamentous attachment, and the posterior smooth surface for the transverse ligament. The neural spine is, however, more produced anteriorly and less so behind

In my 'Osteology of Marsupialia,' I noted, as a result of observations on the skeleton of *Macropus major*, that "in the Kangaroo both the dentata and atlas have the transverse processes grooved merely by the vertebral artery"*. I have since observed in *Macropus laniger* the circumscription of the groove by the development of a slender parapophysis, as in *Diprotodon* A similar vertebrarterial canal occurs in *Phascolomys* and *Phascolarctos* The neural spine of the axis in the Wombat resembles in shape that in the *Diprotodon*, but is rather more produced behind. The hypapophysis is, however, a mere medial low ridge, that in the Kangaroo is significantly more like the process in *Diprotodon*. In both *Macropus*, *Phascolomys*, and *Phascolarctos* a pair of conspicuous foramina near the hind part of the upper (neural) surface of the centrum lead to canals converging as they sink in the osseous substance to a common (venous) passage; these are not present in Proboscidians; a few minute irregular venous foramina may be seen on the corresponding surface of the axis vertebra.

The third (Plate XLIV fig 4) and two consecutive (Cut. fig. 5, *c* 3, *c* 4) cervical vertebræ resemble by their shortness those of the Wombat rather than of the Kangaroo; they are by no means, however, so compressed from before backward as in the Proboscidians

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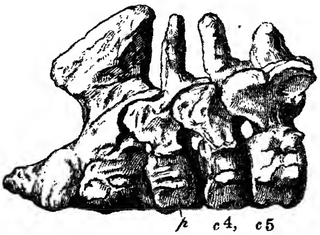
* Trans Zool Soc. vol ii p 394, see also Art *Marsupialia*, Cycl of Anat p 276.

In *Diprotodon* the length of the third cervical centrum is 1 inch 10 lines, the breadth of its hind articular surface is 4 inches 3 lines, the height of the vertebra is 8 inches. The centrum is without hypapophysis, the vertical extent of the hind surface (Plate XLIV. fig. 4, *c*) is 2 inches 9 lines; the two extremes of the transverse ellipse are almost angular. The base of the parapophysis (ib. and cut, fig. 5, *p*) extends from near this angle forward for 1½ inch along the side of the centrum. The upper surface of the centrum shows a large medial venous orifice. Both margins of the rising neurapophyses are deeply notched for the "conjugational foramina," and send off a small diapophysis (ib. *d*) to circumscribe above the vertebrarterial canal. The neural spine (ib. *ns*), 4 inches in height from the roof of the neural canal (ib. *n*), is compressed from before backward, simple, obtusely rounded at the end, strengthened by a low medial ridge, both before and behind, along its basal half. There is no such development of neural spine in the third cervical of Proboscidians; in the larger herbivorous Marsupials it is as conspicuous as in *Diprotodon*, but with altered shape; that in the Wombat most resembling the one in *Diprotodon*, but being relatively lower.

The fourth cervical (Cut, fig. 5, *c* 4) much resembles the third; but, as in the Kangaroo, has a shorter spine, resembling, however, in shape that of the third, being compressed from before backward instead of from side to side as in *Macropus*. The slight increase of size is in breadth, chiefly of the centrum, not in length or height. The neural canal is wider and a little higher; more space is made for the myelon as it traverses the more flexible part of the neck. The large venous foramina and vertical canal are repeated on the upper part of the centrum; the corresponding pair of foramina now also blend into a common fossa, as in the Wombat.

In the fifth cervical (Cut, fig. 5, *c* 5) the neural spine gains in antero-posterior and loses in transverse thickness; the vertical ridges are stronger, especially the one behind; it appears to have been shorter than in the fourth vertebra*. The centrum and neural canal have increased, chiefly transversely; there is very little increase of length. The parapophysis has gained in vertical extent.

Fig. 5.



Second to fifth cervical vertebræ, one-sixth nat. size; *Diprotodon*.

In the series of mutilated vertebræ belonging to Mr. Boyd's specimen of *Diprotodon* are two dorsals (Plate XLIV. figs. 5–8). They show the impressions for the free articulation of the ribs both before and behind (ib. figs. 6 & 7, *pl*, *pl′*), and are remarkable for the retention of the short proportion of the cervicals, and for the terminal bifurcation of the antero-posteriorly compressed spine (ib. figs. 5 & 8, *ns*). They are not consecutive vertebræ, but were not far from one another in the anterior part of the dorsal series.

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* It is so in the sketch sent me by Sir Thomas Mitchell from Sydney (Cut, fig. 5); but, amongst the damages to the specimens in their passage to London, the summit of this spine has been knocked off.

As in this region the vertebræ in many Mammals decrease in breadth before regaining the size which then goes on augmenting to the lumbar region, I first take for description that (Plate XLIV. figs. 5 & 6) which with a broader centrum has a shorter as well as broader spine. The fore-and-aft extent of the centrum is 2 inches at its lower part; it slightly decreases towards its upper surface. The breadth of the centrum is 4 inches 10 lines, above which this dimension is increased by the share contributed by the neurapophyses (ib. fig. 5, *n*, *n*) to the body of the vertebra (*c*); the sutural lines indicative of this share are plainly traceable on the terminal articular surface (ib. fig. 5, *c*), from which the epiphysial plate has become detached. As the ends of both diapophyses and neural spines are broken off, the following dimensions of the vertebra are not the full ones, viz. of breadth 8 inches 6 lines, of height 9 inches 6 lines. The width of the neural canal is nearly 4 inches, its height is fully 3 inches. Both articular surfaces of the centrum are nearly flat, the anterior one in a very slight degree convex; but both surfaces are epiphysial, with coarse furrows and lines affecting a radiate disposition, the extent of which rugosity indicates the complementary plate to have overlapped both elements of the vertebral body, viz. the neurapophysial (fig. 5, *n*, *n*) and the central one (ib. *c*)*.

A prominence on the upper third of the side of the body indicates the lower boundary of the neurapophysis, and this part of the body holds the main part of the impressions (*pl*, *pl'*) for the head of the rib, of which impressions the hinder is the larger. The contour of the articular surfaces of the body is semicircular. The neurapophyses, after contributing their share to the vertebral body, extend upward, outward, and a little forward, contracting into subcylindrical pedicles which suddenly expand to send off the diapophyses (ib. figs. 5 & 6, *d*), prezygapophyses (*z*), and postzygapophyses (*z'*). Before developing the latter processes the neurapophyses begin to bend inward, still ascending; then they contract, especially from before backward, converge, and coalesce to form the base of the antero-posteriorly compressed and laterally expanded spine (*ns*). The base of this spine is strengthened both before and behind by a low broad median ridge; its terminal divisions diverge as they rise. The undivided base forms a low obtuse eminence between them (fig. 5, *ns*).

The prezygapophysis (fig. 6, *z*) projects forward as a semicircular shelf, the flat articular surface looking upward, with a very slight inclination downward and outward; the postzygapophyses (*z'*), of somewhat smaller size, are supported each by a buttress of bone descending from the hinder and outer angle of the spine, and expanding with a prominent convexity to the articular surface. The upper surface of the centrum, between the neurapophysial bases, shows the large venous fossa.

In the other dorsal vertebra (Plate XLIV. figs. 7, 8) the diapophysis is entire on the left side (fig. 8, *d*), and expands into a protuberance with an articular surface (fig. 7, *d*) 1 inch 9 lines by 1 inch 3 lines, for the tubercle of the rib on its lower half. The neurapophysial parts of the centrum are traceable, and make a more definite rising at the upper part on each side the small intervening proportion of the centrum proper (ib. fig. 8, *c*). Posteriorly they also slightly project (fig. 7, *n*) beyond the flat surface of the centrum (ib. *c*); and a smooth tract of the neural canal (fig. 8, *n*′, *n*′) is continued backward upon each of these prominences. The fore surface of the centrum is in a very slight degree convex; both surfaces are epiphysial or rough, with the usual tendency to a radiate disposition of the fine furrows. The postzygapophyses (*z*′) are somewhat more prominent than in the former dorsal (figs. 5 & 6), and the neural spine slopes a little backward. This process is narrower transversely than in fig. 5, and is longer prior to its bifurcation (fig. 8, *ns*). Its strengthening ridges, especially the anterior one, are more developed; the bifurcation of the summit is repeated in this as in the foregoing vertebra, with slight divergence of the terminal prongs, both of which have lost their summits.

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* This is the usual character of epiphyses completing compound bones, as, *e. g.*, at the proximal end of the three confluent metatarsals in the bird, at both ends of the two confluent metacarpals in the ruminant, &c.

In the whole range of the Mammalian series I know of no dorsal vertebræ with characters like the subjects of figures 5–8. Where vertebræ are notable for their shortness and lamelliform type they are confined to the region of the neck, as, *e. g.*, in *Proboscidia* and *Cetacea*; but the dorsal series, in these, promptly resumes the ordinary proportions of length of centrum. Similarly, where the transversely bifurcate character of the neural spine is met with (*e. g.* Elephant, Man), it is restricted to one or two of the cervical series; in *Diprotodon* only is it known to exist in a dorsal vertebra. What modification may ensue or at what distance from the neck in other or posterior dorsal vertebræ my present materials do not enable me to state. I infer that the more usual proportions are acquired in the posterior dorsals from the slight increase presented in the following specimen, and from those which certain of the lumbar vertebræ present.

The specimen referred to, which forms part of the collection in the Museum of the Royal College of Surgeons, consists of the centrum only.

It measures 2 inches 3 lines in antero-posterior diameter, 3 inches in vertical diameter, and 4 inches 9 lines in transverse diameter. Both articular extremities are flat; the epiphysial plates are anchylosed; but where they are broken away the radiating rough lines, characteristic of the epiphysial surface, indicate that the union was tardy and had been recently effected before the animal perished. This vertebra differs by its compressed form and the flattening of the articular ends from the dorsal vertebræ of the ordinary placental Pachyderms, but resembles in these characters the dorsal vertebræ of the Proboscidians; in these, however, the breadth of the vertebral body is not so great as in the fossil. From the cetacean vertebræ the present fossil is distinguished by the large concave articular surface at the upper and anterior part of the side of the body for the reception of part of the head of a rib; this costal surface, which is not quite entire, appears to have been about 1½ inch in diameter. The neurapophyses are anchylosed to the centrum, but the internal margins of their expanded bases are definable, and have been separated by a tract rather less than 1 inch in breadth, of the upper surface of the centrum; at the middle of this surface there is a deep transversely oblong depression. A similar depression is present in some dorsal vertebræ of the *Megatherium* and in the anchylosed lumbar vertebra of the *Mylodon*; but the bodies of the dorsal vertebræ in the great extinct *Bruta* are longer and narrower in proportion to their breadth than in the present fossil. In the Kangaroo the upper surface of the body of the dorsal and lumbar vertebræ is perforated by two vascular canals, which pass down vertically and open below by a single or double outlet. In the Wombat the middle of the upper surface of the bodies of the dorsal and lumbar vertebræ exhibits a single large and deep depression, which in the dorsal vertebræ has no inferior outlet, and in this character they closely resemble the present fossil. The dorsal vertebræ of the Wombat are, however, longer in proportion to their breadth.

Thus the present mutilated vertebra alone would support the conclusion that there had formerly existed in Australia a mammiferous quadruped, superior to the Rhinoceros in bulk, and distinct from any known species of corresponding size. It is interesting and instructive to find one well-marked character in it, viz. the median excavation on the upper part of the body, repeated in the same vertebræ of one of the largest of the existing *Marsupialia*.

The remaining evidences of vertebræ in the Boydian or purchased series of Diprotodont Fossils in the British Museum consist of five centrums and two pairs of detached terminal epiphyses of those elements.

The centrums, in the absence of any costal or hæmapophysial depression, in their increased length and greater expanse of the neural canal, are referable to the lumbar series. Three retain the coalesced bases of the neurapophyses, yet these do not develope diapophyses in the extent to which they are preserved.

The foremost of these lumbar centrums shows a length of 2 inches 4 lines at its lower part, increasing to 2 inches 8 lines at its upper part; the others, with slight general gain of size, show the same proportions. Thus the one which seems the last of the series has a length of 3 inches 3 lines at the lower part, and of 3 inches 8 lines at the upper part of the centrum. Thus we may infer that the part of the spine from which these vertebræ have come was habitually bent in *Diprotodon* with the concavity downward. The degree of increased length in the last over the longest of the other three centrums indicates two or three missing vertebræ intervening between those to hand. The Kangaroo has six lumbar vertebræ, the Koala eight, the Wombats only four (*Phascolomys vombatus*) or five (*Phascolomys latifrons*). Six lumbars is the rule in *Marsupialia*, and I incline to view *Diprotodon* as amenable thereto, rather than as repeating the exceptional formula of *Phascolomys**.

In the foremost of the five fossil lumbar centrums a small protuberance from the upper and fore part of one side indicates the rudiment of a diapophysis; it is not present on the other side; it may be a vertebra, as is sometimes seen in the Kangaroo and other mammals, transitional between the dorsal and lumbar series, having the characters of a rib-bearer on one side and not on the other. A trace of roughness on the side of the fossil centrum corresponding to the protuberance on the other side, may indicate there a ligamentous attachment of a rudiment of the last free rib. The present vertebra, whether interpreted as the last dorsal or first lumbar, shows the small extent to which those vertebræ gained in length as they receded in position. The antero-posterior diameter at the under part of the centrum is 2 inches 5 lines, at the upper part 2 inches 10 lines; the breadth of the anterior surface is 4 inches 10 lines, the vertical diameter of the same surface is 3 inches 9 lines. The epiphysial plate adheres to this surface; it is concentrically marked, thinning off to the centre, where it leaves a vacuity transversely oblong, 1 inch 4 lines by 1 inch in its diameters. From the opposite surface the epiphysis has been detached, showing the radiate disposition of the rugæ of the diaphysial surface, and the proportions contributed by the bases of the neurapophyses to the vertebral body.

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* As I was led to note in my 'Osteology of *Marsupialia*,' *loc. cit.*, p. 396, the number of free trunk-vertebræ is significantly constant in that order, whatever be the difference of costal formula; thus, *Phascolarctos* has 11 costal, 8 lumbar, =19; *Petaurus*, 12 costal, 7 lumbar, =19; *Macropus, Phalangista, Perameles, Myrmecobius, Phascogale, Didelphys, Dasyurus, Sarcophilus, Thylacinus*, have severally 13 costal, 7 lumbar, =19; *Phascolomys vombatus* has 15 costal, 4 lumbar =19.

In the next vertebral body, of similar dimensions, the anterior epiphysis is adherent, but with the line of suture conspicuous; it is from 3 to 4 lines thick at the periphery, and thins off toward the centre, where it leaves a vacuity of about 1 inch in diameter. The surface, for 1 inch at the periphery, is moderately convex, the rest is flat. The free surface of the centrum is greatly and equably concave lengthwise. At the middle of the neural surface is a transversely oblong venous fossa, 9 lines by 6 lines in diameters. This centrum adheres by matrix to the succeeding one, which, repeating the characters above noted, retains about 1 inch of the neurapophysial pedicles or lamellæ. Each at its origin has a fore-and-aft extent of 2 inches, contracting to 1 inch 8 lines at the fractured end; it rises nearer the fore than the hind end of the centrum. The extreme thickness (1 inch) is toward the fore part of the pedicle. The transverse diameter of the neural canal at the broken ends of the pedicles is 3 inches 6 lines. The venous fossa is repeated in this as a single median one; but in another lumbar centrum the entry is divided by a median longitudinal tract of the neural surface, as is commonly the case in the Kangaroo.

In the third of these the left pedicle is preserved to a height of 2 inches, expanding then to an antero-posterior extent of 2 inches 3 lines, and a transverse one of 1 inch 5 lines; at the lower contracted part of the neurapophysis these diameters are, respectively, 1 inch 9 lines and 1 inch. Yet the whole of the outer surface is smooth without trace of outstanding transverse process; whereas in both Kangaroo and Wombat that process comes off at the junction of the neurapophysis with the centrum. We may therefore infer that the neural arch of the lumbar series was loftier in *Diprotodon*, as we have already seen it to have been in the two anterior dorsal vertebræ preserved. The epiphysis is wanting from the hind surface of the third lumbar described, and the sutures of the neurapophyses with the centrum are there exposed. They project a little beyond the epiphysial surface of the centrum. The largest and hindmost of the present series of lumbars (Plate XLIV.

figs. 9, 10), the length of the centrum of which has been noted above, shows a breadth of hind surface (fig. 10) of 5 inches 9 lines, its height being 4 inches; the antero-posterior extent of the base of the pedicle is 2 inches 6 lines; about 9 lines extent of the centrum extends backward beyond it.

The smaller pair of epiphysial vertebral plates (ib. fig. 12), cemented together by the matrix, have come, according to their size and shape, from the cervical series; they are transversely elliptical, 4 inches 6 lines in long diameter, and 3 inches in short diameter. The thicker free or peripheral margins (12, *a*) diverge from each other, and they thin off to a central vacuity (12, *c*). The larger pair (ib. fig. 13) appear to be from the lumbar series; they measure 4 inches 7½ lines across, and 3 inches 8 lines down the middle; their central vacuity is transversely oblong, measuring 1 inch 3 lines by 1 inch. These detached vertebral epiphyses are completely petrified.

The terminal epiphyses of the bodies of dorsal and lumbar vertebræ remain distinct, and come off in pairs attached by intervertebral substance in Kangaroos which have arrived at full growth. I presume that the same circumstance occurred in the course of decomposition or maceration of the carcass and skeleton of *Diprotodon*; hence the presence of such separate pairs of epiphyses receiving co-attachment from the matrix after separation from their proper centrums*.

Of the ribs, though few are entire, so many have reached me as suffice to show that, as in Marsupials and most Mammals, they vary in length, curvature, degree in which the groove for the intercostal vessels and nerve is excavated, distinction of head and tubercle, and relative position of the latter (Plate L. pl_{1-14}).

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* Since the reception of the specimens of vertebræ above described, I have been favoured with two drawings, of the natural size, of a side view and end view of a lumbar vertebra of a *Diprotodon*, from St. Ruth's Station, Condamine River, Queensland, by Dr. Fr. Campbell. In these drawings sufficient of the neural arch is preserved to show the base of the diapophysis extending outward, at 1 inch 6 lines above the level of the upper surface of the centrum. The breadth of the centrum is 5 inches, its vertical diameter 4 inches; the breadth of the neural canal is 3 inches 6 lines, the fore-and-aft extent of the centrum at its upper third is 2 inches 10 lines. An oblique broad low ridge or rising of the outer surface of the pedicle rises to the lower part of the base of the neurapophysis.

The two drawings, of side and front views, of this vertebra have been made carefully, and I believe accurately, as regards admeasurements, by Mr. Campbell's son, who found the vertebra, and whose letter to his father on the subject is as follows:—

(Copy.)

"St. Ruth, 25th May, 1865.

"The enclosed drawings I send to you to amuse you a little till I come down, speculating as to what the huge animal was. I have the bone and some more, now in my possession. A large top jaw with a few pieces of teeth sticking in it, and what looks like a *blow-hole* in the top—some smaller shank bones, or something of the sort—all fossil. They ring like cast iron when knocked together: too heavy to bring down with me:—they are of a dark brick colour. I will try and make drawings of the rest if I have time before I come down. One of the vertebræ of a Bull looks very small indeed alongside this great bone.

(Signed) "Hugh Campbell."

In the letter inclosing his son's drawings Dr. C. writes:—"The bones he mentions in the letter and whose likeness is also inclosed herewith, I regret to say he was induced to part with to a gentleman to whom he was under particular obligations of friendship, and who had expressed a great desire to possess them." Should the present notice ever meet the eyes of the possessor of these fossils he may be assured that it would give me pleasure to make them subservient to the advancement of a knowledge of *Diprotodon*.

The longest specimen measures 2 feet 1 inch, following the convexity of its curve. The tubercle is low, 3½ inches from the head; the intercostal groove is shallow, and chiefly defined by a ridge-like production of the posterior border at the upper fourth of the rib, 3 or 4 inches in extent. Beyond this the rib loses thickness and gains breadth, the latter dimension reaching to 1½ inch about one-third from the broken end.

Another specimen presents a greater degree of curvature. The tubercle is better developed, has a more definite articular surface, extending upon the neck of the rib. The broadest part of the rib (1 inch 9 lines) is at the upper third of the bone. This rib had a more anterior position in the chest than the former; the extent preserved, following the convexity of the curve, is 1 foot 6½ inches.

A third specimen with head, tubercle, and intercostal groove well marked, is less curved than the former, and is larger than either of the above described. A length of 1 foot 4 inches is preserved. The fractured end is elliptic, 1 inch 9 lines in long diameter, 1 inch in short diameter; but the rib midway between the end and the head attains a breadth of 2 inches. This has come from nearer the middle of the chest.

The only entire specimen is a posterior rib, with the tuberosity relatively small and rough; the head large, intercostal groove almost obsolete; body of the rib straight along its distal half, which gradually expands, with loss of thickness to a breadth of 2 inches 2 lines. The length of this rib, following the convex curve, is 1 foot 8 inches. The lower extremity shows the roughened surface for the attachment of the costal cartilage.

The costal fragments yield little more than the character of size. The vertebral end of one, which includes the tubercle, has a circumference below that part of 3½ inches. Another fragment has a circumference of 4½ inches; a third fragment is nearly 6 inches in circumference; a fourth fragment shows a flatter shape.

From the shortness of the costigerous vertebræ and the size of the ribs, their interspaces must have been narrow.

Assuming with much confidence that the dorso-lumbar series in *Diprotodon* included nineteen vertebræ, I assign one more pair of ribs than in the Kangaroo, and reckon fourteen pairs in the dorsal series (Plate L.).

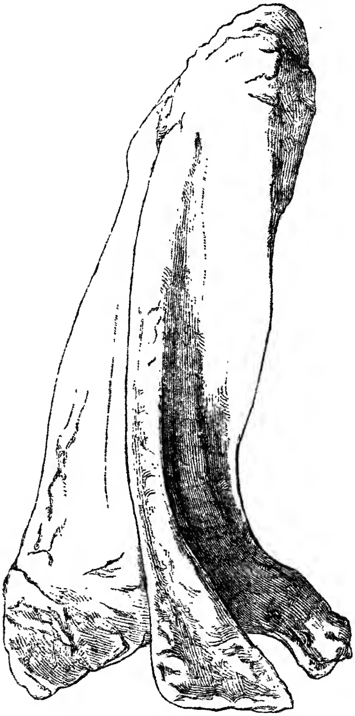
§ 6. *Scapula.*—The scapula is represented in the Boydian collection of Diprotodont remains by an almost entire specimen of that of the left side (Plate XLV.), and by a fragment of the one of the right side.

It is narrow in proportion to its length, and chiefly peculiar by the production of the subspinal plate anteriorly (ib. *a*), whereby the usual proportions of the triangular mammalian scapula are reversed, the part answering to the base (ib. *b*) being the apex, and the articular end of the bone (ib. *a*, *c*) the base of the triangle, which is elongate and irregular.

The articular or glenoid cavity (ib. *d*) presents the usual oval shape with the small end upward (ib. fig. 3); the concavity is deepest lengthwise, and the apical part is most produced. The outer border beneath the acromion (*e*) has been broken off, indicative of its prominence, which is better preserved in the articular cavity of the fragment of the right scapula, showing its resemblance to that part in *Macropus*. This border subsides, becoming thick and convex as it approaches the small or coracoid end of the cavity. The lower border is continued into a rugged triangular surface beneath (fig. 3, *a*) for the attachment of the long head of the triceps; the upper apical part is produced, beak-like, beyond the base of the coracoid (*c*). The inner margin is low near the apex, but less obtuse than the corresponding part of the outer one; it is more produced as it descends; but this margin subsides gradually into the subscapular one.

The spine (ib. fig. 1, *f*) begins by a gradual elevation of the lower or hinder half of the hinder surface of the "base" (*b*), which elevation contracts as it rises from that surface to a thickness of 1½ inch. The free border, of this thickness, is also flat; the spine gradually rising as it advances, describes a slight curve toward the upper or anterior costa (*g*); the lower margin of the free border becomes most produced, and, as the spine expands into the acromion (*e*), this margin also expands and becomes rough for muscular attachments, and in the present specimen forms the most prominent part of the acromion; but the end of this process is broken off. From a pencil-sketch of this scapula made by Sir THOMAS MITCHELL when it arrived at Sydney (Woodcut, fig. 6), the acromion (*e*) continued to expand to an obliquely truncate end, having the upper or fore angle most produced, and, as it were, slightly twisted towards the coracoid (*c*) (indicated by the dotted line in fig. 1, Plate XLV.). As the spine (ib. *f*) rises from the scapular plate, it becomes compressed or thinner beneath the free margin, and presents a smooth concave surface to each scapular fossa (*i*, *j*).

Fig. 6.



Scapula of *Diprotodon*: one-fifth nat. size.

The coracoid process (Plate XLV., *c*), arising from a base of 3 inches in extent, is subcompressed, with the outer surface concave as it extends toward the end of the process, which, however, is broken off. The upper or front "costa" (ib. *g*) describes a strong concave curve as it recedes from the coracoid; the middle third of its extent (ib. *g*′), which was probably convex and produced, has been broken away. Where it is again entire (ib. *g*″) it describes a gentle concavity, and forms the outer border of a sudden thickening of that part of the basal end of the scapula. The upper or basal three-fourths of this anterior border of the bladebone are curved "dorsad," so as to bound or form the (transverse) hollow of the supraspinal fossa (ib. fig. 1, *i*). The part broken from the upper costa (*g*′) may have made the breadth of the fossa, as in the Kangaroo and most Marsupials, greater at its mid part than appears in this fossil.

The subspinal fossa (ib. *j*) increases in breadth from the basal (*b*) to the articular end (*a*) of the scapula, singularly reversing its proportions in other Mammals. In the Koala (*Phascolarctos*) this fossa retains its breadth through an extension of the lower costa nearer to the glenoid cavity than usual*. The corresponding extension is proportionally greater in the scapula of the *Megatherium*; but in *Diprotodon* it is continued as far forward as the neck of the scapula, with an increase of thickness, and a bend toward the "dorsum" of the scapula, increasing the depth of the concavity of this part of the subspinal fossa. The border of the plate (ib. *a*, *a*′) produced below or behind the glenoid cavity (*d*), and having the same aspect, is very thick, concave lengthwise, convex across, with a rough slightly projecting insertional surface at its middle: a more rugged surface appears also at the angle *a*, where it joins the lower or hinder costa, but this is the seat of some mutilation of the fossil.

This costa (*a*, *b*) loses thickness as it recedes from the angle for one-fourth of its extent; it regains a certain thickness and ruggedness for another fourth (*k*), where it is also bent toward the subscapular plane; it then continues drawing nearer to the origin of the spine and finally thickens as it is lost in the obtuse contracted basal end (*b*) of the bladebone.

The long and narrow subscapular surface (Plate XLV. fig. 2 *l*) presents a gentle concavity lengthwise, with a corresponding convexity across the middle, rather increased at the two ends; the mid convexity is changed to a concavity by the in-bending of the part (*k*) of the lower costa above mentioned. The smooth subscapular surface is broken only by the thick short triangular elevation (ib. *m*) extending from near the upper or fore angle of the base.

The singular, not to say unique, development of the "glenoidal" part of the inferior costa (*a*, *a*′) or subspinal plate, was doubtless correlated with some peculiarity of use or application of the fore limb. As to the general shape of the scapula, I cannot suppress expressing the interest with which I have viewed in this old extinct Marsupial or implacental form of Mammal the retention of so much of the archetypal or pleurapophysial proportions which one sees without surprise in inferior Vertebrates such as Monotremes, Birds, Reptiles, and Fishes.

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* This peculiarity is figured in the 'Cyclopædia of Anatomy,' Art. *Marsupialia*, p. 281, fig. 106.

The total length of the above described scapula is 2 feet 3 inches; the extreme breadth is 1 foot 2 inches; the long axis of the glenoid cavity is 6 inches, the short axis 4 inches 2 lines; the height of the spine at the base of the acromion is 4 inches.

Fragments of scapula, from the bed of the Condamine River, west of Moreton Bay, Australia, in the Museum of the Royal College of Surgeons, placed with some doubt in the series of Diprotodont remains in my 'Catalogue of Fossil Mammalia' (4to, 1845, p. 298), can now be certainly referred to *Diprotodon australis*. One of these fragments (no. 1471) includes 4 inches of the interior part of the origin of the spine. "The thickness of the neck of the scapula is 2 inches 9 lines; that of the base of the spine is 1 inch. The indication of the sudden rising of this thick spine from the plane of the scapula distinguishes it from that bone in the Rhinoceros, and its thickness is greater than in the largest Hippopotamus; it is also relatively greater in comparison with the neck of the scapula than in the Elephant" (p. 298). The specimen was thus differentiated, in 1845, from all known Mammals of corresponding or approximate bulk, and is now seen to conform in the particulars cited with the bladebone of *Diprotodon*.

A portion of the glenoid cavity and neck of the scapula of a large Mammalian quadruped (no. 1472), from the same Australian deposits, shows similar dimensions to those in the entire scapula of *Diprotodon australis*.

§ 7. *Humerus*.—In *Macropus* the articular head of the humerus is subhemispherical, looks a little backward as well as upward (the bone being held vertically), and overhangs the back part of the shaft. The inner and outer tuberosities rise above the head, in front of it. The inner tuberosity is thicker and shorter than the outer one, which extends ridge-like obliquely from without inward and forward where that end projects, forming the outside of the deep groove, dividing it from the inner tuberosity; the groove expands and shallows as it descends, and is soon lost in the fore part of the shaft.

The inner tuberosity is supported on a columnar development of the fore and inner part of the shaft. From the fore end of the oblong outer tuberosity the "deltoid" ridge extends halfway down the middle of the fore part of the shaft, being more or less prominent in different species. In all the ridge attains its greatest breadth and prominence at its lower part before its sudden subsidence. At the outer side of the shaft above the developed termination of the deltoid ridge, projects a short, thick, longitudinal ridge, with a rough obtuse surface. Thus the fore part of the upper half of the humerus is divided into two facets, the inner one deepening upwards to the inter-tuberous or bicipital groove; the outer one broader and flatter, between the outer and the deltoid ridges. The back part of the upper half of the humerus is also, but less definitely, divided into two longitudinal tracts; the outer one flattened or slightly concave transversely where bounded by the outer ridge; the inner one gradually contracting, with increased transverse convexity, to be continued into the ridge leading to the ento-condyloid tuberosity. The shaft of the humerus is more bent, with the concavity backward, than usual; the distal end not being turned forward in the degree which gives the ordinary sigmoid shape to this bone in unguiculate mammals.

The humerus in Marsupials is not described in either editions (1805, 1835) of the 'Leçons d'Anatomie Comparée.' But in the 'Ossemens Fossiles' (4to, tom. iv. p. 284) CUVIER notes the precaution requisite in the examination of the distal articular surface of the humerus in Marsupials on account of its resemblance to that in *Carnivora*. In the posthumous 8vo edition of the 'Ossemens Fossiles,' tom. vii. p. 276, after the generalization as to the perforation of the inner condyle in *Carnivora*, is added: "ainsi que chez les Didelphes et dans tous les animaux à bourse." So likewise DE BLAINVILLE states that the inner condyle of the humerus is perforated, "chez tous les Didelphes sans exception," using the term in his peculiar taxonomic sense as equivalent to the *Marsupialia* of other zoologists. I have, however, pointed out exceptions to this rule in certain Dasyures (*Dasyurus Maugei*), Phalangers (*Ph. Cookei*), and Petaurists*.

So much it seemed requisite to premise, because the imperforate condition of the inner condyle also characterizes the bone in *Diprotodon*, differentiating it from the humerus in *Macropus* and *Phascolomys*, without, however, affecting the marsupiality of the great extinct Herbivore. To the description of this bone in *Diprotodon* I now proceed.

The humerus (Plate XLVI.) is more nearly straight than in other Marsupials, and is remarkable for the feeble development of the ridges for muscular attachments. At a glance one sees its relations to the restricted offices of support and locomotion with much less subserviency than in the smaller existing Marsupials to more varied applications of the fore limb.

The head of the bone (ib. figs. 1 & 2, *a*, and fig. 3) rises above the tuberosities (*b*, *c*), forming a very large proportion of the upper end (fig. 3). It has the usual degree of convexity, with a full oval contour, the long axis being transverse, and the smaller end next the outer tuberosity; it overhangs the back part of the shaft at its inner two-thirds (fig. 1), but in a less degree than in the Kangaroo. The inner tuberosity (*b*) is represented by a low broad, rough ridge, extending from the inner side along the fore part of the periphery of the head to near the small outer end of the articular ball; here it is interrupted by a wide but very shallow representative of the "bicipital groove." The outer tuberosity (*c*) projects in a greater degree from the outer side of the base of the head.

The broad, low, rounded angle between the fore and outer sides of the humeral shaft, continued from the fore end of the outer tuberosity (fig. 2, *c*), representing at first the outer side of the bicipital groove, descends and assumes rather more of the character of a muscular ridge at the mid length of the shaft (fig. 2, *d*) before subsiding.

The homologue (ib. *e*) of the short external ridge in *Macropus* here projects as strongly from that side of the bone, but on the same transverse parallel with the lower, best developed part of the deltoid ridge (*d*). Consequently the external ridge is relatively lower placed than in the Kangaroos; it is also relatively shorter, lengthwise, and stands out more abruptly at its upper part.

The representative of the deltoid ridge divides the fore part of the shaft unequally, and the broader division or tract (fig. 2, *f*) is internal, the narrower division or tract (*ib. g*) being turned so as to form rather the outer facet or side of the upper half of the humeral shaft. Another peculiarity of the present humerus is a well-defined oval rough surface (fig. 1, *h*) at the outer side of the back part of the shaft, one-fourth of the way down; this surface measures 2 inches 2 lines lengthwise, by 1 inch 2 lines across; the lower half of its periphery is most prominent. A low ridge, about an inch in length, in *Macropus major*, seems to answer to this process.

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* Osteology of Marsupialia, *loc. cit.* p. 400.

The shaft expands transversely and becomes flattened from before backward at its lower third as far as the distal articulation (fig. 4), which resumes antero-posterior thickness with reduction of transverse extent. The ento-condylar ridge (*i*) is much produced, though relatively less than in *Macropus*; the upper and lower borders meet at an open angle; the ridge is very thick; it extends more than 2 inches from the ulnar condyle; it is imperforate.

The ectocondylar ridge (*k*) is longer than the inner one (*i*), but is less prominent; it is also angular in form, but more openly so. The upper and longer side, commencing 6 inches above the radial condyle, is narrow and slightly turned forward; the lower side rapidly expands to nearly the fore-and-aft breadth of the radial condyle, along the radial or outer border of which the ectocondylar ridge subsides. The middle of the distal expansion of the shaft, above the articulation, is remarkable for the size and depth of the anterior fossa (fig. 2, *l*); the posterior or olecranal depression (fig. 1, *m*) is comparatively feebly marked.

The radial (*n*) and ulnar (*o*) condyles are more convex and more equal than in *Macropus*; they are divided by a narrower and deeper trochlear channel.

The radial condyle is the longest; its outer and hinder marginal contour describes part of a circle, and is not encroached upon, as in *Macropus*, by the rough surface from the ectocondylar tuberosity. The ulnar condyle, which begins in *Macropus* to subside or give way to augment the intermediate concavity, here retains its hemispheroid form (ib. fig. 4, *o*). It is interesting to note this resemblance to the distal articulation of a femur in the humerus of an animal low in the Mammalian scale.

The hind or "olecranal," supracondylar depression (fig. 1, *m*) is shallow, limited in situation to above the ulnar condyle, and the narrow intercondylar channel, and not extending to above the radial condyle.

I have not been able to find the orifice of a medullary artery: the distal portion of the left humerus, broken from the rest of the bone near the middle of the shaft, 11½ inches from the distal end, shows a depression near the middle of the fractured surface half an inch across and not quite an inch deep; and this, if it be not an accidental excavation in the dense cancellous structure, is the sole indication of a medullary cavity. Such cavity is wanting or small in the gigantic extinct Sloths. It is, again, with interest that I view this sign of low organization in the great extinct Marsupial mammal*.

Ulna.—Of the ulna I have hitherto received only the proximal half, and with the olecranon not quite entire; it is a strong but low trihedral process, smooth and concave on the inner side, roughish and flattened behind; with a smooth almost horizontal triangular surface at the upper part. The breadth of the base of the olecranon is 3 inches 10 lines; the circumference of the base is 13 inches. The articular surface answering to the "greater sigmoid cavity" is concave, longer and deeper from before backward than from side to side. From its upper and outer part a less concave articular surface is continued upon the inner side of the base of the olecranon. The lower part of this surface, which may have afforded the "lesser sigmoid cavity" to the radius, is broken away. The rough tract for syndesmotic junction with the radius extends down the outer side of the shaft inclining obliquely forward: it is about an inch in breadth. There is a small but well-marked tuberosity and depression, on the outer or radial side of the ulna, 1½ inch below the "greater sigmoid cavity," answering to a corresponding process in the Wombat. The elongation of the olecranon in that burrowing Marsupial, augmenting the lever for working the fore paw, does not exist, and was not needed in the gigantic gradatorial *Diprotodon.* The portion of ulna above described indicates a massive and powerful fore arm, and has encouraged me to indicate the continuation of the ulna, as a distinct bone, to the carpus, in my restoration of *Diprotodon* (Plate L.). The canal for the medullary artery enters the bone on the inner side (that next the radius) below the "sigmoid" articular cavity, and the canal is directed inward and a little upward. This fossil was obtained in the bed of the Condamine River, west of Moreton Bay, by Sir Thomas Mitchell, C.B.

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* Description of the Skeleton of *Mylodon robustus*, 4to, 1842, p. 82. Memoir on the Megatherium, 4to, 1860, p. 49.

§ 8. *Pelvis.*—In a collection of bones from fluviatile freshwater deposits at Eton Vale, Darling Downs*, in the usual massive or weighty, semipetrified condition of fossils from those beds, were fragments of a large pelvis, readjustible to the extent of giving a great part of the sacrum and ilia, both acetabula, the acetabular portion of each ischium to the extent of 7 or 8 inches, and about 5 inches of the acetabular end of each pubis.

The sacrum consists of two vertebræ (Plate XLVII. s_1, s_2), uniting with the ilia (ib. $_{62}$) by a terminal expanse of the transverse processes (ib. fig. 1, pl_1, pl_2), coequal with the antero-posterior extent of the entire sacrum, and giving to that bone a subquadrate form one-third broader than it is long. Much of the anterior articular surface of the body of the first sacral (fig. 1, s_1) is preserved, and a smaller proportion of the posterior surface of that of the second sacral (ib. s_2). Both surfaces show the usual mammalian flatness and concentric lineation for union by intervertebral sclerous substance with contiguous centrums: the rougher surface shows the loss of the epiphysial plate. The transverse diameter of the fore part of the first centrum is 5 inches; the vertical (neurohæmal) diameter is 3 inches. The transverse diameter of the hind end of this centrum, giving that of the fore end of the succeeding anchylosed centrum (s_2), is 3 inches 5 lines. The hæmal† surface of both centrums (Plate XLVII. fig. 1, $s_{1,2}$) is flat, subquadrate, the contour being straight lengthwise, the coalesced ends making no projection forward or hæmad.

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* These fossils, collected in the above-named locality by Edward S. Hill, Esq., were liberally presented to the British Museum by Sir Daniel Cooper, Bart., in 1864.

† In noting the position and aspect of the parts of this pelvis according to anthropotomical description,

The upper and anterior border of the first centrum, however, is abraded, as is the lower and anterior border of the second centrum; both being entire, would give a slight concavity to the longitudinal contour of the hæmal surface. The length of the sacrum is 5 inches 8 lines, that of the anterior centrum being 3 inches 3 lines. The rib-part (pl_1) of the broad and thick transverse process of the first sacral recedes slightly as it expands, passing outward to join the ilium ($_{62}$), with a slight curvature convex hæmad. The greater part of the hæmal surface of the process is flat transversely, becoming slightly concave at its hind part; lengthwise it is here convex from before backward. The line of the confluence with the ilium, indicated by a slight eminence (ib. fig. 1, *p, p*), is feebly curved with the convexity outward. The sacro-iliac symphysis is 8 inches in length following the curve; the origin of the sacral rib or transverse process has a fore-and-aft extent of 3 inches 9 lines. The direction of the origin of the process in both vertebræ is oblique, from near the neural surface of the centrum anteriorly to the hæmal one posteriorly. The hind border of the origin of the first transverse process forms the fore margin of the wide anterior outlet of the intervertebral canal (*i*), which expands into an infundibuliform channel backed by the succeeding transverse process (pl_2).

The neural position of the fore part of the transverse process (*pl* 2) of the second sacral makes the aspect of the nerve-outlet (*i*) obliquely hæmad and backward; the long diameter of the outlet is $1\frac{1}{4}$ inch; the mass of nerves therefrom emerging has deeply grooved the upper or fore part of the great sciatic, or sacro-ischiatic notch (*m*). The outlet thus intervening between the bases of the transverse processes is circumscribed externally by the confluence of the expanded ends of those processes, forming the articular surface of the sacrum with the ilium.

The tuberosity (Plate XLVII. figs. 2, 3, z') representing the confluent zygapophyses of the first and second sacrals is on the inner side and anterior to the posterior outlet (ib. fig. 2, *j*) of the intervertebral nerve-canal, and partly overhangs a smaller outlet (*g*) of a canal passing backward to open into the large intervertebral canal. The fractured base of the left postzygapophysis (ib. z'') of the second sacral is preserved, close to the outer end of the hind margin of the neural arch.

The neural spine of the first sacral (ib. figs. 2 & 3, ns_1) is coextensive at its base with the summit of the arch, has a fore-and-aft extent of 2 inches, a transverse breadth of 1 inch 2 lines. The summit of the spine is broken away at the height of about 1 inch,

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the surface of the sacrum and ilium turned toward the pelvic cavity is "anterior" or "forward;" but in the ordinary posture of the quadruped it would be "inferior" or "downward." In the nomenclature of Dr. BARCLAY such surface would be "central" and look "centrad," the opposite surface "peripheral" and looking "peripherad;" or those surfaces might be "sternal" and "dorsal" respectively. I shall use the term "neural" and "neurad" in the sense of BARCLAY's "dorsal" and "dorsad" and of the anthropotomist's "posterior," and the term "hæmal" and "hæmad" in the sense of "sternal" and "sternad." Fore or anterior, and aft, hind, or posterior will be used to denote the relations of the parts toward the head or the tail of the quadruped.

and there the spine is 1 inch in thickness and 1 inch 7 lines in fore-and-aft extent. The anterior orifice of the neural canal (Plate XLVII. fig. 3, *n*) is 3 inches 5 lines in transverse diameter, but only 1 inch vertically between the centrum and summit of the neural arch; and this dimension is rather lessened at the middle of the canal by a slight rising along that part of the neural surface of the centrum. A medial vertical ridge with a depression on each side marks the fore part of the base of the thick neural spine (ns_1,). The fractured base of the prezygapophysis (*z*) of the first sacral measures 2 inches 4 lines by 1 inch 5 lines.

A partial ossification extends from the base of the neural spine of the first to that of the second sacral (ib. fig. 2, ns_1 & $_2$), bisecting a deep triangular depression, which, however, does not communicate with the neural canal; a continuous ossification from the first to the second sacral neural arch forms a smooth unbroken ceiling to the canal. The posterior outlet of the neural canal (ib. *n'*) of the second sacral is transversely extended, 3 inches 6 lines across, 10 lines in height on each side the mid rising of the centrum. A greater proportion of the neural spine (ns_2) of the second sacral than of the first is broken away; the remaining base gives 1 inch 8 lines in fore-and-aft extent, 10 lines in greatest breadth, which is at the hind part; the fore part is ridged with a small depression on each side. The irregular ossification is continued from the median ridge to the antecedent spine.

Of both ilia a large proportion has been preserved. The acetabular part (*o*) swells out from the body of the bone, before (fig. 1) and behind (fig. 2), and developes a tuberosity (*d*) oblong lengthwise, triangular transversely, at the upper or anterior part of the brim of the cavity. In advance of the acetabulum the ilium contracts, especially from the neural to the hæmal aspect, or is depressed and lamelliform; but continues thickest medially to form the junction with the sacrum, and contracts laterally to a smoothly rounded concave margin (ib. figs. 1 & 2, *n*). About the sacro-iliac symphysis the medial and anterior border, or "crista" of the ilium (*c*) contracts to a thickness of 1½ inch, and where it is entire is convex and roughened. At the fore part of this symphysis is an oval foramen, 1 inch by 9 lines in diameters (ib. fig. 3, *f*), the outlet of a canal communicating with the capacious intervertebral canal. The free portion of the ilium is lamelliform, arches outwardly, the thin outer or hinder border (*n*) describing a bold concave curve. Save for two inches near the symphysis of the right ilium, the crista is broken away. The hæmal surface of the iliac plate (fig. 1, $_{62}$) is almost flat. Transversely, it is convex one-third of the extent from the fractured margin (*c*), concave to the opposite outer margin (*a*, *n*), both curves being feeble; lengthwise it becomes concave toward the acetabulum. The hæmal tract (*p*, *p*) of the sacro-iliac symphysis forms a low broad smooth convex ridge, enlarging and slightly rising as it approaches the acetabular part of the ilium, but subsiding before this begins to expand; this ridge, or tract, feebly represents the "linea ileo-pectinalis." It seems to be suddenly resumed by a process (*e*) at the junction of the ilium with the acetabular end of the pubis ($_{64}$). I infer, at least from its being broken off on each side of this pelvis, that it projected far enough to be called a "process" rather than a "tuberosity." The fractured base is oblong, 1½ inch by 9 lines. The process so indicated answers to that called "ilio-pubic" in *Poëphaga**.

The neural surface (Plate XLVII. fig. 2, $_{62}$) of the free part of the ilium is almost as flat as the hæmal surface; lengthwise the general slight convexity changes to a concavity as it approaches the acetabulum; transversely the surface becomes gently convex towards the thin concave border (*n*).

The neural surface of the ilium is divided into a rough and a smooth part; the latter is exterior, narrow, extending about 1½ inch from the external border (*n*), becoming gradually narrower to within 4 inches of the precotylar tuberosity (*d*), where the smooth tract ends; the rest of the neural surface of the iliac plate is chiefly roughened by coarse grooves and low ridges, mostly inclining lengthwise with more or less obliquity, indicative of coarse and strong muscular attachments.

At the inner and back part of the sacro-iliac symphysis an angular tuberosity, answering to the "posterior inferior spine," unites with a larger rough tuberosity from the transverse process of the second sacral vertebra, together forming a large "sacro-iliac" tuberosity (ib. *u*), overhanging the deep and wide groove at the fore or upper part of the great sacro-sciatic notch (*m*). The plane of the long curved lamelliform ilium is thus almost horizontal, or with surfaces looking neurad and hæmad; the long axis of the bone forms with that of the sacrum an angle of 35° (Plate L. $_{62}$, *s*).

Of the ischium (ib. figs. 1, 2, 4, $_{63}$) the spine is represented by a slightly prominent surface (*l*), roughened at its upper and lower margins, of an oval form, 2 inches by 1 inch in diameters, the long one being in the direction or axis of the ischium, and the small end of the oval is forward. Between the upper part of the "spina ischii" and the neural margin of the acetabulum is a low subcircular rugous tuberosity (fig. 2, *q*) 1 inch in diameter. The back or neural wall of the acetabulum contracts as it retrogrades, the part contributed by the ilium being broader than that by the ischium.

The acetabular part of the innominatum contracts transversely, and expands in the neuro-hæmal direction from the line *m*, *d* to the line *l*, *t*.

The ischium as it is produced backward beyond the acetabulum contracts, but is rounded and thick posteriorly, and is thinned off only anteriorly where it forms part of the margin of the "foramen ovale." The ischia diverge from each other at this part, instead of retrograding parallel with each other as in *Macropus*; but to what extent is not shown in the present specimen.

The great sacro-sciatic notch (figs. 1 & 2, *m*, *l*) presents a deep and wide groove (*m*) at the fore part, overhung by the produced hind part of the sacro-iliac symphysis (*u*), which symphysis is here obliterated by anchylosis. Below the groove the back part of the acetabulum makes convex that part of the margin of the notch, which margin is again concave slightly to the tuberosity representing the ischial spine (*l*).

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* Osteol. of Marsupialia, *tom. cit.* p. 403: the shares taken respectively by the ilium and pubis in the formation of the ilio-pubic process is shown in the 'Cyclopædia of Anatomy,' 8vo, vol. iii. (1841), Art. *Marsupialia*, p. 284, fig. 110 (*Hypsiprymnus*).

In both "innominata" the pubis (64) is broken off close to the acetabulum. The diameters of the fractured surface are 2 inches 6 lines and 1 inch 5 lines, the latter breadth being near the back part of the bone which gives a subtriangular section. The anterior apex is formed by a rough ridge, which rises from the hæmal part of the pubis about 2 inches from the ilio-pubic process (Plate XLVII. fig. 1, *e*), leaving a shallow groove between the ridge and the acetabular margin.

The acetabulum (ib. fig. 4, *t*) is a nearly hemispherical depression, 5½ inches by 5¼ inches across the opening, nearly 3 inches in depth; its rim is smoothly rounded and less thick between the pubic ridge (64) and the "antero-inferior" iliac spine (*d*); thicker and rough from this to the posterior or ischial part (63'); this, as it bounds the acetabulum posteriorly, curves upward, gradually subsiding to form the outer wall of the "cotyloid notch" or groove (*y*) conducting the vessels to the synovial and adipose mass about the expanded, rough, slightly depressed surface for the origin of the "ligamentum teres." This surface (*x*) is oval, 2 inches 8 lines by 1 inch 3 lines in diameters; the cotyloid groove is 10 lines wide. The aspect of the acetabulum is outward and more obliquely downward and backward than in *Macropus*, through a greater development of the iliac, and especially of the pubic, walls.

The sacrum is in the line of the lumbar vertebræ, upon which line the ilia are directed obliquely forward and neurad at the angle above given (Plate L.).

The condition already noted of the materials for the recomposition of the present pelvis allows not of determination of the form and extent of the "brim of the pelvis," assuming, as is most probable, that this was naturally entire; nor does it give the extent, form, and direction of the ischio-pubic symphysis which I conclude to have existed. The transverse diameter of the pelvic cavity between the acetabular origins of the pubic bones (ib. fig. 1, 64) is 1 foot, between the ischial spines (*l*, *l*) 7 inches. From the portion traceable of the "foramen ovale" I infer it to have been relatively large, as restored in Plate L.

The ischia are divergent in the extent to which they are preserved. Although the tuberosity and terminal part of the ischium are wanting, the hinder articular surface of the second sacral centrum (Plate XLVII. fig. 2, *s* 2) permits a conclusion that the ischia were free from any direct union with the vertebral column.

The remains of no quadruped so large as that indicated by the above-described pelvis, save those of *Diprotodon*, have been discovered in the freshwater deposits of Darling Downs. Yet it would betray an undue confidence in the proportion of present acquisitions of fossil remains to the entire extinct mammalian population of Australia, to infer specific relationship from sameness of locality, or even some degree of juxtaposition of parts of a skeleton. It is incumbent, therefore, to state the results of the comparison of the pelvis in question with those of known genera of Mammals which have led me to the conclusion that it is marsupial and referable to the largest known species of the pouched order.

The most conspicuous feature of the pelvis, without doubt, is unlike the corresponding part in any known marsupial, and so much more resembles that in the Elephant as to have supported the view first suggested by the flattened form of the femur next to be described, if more instructive characters had not been shown deducible from the pelvis in question.

The ilia, though not quite entire along the "labrum" (Plate XLVII. figs. 1 & 2, *c*), are sufficiently so to support the inference that they were short, broad or expanded, with a flattened surface rather than a fossa, directed hæmad or downward, and in a minor degree forward. Such a lamelliform ilium is not presented by any existing genus of Marsupial, but is found, besides the Proboscidians, in Megatherioids, Sloths, Apes, and Man.

From the Elephant's the ilia of the present species differ in the much less production of the angle terminating in the antero-superior spine (*a*, *a*), which, in Proboscidians, extends outward and bends down in an almost hooked form to near the parallel of the acetabular outlet.

In the Megatherium and Mylodon the ilia are proportionally more expanded and outwardly extended than in the Elephant. The ilia of the Sloths (*Bradypus*, *Cholœpus*) come nearer to the proportions of those in Plates XLVII.; but the antero-superior angle is rounded off, and the position and aspect of the iliac planes are different. There is, however, a more marked, definite, and weightier distinction between the present pelvis and that of other Mammals with expanded lamelliform ilia. Leaving the human and simial pelves out of the comparison, that of the Elephant includes four sacral vertebræ, and the Sloths, both arboreal and terrestrial, have the sacrum unusually prolonged to effect the second junction with the innominate bones at the ischial tuberosities, thus converting the "great sciatic notch" into a foramen.

In the present pelvis the sacral vertebræ are but two in number. Now this, as a rule, is the number to which the sacral vertebræ are restricted in *Marsupialia*; and it strikes me as the more significant of the affinity, so indicated in the present pelvis, because it is associated with a modification of the ilium which, in the placental series, goes with at least double that number, and commonly with many more sacral vertebræ, five or six, *e.g.*, in the Sloths and Megatherium, and as many as eleven vertebræ anchylosed in a mass in the Mylodon. A still more decisive mark of Marsupial affinity in the pelvis in question is the evidence of an ilio-pubic process (Plate XLVII. fig. 1. *e*, *e*); and this also points to the particular family of *Marsupialia* to which the large quadruped under consideration is more nearly related. Only in the Kangaroos is this process so developed as to be subject to such violence as has broken it away on both sides of the present pelvis. In all other *Marsupialia* it is indicated, if at all, by a mere tuberosity. The concurrence, therefore, of a bisegmental sacrum with the ilio-pubic process decides me to restrict further comparison with the pelvis of the Kangaroos (*Macropus*).

I take the difference of form of the iliac bones, which is very great, between *Macropus* and *Diprotodon*—for if we arrive at the Marsupial genus with a diprotodont dental formula by the pelvic route we may be absolved of rashness in drawing the obvious conclusion—to depend on the corresponding differences in the mode of locomotion deducible from the structure and proportions of the limb bones. *Diprotodon*, by the equable and massive development of fore and hind limbs, must have progressed on dry land, like the Elephants and Megatherioids, with a regular, quadrupedal, gravigrade pace, though no doubt less sluggishly than either *Mylodon* or *Megatherium*. It is evident that it could not depend on the hind limbs alone for rapid escape from enemies as do the Kangaroos. The powerful exertions those singular marsupial animals impose upon their long legs in the successive bounds by which they rapidly traverse the plain, call for the provision of long muscles and of strongly contracting ones, indicated by the long, strong, three-sided, and three-ridged ilia, in which both sides of the prism destined for muscular attachments are deeply hollowed. The corresponding pelvic muscles in *Diprotodon* must have been relatively shorter, less thick, but broader, and, in relation to the thigh bone, arranged and disposed more or less as in the Elephant.

Amongst minor differences between *Macropus* and *Diprotodon* in the anatomy of a part of the skeleton in which they agree in more essential characters, I note that the outer margin of the sacral apophyses (Plate XLVII. fig. 1, pl_1, pl_2), uniting with the ilia at *p*, *p*, do not curve hæmad as in *Macropus*, making that surface transversely concave.

The outlet of the anterior canal communicating with the wide intervertebral nerve-passage, answering to that marked *g* in *Diprotodon* (ib. fig. 2), is relatively smaller and more in advance of the soldered zygapophyses uniting together the two sacrals in *Macropus*. The "spine of the ilium" in *Macropus* is represented by a relatively narrower and less prominent surface than in *Diprotodon*, is further from the ischial spine, nearer the middle of the back wall of the acetabulum in *Macropus*. The breadth of this wall is almost equal in the Great Kangaroo, and the hind contour of the acetabular brim is almost parallel with the coextensive inner and hinder border of the innominatum.

The ischium, as it is produced backward beyond the acetabulum, is relatively more compressed and lamelliform in *Macropus* than in *Diprotodon*, and, most probably, is relatively longer. In the acetabulum itself the vascular groove and the ligamentous depression are relatively deeper in *Macropus* than in *Diprotodon*.

§ 9. *Femur*.—The femur is remarkable for the length, breadth, and depth of the proximal end, including the "head," "neck," and "trochanters," for the rise of the head above the great trochanter, for the fore-and-aft flattening of the shaft, and for the extent in the same direction of the inner condyle chiefly due to the prominence of its narrow anterior tuberous end.

The chief dimensions of this bone are given in the 'Table of Admeasurements,' p. 574.

The "head" (Plates XLVIII. & XLIX. fig. 1, *a*) is egg-shaped, the great end hemispherical with the articular surface produced upon the upper part of the neck, contracting and representing the small end of the egg (Plate XLVIII. fig. 1, *b*). There is no pit for attachment of a ligamentum teres; the sole indication of any special addition to the fibres of the capsule of the joint is a rough shallow indent of an angular form, encroaching on the ball from the under part of its periphery (Plate XLIX. fig. 1, *c*). The fore-and-aft diameter of the head is 4½ inches; the transverse extent to the end of the supracervical apex is 6 inches: this production is more conspicuous at the fore (Plate XLVIII. fig. 1, *b*) than at the back part (Plate XLIX. fig. 1). The margin of the articular surface is slightly prominent, through the sudden contraction of the rough surface of the neck; but this is chiefly at the fore part (Plate XLVIII. fig. 1, *d*), towards which aspect the head slightly inclines. At the back part of the neck, just beyond the head, there is a low ridge 1½ inch long (Plate XLIX. fig. 1, *e*) parallel to the margin of the articular surface.

The rugged surface of the great trochanter commences at the middle of the upper part of the neck, with a moderate elevation and a border convex towards the head (Plate XLVIII. fig. 2, *f*); its fore-and-aft breadth here is 3½ inches, but the process expands as it recedes from the head, sloping downward to a breadth of 4 inches 10 lines; its outer expanded termination is subbilobed, the posterior lobe (ib. *h*) being most produced outward; the anterior one (ib. *g*) is continued furthest down the shaft.

Anteriorly the great trochanter is defined by the abrupt rising of the rugged surface from the smooth surface of the neck along a curved line (ib. fig. 1, *i*, *i*), which bends round the lower part of the anterior lobe (ib. fig. 1, *g*),: this is continued upon the fore part of the shaft near its outer margin for an extent of 5 inches from the upper surface of the lobe; the posterior lobe (Plate XLIX. fig. 1, *h*) extends a shorter way down the outer surface of the femur, and is defined, or rises, very abruptly from the smooth tract of that part of the shaft. The posterior part of the trochanter projects as a thick oblong tuberosity (ib. fig. 1, *k*) above the trochanterian depression (ib. *l*): the mouth of this depression is 3 inches 8 lines in length, 1 inch in breadth, opening parallel with the lower margin of the neck, and is 3 inches in depth. Beyond the depression the posterior margin of the trochanter is less defined from the femoral shaft than is the anterior one. The neck of the femur (Plates XLVIII. & XLIX. *m*) begins by hardly a less diameter than the head from above downward, and augments in that direction as it extends outward; it is rapidly compressed from before backward, as it recedes, especially where it is continued into the trochanterian fossa (*l*); its upper margin is slightly concave from within outward, convex from before backward, 2 inches broad in that direction; the lower margin is 2 inches 6 lines. This margin is not uniformly convex across, but is remarkable for the production of its hinder half into a long narrow elliptical rough ridge, representing the small trochanter, which is 6 inches in length and 1½ inch across the middle (Plate XLVIII. fig. 1, *n*).

The lower ends of this and of the anterior tuberosity of the great trochanter are on the same transverse parallel, at which the proper shaft of the femur may be said to commence. This is defined by a gentle concave curve in both outer and inner sides, the least transverse diameter being 4 inches 5 lines. At the upper half of the shaft the fore-and-aft thickness decreases from the outer to the inner border, which is reduced to 2 inches before rounding off. This border gains in thickness as it approaches the lower end.

The outer side of the compressed shaft preserves a thickness of about 2½ inches along its middle two-thirds, expanding above and below to the ends of the bone. The fore surface of the shaft is smooth, the hind surface shows a shallow narrow longitudinal depression (Plate XLVIII. fig. 2, *o*; Plate XLIX. fig. 1, *o*), and near the outer border, 3½ inches above the outer condyle, it is 2½ inches in length. A subcircular feebly marked rough surface or patch (Plate XLIX. fig. 1, *p*) is discernible near the middle of the back surface, not quite halfway down the shaft.

The rotular surface (Plate XLVIII. figs. 1, 2, 4, *r*) of the distal end, defined by a low rising from a slightly depressed fore part of the lower end of the shaft (ib. fig. 1, *q*), is made strongly concave transversely by the forward production of the narrow tuberous end (ib. *s*) of the fore part of the inner condyle (*t*), from which it is divided by a channel ½ an inch wide (ib. fig. 4, *x*) continued to the intercondylar pit (*u*) from the inner surface of the distal end of the shaft. The large rotular surface, thus concave transversely, convex from before backward, is broadly continuous with the articular surface of the outer condyle (ib. figs. 1 & 4, *v*). The fore-and-aft extent of the inner condyle, including the rotular part, is 8 inches; the same diameter of the outer condyle is but 4 inches 7 lines. The transverse diameter of the back part of the inner condyle is 3 inches 6 lines; that of the outer condyle is the same; the transverse diameter of both condyles (Plate XLIX. fig. 1, *t*, *v*), including the intervening depression (*u*), is 7 inches 6 lines. The form of the articular surface is very different here, in the two condyles; the inner one (*t*) shows a full convexity in both directions, the transverse contour becoming flattened toward the outer border. The outer condyle (*v*) is slightly concave transversely along two-thirds of its middle part, the outer convex border being somewhat produced; the outer condyle is also less convex from before backward than the inner one. There can hardly be said to be a popliteal depression; the vertical line of the back of the shaft is continued directly into the intercondyloid groove (*u*), the sides of which are formed by the production of the back parts of the condyles.

The inner surface of the distal end of the shaft developes a strong ridge (Plate XLVIII. fig. 5, *w*), extending above 4 inches from the back part of the inner condyle toward the rotular division of the same. The outer supracondylar surface is more even and is slightly concave, divided by a moderate rough prominence (ib. fig. 1, *y*) from the smooth outer part of the shaft.

The outer side of the shaft, for a short way below the great trochanter, joins the hind surface at an angle, simulating a low ridge continued from the end of the hind lobe of that process, and subsiding into the rounded smooth convexity of the outer part of the shaft; but there is no "linea aspera." I cannot detect in this femur any orifice of a medullary artery. The fractured surface of the shaft of a left femur does not indicate any medullary cavity. But in the shaft of another femur, corresponding with the above in size and shape, the transverse being to the antero-posterior diameter as two to one, there is a conspicuous orifice for the medullary artery, at the back part and a little above the middle of the shaft, toward the inner side; the canal slopes upward, to a small spheroid medullary cavity, with dense walls 1 inch in thickness (Plate XLVIII. fig. 3).

The femur, like the pelvis, of *Diprotodon* presents the greatest resemblance in general form and characters to the corresponding bone in *Proboscidia.* The head is devoid of the ligamentous pit; the shaft is straight and antero-posteriorly compressed; there is little* or no medullary cavity.

Passing to particulars of structure, there appear several more or less well-marked differences.

The head rises higher above the neck and trochanter in *Elephas*; it has a more directly upward aspect, the neck is shorter, the great trochanter is absolutely, as well as relatively, of less extent in *Elephas.* The trochanterian depression is less deep, and opens nearer the exterior surface of the trochanter. A ridge is continued downward from the border of the depression, which, with a second ridge continued downward from the outer part of the trochanter, bounds a flat facet forming the outer surface of the upper half of the shaft: there is no such definite facet in *Diprotodon.* At the upper and fore part of the trochanter in *Elephas* there projects a tuberosity midway between the head of the femur and the outer part of the trochanter, and the neck rising to support this tuberosity is somewhat convex transversely: there is no trace of this tuberosity in *Diprotodon*, and the fore part of the neck is concave transversely. The small trochanter, which in *Elephas*, as in *Diprotodon*, is a long longitudinal ridge, is situated lower down in *Proboscidia.* The ridges continued from the great trochanter upon the shaft represent, in that order, "lineæ asperæ," of which there is no trace in *Diprotodon.*

The differences become more marked at the lower end of the bone. The rotular surface or pulley is absolutely as well as relatively narrower in *Elephas*; it has a more anterior aspect, is supported on an anterior production or expanse of the femur; the sides of the pulley are parallel, subequally developed; there is no production of the inner one as in *Diprotodon.* The outer as well as the inner part of the rotular articular surface is distinct from that surface in the corresponding condyle. Both condyles are convex and equally prominent behind in *Elephas*: in which genus there is no transverse hollowing of the outer condyle, giving a trochlear character with production of the outer border of that condyle as in *Diprotodon.* The intercondylar groove is deeper and narrower in *Elephas* than in *Diprotodon.*

Omitting the notes of comparisons of the femur of *Diprotodon* with that in other large quadrupeds, the essential correspondence throwing true light on the determination of the species to which it belongs, and the affinity of that species, is found in the *Poëphaga* or *Macropodidæ* exclusively. It is there only that one finds the transverse excavation of the surface of the outer femoral condyle, producing the contrast of a trochlear character of surface with the uniform convexity or ball-like prominence of the inner condyle. In the Great Kangaroos (*Macropus major*, *M. laniger*) the character is exaggerated, the channel is deeper, and its outer border is more produced. The rotular surface is broad, with unsymmetrical sides in *Macropus*, the inner border being sharpest, though less produced anteriorly than in *Diprotodon.*

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* See Osteological Catalogue of the Museum of the Royal College of Surgeons, 4to, p. 481.

Macropus further agrees with *Diprotodon* and differs from *Elephas* in the continuity of the articular surface, giving that of the synovial cavity, of the rotular and condylar joints; but in *Macropus* the inner as well as the outer condyle is so continuous. The intercondylar groove is relatively wider in *Macropus* than in *Diprotodon*, and, the condyles being more backwardly produced, it is deeper. The longitudinal ridge-like small trochanter is placed higher in *Macropus* as in *Diprotodon* than it is in *Elephas*. The attachment of that muscle, which leaves a circular rough patch at the back of the femoral shaft in *Diprotodon*, is developed into a tuberosity in *Macropus*. The trochanterian depression is very deep in *Macropus* as it is in *Diprotodon*. The great trochanter is bilobed exteriorly in *Macropus*, but with a deeper and differently directed dividing channel than in *Diprotodon*.

I discern in the foregoing correspondences the essential marks of affinity, and view the greater elevation of the trochanter major in *Macropus*, the greater length and cylindroid form of the shaft of the femur, the greater relative antero-posterior extent of the distal end, and especially of the outer condyle, with the stronger indications of muscular attachment, as adaptive characters in the smaller Marsupial related to its more rapid and vigorous modes of locomotion.

§ 10. *Tibia.*—The tibia, of the general dimensions given in the Table of Admeasurements, p. 574. I conclude,by the agreement in size and character of the upper articular surface (Plate XLIX. fig. 4) with the lower one of the femur (Plate XLVIII. fig. 4), to belong to the same leg, viz. the right one, of *Diprotodon*.

The external upper articular surface (Plate XLIX. figs. 3 & 4, *a*) is gently undulated, with a transverse convexity adapted to the corresponding concavity on the outer femoral condyle, and with the concavities, though slight, answering to the convexities of that condyle. The inner articular surface (ib. fig. 4, *b*) is larger, and presents a uniform concavity to the convex inner femoral condyle. The ridge or spine (ib. figs. 2, 4, *c*) between the articular surfaces extends 2 inches from behind forward and a little outward; it is from 6 lines to 10 lines thick, and is roughly excavated above. The rough part of the upper surface of the tibia (ib. *d*), in advance of the articulations, is more extended transversely from behind forward. The breadth of the upper surface exceeds by two-fifths the fore-and-aft diameter. The articular surface for the head of the fibula (ib. fig. 3, *e*) is continuous with the outer articular surface (*a*), and extends from its outer and hinder margin at a rather acute angle or nearly a right angle, 1 inch 4 lines down the shaft, the breadth of the surface being 1 inch 6 lines. In advance of this surface projects the external tuberosity (*f*), from which a thick (epicnemial) ridge-like rising of bone (ib. fig. 2, *g*, *g*) extends in front of the upper end of the tibia for about 5½ inches. This ridge or prominent tract is longitudinally striate or scratched, as if it were an ossified ligamentous attachment. At the middle of the fore part of the tibial head a narrow ridge (ib. fig. 2, *h*) is continued from the transverse one 2 inches down the shaft, like the procnemial ridge in birds, but much less produced. From the junction of the "epicnemial" ridge with the outer tuberosity a narrow "ectocnemial" ridge (ib. *i*) extends for an inch or two down the shaft. Beneath the outer tuberosity is a rough shallow cavity (ib. fig. 3, *k*), and a similar but smaller one (ib. *l*) impresses the shaft a little way below the fibular articular surface. An internal tuberosity (figs. 2 & 4, *m*) is feebly marked below the contracted inner end of the inner articular surface.

The shaft of the tibia rapidly contracts to a transverse diameter of 2½ inches at the middle third of its extent, where it is trihedral, with the angles rounded off. It appears to be twisted with the inner malleolus turned forward; but this is very feebly marked, not projecting below the distal articular surface. At the outer and back part of the lower half of the shaft is a rough longitudinal prominence (fig. 3, *n*), 4 inches by 1 inch, seemingly for ligamentous attachment of the corresponding part of the shaft of the fibula. At the inner and back part of the shaft a low narrow fibrous ridge runs parallel with the inner border of the fibular ridge, defining therewith an oblique shallow canal, 9 lines in width.

A slightly raised border of bone (figs. 2, 3, *p*), from 1 to 2 inches distant from the lower articulation, seems to indicate the original line of junction of the epiphysis. Malleoli cannot be predicated of the distal end of this tibia (ib. fig. 5). At the inner periphery of the articular surface, instead of a prominence there is a notch (ib. *q*), from which a groove 1½ inch long and 5 inches wide extends outward and forward into the joint; the rough convex border of the articulation external to this, corresponding in position to the fore part of the upper end of the tibia, appears to represent an internal malleolus. On each side the entering groove (*q*) the distal articular surface is slightly convex; in the rest of its extent it is nearly flat; its form is oblong, with the long axis at right angles to that of the upper articular surface, *i. e.* from before backward instead of from side to side.

In a portion of the shaft of a tibia, obtained by Sir THOMAS MITCHELL from the bed of the Condamine River, the upper part of the ridge between the outer and hinder surfaces shows the orifice of a medullary arterial canal, which expands as it slightly descends. No medullary cavity, however, is shown in this fragment. The compact part of the wall of the shaft is half an inch thick, and a moderately close cancellous structure extends inward to the centre of the shaft.

A subtrihedral portion of bone, including the distal end and accompanying the above portion of tibia, I believe to be part of the fibula; it is 7 inches in circumference. The centre of the shaft is occupied by a close cancellous texture. The articular extremity is much abraded; a trace of the epiphysial suture remains; and I find that this is long conspicuous in the fibula of the Wombat. I have given what I conjecture to be the proportions of the fibula in my restoration of the skeleton of *Diprotodon*, Plate L.

In the singular form of the tibia of *Diprotodon* are presented Marsupial characters exclusively. "The outer articular surface is continuous with that of the head of the fibula"*, as in the Wombat and Koala; "the shaft of the tibia is twisted as in Opossums, Dasyures, Phalangers, and Petaurists, as well as in the Koala and Wombat"*.

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* Osteology of the Marsupialia, *loc. cit* p. 405.

"The internal malleolus is very slightly produced in any Marsupial"*, save in the Wombat and Kangaroo.

"The fibula is complete and forms the external malleolus in all Marsupials;" and such may be inferred to be its condition from the evidences of attachment shown in the tibia of *Diprotodon*. Only, instead of "the close contact and attachment ensuring a due degree of fixity and strength"* in the Kangaroos, I infer from the articular surfaces on the tibia for the fibula and for the foot that this "enjoyed a movement of rotation analogous to the formation and supination of the hand"*, as in all save the saltatory Marsupials; and we may infer a corresponding modification of the foot approximating *Diprotodon* to *Phascolomys*.

§ 11. *Conclusion*.—Thus in the series of *Mammalia* which characterizes the Australian continent we have evidence of the former existence of a species as large as the *Megatherium*—that strange extinct animal which similarly crowns the series of *Bruta* correspondingly characteristic of the South American Continent.

It is interesting to note the similarity in size, number, and form of working-surface of the molar teeth in the extinct Marsupial and Bradypodal giants; so much so that, notwithstanding the different dental structures and conditions of growth, one cannot resist the inference of a correspondence of diet. But whereas in *Megatherium* the front teeth are wanting, and the prehension of the vegetable food was allotted to limbs and tongue, in *Diprotodon* instruments allied to those by which the Beaver and Wombat gnaw the ligneous fibre were magnified to the proportions of the body to be provided for. The Marsupial monster brought down the tempting foliage by erosion of the trunk, not by the strong hawl of a forcible grasp. Accordingly, the limbs show not those abnormal proportions which distinguish the Megatherioids; they manifest, especially the hind ones with the pelvis, in the Diprotodon, forms and proportions recalling those of the Elephant, and suggestive of analogous uses and mode of progression. The fore limbs may be inferred by the modifications of the distal articulation of the humerus, and by what is known of the ulna, to have enjoyed the rotatory as well as flexile movements. Herein the Diprotodon resembles the Megatherium rather than the Elephant; but the truer inference from the retention of the rotatory and probably unguiculate structure of the fore paws is that, as in the existing herbivorous Marsupials, they were needed for the manipulations of the pouch.

The resemblance in the Diprotodont and Megatherioid dentitions, guiding to the recognition of function or adaptive purpose, are underlain, as above hinted, by differences of textural and genetic conditions. *Diprotodon* combines enamel with dentine, osteodentine, and cement; *Megatherium* has no enamel. In *Diprotodon* the molars have a limited period of growth; in *Megatherium* that period was limited only with life.

The Australian giant adds to number, relative size, and shape of crown, of its molars, in which it accords with the existing macropodal marsupial dwarf of that land, the further correspondence in the coronal enamel and the divergent roots of the grinders.

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* Osteology of the Marsupialia, *loc. cit.* p. 405.

Similarly, the American phytophagous giant added to a bilophodont working-surface of its few and small molars, the peculiar texture and rootless condition of the long deeply implanted ever-growing dental mass, characteristic of the molars of the existing dwarf-sloths of its continent.

When only the large curved pair of upper scalpriform incisors of *Diprotodon* were known, to which the subcompressed lower pair are opposed, an alliance of *Diprotodon* to *Phascolomys* was suggested. The subsequent evidence of a nearer affinity to *Macropus* instructively exemplifies the superior value of the molar teeth as indicators of the nature of an extinct animal*.

It is true that in the proportions of the limbs, especially in those of the tibia and its distinction from the fibula, as in some other particulars of the osteology of *Diprotodon*, it resembles more the Wombats than the Kangaroos; but the more weighty and essential correspondences are with the *Macropodidæ*; the equipedal modifications are adaptive and necessitated by the bulk of the extinct marsupial herbivore. The most elastic imagination could hardly stretch to the association of the disproportionate hind limbs of the Kangaroo with a trunk equalling that of a Rhinoceros; for according to that pattern, *Diprotodon* must have towered to a height of 30 feet. The departure from the type of its diminutive modern allies is, again, interestingly analogous to that which occurs in the herbivorous *Bruta*. The bulk and weight of body in *Megatherium* precluded the proportions of length and slenderness, with terminal prehensile instruments, in the limbs, by means of which its diminutive congeners and contemporaries have been enabled to withdraw themselves from an unequal conflict into the safe shelter of lofty trees. In like manner the bulk and weight of *Diprotodon* militated against its enjoying the privilege of the elongate saltatory limbs to which its small congeners and contemporaries the Kangaroos have owed their safety, or the scansorial ones by which the Koala climbs out of danger.

The analogies traceable between the extinct herbivorous giants of the two remote tracts of dry land are full of interest and instruction. I may add that as swift and continuous course and power of climbing are privileges checked or regulated by the mass and weight to be hurried along or dragged aloft, so likewise is the faculty of burrowing and concealment under ground. The *Diprotodon* was as impotent to avail itself of the means of escape to which the comparatively diminutive Wombats owe their present existence, at it was of the interposition of space, which the Kangaroo by a succession of long leaps, rapidly puts between itself and its pursuers.

Subject to this explanation the combination of Wombat- and Kangaroo-characters may be adduced as exemplifying that more generalized structure in *Diprotodon* from which, or from some earlier still more generalized marsupial type, have diverged the three existing families with special modifications, respectively, for burrowing, climbing, and leaping; or, to borrow a figure from another hypothetical school, I might remark that the orders, or other natural groups, of placental Mammalia represented by members of the marsupial series have been indicated by different zoologists[*], and with no material divergence of opinion; but not until now has evidence been received of a Marsupial representative of the proboscidian group.

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* Agreeably with the rule laid down by the great Founder of Palæontology; "La première chose à faire dans l'étude d'un animal fossile, est de reconnoître la forme de ses dents molaires: on détermine par-là s'il est carnivore ou herbivore, et dans ce dernier cas, on peut s'assurer jusqu'à un certain point de l'ordre d'herbivores auquel il appartient," CUVIER, Ossemens Fossiles, 4to, vol. iii. 1812, p. 1 (Premier Mémoire).

In certain Mastodons there is a pair of incisive tusks below as well as above: the proboscidians maximize the rodent type as *Diprotodon* does the Marsupial one. The brain of the Elephant is essentially "lissencephalous," inasmuch as the cerebrum does not extend upon the cerebellum. This position of one primary mass of brain in front of the other is, as stated in the definitions of the subclasses of *Mammalia* in the cerebral system[†], a more constant and important character than convolution of surface, which in both *ly-* and *liss-encephala* relates to the bulk of the species and of the brain. So, on the other hand, if a smooth cerebrum overlaps the cerebellum as in the Marmoset, it is essentially "gyrencephalous." The terms suggested by the superficial character which prevails, with exceptions, are arbitrary, but the most convenient for expressing the more constant characteristics of the brain therewith associated.

In the extinction of *Diprotodon*, as of *Megatherium*, there seems to be an additional exemplification of the fruitful and instructive principle which, under the phrases "contest for existence," or "battle of life," embodies the several circumstances, such as seasonal extremes, generative power, introduction of enemies, &c., under the influence of which a large and conspicuous quadruped is starved out, or falls a prey, while the smaller ones migrate, multiply, conceal themselves, and escape.

We infer from the fact of remains of young and inexperienced Diprotodons occurring in Australian Caverns with those of *Thylacoleo*, that the large Marsupial herbivore had its enemy in, and occasionally fell a victim to, the large Marsupial Carnivore[‡]; as at the present day the Kangaroo is laid in wait for by the Thylacyne, or 'Native Wolf', and the Dasyure, or 'Native Cat.'

We may speculate upon the possible relation of the first introduction of the Human kind into Australia, and of the subsequent insulation of that land from the rest of the Papuan Continent, to the final extinction in the so restricted territory of all the characteristic Mammals which happened to surpass in bulk the still existing, swift-retreating, saltatorial and nocturnal Kangaroos.

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[*] "On dirait, en un mot, que les marsupiaux forment une classe distincte, parallèle à celle des quadrupèdes ordinaires et divisible en ordres semblables; en sorte que si on plaçait ces deux classes sur deux colonnes, les sarigues, dasyures et pérameles seraient, vis-à-vis des carnassiers insectivores à longues canines, tels que les tenrecs et les taupes; les phalangers et les potoroos, vis-à-vis des hérissons et des musaraignes; les kangaroos proprement dits, ne se laisseraient guère comparer à rien; mais les phascolomes devraient aller vis-à-vis des rongeurs."—CUVIER, Règne Animal, ed. 1817, tom. i. p. 171. "Les ornithorhynches et les échidnés y formeraient un groupe parallèle à celui des édentés."—*Op. cit.* ed. 1829, tom. i. p. 174.

[†] OWEN, "On the Characters &c. of the class Mammalia," Proceedings of the Linnean Society, 1857.

[‡] I shall return to the question of the evidence of the carnivority of *Thylacoleo* in a subsequent communication.

It is true that reliable evidence has not reached us of the contemporaneity of Man with *Diprotodon* in Australia. No human tooth or bone, no weapon fashioned by man, has hitherto been detected in the breccia-caves, or has been picked up in the lacustrine beds in South Australia, Queensland, and Melbourne, from which the largest and oldest Diprotodonts have been exhumed, mostly under conditions of petrifaction, suggestive of interment in those deposits during a vastly longer period than the Mammoths and Rhinoceroses have lain in our own brick-fields.

A human skeleton, or part of it, picked out of the deposits forming the bed of a tributary of the Condamine, and yielding the same results of chemical analysis as are recorded of a Diprotodont fossil at p. 572, would be one of the much needed decisive and satisfactory evidences of the antiquity of Man. To promote the investigation in the Australian Continent which the present phase of the ancient history of our own species so much requires, I ventured some time ago to address the Legislature of New South Wales, and with results, as respects the aid and encouragement of such researches, which are given in the subjoined notes*.

The range of *Diprotodon australis*, during the period of its existence, in the Australian Continent is shown by the evidence at present possessed to have been wide.

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* "London, British Museum,
23rd February, 1867.

"Sir,—The enlarged and liberal views of your administration embolden me to suggest that a careful and systematic exploration of the Limestone-caves of Wellington Valley, discovered by the Colonial Surveyor in or about 1832, would be a work worthy of your encouragement.

"The fossil remains which were then obtained from the caves revealed the important and suggestive fact that the marsupial type of structure prevailed in the ancient and extinct as in the existing quadrupeds of Australia.

"Besides the great accession of such evidences as would accrue to the Museum at Sydney from such exploration, most instructive evidence may be expected bearing upon the antiquity and origin of the aboriginal races of Australia. Such contribution to human knowledge, initiated and supported by New South Wales, would be gratefully appreciated by all who in this hemisphere are devoted to the progress of science, and would redound to the honour of your present constitutional Government.

"I would willingly devote time to the determination and description of such specimens, or duplicates, as, so acquired, might be transmitted to me for that purpose, or be liberally sent for deposition in the British Museum; and these descriptions would be punctually transmitted to the Museum at Sydney, as materials of its Catalogue, or to such address as you might be pleased to indicate, in reference to a systematic description of the Wellington Valley Bone-caves.

"I feel confident, from personal conference on the subject with the late Sir Thomas Mitchell, who confided to me the fossils he was able to bring over for description in his work published in 1838, that the results of the proposed exploration, in the hands of one qualified, would amply repay a grant, say of £200 or £300, if placed on the estimates and sanctioned by the Assembly.

"I have, &c.,
(Signed) "Richard Owen, F.R.S."

"The Hon. Henry Parkes, &c.,
Colonial Secretary, New South Wales."

Table of Localities of *Diprotodon australis*, showing:—

Where found.	By whom.	Date.
Breccia-cavern, Wellington Valley[*]	Sir Thomas Mitchell, C.B.	1836
Tributaries of Condamine River, Darling Downs[†]	Sir Thomas Mitchell, C.B.	1842
Quaternary gravel, Mount Macedon, Melbourne[‡]	E. C. Hobson, M.D.	1843–45
Tributaries of Condamine River, Darling Downs[§]	Ludwig Leichhardt, M.D.	1844
Freshwater beds, Mount Macedon, Melbourne[‖]	Patrick Mayne, Esq.	1844
Bone-caves, Wellington Valley	Count Strzelecki	1844
King's Creek, Darling Downs	Mr. Turner	1847
Gowrie, Darling Downs[¶]	Fred. Neville Isaac, Esq.	1849
Galtendaddai, Melville Plains	Wm. Buchanan, Esq.	1851
Creeks, Darling Downs	Henry Hughes, Esq.	1856
Gowrie, Darling Downs	John E. Allport, Esq.	1860
Welcome Springs, South Australia[**]	Fred. Geo. Waterhouse, Esq.	1861
Valley of Condamine[††]	J. H. Hood, Esq.	1861
Portland Bay, South Australia	James S. Wilson, Esq.	1860
Hergott's Springs, Mount Attraction, 500 miles N. of Adelaide[‡‡]	William Burrett, Esq.	1861
Eton Vale, middle of Darling Downs	Edward S. Hill, Esq.	1863
St. Ruth Station, Tributary of Condamine River	Hugh Campbell, Esq.	1865
St. Jean Station, Queensland	M. Satche St. Jean	1865
Clifton Plains, Darling Downs	F. Nicholson, Esq.	1866
Breccia-cavern, Wellington Valley	Gerard Krefft, Esq.	1866

To this letter I was favoured with the following reply:—

(Copy.)

"Colonial Secretary's Office, Sydney, New South Wales,
16th June, 1869.

"Sir,—With reference to your letter of the 23rd of February, 1867, recommending that the Government of this Colony should cause a careful and systematic exploration to be made of the Limestone-caves at Wellington Valley, I have now the honour to inform you that the sum of £200 has been voted by the local Parliament for carrying out your suggestion, and that the Curator of the Australian Museum has been charged with the duty of making the necessary exploration.

"I have the honour to be, Sir,
"Your most obedient Servant,
(Signed) "John Robertson, Colonial Secretary."

"To Professor Owen, F.R.S."

I was gratified by reading in 'The Times' of December 1st, 1869, a notice from the Sydney Correspondent of that Newspaper to the effect that "the Wellington Bone-caves have been explored by Dr. A. M. Thomson and Mr. Krefft of the (Sydney) Museum, with astonishing and unexpected results."

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[*] See description of these Bone-caves by their discoverer in his 'Three Expeditions into the Interior of Eastern Australia,' 8vo, vol. ii. 1838.

[†] Sir T. Mitchell places the locality in lat. 28° S., long. 150° E.; and of the Condamine, he writes: "This stream is remarkable from forming large basins at some places and losing its course in swamps at others, and at other parts again cutting its course in a deep channel, through deep beds of alluvium, in which these bones [of *Diprotodon*] are thus brought to light."—Letter dated January 3, 1842.

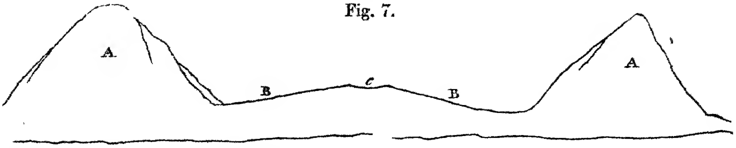
[‡] Dr. Hobson, in transmitting these specimens, sent a sketch of the locality with the following Note:—

"The country from Melbourne is volcanic the whole way; indeed the bank which borders the estuary on which the town is built is the commencement of vast elevated plains of volcanic origin, covered with vesicular lava, scoriæ, interrupted by deep ravines, which show on their steep sides, in many places, a regular columnar basaltic arrangement. Of this nature is the entire country betwixt Mount Macedon and Melbourne, gradually rising, but so slightly as to be almost imperceptible, till you arrive at the volcanic hills that immediately surround Mount Macedon, which is, I believe, composed of granite on its top, and of schistose slates on its side.

The fossils from Mount Macedon are less petrified than those from Darling Downs: the osseous substance crumbles away if not supplied with gelatine, like the fossil bones and tusks from our brick-earth in England.

"Amongst the secondary hills which skirt the base of Mount Macedon there is a considerable circular plane, which is more elevated in its centre than at its circumference, and which will be better explained by a diagram:—

Fig. 7.



A, A. Volcanic Hills surrounding the plain.

B B. The plain.

c. A swamp or bog in which are found the bones at a depth of 4½ feet. After digging through a solid peaty soil for 3 feet you then arrive at a stratum of gravel about 18 inches thick, in which the bones are deposited. This layer of gravel rests upon a bed of firm clay, which is unfossiliferous.

"The bog or marsh in which the bones are found is about four acres in extent, and appears to contain bones at every point. I opened two pits at 150 yards distance from each other and found bones in both, in the same stratum of gravel."—Letter dated January 1st, 1845.

§ The following valuable Note on the formations of the locality accompanied the transmission of the fossils, by the gifted and unfortunate explorer of that part of Australia:—

"The Darling Downs are extensive plains, formed by broad shallow valleys, without trees, covered only with grass and herbage, which grows luxuriantly on the rich black soil, in which concretions of carbonate of lime are frequently found. Ranges of low hills, forming long simple lines with sudden slopes and flat-topped cones, accompany these valleys, and have an open forest formed of various species of rather stunted *Eucalyptus*. All these hills are formed by a basaltic rock, containing frequently crystals of peridote, and being often cellular, sometimes real scoria. The base of the rock is, however, feldspathic; and, as the peridote is frequently absent, the rock becomes uniformly grey, forms a white globule before the blowpipe, and is therefore to be classed amongst the trachytes or phonolites. The plains are filled by an alluvium of considerable depth, as wells, dug 50-60′ deep, have been sunk within it. The plains and creeks in which the fossil bones have been found are 'Mr. Hodgson's Creek,' 'Campbell's Creeks,' 'Mr. Isaac's Creek,' and 'Oaky Creek." They pass all into and through immense plains on the west side of the Condamine, into which they fall. The bones are either found in the bed of the creek, particularly in the mud of dried up water-holes, or in the banks of the creeks in a red loamy breccia, or in a bed of pebbles, containing many trachytic pebbles of the coast range from the west side of which these creeks descend.

"In the banks of the creeks you find at first the rich black soil of the plain, about 3′ thick, then layers of clay and of loam, here and there, particularly at 'Isaac's Creek,' with marly concretions of strange irregular forms. The masses of these concretions are often of considerable thickness, though not extending far horizontally. The loam contains small broken pieces of ironstone (breccia) and is equally local. Below these the bed of pebbles lies. The bones found in the breccia are generally near the concretions, but not with them, or they occur amongst the pebbles. A very interesting fact is the presence of univalve and bivalve shells, which live still in the neighbouring water-holes, in the same beds, in which the bones are found. They are either intimately united with the bones by a marly cement, or they occur independently. The greatest depth in which bones are found is 12′. At 'Oaky Creek' we found them at the surface. Besides the bones of the gigantic animal, there are lower jaws and different parts of the skeleton of four other Kangaroos, many of them little different from the living ones, and probably identical with those of Wellington Valley. It seems to me that the conditions of life can have very little changed, as the same shells live still in similar water-holes. The want of food can

I subjoin the analysis of bones and teeth of *Diprotodon australis*, from the beds of the creeks, Darling Downs, by WALTER FLIGHT, D.Sc., Assistant in the Department of Mineralogy of the British Museum.

"Laboratory, Mineral Department, British Museum,
21st January, 1870.

"DEAR SIR,—I have the honour to lay before you a Report on the chemical examination of a portion of a jaw-bone of the Diprotodon, which was received from Professor OWEN at the end of last year.

"The method, devised by M. SCHEURER-KESTNER, and described by him in the 'Comptes Rendus,' vol. lxix. p. 1207, was employed in this inquiry, and the following analytical determinations were made.

"1·2462 gramme of bone, treated with 108 cub. centims. of hydrogen chloride, of specific gravity 1·04 (from 5° to 6° BAUMÉ), for twenty-two hours at ordinary temperatures, left a residue which, after having been dried at 100° C. till it ceased to lose weight, amounted to 0·1953 gramme, and, after ignition, to 0·1772 gramme. By this treatment the calcium phosphate, carbonate, and fluoride iron phosphate, &c., as well as the 'soluble osseine' of SCHEURER-KESTNER, are taken up by the acid, and there remain 14·259 per cent. of insoluble mineral matter (chiefly silica coloured red with iron peroxide) and 1·452 per cent. loss of weight by ignition, which in the memoir alluded to is taken to be insoluble osseine.

"The pounded bone, however, when heated did not change in colour to any great degree, nor emit the expected characteristic odour. A nitrogen determination was next made with the following result. 0·541 gramme of bone, heated with soda-lime, and the resulting platinum ammonium chloride ignited, gave 0·002 gramme of platinum. This amount of the metal corresponds with 0·00028 gramme of nitrogen. Assuming that gelatine contains 17·5 per cent. of nitrogen, the above nitrogen corresponds with 0·0016 gramme of osseine, or 0·295 per cent.

"The bone therefore contains about ¼ per cent. of osseine altogether; and the loss of weight, amounting to 1·452 per cent., and regarded by SCHEURER-KESTNER as due to insoluble osseine, must be ascribed to a further loss of water which was not expelled at the temperature of 100° C.

"As the amount of matter insoluble in hydrogen chloride appeared unusually large (14·259 per cent.), it was thought that the action of the acid might not have been complete. In spite of my failing to detect phosphoric acid in this portion, I nevertheless thought it advisable to subject a further quantity of the bone to the action of a corresponding amount of acid of the same strength for a longer period. 1·0468 gramme of bone digested with 90·5 cub. centims. of hydrogen chloride for sixty-eight hours at ordinary temperatures left a residue weighing 0·1418 gramme, which, when ignited, was reduced to 0·1341 gramme. In this case, then, 12·81 per cent. of mineral matter remained undissolved, whilst the loss by ignition amounted to 0·735 per cent.

"With a view to determine the water present in this bone, 2·5046 grammes were heated for several hours, first at 100° C. and after at 120° C.; the water lost amounted to 0·0932 gramme, or 3·721 per cent. After ignition and treatment with ammonium carbonate, the total loss was 0·1446, or 5·774 per cent. Subtracting

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scarcely be the cause of their disappearing, as flocks of sheep and cattle pasture over their fossil remains. But as such an herbivore must have required a large body of water for his sustenance, the drainage of these plains, or the failing of those springs, the calcareous waters of which formed the concretions in the banks of the creeks, has been probably the cause of their retiring to more favourable localities, and I should not be surprised if I found them in the tropical interior, through which I am going to find my way to Pt. Essington."—Letter dated "Sydney, 10th July, 1844."

"Found about 6 feet below the surface in sinking a well."—Note by Mr. MAYNE.

¶ "In the 'red bank' of this [Isaac's] creek."—Note by Mr. ISAAC.

** Found about 8 feet below the surface in sinking a well, lat. 137′ 50″ S., long. 39′ 35′ E.—Note by Mr. F. G. WATERHOUSE.

†† "From 100 feet below the surface, in digging a well, in the valley of the Condamine River."—Note by Mr. HOOD.

‡‡ "They were taken from a bed of sand and quartz conglomerate, at a depth of about 5 feet."—Note by Mr. BURRETT.

from this number the amount of osseine present (0·295 per cent.), there remain 5·479 per cent., the proportion of water present in the bone.

"It may be mentioned that the bone contained a considerable quantity of vivianite, which gave to portions of its interior a dull blue colour.

"*Analysis of Substances filling cavity of tooth of* Diprotodon [Pulp-cavity of lower incisor, Plate XLII. fig. 5, *i*.—R. O.].

"The nitrogen in a portion of the above, which had been dried for many hours at 100°, was determined by the usual method.

"0·9814 gramme of substance, heated with soda-lime, gave 0·0071 gramme platinum, which corresponds with 0·001 gramme nitrogen.

"Regarding this nitrogen as present in the form of osseine, and employing the number obtained by Frémy (17·5 per cent.) in his analysis of the gelatine of beef-bone as correctly representing the percentage of nitrogen in osseine, we find the substance from this fossil tooth to contain 0·58 per cent. of osseine.

"When heated in a tube it became slightly darker in colour, and emitted a faint empyreumatic odour.

"It was found to be composed of

Much carbonate lime,
Some phosphate lime,
Much silica, iron oxide, &c.,
Some water not removable at 100°,
The small portion of organic matter already mentioned.

"I am, dear Sir,
"Your obedient Servant,
"Walter Flight."

"To Professor Story-Maskelyne,
Keeper of the Mineral Department, British Museum."

Table of Admeasurements.

Skull.

	Feet.	Inches.	Lines.
Length from occipital condyle to fore end of premaxillary	3	0	0
Breadth between outer sides of zygomata	1	6	0
Depth to lower border of mandible at the orbit	1	10	0
Depth to outlet of socket of upper *m* 1, at the orbit	1	0	0
Breadth of occiput	1	1	0
From lower border of foramen magnum to middle of superoccipital ridge	0	9	9
Breadth of cranium at middle of temporal fossa and at the level of the upper part of the zygomata	0	6	6
Depth of facial part anterior to the orbit	0	8	0
Depth of facial part at fore end of nasals	0	10	0
Depth of zygoma at the lower end of the malomaxillary suture	0	4	0
Length of mandible from back of condyle to outlet of incisive sockets	2	0	0
Depth from summit of coracoid process	1	1	0
Fore-and-aft breadth of ascending ramus	0	7	0
Depth of horizontal ramus at the socket of *m* 1	0	5	0
Length of symphysis	0	6	0
Greatest depth of symphysis	0	3	6
Length of upper incisive alveolar series	0	4	6
Length of upper molar alveolar series	0	8	6
Length of lower molar alveolar series	0	9	0

Phil. Trans. MDCCCLXX. Plate

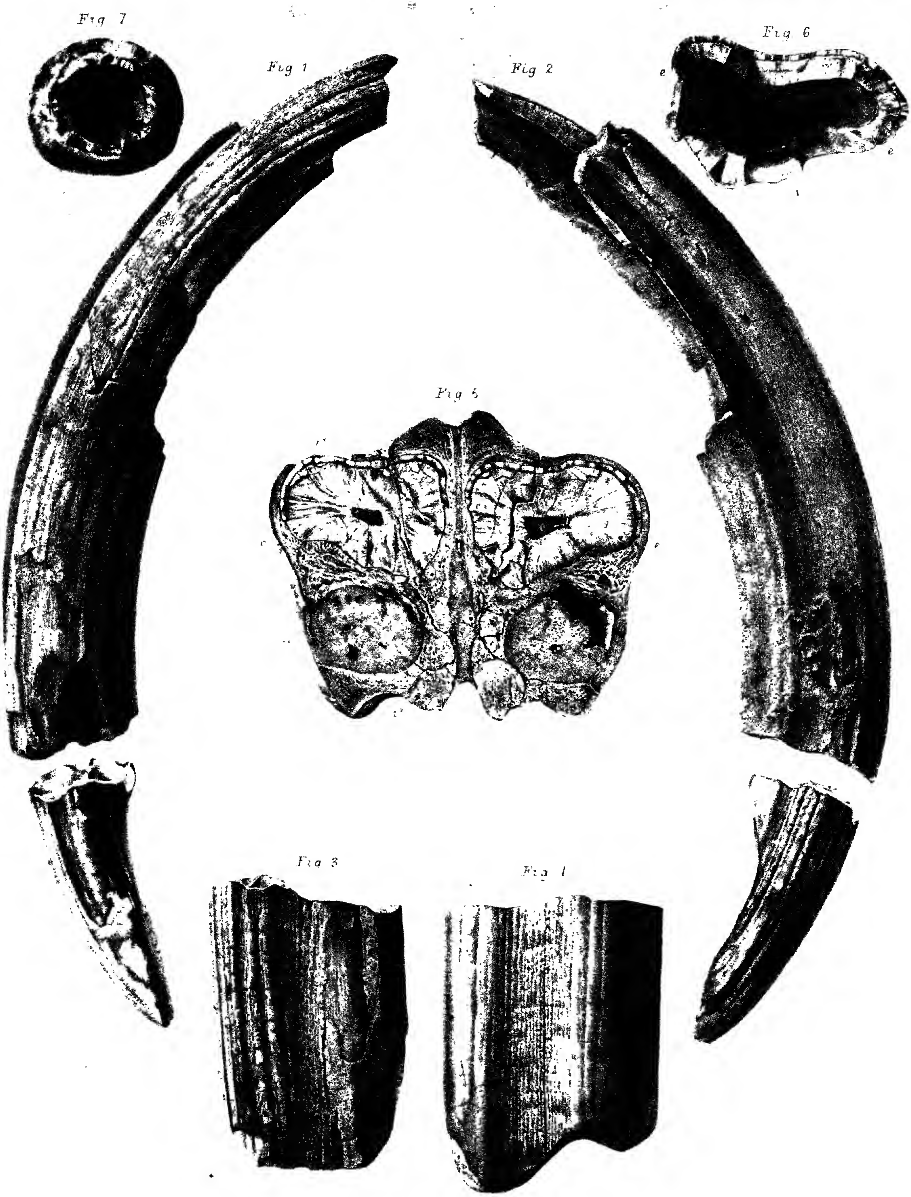
Fig 7
Fig 1
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Fig 1

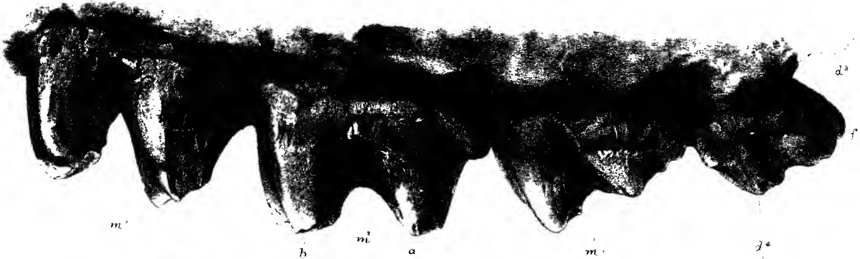
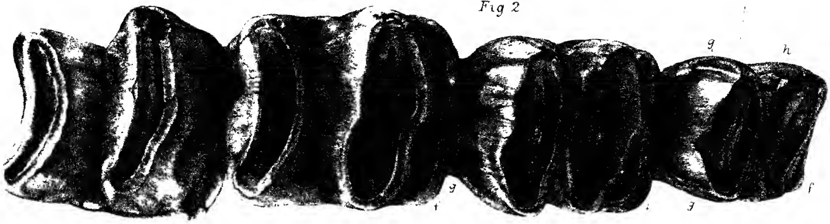

d4
f
m3
m2
b
a
m1
d4

Fig 2



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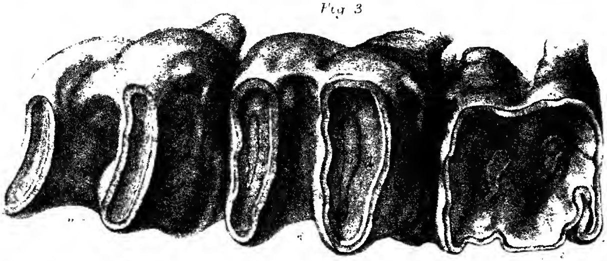

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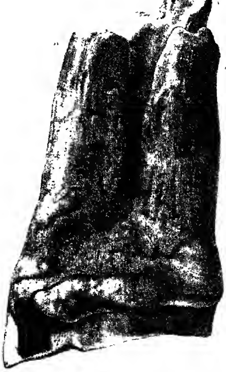
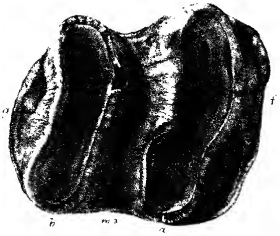


Fig 4



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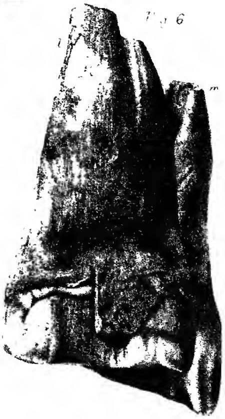

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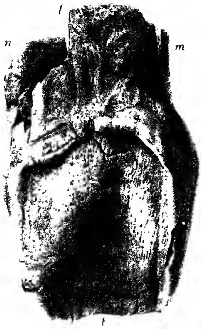
Fig 3



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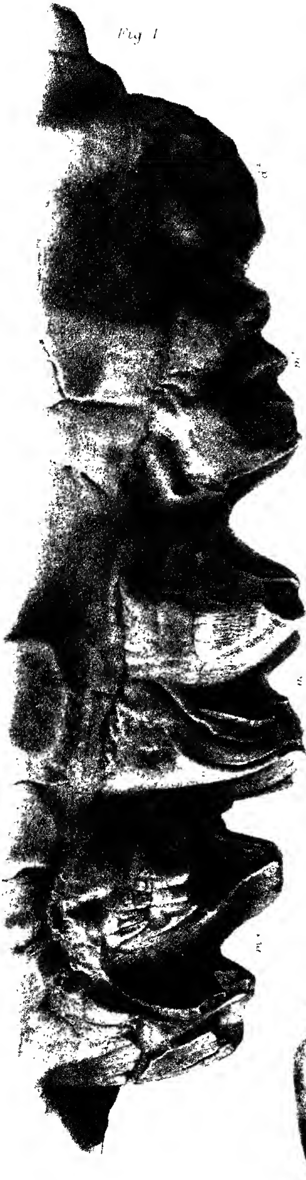

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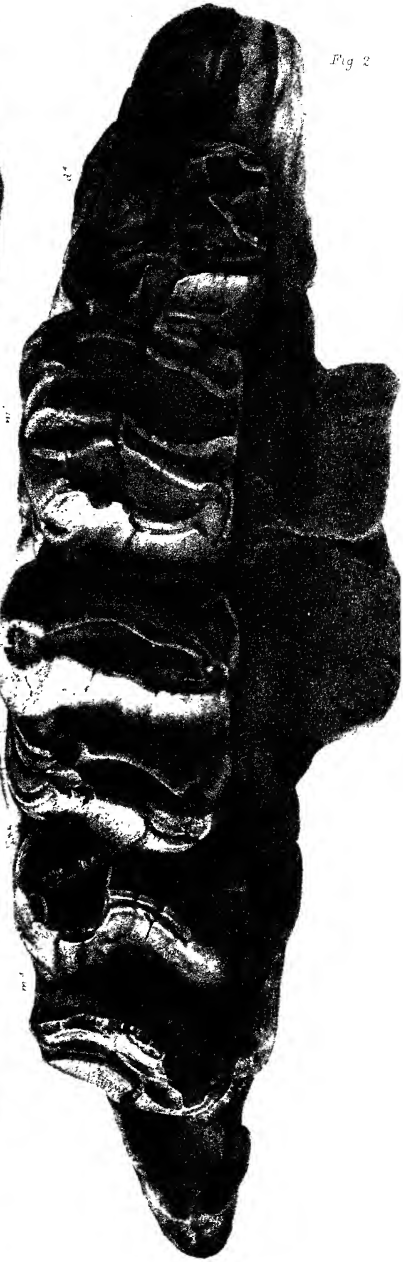

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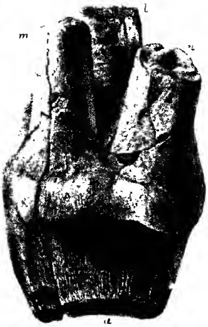


Description of the Plates.

PLATE XXXV.

Fig. 1. Side view of skull of *Diprotodon australis*:—one-fifth nat. size.
Fig. 2. Back view of skull of *Diprotodon australis*:—one-fifth nat. size.
Fig. 3. Front view of skull of *Diprotodon australis*:—one-fifth nat. size.
Fig. 4. Proportions of malar and squamosal forming glenoid cavity:—one-fifth nat. size.
Fig. 5. Side view of skull, in outline, of *Macropus laniger*:—one-fifth nat. size.

PLATE XXXVI.

Fig. 1. Outer side of first incisor. upper jaw:—nat. size.
Fig. 2. Inner side of first incisor, upper jaw:—nat. size.
[An extent of an inch of the tooth is wanting at the place of fracture.]
Fig. 3. Outer or enamelled side of working extremity of a first incisor.
Fig. 4. Outer or enamelled side of working extremity of a first incisor from another individual.
Fig. 5. Section, vertical and transverse, of premaxillary alveoli, with the first incisors, *i* 1, *in situ*: *i* 2. end of the socket of the second incisor; *i* 3, part of that of the third incisor.
Fig. 6. Basal or inserted end of first incisor; the margin mutilated.
Fig. 7. Basal or inserted end of second incisor.

All the figures are of the natural size.

PLATE XXXVII.

Fig. 1. Outer side view of the right upper molars, *in situ*, of probably a female *Diprotodon*.
Fig. 2. Grinding-surface of ditto.
Fig. 3. The three last molars of another *Diprotodon*.
Fig. 4. Grinding-surface of crown of last molar (*m* 3) of a larger *Diprotodon*
Fig. 5. Front view of the same tooth.
Fig. 6. Back view of the same tooth.

All the figures are of the natural size

PLATE XXXVIII.

Fig. 1. Outer side view of the right upper molars, *in situ*, of a large, probably male, *Diprotodon*.
Fig. 2. Grinding-surface of ditto, with part of bony palate.
Fig. 3. Hind surface of last molar (*m* 3) of another *Diprotodon*.
Fig. 4. Front surface of the same tooth.

All the figures are of the natural size.

PLATE XXXIX.

Fig. 1. Outside view of last upper molar (*m* 3) of a small, probably female, *Diprotodon.*
Fig. 2. Outside view of last upper molar (*m* 3) of a larger, probably male, *Diprotodon.*
Fig. 3. Sockets and ends of roots of penultimate and last upper molars.
Fig. 4. Outside view of lower incisor [an extent of an inch is wanting at the place of fracture].
Fig. 5. Working surface of exposed end of lower incisor
Fig. 6. Transverse section at its emergence from the socket.
Fig. 7. Side view of second upper incisor (*i* 2).
Fig. 8. Working surface of the same tooth.

PLATE XL.

Molar teeth of the lower jaw.

Fig. 1. Outer side view of second molar (*d* 4) of a young *Diprotodon.*
Fig. 2. Working surface of second molar (*d* 4) of a young *Diprotodon.*
Fig. 3. Working surface of second molar of an older, probably female, *Diprotodon.*
Fig. 4. Working surface of second molar of an old, probably male, *Diprotodon*
Fig. 5. Outer side view of third molar (*m* 1) of a young *Diprotodon.*
Fig. 6. Working surface of third molar (*m* 1) of a young *Diprotodon*
Fig. 7. Working surface of third molar (*m* 1) of an older, probably female, *Diprotodon.*
Fig. 8. Working surface of third molar (*m* 1) of an old, probably male, *Diprotodon.*
Fig. 9. Outer side view of fourth molar (*m* 2) of a young *Diprotodon*
Fig. 10. Working surface of fourth molar (*m* 2) of a young *Diprotodon*
Fig. 11. Working surface of fourth molar (*m* 2) of mature, probably male, *Diprotodon.*
Fig. 12. Outer side view of crown of germ of fifth molar of a young *Diprotodon*
Fig. 13. Front view of fifth molar of a mature *Diprotodon.*
Fig. 14. Working surface of penultimate lower molar of *Macropus Atlas*
Fig. 15. Working surface of penultimate lower molar of a Malayan Tapir
Fig. 16. Working surface of crown of incomplete fifth molar of a young *Diprotodon.*
Fig. 17. Working surface of penultimate molar (*m* 2) of an old *Diprotodon.*
Fig. 18. Working surface of last molar (*m* 3) of an old *Diprotodon.*

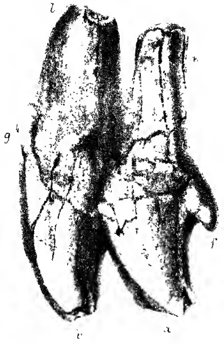
Figures 1, 2, 5, 6, 9, 10, 12, 16 are from the same individual.

All the figures are of the natural size.

PLATE XLI.

Fig. 1. Outside view of anterior part of mandible and teeth (*i* 1, *d* 4, *m* 1) of an immature *Diprotodon*: the first molar (*d* 3) is given in outline.
Fig. 2 Inside view of anterior part of the same mandible and teeth (*i* 1, *d* 3 restored in outline, *d* 4, *m* 1).

Fig 1



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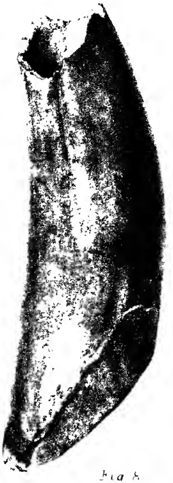

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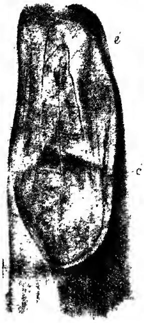
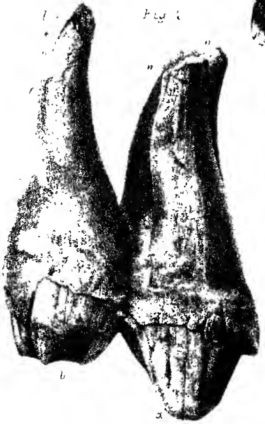


Fig 4

Fig 2



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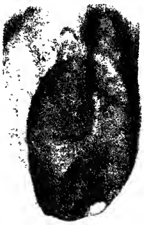

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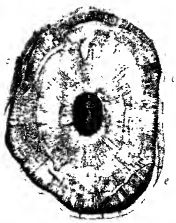

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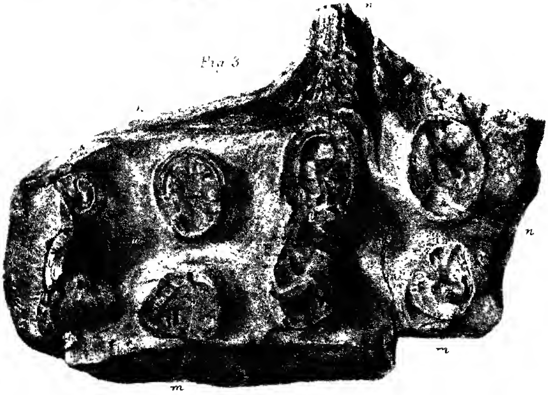

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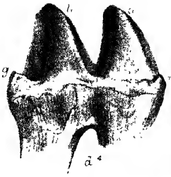

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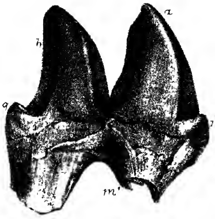

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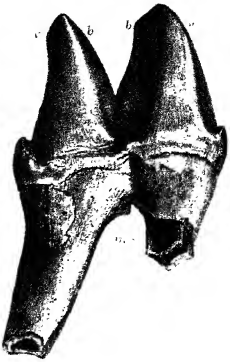

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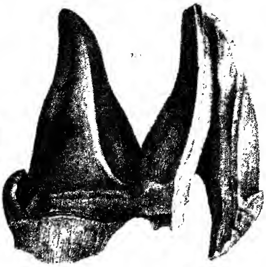

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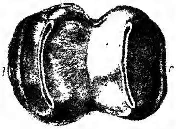

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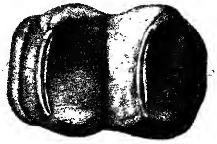

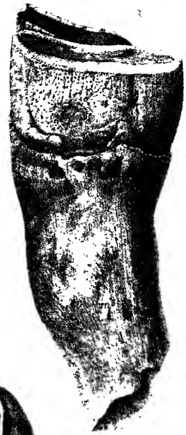

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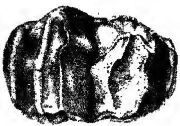

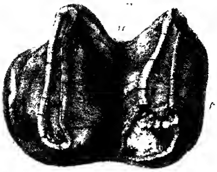

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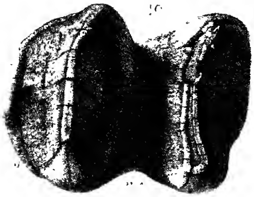

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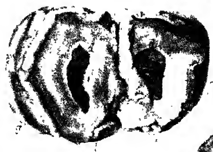

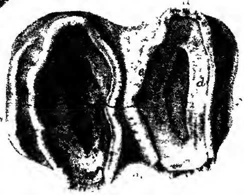

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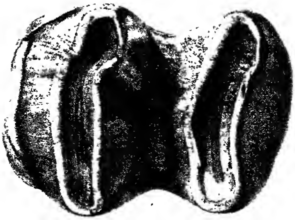

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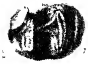

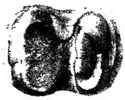

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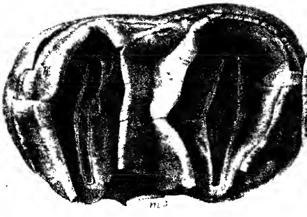

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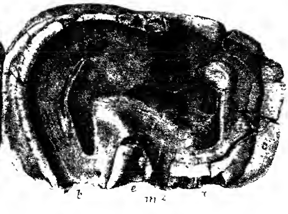

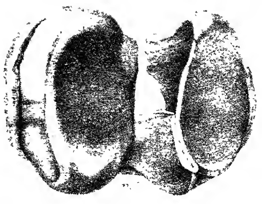

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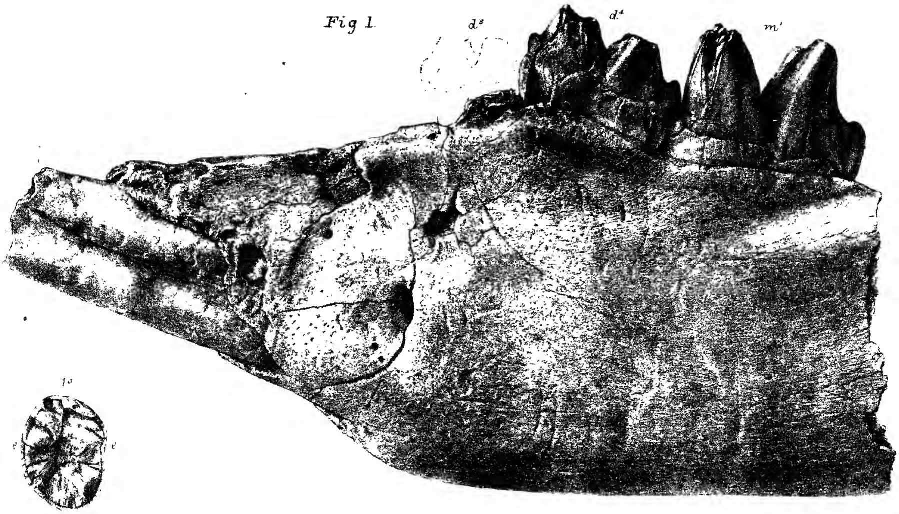
d^3
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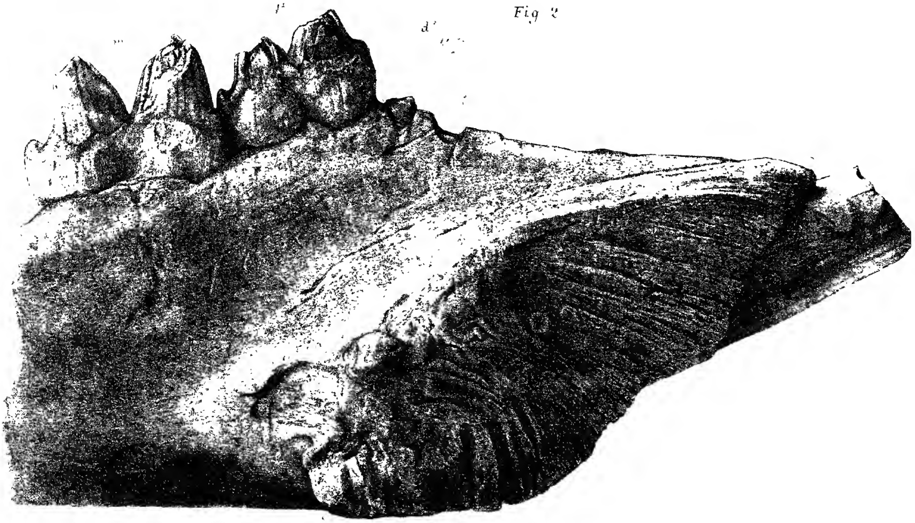
Fig 2



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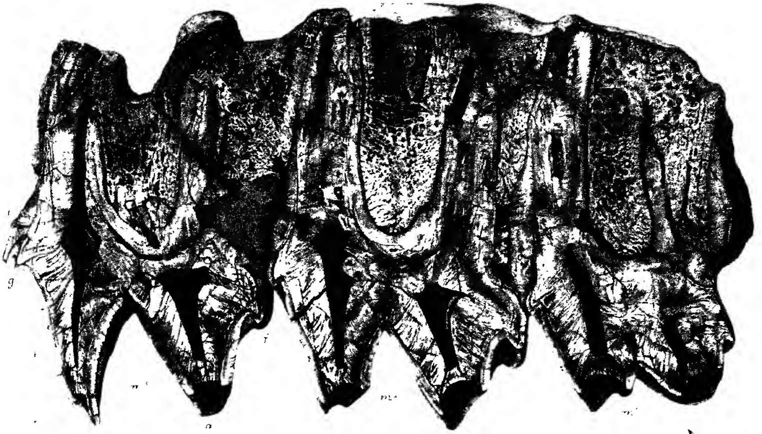
LOWER TEETH, YOUNG DIPROTODON

Fig 1



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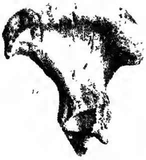


Fig 4

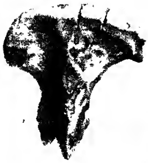


Fig 2

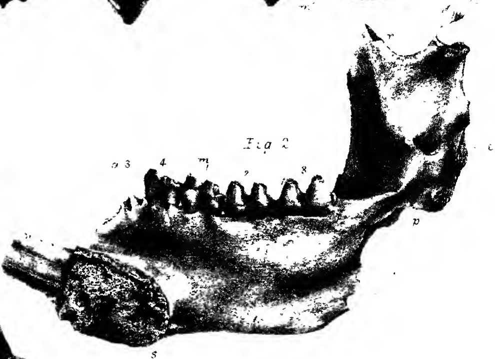
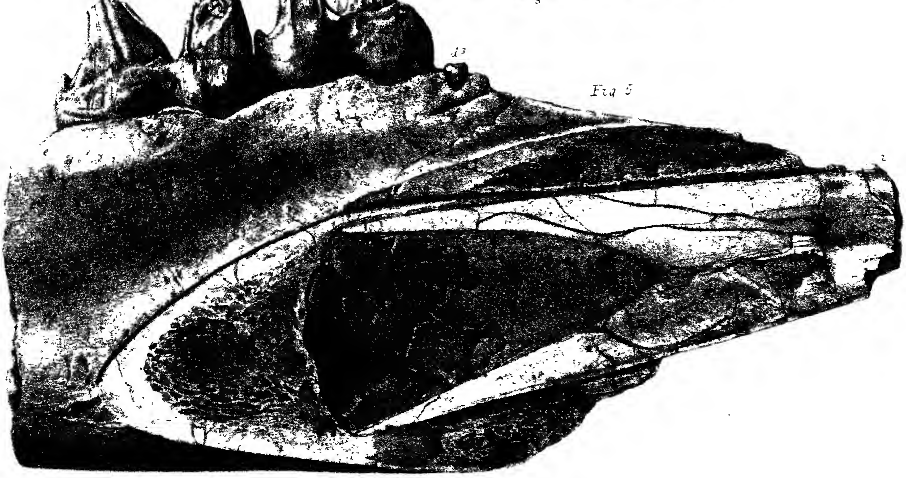


Fig 5



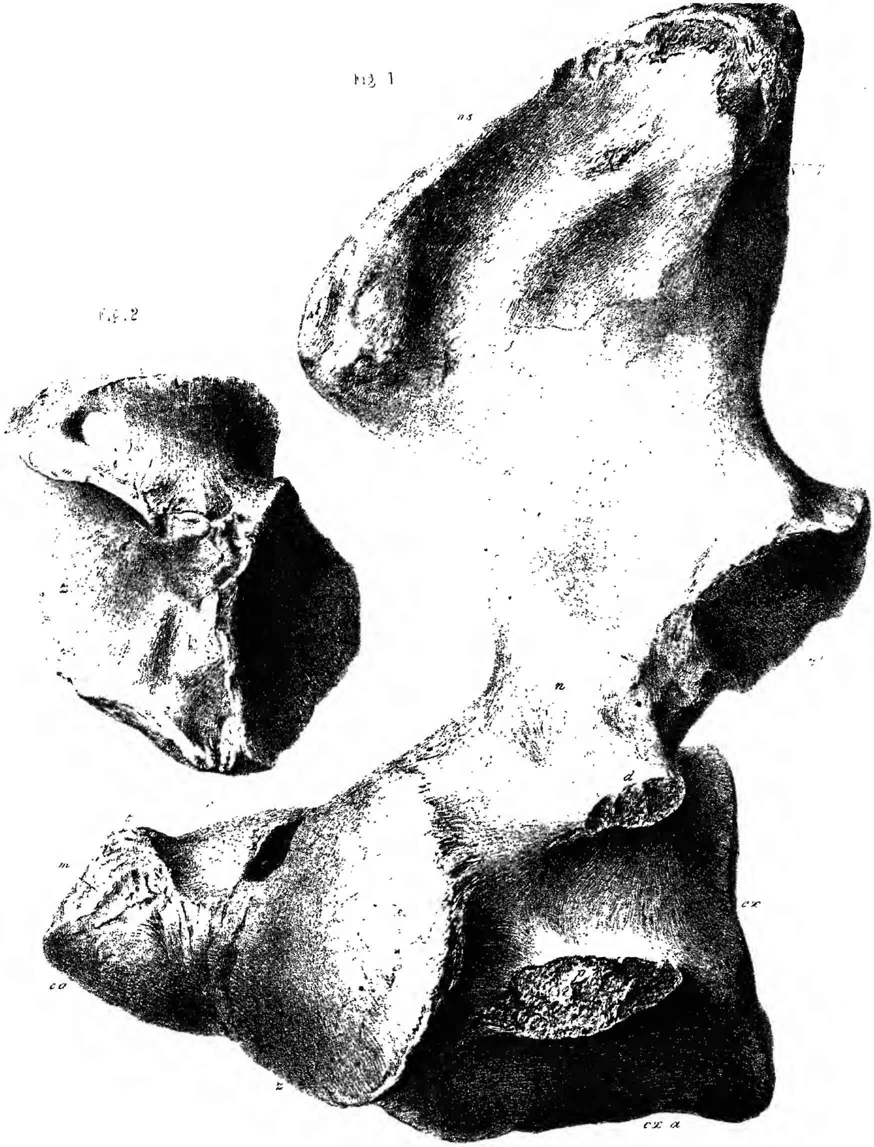


Fig. 1
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W. H. Wesley lith.

W. West imp.

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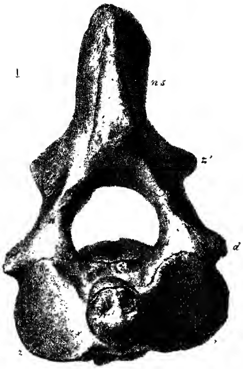

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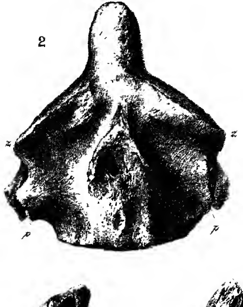

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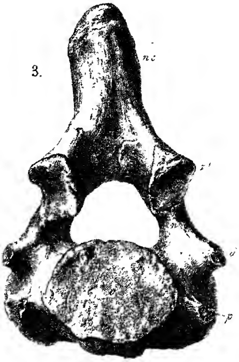

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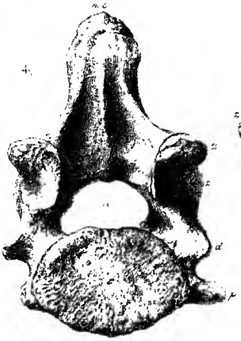

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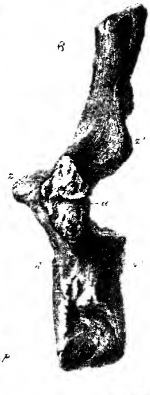

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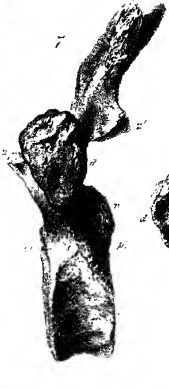

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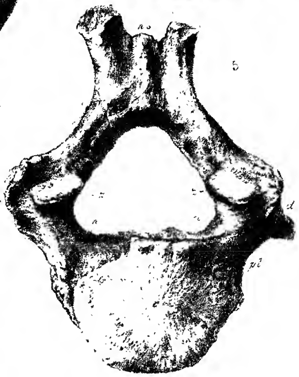

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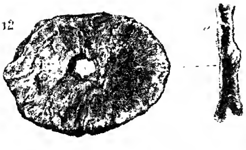

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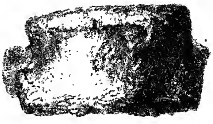

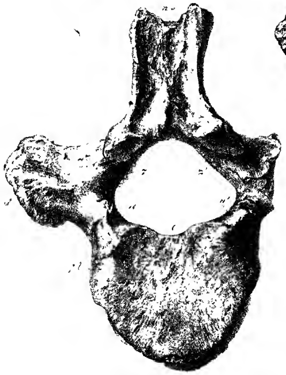

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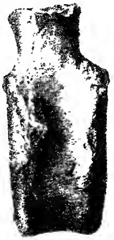

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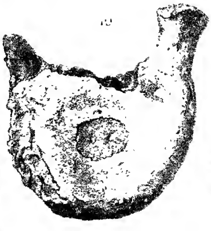

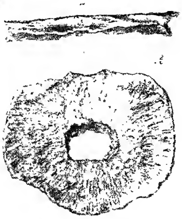

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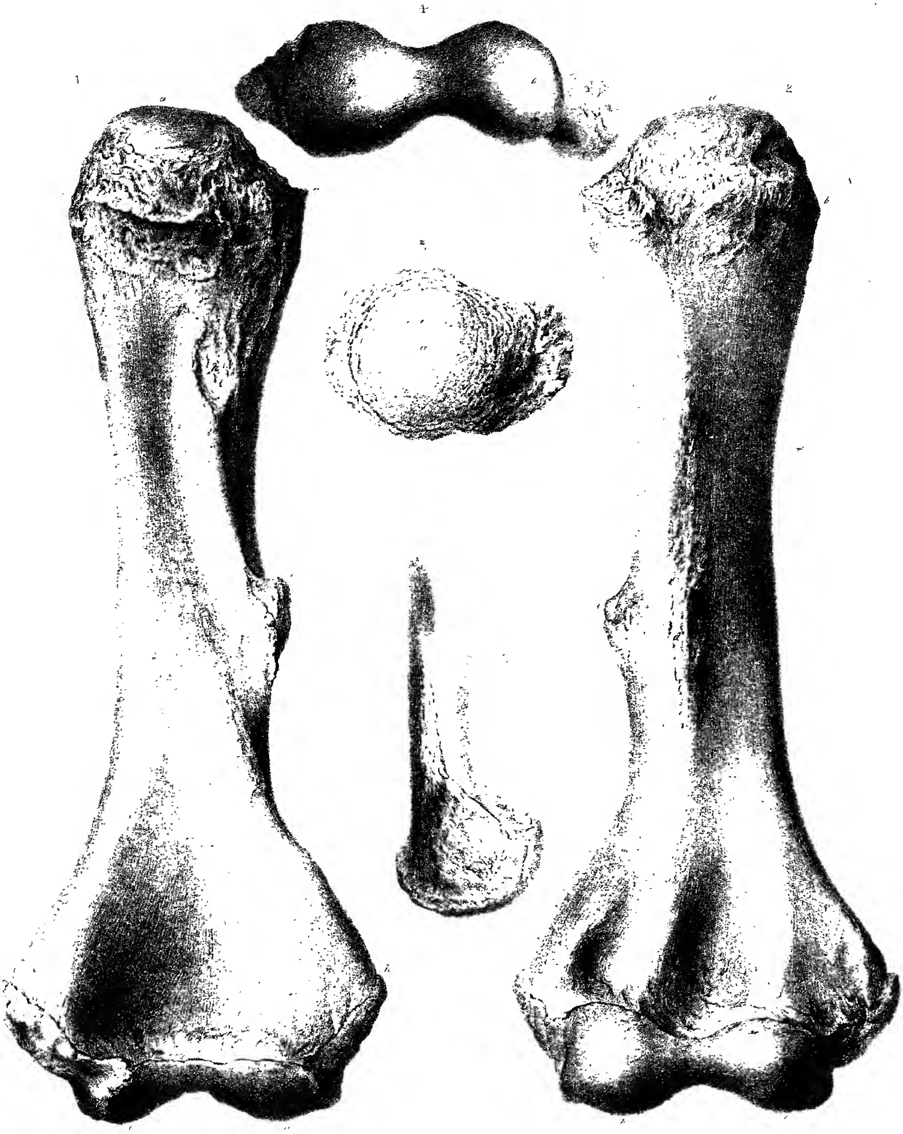
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G Wesley lith

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PLATE XLII.

Fig. 1. Section of upper molars (m 1, m 2, m 3) of a mature, not old, *Diprotodon*.
Fig. 2. Inside view of mandible and teeth of ditto: reduced to one-sixth.
Fig. 3. Back view of mandibular condyle: reduced to one-sixth.
Fig. 4. Front view of ditto.
Fig. 5. Section of anterior part of mandible and incisor tusk of an immature *Diprotodon*.

PLATE XLIII.

Fig. 1. Axis, or vertebra dentata:—nat. size.
Fig. 2. Portion of atlas vertebra:—nat size.

PLATE XLIV.

Fig. 1. Front view of axis.
Fig. 2. Under view of axis.
Fig. 3. Back view of axis.
Fig. 4. Back view of third cervical vertebra.
Fig. 5. Back view of an anterior dorsal vertebra.
Fig. 6. Side view of the same dorsal vertebra.
Fig. 7. Side view of a succeeding dorsal vertebra.
Fig. 8. Back view of the same vertebra.
Fig. 9. Side view of the body of a lumbar vertebra.
Fig. 10. Back view of ditto.
Fig. 11. Under view of ditto.
Fig. 12. Epiphyses of the body of a cervical vertebra *a*, edge- or side-view.
Fig. 13. Epiphyses of the body of a lumbar vertebra· *a*, edge-view.

All the figures are one-third the natural size

PLATE XLV.

Fig. 1. Outer surface or "dorsum" of the left scapula.
Fig. 2. Inner or subscapular surface of the left scapula.
Fig. 3. Articular end, with glenoid cavity of the left scapula

One-third the nat size

PLATE XLVI.

Fig. 1. Back view of left humerus.
Fig. 2. Front view of left humerus.
Fig. 3. Proximal end of left humerus.
Fig. 4. Distal end of left humerus.
Fig. 5. Inner side view of distal third of a right humerus.

One-third the nat. size.

PLATE XLVII.

Fig 1. Under or "hæmal" view of the pelvis.
Fig 2. Upper or "neural" view of the sacrum and right half of the pelvis.
Fig. 3. Front view of the sacrum.
Fig. 4. Acetabular cavity.

[See 'Admeasurements' in text for scale of reduction.]

PLATE XLVIII.

Fig. 1. Front view of femur.
Fig. 2. Outside view of femur.
Fig. 3. Section of middle of shaft of femur.
Fig. 4. Lower articular end of femur.
Fig. 5. Inside view of lower end of femur.

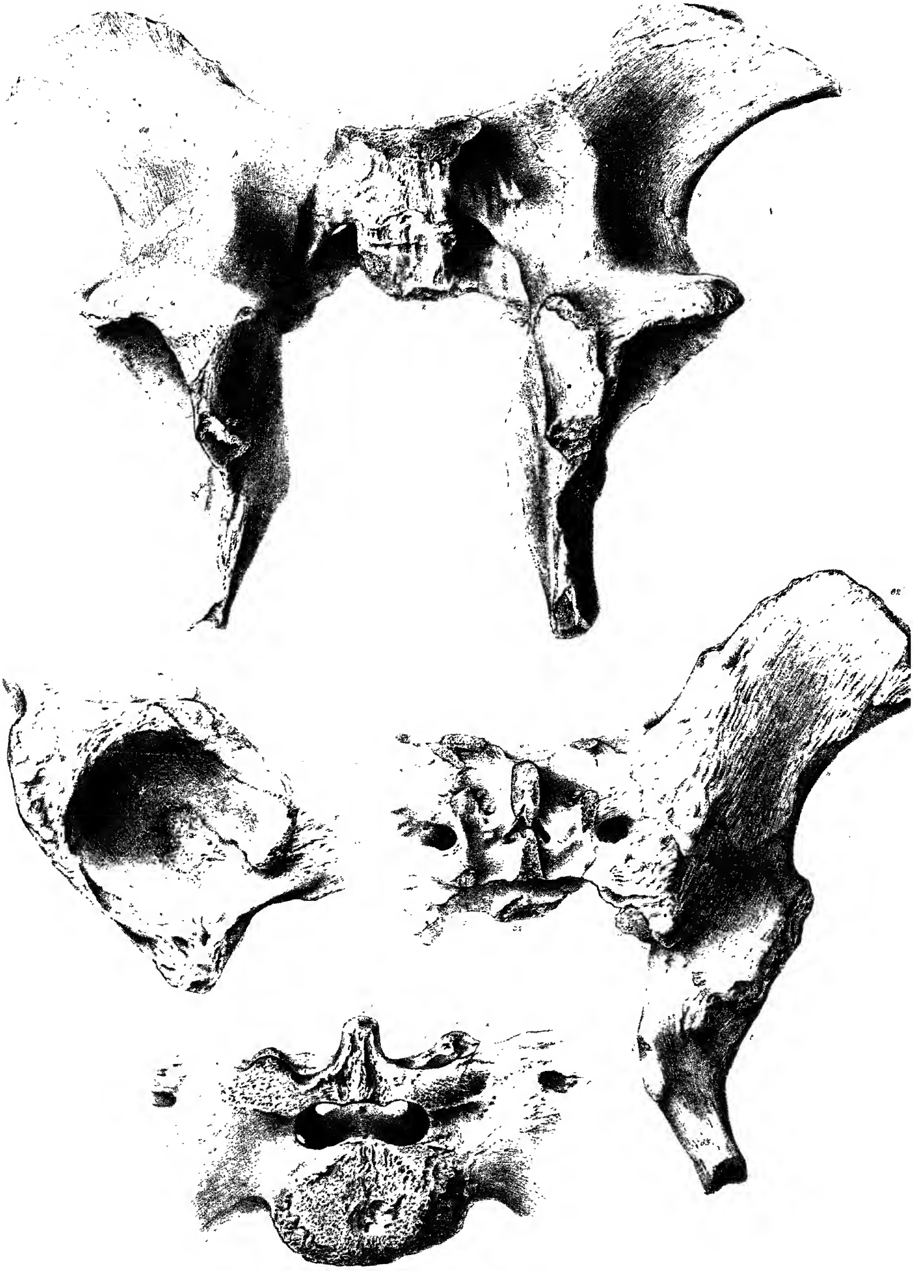
One-third nat. size.

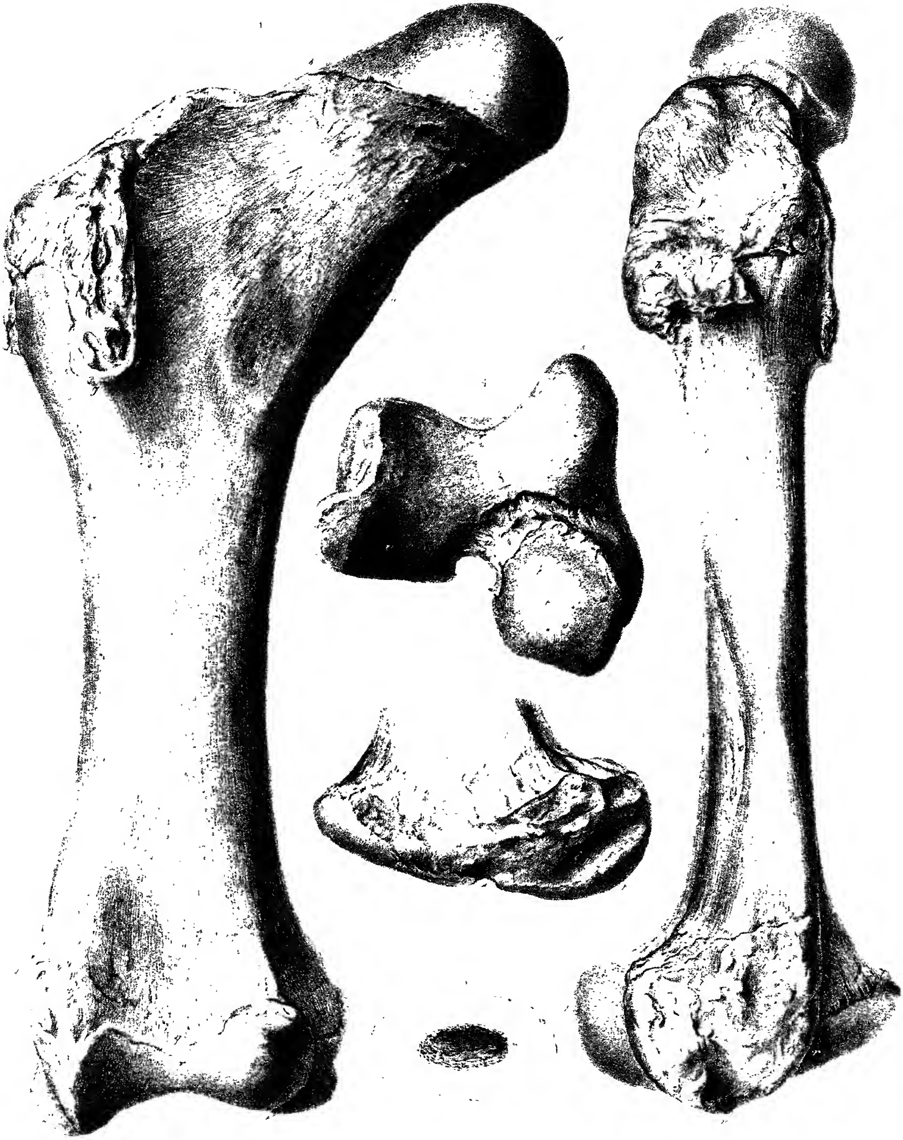
PLATE XLIX.

Fig. 1. Hind view of femur
Fig. 2. Front view of tibia.
Fig. 3. Outside view of tibia.
Fig. 4. Upper end of tibia.
Fig. 5. Lower end of tibia.

Plate L.

Restoration of the skeleton of *Diprotodon australis*:—one-twelfth nat. size.





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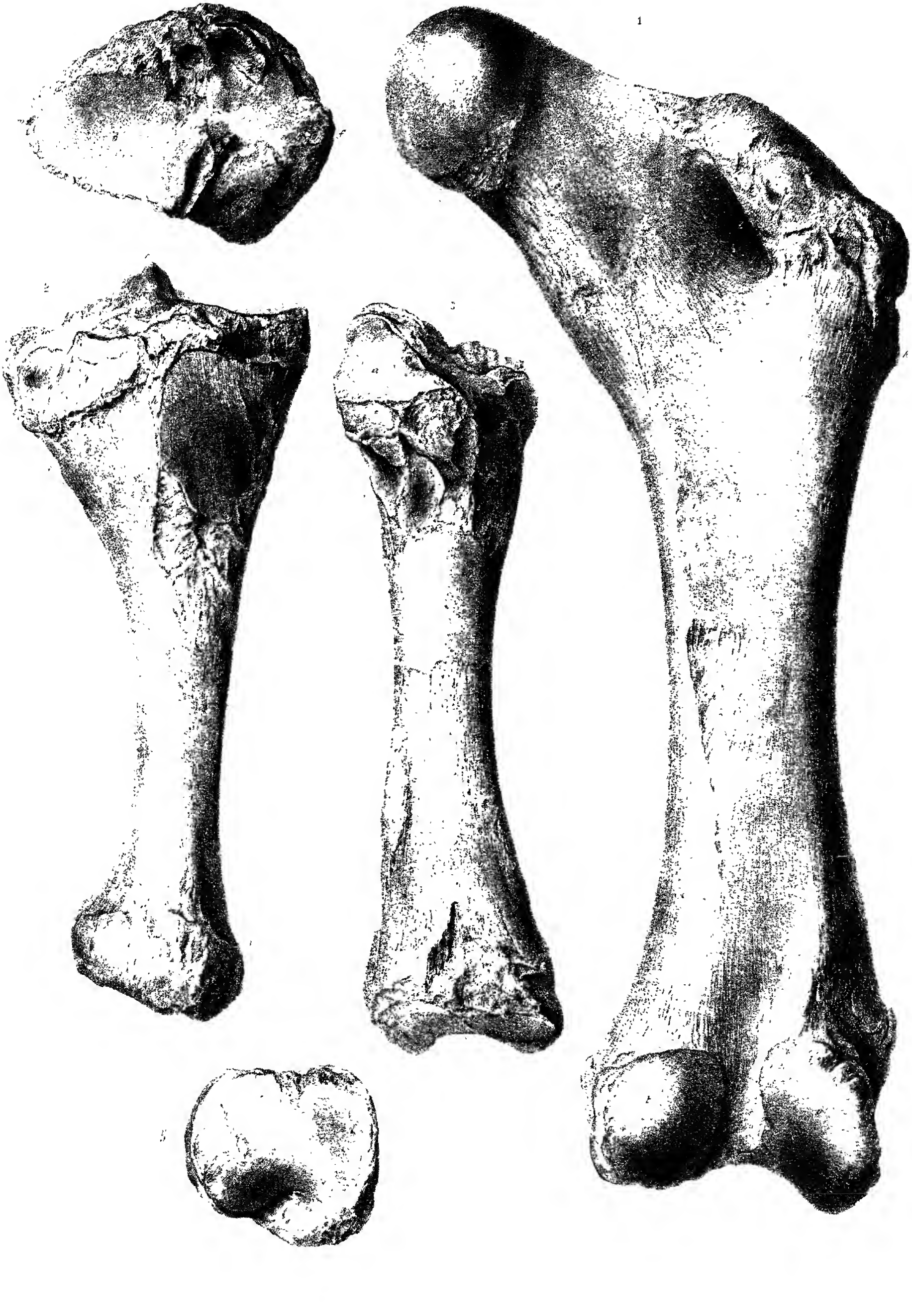
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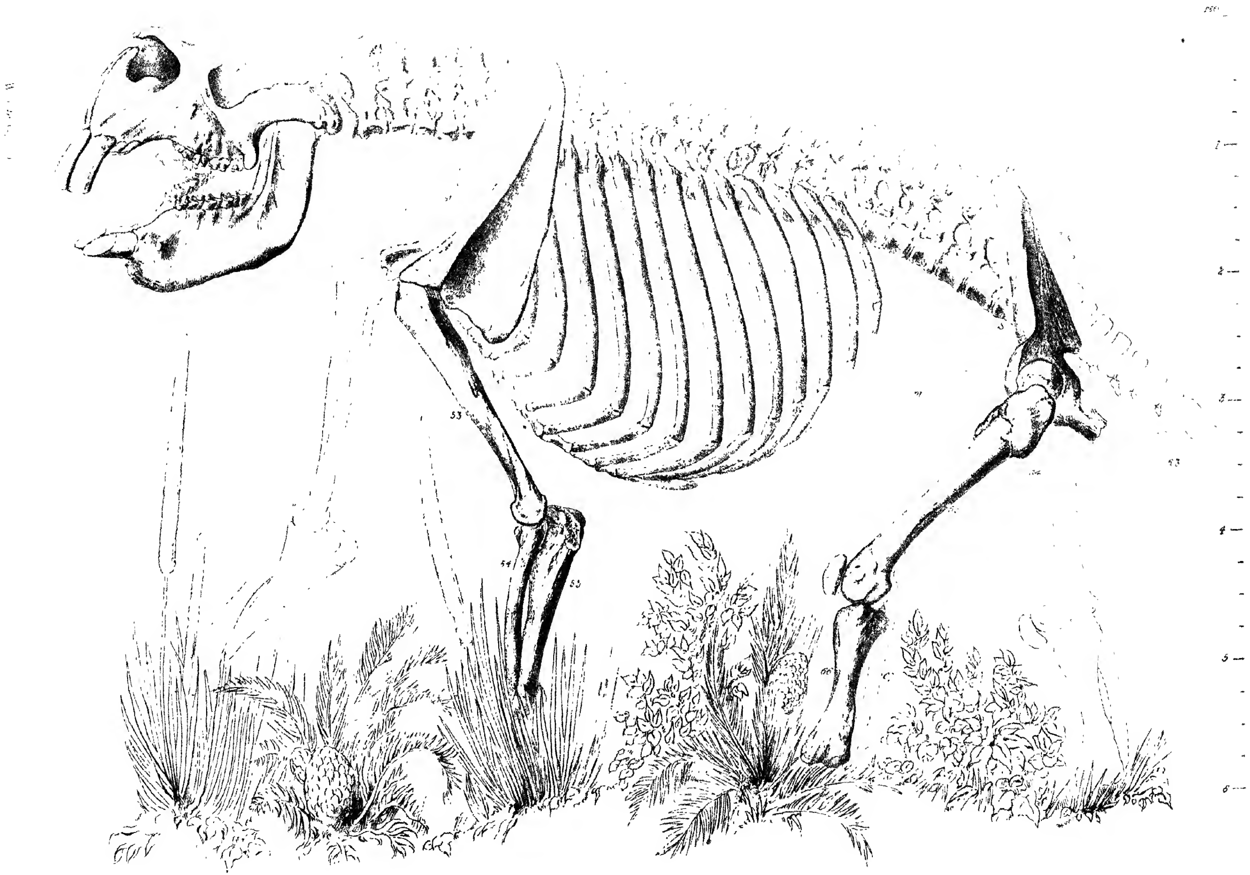
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XXIV. *On the Values of the Integral* $\int_0^1 Q_n Q_{n'}\, d\mu$, $Q_n, Q_{n'}$ *being* LAPLACE'S *Coefficients of the Orders* n, n', *with an application to the Theory of Radiation. By the Hon.* J. W. STRUTT, *Fellow of Trinity College, Cambridge. Communicated by* W. SPOTTISWOODE, *F.R.S.*

Received May 17,—Read June 16, 1870.

IN the course of an investigation concerning the potential function which is subject to conditions at the surface of a sphere which vary discontinuously in passing from one hemisphere to the other, it became necessary to know the values of the integral

$$\int_0^1 Q_n Q_{n'} d\mu,$$

Q_n, $Q_{n'}$ being LAPLACE's coefficients of the orders n, n' respectively. The expression for Q_n in terms of μ is

$$Q_n=\frac{1.3.5\ldots.(2n-1)}{1.2.3..n}\left\{\mu^n-\frac{n(n-1)}{2.(2n-1)}\mu^{n-2}+\frac{n(n-1)(n-2)(n-3)}{2.4(2n-1)(2n-3)}\mu^{n-4}-\ldots\right\}^*;$$

but the multiplication of two such series together and subsequent integration with respect to μ would be very laborious even for moderate values of n and n'.

By the following method the values of the integrals in question may be obtained without much trouble. According to the definition of the functions Q,

$$\frac{1}{\sqrt{1+e^2-2e\mu}}=1+Q_1 e+Q_2 e^2+\ldots+Q_n e^n+\ldots$$

so that

$$\int_0^1\frac{d\mu}{\sqrt{1+e^2-2e\mu}\,\sqrt{1+e'^2-2e'\mu}}=\sum_{n=0}^{n=\infty}\sum_{n'=0}^{n'=\infty}\int_0^1 Q_n Q_{n'} d\mu\,.\,e^n e'^{n'},$$

which shows that $\int_0^1 Q_n Q_{n'} d\mu$ is the coefficient of $e^n e'^{n'}$ in the expansion of the integral on the left in powers of e and e'.

On effecting the integration and reducing, we obtain as the quantity to be expanded,

$$\frac{1}{\sqrt{ee'}}\log\frac{(1+\sqrt{ee'})(\sqrt{e}-\sqrt{e'})}{\sqrt{e}\sqrt{1+e'^2}-\sqrt{e'}\sqrt{1+e^2}}=\frac{1}{\sqrt{ee'}}\log(1+\sqrt{ee'})-\frac{1}{\sqrt{ee'}}\left\{\log\sqrt{1+e^2}+\log\frac{1-\sqrt{\frac{1+e'^2}{1+e^2}}\sqrt{\frac{e}{e'}}}{1-\sqrt{\frac{e}{e'}}}\right\}$$

$$=\frac{1}{\sqrt{ee'}}\left\{(ee')^{\frac{1}{2}}-\frac{ee'}{2}+\frac{(ee')^{\frac{3}{2}}}{3}-\frac{(ee')^2}{4}+..\right\}$$

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* THOMSON and TAIT's Natural Philosophy, p. 624.

$$+\frac{1}{\sqrt{ee'}}\left[\begin{array}{l}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{1}{2}}-1\right\}\left(\frac{e}{e'}\right)^{\frac{1}{2}}+\frac{1}{2}\left\{\frac{1+e'^2}{1+e^2}-1\right\}\frac{e}{e'}\\ \quad+\frac{1}{3}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{3}{2}}-1\right\}\left(\frac{e}{e'}\right)^{\frac{3}{2}}+\frac{1}{4}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{2}-1\right\}\left(\frac{e}{e'}\right)^{2}\\ \quad+\cdot\cdot\cdot\cdot\cdot\cdot\cdot\quad+\cdot\cdot\cdot\cdot\cdot\cdot\cdot\\ \quad-\frac{1}{2}\log\left(1+e^2\right).\end{array}\right]$$

Since we know *à priori* that the expansion will only contain whole positive powers of e, we may leave out all the terms in $e^{\frac{1}{2}}\,e^{\frac{3}{2}}\ldots\ldots$ (no negative powers of e occur)*. We thus obtain

$$\begin{array}{l}1+\frac{ee'}{3}+\frac{(ee')^2}{5}+\frac{(ee')^3}{7}+\frac{(ee')^4}{9}+\ldots\ldots\\ \quad+\frac{1}{e'}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{1}{2}}-1\right\}+\frac{1}{3}\frac{e}{e'^2}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{3}{2}}-1\right\}\\ \quad+\frac{1}{5}\frac{e^2}{e'^3}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{5}{2}}-1\right\}+\frac{1}{7}\frac{e^3}{e'^4}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{7}{2}}-1\right\}\\ \quad+\frac{1}{9}\frac{e^4}{e'^5}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{9}{2}}-1\right\}+\frac{1}{11}\frac{e^5}{e'^6}\left\{\left(\frac{1+e'^2}{1+e^2}\right)^{\frac{11}{2}}-1\right\}\\ \quad+\ldots\ldots\qquad\qquad\qquad\qquad+\ldots\ldots\end{array}$$

In this all terms containing negative powers of e' may be thrown away, as they must finally disappear even if retained. The terms on the left *after the first line* contain only even powers of e, and those on the right only odd. It appears, too, that with the even powers of e go the odd of e', and conversely. Hence if n, n' be both even or both odd there is no part of the coefficient of $e^n e'^{n'}$ to be found after the first line, and none in the first line unless $n=n'$. Thus

$$\int_0^1 Q_n Q_{n'}\,d\mu=0$$

if n, n' are both odd or both even, unless they are the same, in which case

$$\int_0^1 (Q_n)^2\,d\mu=\frac{1}{2n+1}.$$

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* Professor CAYLEY has remarked that the finite expression itself may be modified so as to get rid of these terms, and then becomes

$$\frac{1}{2\sqrt{ee'}}\log\frac{1+\sqrt{ee'}}{1-\sqrt{ee'}}+\frac{1}{2\sqrt{ee'}}\left\{\log\frac{1+\sqrt{\frac{e(1+e'^2)}{e'(1+e^2)}}}{1-\sqrt{\frac{e(1+e'^2)}{e'(1+e^2)}}}-\log\frac{1+\sqrt{\frac{e}{e'}}}{1-\sqrt{\frac{e}{e'}}}\right\}.$$

For so far as the terms containing fractional powers of x are concerned,

$$\log(1+\sqrt{x})\text{ and }\tfrac{1}{2}\log\frac{1+\sqrt{x}}{1-\sqrt{x}}$$

are identical.—(Nov. 1870.)

These results* are immediate consequences of what is known with respect to the values of the integrals

$$\int_{-1}^{+1} Q_n Q_{n'}\, d\mu,$$

in which the integration extends over the *whole* sphere; for if n, n' are both odd or both even, $Q_n Q_{n'}$ is an even function of μ, and so

$$\int_{-1}^{0} Q_n Q_{n'}\, d\mu = \int_{0}^{1} Q_n Q_{n'}\, d\mu.$$

The peculiar character of the integrals over the hemisphere only shows itself when one of the quantities n, n' is even and the other odd.

The coefficient of e^0 in the expansion is $1+\frac{(1+e'^2)^{\frac{1}{2}}-1}{e'}$;

$$\text{coefficient of } e^2=\frac{e'^2}{5}-\frac{1}{2}\frac{(1+e'^2)^{\frac{1}{2}}}{e'}+\frac{1}{5}\frac{(1+e'^2)^{\frac{5}{2}}-1}{e'^3};$$

$$\text{coefficient of } e^4=\frac{e'^4}{9}+\frac{1.3}{2^2\,\underline{|2}}\frac{(1+e'^2)^{\frac{1}{2}}}{e'}-\frac{5}{2}.\frac{1}{5}\frac{(1+e'^2)^{\frac{5}{2}}}{e'^3}+\frac{1}{9}\frac{(1+e'^2)^{\frac{9}{2}}-1}{e'^5};$$

$$\text{coefficient of } e^6=\frac{e'^6}{13}-\frac{1.3.5}{2^3\,\underline{|3}}\frac{(1+e'^2)^{\frac{1}{2}}}{e'}+\frac{5.7}{2^2\,\underline{|2}}.\frac{1}{5}\frac{(1+e'^2)^{\frac{5}{2}}}{e'^3}-\frac{9}{2}.\frac{1}{9}\frac{(1+e'^2)^{\frac{9}{2}}}{e'^5}+\frac{1}{13}\frac{(1+e'^2)^{\frac{13}{2}}-1}{e'^7};$$

$$\begin{aligned}\text{coefficient of } e^8=\frac{e'^8}{17}&+\frac{1.3.5.7}{2^4\,\underline{|4}}\frac{(1+e'^2)^{\frac{1}{2}}}{e'}-\frac{5.7.9}{2^3\,\underline{|3}}.\frac{1}{5}\frac{(1+e'^2)^{\frac{5}{2}}}{e'^3}\\&+\frac{9.11}{2^2\,\underline{|2}}.\frac{1}{9}\frac{(1+e'^2)^{\frac{9}{2}}}{e'^5}-\frac{13}{2}.\frac{1}{13}\frac{(1+e'^2)^{\frac{13}{2}}}{e'^7}+\frac{1}{17}\frac{(1+e'^2)^{\frac{17}{2}}-1}{e'^9}.\end{aligned}$$

The law of formation of these series is obvious, and the coefficient of e^{2n} could, if necesssary, be written down.

From the symmetry of the original expression in e and e' we know that the coefficient of $e^n e'^{n'}$ must be the same as that of $e^{n'} e'^{n}$; so that, in order to obtain all the integrals required, it is not absolutely necessary to consider the coefficients of odd powers of e. Nevertheless, in the calculation for instance of $\int_0^1 Q_1 Q_{10}\, d\mu$, it would be much easier to obtain it as the coefficient of e'^{10} in the series which multiplies e, than as the coefficient of e' in the series which multiplies e^{10}.

The coefficient of e

$$=\frac{e'}{3}+\frac{1}{3}\frac{(1+e'^2)^{\frac{3}{2}}-1}{e'^2};$$

coefficient of e^3

$$=\frac{e'^3}{7}-\frac{3}{2}.\frac{1}{3}\frac{(1+e'^2)^{\frac{3}{2}}}{e'^2}+\frac{1}{7}\frac{(1+e'^2)^{\frac{7}{2}}-1}{e'^4};$$

---

* They would, of course, be more simply obtained by taking the integration in the first instance from $\mu=-1$ to $\mu=+1$.—(Nov. 1870.)

coefficient of e^5

$$=\frac{e'^5}{11}+\frac{3.5}{2^2\underline{|2}}\cdot\frac{1}{3}\frac{(1+e'^2)^{\frac{3}{2}}}{e'^2}-\frac{7}{2}\cdot\frac{1}{7}\frac{(1+e'^2)^{\frac{7}{2}}}{e'^4}+\frac{1}{11}\frac{(1+e'^2)^{\frac{11}{2}}-1}{e'^6};$$

coefficient of e^7

$$=\frac{e'^7}{15}-\frac{3.5.7}{2^3\underline{|3}}\cdot\frac{1}{3}\frac{(1+e'^2)^{\frac{3}{2}}}{e'^2}+\frac{7.9}{2^2\underline{|2}}\cdot\frac{1}{7}\frac{(1+e'^2)^{\frac{7}{2}}}{e'^4}$$

$$-\frac{11}{2}\cdot\frac{1}{11}\frac{(1+e'^2)^{\frac{11}{2}}}{e'^6}+\frac{1}{15}\frac{(1+e'^2)^{\frac{15}{2}}-1}{e'^8};$$

and so on. From these series the coefficients of $e^n e'^{n'}$ for moderate values of n and n' may be calculated with facility.

It is desirable to know the limit of the integral $\int_0^1 \mathrm{Q}_n\mathrm{Q}_{n'}d\mu$ when n becomes very large, n' remaining finite. A distinction is necessary according as it is the even or the odd suffix which is supposed to increase without limit.

The whole coefficient of e^{2n} is a sum of terms of the form $\frac{(1+e'^2)^{\frac{4m+1}{2}}}{e'^{2m+1}}$, where m is zero, or any positive integer, each term multiplied by a numerical factor, which may be regarded as a function of n and m. The general term in the expansion of $\frac{(1+e'^2)^{\frac{4m+1}{2}}}{e'^{2m+1}}$ in powers of e' is

$$e'^{2r-2m-1}\frac{(4m+1)(4m-1)\ldots\ldots3.1.1.3\ldots(2r-4m-3)}{2^r\underline{|r}},$$

irrespective of sign.

If we put $2r-2m-1=2n'-1$, it becomes

$$e'^{2n'-1}\frac{(4m+1)(4m-1)\ldots3.1.1.3\ldots(2n'-2m-3)}{2.4.6\ldots(2n'+2m)}.$$

The coefficient of $e^{2n}e'^{2n'-1}$ is thus a series of terms of the form

$$\frac{(4m+1)\ldots\ldots1.1\ldots\ldots(2n'-2m-3)}{2.4.6\ldots(2n'+2m)},$$

each term multiplied by a factor depending on n and m but *independent of n'*. The question is which term has the predominance when n' increases without limit? It appears that it is the one corresponding to $m=0$; for the ratio of this to the general term is

$$\frac{1.1.3\ldots\ldots(2n'-3)}{(4m+1)\ldots\ldots1.1\ldots\ldots(2n'-2m-3)}\cdot\frac{2.4\ldots\ldots(2n'+2m)}{2.4\ldots2n'}$$

$$=\frac{(2n'-2m-1)\ldots\ldots(2n'-3)\times(2n'+2)\ldots\ldots(2n'+2m)}{(4m+1)(4m-1)\ldots1},$$

a fraction which increases without limit with n'.

The value of $\int_0^1 Q_{2n}Q_{2n'-1}\,d\mu$, when n' is indefinitely great, is therefore identical with the coefficient of $e'^{2n'-1}$ in the expansion of

$$\frac{1.3.5.....(2n-1)}{2.4.6....2n}\,\frac{(1+e'^2)^{\frac{1}{2}}}{e'}.$$

Now

$$\frac{(1+e'^2)^{\frac{1}{2}}}{e'}=\frac{1}{e'}+\frac{1}{2}e'+....\pm\frac{1.1.3....(2n'-3)}{2.4.6....2n'}e'^{2n'-1};$$

and by a known theorem, when n' is indefinitely great,

$$\frac{1.1.3.....(2n'-3)}{2.4.6...2n'}=\frac{1}{2n'-1}\,\frac{1}{\pi^{\frac{1}{2}}n'^{\frac{1}{2}}}.$$

Finally, therefore, $\int_0^1 Q_{2n}Q_{2n'-1}\,d\mu$, when n' increases without limit, takes ultimately the form $\pm\frac{1.3.5....(2n-1)}{2.4.6...2n}\,\frac{1}{2\pi^{\frac{1}{2}}n'^{\frac{3}{2}}}$.

If n be very great (though infinitely small, perhaps, compared to n') this becomes $\pm\frac{1}{2\pi n^{\frac{1}{2}}n'^{\frac{3}{2}}}$. In a similar manner it may be proved that the limit of $\int_0^1 Q_{2n-1}Q_{2n'}d\mu$, when n' increases indefinitely, is

$$\pm\frac{3.5.7...(2n-1)}{2.4.6....(2n-2)}\cdot\frac{1}{4\pi^{\frac{1}{2}}n'^{\frac{5}{2}}}.$$

If, now, n increases without limit, we obtain

$$\frac{n^{\frac{1}{2}}}{2\pi n'^{\frac{5}{2}}}.$$

There is no inconsistency in the non-agreement of the values found when n and n' are indefinitely great, for the limiting circumstances contemplated in the two cases are in reality quite different. It may be convenient for the sake of comparison to repeat here the equation

$$\int_0^1 (Q_n)^2 d\mu=\frac{1}{2n+1},$$

which is true whether n be great or small.

The annexed Table contains the exact numerical values of the integrals for which neither suffix is greater than eleven. If we fix our attention on a given value of n (say 6), while n' varies, we see that the integrals, or, rather, those of them which do not vanish, begin by being alternately opposite in sign, and increase in value up to $n'=n-1$; that when $n'-n$ a change of sign is missed, and that for greater values of n' the regular alternation of sign is reestablished in conjunction with a steady diminution in numerical value.

This is in accordance with what might have been expected from the general character of the functions Q. They have their maximum (arithmetical as well as algebraical) value, namely unity, when $\mu=1$, an even function, Q_{2n}, vanishing n times, and an odd function, Q_{2n+1}, vanishing $n+1$ times for values of μ ranging from 0 to 1 inclusive.

$\int_0^1 Q_n Q_{n'} d\mu$.	$Q_{0'}$.	$Q_{1'}$.	$Q_{2'}$.	$Q_{3'}$.	$Q_{4'}$.	$Q_{5'}$.	$Q_{6'}$.	$Q_{7'}$.	$Q_{8'}$.	$Q_{9'}$.	$Q_{10'}$.	$Q_{11'}$.	$Q_{12'}$.	$Q_{13'}$.	$Q_{14'}$.	$Q_{15'}$.
Q_0	1	$\frac{1}{2}$	0	$-\frac{1}{8}$	0	$\frac{1}{16}$	0	$-\frac{5}{128}$	0	$\frac{7}{256}$	0	$-\frac{21}{1024}$	0	$\frac{33}{2048}$	0	$-\frac{13.33}{2^{15}}$
Q_1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{8}$	0	$-\frac{1}{48}$	0	$\frac{1}{128}$	0	$-\frac{1}{2^8}$	0	$\frac{7}{3.2^{10}}$	0	$-\frac{3}{2048}$	0	$\frac{33}{2^{15}}$	0
Q_2	0	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{8}$	0	$-\frac{5}{128}$	0	$\frac{7}{5.2^5}$	0	$-\frac{15}{2^{10}}$	0	$\frac{11}{2^{10}}$	0	$-\frac{273}{2^{15}}$	0	
Q_3	$-\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{9}{128}$	0	$-\frac{1}{2^6}$	0	$\frac{7}{2^{10}}$	0	$-\frac{54}{7.2^{11}}$	0	……	0	……	0
Q_4	0	$-\frac{1}{48}$	0	$\frac{9}{128}$	$\frac{1}{9}$	$\frac{9}{128}$	0	$-\frac{70}{3.2^{10}}$	0	$\frac{27}{2^{11}}$	0	$-\frac{297}{2^{15}}$	0	$\frac{1001}{2^{14}.9}$	0	
Q_5	$\frac{1}{16}$	0	$-\frac{5}{128}$	0	$\frac{9}{128}$	$\frac{1}{11}$	$\frac{25}{2^9}$	0	$-\frac{25}{2^{11}}$	0	$\frac{189}{2^{15}}$	0	……	0	……	0
Q_6	0	$\frac{1}{128}$	0	$-\frac{1}{2^5}$	0	$\frac{25}{2^9}$	$\frac{1}{13}$	$\frac{25}{2^9}$	0	$-\frac{525}{2^{15}}$	0	$\frac{77}{2^{13}}$	0	……	0	
Q_7	$-\frac{5}{128}$	0	$\frac{7}{5.2^6}$	0	$-\frac{70}{3.2^{10}}$	0	$\frac{25}{2^9}$	$\frac{1}{15}$	$\frac{1225}{2^{15}}$	0	$-\frac{383}{2^{11}.3}$	0	……	0	……	0
Q_8	0	$-\frac{1}{2^8}$	0	$\frac{7}{2^{10}}$	0	$-\frac{25}{2^{11}}$	0	$\frac{1225}{2^{15}}$	$\frac{1}{17}$	$\frac{1225}{2^{15}}$	0	$-\frac{1617}{2^{17}}$	0	……	0	
Q_9	$\frac{7}{256}$	0	$-\frac{15}{2^{10}}$	0	$\frac{27}{2^{11}}$	0	$\frac{525}{2^{15}}$	0	$\frac{1225}{2^{15}}$	$\frac{1}{19}$	$\frac{3969}{2^{17}}$	0	……	0	……	0
Q_{10}	0	$\frac{7}{3.2^{10}}$	0	$-\frac{54}{7.2^{11}}$	0	$\frac{189}{2^{15}}$	0	$-\frac{383}{2^{11}.3}$	0	$\frac{3969}{2^{17}}$	$\frac{1}{21}$	$\frac{3969}{2^{17}}$	0	……	0	
Q_{11}	$-\frac{21}{1024}$	0	$\frac{11}{2^{10}}$	0	$-\frac{297}{2^{15}}$	0	$\frac{77}{2^{13}}$	0	$-\frac{1617}{2^{17}}$	0	$\frac{3969}{2^{17}}$	$\frac{1}{23}$	……	0	……	0

When, therefore, one of the quantities n, n' is large and the other not nearly equal to it, $Q_n Q_{n'}$ is affected with a sign rapidly alternating, and consequently the value of the $\int_0^1 Q_n Q_{n'} d\mu$ is comparatively very small. But if n and n' are nearly equal, the functions Q_n, $Q_{n'}$, in spite of the rapid alternation, keep together as it were in sign for a considerable fraction of the range of integration, and so the value of the integral is largely increased.

Again, for all cases included in the Table it will be found that

$$\int_0^1 Q_{2n} Q_{2n-1} d\mu = \int_0^1 Q_{2n} Q_{2n+1} d\mu,$$

a relation which is evidently general, although not very easily proved to be so. After a good deal of trouble I arrived at the following demonstration:—

If in the expression

$$\frac{1}{\sqrt{ee'}} \log \frac{(1+\sqrt{ee'})\left(1-\sqrt{\frac{e}{e'}}\right)}{\sqrt{1+e^2}-\sqrt{\frac{e}{e'}}\sqrt{1+e'^2}},$$

whose expansion gives the integrals under consideration, we put

$$ee'=x, \quad \frac{e}{e'}=y,$$

we obtain

$$\frac{1}{\sqrt{x}} \log \frac{(1+\sqrt{x})(1-\sqrt{y})}{\sqrt{1+xy}-\sqrt{y}\sqrt{1+\frac{x}{y}}}.$$

In consequence of the symmetry of this in respect to y and $\frac{1}{y}$, it may be expanded in a series of positive and negative powers of $\sqrt{y}$ of the form

$$A_0 + A_1\left(\sqrt{y}+\frac{1}{\sqrt{y}}\right) + A_2\left(y+\frac{1}{y}\right) + A_3(y^{\frac{3}{2}}+y^{-\frac{3}{2}}) + \ldots,$$

A_0, A_1, ... being functions of x.

The terms that we are engaged in examining are those in $\sqrt{y}$ or $\frac{1}{\sqrt{y}}$, so that the question reduces itself to the determination of A_1 as a function of x, or at least an examination of its nature.

Now A_1 is the term independent of y after differentiation of the series with respect to $\sqrt{y}$. Hence

$$A_1 = \text{term independent of } y \text{ in } \frac{d}{d\sqrt{y}} \cdot \frac{1}{\sqrt{x}} \log \frac{(1+\sqrt{x})(1-\sqrt{y})}{\sqrt{1+xy}-\sqrt{y}\sqrt{1+\frac{x}{y}}}.$$

On differentiation and reduction we arrive at the expression

$$-\frac{1}{\sqrt{x}(1-y)} + \frac{1+x}{\sqrt{x}}(1-y)^{-1}\left\{1+x^2+x\left(y+\frac{1}{y}\right)\right\}^{-\frac{1}{2}},$$

from which the part independent of y has to be selected.

From the first term we have simply $\frac{-1}{\sqrt{x}}$. As for the second,

$$\left\{1+x^2+x\left(y+\frac{1}{y}\right)\right\}^{-\frac{1}{2}}$$

$$=(1+x^2)^{-\frac{1}{2}}\left\{1-\frac{1}{2}\frac{x}{1+x^2}\left(y+\frac{1}{y}\right)+\frac{1.3}{2^2\underline{|2}}\frac{x^2}{(1+x^2)^2}\left(y+\frac{1}{y}\right)^2\right.$$

$$\left.-\frac{1.3.5}{2^3\underline{|3}}\frac{x^3}{(1+x^2)^3}\left(y+\frac{1}{y}\right)^3+\ldots\right\}.$$

The term in $\left(y+\frac{1}{y}\right)(1+y+y^2+y^3+\ldots)$ independent of y is 1.

The term in $\left(y+\frac{1}{y}\right)^3(1+y+y^2+\ldots)$ is $1+3$, and generally, *if n be odd*, the part independent of y in

$$\left(y+\frac{1}{y}\right)^n(1-y)^{-1} \text{ is } \frac{1}{2}(1+1)^n=\frac{1}{2}.2^n.$$

Thus the term in $\left\{1+x^2+x\left(y+\frac{1}{y}\right)\right\}^{-\frac{1}{2}}(1-y)^{-1}$ independent of y

$$=(1+x^2)^{-\frac{1}{2}}\left\{\begin{array}{l}-\frac{1}{2}\frac{x}{1+x^2}\frac{1}{2}.2-\frac{1.3.5}{2^3\underline{|3}}\frac{x^3}{(1+x^2)^3}\frac{1}{2}.2^2\\ -\ldots\ldots+\text{ an } \textit{even} \text{ function of } x\end{array}\right\}$$

$$=\frac{(1+x^2)^{-\frac{1}{2}}}{2}\left\{\begin{array}{l}1-\frac{1}{2}.\frac{2x}{1+x^2}+\frac{1.3}{2^2\underline{|2}}\frac{(2x)^2}{(1+x^2)^2}-\frac{1.3.5}{2^3\underline{|3}}\frac{(2x)^3}{(1+x^2)^3}\\ +\ldots\ldots+\text{ an even function of } x\end{array}\right\}$$

$$=\frac{(1+x^2)^{-\frac{1}{2}}}{2}\left[\left\{1+\frac{2x}{1+x^2}\right\}^{-\frac{1}{2}}+\text{ even function of } x\right]$$

$$=\frac{1}{2}\frac{1}{1+x}+\text{ an even function of } x.$$

Finally, therefore,

$$A_1=-\frac{1}{2\sqrt{x}}+\frac{1+x}{\sqrt{x}}\left\{a_0+a_2x^2+a_4x^4+\ldots\right\},$$

where a_0, $a_2\ldots$ are unknown coefficients, of which we will only determine a_0 by a reference to the expression from which A_1 was obtained, which shows that

$$-\tfrac{1}{2}+a_0=0,$$

so that

$$A_1=\tfrac{1}{2}\sqrt{x}+a_2x^{\frac{3}{2}}+a_2x^{\frac{5}{2}}+a_4x^{\frac{7}{2}}+a_4x^{\frac{9}{2}}+\ldots.$$

Multiplying by $\sqrt{y}$ and replacing e and e',

$$A\sqrt{y}=\tfrac{1}{2}e+a_2e^2e'+a_2e^3e'^2+a_4e^4e'^3+a_4e^5e'^4+a_6e^6e'^5+a_6e^7e'^6+\ldots.$$

These are the only terms in the expansion of $\frac{1}{\sqrt{ee'}}\log\frac{(1+\sqrt{ee'})\left(1-\sqrt{\frac{e}{e'}}\right)}{\sqrt{1+e^2}-\sqrt{\frac{e}{e'}}\sqrt{1+e'^2}}$ in which the index of e is *one* higher than that of e'.

Having regard, now, to the symmetry in e and e', we see that generally

$$\int_0^1 Q_{2n}Q_{2n-1}d\mu=\int_0^1 Q_{2n}Q_{2n+1}d\mu.$$

As an application of some of the results of this investigation I will take the following physical problem. A spherical ball of uniform material is exposed to the radiation from infinitely distant surrounding objects. It is required to find the *stationary* condition. For the sake of simplicity, the surface of the sphere will be supposed to be perfectly black, that is, to absorb all the radiant heat that falls upon it, and NEWTON'S law of cooling will be employed, at least provisionally.

If V denote the temperature, it is to be determined by the equations

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)V=0, \quad \ldots\ldots \quad \text{(A)}$$

$$k\frac{dV}{dr}=F(E)-hV, \quad \ldots\ldots \quad \text{(B)}$$

where F(E) is a function of the position of the point E on the surface, and denotes the heat received per unit area at that point, k is the conductivity, and h the coefficient of radiation. Equation (A) is to be satisfied throughout the interior and (B) over the surface of the sphere.

If V be expanded in LAPLACE'S series,

$$V=S_0+S_1\frac{r}{a}+S_2\frac{r}{a^2}+\ldots; \quad \frac{dV}{dr=a}=\frac{1}{a}(S_1+2S_2+3S_3+\ldots);$$

and if

$$F=F_0+F_1+F_2+\ldots.$$

be the expansion of F in a similar series of surface harmonics, we obtain, on substituting in (B) and equating to zero the terms of any order,

$$S_0=\frac{F_0}{h},$$

$$S_1=\frac{F_1}{h+\frac{k}{a}},$$

$$S_2=\frac{F_2}{h+\frac{2k}{a}},$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$S_n=\frac{F_n}{h+\frac{nk}{a}}. \quad \ldots\ldots \quad \text{(C)}$$

The mean temperature S_0 is seen to be independent of the conductivity and of the size of the sphere.

The case where the heat which falls on the sphere proceeds from a single radiant-point is not only important in itself, but may be made the foundation of the general solution in virtue of the principle of superposition. Taking the axis in the direction of the radiant-point, we have

$$F(E)=\mu$$

over the positive hemisphere, that is, from $\theta=0$ to $\theta=\frac{\pi}{2}$,

while over the negative hemisphere $F(E)=0$.

It is required to expand F in a series of spherical harmonics.

Let $F=\frac{1}{2}\mu+f$, then f is a function of μ, which is equal to $\frac{1}{2}\mu$ over the positive hemisphere and to $-\frac{1}{2}\mu$ over the negative. The problem therefore reduces itself to the expression of $\frac{1}{2}\mu$ over the positive hemisphere in a series of functions Q *of even order*. The same series will then give $-\frac{1}{2}\mu$ over the negative hemisphere.

Assume

$$\tfrac{1}{2}\mu=A_0+A_2Q_2+A_4Q_4+\ldots\ldots$$

Multiplying by Q_{2n}, and integrating with respect to μ from $\mu=0$ to $\mu=1$,

$$\tfrac{1}{2}\int_0^1 Q_1Q_{2n}d\mu=A_{2n}\int_0^1(Q_{2n})^2d\mu,$$

all the other terms on the right vanishing.

Now $\int_0^1(Q_{2n})^2d\mu=\frac{1}{4n+1}$.

$\int_0^1 Q_1Q_{2n}d\mu=$coefficient of e^{2n} in the expansion of

$$\frac{1}{3}\,\frac{(1+e^2)^{\frac{3}{2}}-1}{e^2}$$

$$=-(-1)^n\,\frac{1.1.3.5\ldots.(2n-3)}{2.4.6\ldots.(2n+2)},$$

or

$$A_{2n}=-(-1)^n\,\frac{4n+1}{2}\,\frac{1.1.3\ldots.(2n-3)}{2.4.6\ldots.(2n+2)}.$$

Accordingly

$$F(E)=\tfrac{1}{4}+\tfrac{1}{2}Q_1+\tfrac{5}{16}Q_2-\tfrac{3}{32}Q_4+\ldots-(-1)^n\frac{4n+1}{2}\,\frac{1.1.3.5\ldots.(2n-3)}{2.4.6\ldots.(2n+2)}\,Q_{2n}+\ldots\ldots$$

When n is great the coefficient of Q_{2n} approximates to $\frac{1}{2\sqrt{\pi.n^{\frac{3}{2}}}}$.

This completes the solution for a sphere exposed to the radiation from an infinitely distant source of heat situated over the point $\mu=1$.

If its coordinates are μ', ϕ', it is only necessary to replace μ in Q_{2n} by

$$\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi-\phi').$$

Hence if H denote the intensity of the radiation which comes in *direction* μ', ϕ', the general value of S_n is

$$S_{2n}=-(-1)^n\frac{4n+1}{2\left(h+\frac{nk}{a}\right)}\frac{1.1.3.5\ldots(2n-3)}{2.4.6\ldots(2n+2)}$$

$$\times\iint \mathrm{H}Q_{2n}(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi-\phi'))d\mu'd\phi',$$

the integration going *all* round the sphere.

Now $(4n+1)\iint \mathrm{H}Q_{2n}d\mu'd\phi'$ is the same as $4\pi\mathrm{H}_{2n}$, where H_{2n} is the harmonic element of H of order $2n$; so that

$$S_{2n}=-(-1)^n\frac{1.1.3\ldots(2n-3)}{2.4.6\ldots(2n+2)}\frac{2\pi\mathrm{H}_{2n}}{h+\frac{2nk}{a}},$$

$$S_0=\frac{\pi}{h}\mathrm{H}_0,$$

$$S_1=\frac{2\pi}{3\left(h+\frac{k}{a}\right)}\mathrm{H}_1,$$

$$S_2=\frac{\pi}{4\left(h+\frac{2k}{a}\right)}\mathrm{H}_2,$$

$$S_4=-\frac{\pi}{24\left(h+\frac{4k}{a}\right)}\mathrm{H}_4.$$

It is remarkable that the odd terms in H (except H_1) are altogether without influence. The reason is simply that they do not affect the total heat falling on any point of the surface.

For this is expressed by

$$\int_0^1\int_0^{2\pi}\mu\mathrm{H}_n d\mu d\phi,$$

the point considered being taken as pole of μ, which involves no loss of generality.

Now (THOMSON and TAIT, p. 149)

$$\mathrm{H}_n=\sum_{s=0}^{s=n}(\mathrm{A}_s\cos s\phi+\mathrm{B}_s\sin s\phi)\Theta_n^s,$$

where Θ_n^s is a function of μ not containing ϕ.

When the integration with respect to ϕ is effected, all the terms will vanish except that whose coefficient is A_0. For this purpose, therefore, we may take

$$\mathrm{H}_n=\mathrm{A}_0\Theta_n^0 \text{ or } \mathrm{A}_0'Q_n,$$

and we know that $\int_0^1\mu Q_n d\mu$ vanishes if n be odd and different from unity*.

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* The proof given is sufficient for the object in view; but it may be well to notice that the essential thing is that the two surface harmonics which are multiplied together are either both odd or both even. A harmonic

The same thing is true for an ellipsoid or body of any figure which lies altogether on one side of every tangent plane, namely, that the terms of odd order in H (except one) are wholly without influence on it, and for the same reason.

We saw that in the case of a sphere the mean temperature was independent of the conductivity, and also of the size of the sphere; but this depends on NEWTON'S law of cooling. A comparison, however, may be made which shall hold good whatever be the law of variation of radiation with temperature; for if the conducting-power of any uniform body (which need not be oval) be increased in the same proportion as its linear dimensions, a corresponding distribution of temperature will satisfy all the conditions. Conclusions of interest from a physical point of view may be deduced from the foregoing considerations, but I refrain from pursuing the subject at present, as the physical problem was only brought forward in illustration of the mathematical results developed in this paper.

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of even order has identical values at opposite points of the sphere, and one of odd order has contrary values. The product of two harmonics which are either both even or both odd has therefore the same value when integrated over any portion of the sphere, or over what may be called the opposite portion, or as a particular case over two opposite hemispheres. The last two integrals are the halves of the integral over the whole sphere, which vanishes by a well-known property of these functions.

XXV. *On a Searcher for Aplanatic Images applied to Microscopes, and its effects in increasing Power and improving Definition.*

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In the observations which I have the honour to submit to the Royal Society, I purpose at present* to describe as briefly as possible—

I. Some experiments which suggested an inquiry into a method of raising microscopic power consistent with a corresponding improvement in the precision of definition, so generally destroyed by excessive amplification.

II. I next purpose to give some account of the inquiries by which the construction of an aplanatic-image searcher was gradually arrived at; the object of which was to search for aplanatic foci, to compensate residuary errors by new spherical and chromatic corrections whilst amplifying power, and to increase the small interval existing between a deep objective and its object, whilst the focal perspective or depth was also increased.

Such an inquiry,—in the present elaborated delicacy of adjustment accomplished in microscopes of the highest quality, especially when armed with "immersion sixteenths" which have alone succeeded in resolving Nobert's most delicate bands, embracing lines 112000 to the English inch,—would seem either superfluous or futile.

The research was originally suggested by the accidental resolution of the Podura scale. This exquisite object, so justly prized by the optician for the trial of microscopes, affords peculiar markings resembling notes of admiration, of sufficient delicacy to put even the defining-power of objectives of one-fiftieth of an inch to a severe ordeal. I had observed these markings to disappear and be resolved into black beads. The objective employed had nearly one-seventh of an inch focal length, and an aperture of 50°. The object was illuminated by solar rays reflected obliquely by a plane mirror. Having related this effect to eminent opticians, I was informed that no objectives (at that time 1862) could resolve this test. I prevailed on them, however, to construct a "very fine" one-eighth expressly for this resolution; as this totally failed, a one-sixteenth was carefully constructed with no better success, and finally a one-fourth of very large aperture; all these failed to exhibit the Podura beading*. Some unsuspected cause of this failure evidently remained to be investigated. The evidently delusive character of the standard test, so much relied upon for the construction of microscopic object-glasses, suggested the necessity of a search for other less uncertain methods of testing. The principle of proceeding from the known to the unknown appeared to offer the only sound basis of inquiry.

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* The writer may here perhaps be allowed to offer an explanation of the delay in presenting this Memoir, notwithstanding the substance had been verbally communicated to Professor Stokes so early as the summer session of 1869.

It had been intended to give a much more extended account of the results obtained, and for this purpose, during the following autumn and winter, an extended paper was being prepared. In 1870 further delay seemed undesirable, and accordingly a new and brief memoir was drawn up of which the present paper forms a portion.

Simple objects were now examined. The finest glass threads presented linear images of any conceivable degree of proximity, whilst their fused extremities, when selected as forming refracting spherules one-thousandth of an inch in diameter, presented miniature landscapes and points of light of remarkable precision, the spherical aberration of which could be easily calculated to be of insignificant amount for limited apertures. Even a plano-convex lens of one-thirtieth of an inch focal length and three-hundredths aperture displayed, though uncorrected, miniature pictures of marvellous beauty, bearing considerable amplification; whilst a combination of achromatic lenses corrected with all the resources of modern art, seemed capable of forming an exquisitely small image of any given object placed at a distance from it, the appearance of which, when examined by the microscope to be tested, could at once be verified by the object producing the miniature test. When suitable precautions are taken—such as (1) axial coincidence of the objectives; (2) proper corrections for an "uncovered" or aërial, or for an aqueous image when immersion lenses are employed, and for the distance of the object from the image-forming objective,—these miniature test-images bear an extraordinary amount of amplification by the microscope, displaying at once the erroneous corrections. I have found it convenient in general to use the image or *miniature*-forming objective of a deeper focus than the observing, generally one-half.

The following experiments, as elucidating the operation of this testing, will, it is hoped, explain its critical powers. The mechanical arrangements are shown by diagrams, figs. 1, 1 *a*, Plate LII.

Experiment 1.—Miniature of a small thermometer, the ivory scale being graduated 24° to the inch. A power of 300 diameters, gained by a low eyepiece "A" and the objective of one-eighth focal length (made expressly for Podura beading-test), was applied to view the miniature formed by a one-sixteenth objective of excellent quality; and the following appearances were carefully noted at the time of observation.

Result.—The sparkle of light on the bulb of the instrument, the graduation, and the metallic thread within the glass tube are invisible, obscured by a nebulous yellow fog which no objective adjustments are able to dissipate. Fig. 3, Plate LII. (fig. 5 shows improving definition).

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* Messrs. POWELL and LEALAND spared neither skill nor time in endeavouring to assist my inquiry; but with these glasses I signally failed to exhibit to them the new test (1864).

In consequence of this unexpected discovery, regarding the quality of a "very fine" one-eighth, it was returned to the opticians to their surprise for better compensation. It was then, after more accurate compensation by them, again submitted to precisely the same testing conditions.

New results.—Appearance of a slight nebulous yellow cloud through which could be distinctly seen the ivory scale finely graduated, the bulb sparkle, and even minute separated mercurial particles scattered within the glass stem (fig. 7).

The definition had been therefore decidedly reformed. Previously, however, to the alteration, experiments had been tried for the purpose of ascertaining whether a defective glass would still form a fine miniature. It might be reasonably expected that such slight errors as had escaped the notice of eminent opticians would not materially injure a miniature image in which the aberration would probably be reduced in the miniature itself. The image of the thermometer now formed with the imperfect eighth was viewed with a fine sixteenth (at about 800 diameters), when it was gratifying to observe a very beautiful display of the picture well defined in all respects (fig. 4).

These and other experiments appeared to warrant an important conclusion—that an image-test miniature formed by an objective of fair quality enjoyed sufficient accuracy of definition in miniature (even when the object was placed at varying distances from the stage or focal point of vision) to form a trustworthy test of microscopical definition, provided the aperture of the miniature-forming objective was equal to that of the objective to be tested.

To estimate the size of a miniature (α) of a given object (θ) placed at a considerable distance (d) from the miniature (fig. 1 a), it is necessary to consider that the conventional focal length F of an objective may be defined to be 10 inches divided by the micrometric ratio of amplification (m) when the image is thrown on a screen 10 inches from the object, so that

$$\mathrm{F}=\frac{10}{m},\ \text{when } d=10,$$

or

$$m=\frac{10}{\mathrm{F}},$$

$$\propto\frac{1}{\mathrm{F}},\ \text{when } d \text{ is constant.} \qquad (1)$$

When very deep objectives are used (fig. 1) the position of the plane of focal vision varies very slightly for a considerable increase of the length of the microscope, so that if the draw tube be graduated, the increase of power is nearly proportionate to the increased length or reading, because the focal plane being nearly fixed, the image will appear upon a screen enlarged proportionally as its distance (d) increases. On reversing the rays, the miniature diminishes in proportion as a given object is removed further from it: from which it follows that approximately, and sufficiently near for the purpose in hand,

$$m \propto d \ \text{when F is constant.} \qquad (2)$$

Compounding these expressions (1) and (2),

$$m \propto \frac{d}{\mathrm{F}} = p \cdot \frac{d}{\mathrm{F}}.$$

The constant $p=1$; for if m be made equal 160, the conventional focus $\mathrm{F}=\frac{1}{16}$, when $d=10$, and hence (d being large and F small) approximately,

$$m=\frac{d}{\mathrm{F}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If (f) be the focal length of a small lens whose thickness is neglected, a very similar approximate result can readily be obtained from the optical formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}.$$

For by construction

$$u+v=d,$$

and by similar triangles

$$mv=u.$$

Eliminating the unmeasured distances u and v of the object and image from the "centre" of the lens, it will be found that

$$f=\frac{d}{m+2+\frac{1}{m}},$$

and m being very large in these experiments,

$$f=\frac{d}{m+2} \text{ nearly*}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or

$$m=\frac{d}{f}-2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But in the case of the miniature images employed, m is so large that (-2) may be neglected, so that $m=\frac{d}{f}$ is sufficiently near for their measurement.

The thermometer was now placed 100 inches distance from the microscopical focus; the one-sixteenth being employed to form the image, $f=\frac{1}{16}$, $d=100$; hence

$$m=100 \div \tfrac{1}{16}=1600 \text{ very nearly.}$$

The divisions on the thermometer would be therefore reduced in the image to a miniature 1600 times less than the original, or about 40,000 to the inch, whilst the breadth of a single line would be only the 400,000th.

The means being thus obtained of readily estimating the size of images of known objects at known distances, the examination of immersion objectives next occupied my attention. Double stars were artificially produced in thin brass ($\frac{3}{100}$ of an inch thick) by placing minute apertures ($\frac{12}{1000}$ in diameter) in front of a brilliant flame, at the distance of 100 inches from the focal point of observation (fig. 9, Plate LII.). The apertures were so arranged as to gradually exhibit closer double disks (as shown roughly in fig. 9), which were carefully drawn on brass under the microscope and then accurately pierced. The miniature effect of the star-doublets is represented in the following Table, the immersion one-sixteenth objective being converted into one-twentieth*, so that f here$=\frac{1}{20}$, $m=2000$ at 100 inches distance (nearly).

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* This method also gives the focal length of a minute lens, to determine which accurately is attended with no little difficulty.

Doublet.	Size of disks.	Calculated size of images nearly.	Distance between their centres.
No. 2.	$\frac{12}{1000}$	$\frac{1}{166000}$	$\frac{1}{40000}$
No. 3.	$\frac{12}{1000}$	$\frac{1}{166000}$	$\frac{1}{20000}$

It will be readily seen from the diagram (fig. 8) that, in No. 2 the disks being $\frac{12}{1000}$ and the separating interval between centres being $\frac{1}{20}$, the actual dividing interval is $\frac{38}{1000}$, or above three times the real diameter of each disk.

Experiment 2.—A drop of distilled water being suspended between the objectives, both of which were fitted with single front "immersion lenses," I was astonished to find that the separating interval (accurately measuring $\frac{38}{1000}$ of an inch) between the centres of the disks had totally disappeared in the miniature image. The disks now resembled a finely divided double star just separated by a black line, yet this minute interval should have appeared above three times the diameter of a disk, as at B in fig. 6 & BC fig. 8. Apparently, therefore, in the eyepiece, spurious disks had been formed four times and one-sixth larger than a true aplanatic representation by the microscope†.

It follows from this experiment that if the disks be supposed to gradually diminish to points, the limiting value of the residuary spurious disks would give nearly $\frac{1}{52000}$ of an inch for the diameter of the least circle of confusion, representing the actual amount of residuary lateral aberration.

This appears from a diagram, where in the limit (fig. 8) EF$=4$AE, when AB, CD both vanish in the case of the disks being reduced to points.

The phenomena presented by these artificial doublet-image tests, gave fine evidence of the skill employed in the construction of the glasses, and of the accuracy with which the axes of the optical parts had been made to coincide in this delicate experiment.

All the disks appeared sharply cut and planetary (fig. 10), surrounded with a black ring supplemented by accurately formed diffraction rings, which enlarged and glowed with

---

[*] By the adaptation of a "water-lens" one-thirtieth inch focus.

[†] It will be observed in this experiment that the standard distance of 9 inches at which the object should be placed from the objective was increased to 100. It was found that only a very slight adjustment of the screw collar of the image-objective was necessary to compensate for this great increase of defining distance. It is hardly necessary to remark further that in a minute miniature image the aberration is insignificant compared with that taking place in a greatly magnified image of an object placed in the focus. This distinction is inseparable from this experiment, as already explained.

The minute apertures, made accurately with Swiss watchmaking-tools, were carefully blackened, to prevent internal reflexion, with a solution of perchloride of platinum.

prismatic colours, both within and without the sharpest focal point or image, forming concentric intersections, displaying coloured pencils passing either to or from their finest point of focal combination where colour should be destroyed (fig. 6).

Experiment 3.—The disks (λ) shown by the apertures $\frac{12}{1000}$ of an inch in diameter, and separated between centres $\frac{1}{20}$, were now brought nearer to the objective O′. It was then observed that the image-disks (of above four times their proper size) began to separate; and since the spurious disk retains its false *annular* expansion independent of the true magnitude, it became evident that the exact distance at which the test doublet was first divided, gave for other objectives a comparative measure of their aberration; the very slight aberration in the image (of the sixteenth) being scarcely appreciable, especially when favoured by the advantage of the water-film to enhance the precision of definition on the immersion system.

By such experiments, with the finest glasses obtainable, the existence of an aberration of material and measurable amount being thus established, the next question to be settled assumed the following character, viz.

What was the nature of the aberration produced by displacement of the final focal image viewed by the eye-lens; and whether better effects could be produced by a different distribution of the magnifying-powers.

It was now found that increasing the distance between the eye-lenses and the objective, gained power indeed, but caused the aberration to increase faster than the power gained.

Intermediate Huyghenian eyepieces, inverted, were found to increase power but sacrifice definition; the apparent aberrations seemed incorrigible, so that this plan was finally abandoned in 1864. Although by this means the *Pleurosigma rhomboides* was fairly shown to Messrs. POWELL and LEALAND, with their one-sixteenth objective (dated 1862), they stated this method had been tried long before and relinquished as useless to improve definition.

Sliding-tubes (made by them at my request) were now furnished with a "universal screw," in order to admit a great variety of single and compound cemented lenses (more or less chromatically and spherically corrected) being inserted within the draw tube midway between the eyepiece and the objective. So, also, whole or parts of objectives were similarly applied, thus forming a microscope within a microscope admitting endless combinations of compensations.

It now seemed perfectly clear that any attempt to improve high-power definition must be preceded by the attainment of a ready and decisive method of ascertaining whether the balance of compensations was equal, or, on the other hand, over- or undercorrected. The *Image-test* already described appeared to effect this object in the following manner. The finest glasses, it is well known, are constructed upon the principle of balancing compensations, the effect of the posterior combinations when overcorrected compensating that of the anterior glasses which are undercorrected. To ascertain therefore the indications of the character of a given correction (still employing the exquisite images formed by "the sixteenth") wire gauze, forty meshes to the inch, was placed in front of a brilliant light; the image of the gauze was distinctly visible under the one-quarter objective (×250 diameters) finely corrected for an uncoverd object.

To ascertain the appearances due to *overcorrection*, the front glasses were removed; whilst to examine those of undercorrection the front set alone was employed, the inner glasses being removed.

First result.—The image no longer appeared like gauze, but displayed (unless the aperture was reduced) extraordinary patterns, prismatic, translucent, and, as it were, chequered or plaid-like; all of which were situated entirely *above* the best focal point, and nothing but a confused nebulous field *below it.*

Second effect.—The employment of the front lenses alone now reversed the position of these appearances.

Readjusting all the glasses, it was then discovered that the false images were developed principally below the best focal image (ascertained by reducing the aperture of the microscope[*]) when the objective was undercorrected, and above it when overcorrected. Brilliant images of glittering particles of mercury scattered on black cloth *nearly vertically* illuminated, fine gauze 80 meshes to the inch, perforated metal, gold-leaf displaying against a brilliant light immeasurably small perforations exposed on a rich malachite green ground[†], were submitted to be examined in miniature as test-objects. From a variety of experiments of this kind the following data were arrived at, to guide preliminary observations:—

That when any well-defined structure is viewed by the best microscopes, there exist *eidōla*[‡] or false images on each side of the best focal point.

That they are placed principally above or principally below the focal point of centrical pencils, according as the glasses are over or undercorrected; and that for a *single* stratum sufficiently thin, these eidola are nearly symmetrically exhibited on both side, of the best focal point only when the compensations are perfectly balanced.

It follows from these results that when a structure consists of two superimposed strata, in such close contiguity as to come within the optical limits of the *eidola*, the false images of the lower stratum are liable to be confused and commingled with the true image of the upper stratum when the objective is overcorrected, and when it is undercorrected the false images of the upper are confused with the true of the lower stratum.

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[*] The true image is at once seen by reducing the aperture; for this purpose a system of circular stops was applied to the microscope at the part where the objective is attached, admitting an instantaneous change in the aperture, and showing remarkable effects produced by change in the excentric aberration. Its mode of attachment is shown at β, fig. 1, Plate LII., where it is marked *aberrameter*.

[†] The gold-leaf is mounted on a slide in the ordinary way, and exhibits interesting and instructive phenomena.

[‡] Εἴδωλον.

These coincidences of eidola with true focal images may in both cases equally delude the observer.

The next question was the most favourable distribution of the elements of magnifying-power. According to a well-known optical principle, it seemed desirable to bend the rays by less sudden refractions. It is a peculiar result that when the incident and emergent pencils are equally bent so as to be equally inclined to the axis of an equi-convex lens, that then only is the aberration a minimum.

The effects of different distribution of power are well shown by the following experiments, in both of which the same amplification was employed of 400 diameters.

Experiment 4.—A miniature landscape formed by a convexo-plane lens $\frac{1}{30}$ focal length and $\frac{3}{100}$ aperture was examined with the one-eighth and an "A" eyepiece: axis horizontal and window open.

Result.—Landscape dark and hazy, as seen in the microscope.

The deficiency of light was most remarkable.

The same power (400) was now obtained with the half-inch objective and a D eyepiece.

Experiment 5.—The miniature being formed as before by the small lens, the microscope was now again brought into operation on the minute image horizontally.

Result.—Exquisite picture brilliantly lit up; even the foliage glittering in the sunlight was sharp, clear, and decisive, so that the details of the garden picture were marvellously displayed.

The difference appeared truly surprising as regards the two methods of obtaining the same magnifying-power, especially the increased light *with diminished aperture.*

In both these cases the greatest pains were taken to properly adjust the index collars of the objectives for the finest possible definition of an uncovered object.

A new fact had appeared highly suggestive of further inquiry. Accordingly, distribution of power was now varied by employing differently constructed eye-lenses, especially "crossed lenses"*, and *inserting, midway between the objective and eyepiece, convex lenses* of great variety. It was now seen that these lenses, intermediately placed, developed an entirely new aberration of a negative kind†. It became important to decide

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* Crossed lenses, well known to give a minimum aberration having the radii of their curved surfaces as 6 : 1.

† It is convenient to define the aberration to be positive or negative, or the lens to be *over-* or *undercorrected,* by the simple fact that a convex lens causes the excentrical rays to cross the axis at a point nearer the centre of the lens than the centrical rays, in which case, and in all analogous cases, it may be said that the lens is undercorrected and afflicted with a negative aberration. English objectives are now constructed on the principle of having the posterior sets overcorrected and the anterior undercorrected so skilfully as to destroy, by opposite errors nearly, the residuary aberration; but the opinion may be hazarded that future combinations will yet be found which will completely throw into the shade the present powers of the microscope, when perhaps we shall be in a better position to attempt to determine the microscopical features of molecular life, at present probably beyond its grasp, as no single particle so small as the *sixty-thousandth* of an inch in diameter can be clearly defined if isolated, until residuary error is very much reduced.

It is to be regretted that the precise nature of the marvellous combinations invented by Professor AMICI for whether compensations of aberration could be effected by attending to some definite principle or law.

The previously ascertained properties of *eidola* enabled many experiments to be made with rapidity and certainty. The following principles were, in short, patiently arrived at by experiments extending over several years:—

I. Displacement of the final focal image towards the eye-lenses, provided the front lens or facet of the object-glass is kept at the same distance from the object under observation, is caused by approximating even slightly the component adjusting lenses of the objective, and this movement causes a negative aberration, and *vice versâ.*

II. With test-images, both observing and miniature or image-forming objective follow the same law of compensation. If one be overcorrected the other must be similarly adjusted, and *vice versâ.*

III. Using additional compensating lenses to gain increase of power, intermediately placed between eyepiece and objective, the finest definition is obtained when each of the three sets, viz. lenses, observing and image-objective, are similarly though slightly overcorrected, as compared with a standard defining distance of 9 inches.

Although a fine definition seemed now attainable by means of supplementary compensating lenses, if judiciously introducing balancing compensations, yet their practical adjustments were innumerable and tediously accomplished*. At this stage of the research, frequent consideration of the well-known optical equations for a vanishing aberration fortunately suggested to me the idea of searching the axis, mechanically, for aplanatic foci. In reference to these equations, which would be out of place here, it has been observed by Dr. PARKINSON, F.R.S.†, "If the aberration for rays parallel at incidence of a compound lens of given focal length—consisting of several thin lenses in contact—be examined, it will consist of a series of terms similar to that in Art. (129), one term for each lens, and the condition that the aberration shall vanish will lead to an equation involving more than one unknown quantity, and consequently admitting an unlimited number of solutions."

In the distribution of the power-lenses, and in the application of a traversing searcher, it was indispensable that the object should be kept distinctly visible in the

---

objectives remain unknown. As one of the Jurors in the Paris Exposition, his microscope necessarily remained both uncelebrated and unelucidated in the Reports.

* During 1865–1869 many experiments were tried with complete objectives and various parts of them, either over- or undercorrected by means of a sliding-tube carrying them and fitting into the "draw tube."

Professor LISTING of Göttingen has confirmed the value of this method of amplification quite independently in two papers published in 1869. Nachr. d. kgl. Gesell. der Wissensch. 1869, No. 1, and Poggend. Annalen, 1869, vol. xvi. p. 467 ('Nature,' Jan. 27, 1870).

In the first he recommended an inverted Huyghenian eyepiece, and in the second intermediate achromatic lenses.

As regards intermediate lenses, the writer has ascertained (Nov. 1870) that Dr. GORING (Micrographia, ed. 1837) has anticipated both these methods.—Note added Nov. 1870.

† GRIFFIN's 'Optics,' by PARKINSON, p. 122, 2nd ed. 1866.

field of view, by a proper selection of lenses whilst the optical compensations were being adjusted. The form finally adopted is simply this:—

A pair of slightly overcorrected achromatic lenses, admitting of further correction by a separating adjustment, are mounted midway between a low eyepiece and the objective, so as to admit of a traverse of 2 or 3 inches by means of a graduated milled head. These lenses are conveniently traversed within the draw tube; and can be brought to bear within 4 inches of the objective, or at a distance of 10 inches.

The focal length of the combination forming the aplanatic image-searcher may vary from $1\frac{1}{2}$ inch to $\frac{3}{4}$ of an inch. The latter applies more effectively to low objectives when it is desirable to obtain extraordinary depth of focal penetration, and vision through very thick glass*—as with a half inch giving 700 diameters with a C eyepiece. I possess a WRAY half-inch objective† which bears an E eyepiece and searcher. It should now be stated that the searcher may be employed with very different intentions. Thus—

When it is desirable to view an object through a very thick refracting medium, the searcher is brought as close as possible to the objective, which action lengthens the focus of the objective; and the same thing is necessary when the observer wishes to throw the *eidola* of an upper structure above and away from the true image of the lower but contiguous stratum—as when the lower beads of the Podura are required, or when it is required to give additional negative aberration to an objective too positively corrected in which the front glasses are already forced into a dangerous proximity.

On the contrary, when the searcher is traversed the opposite way, the *objective lenses* require to be brought nearer together; the instrument is then more adapted for viewing objects or particles lying in the upper plane of a complex structure, throwing the *eidola* of the lower layer below that layer itself, and so leaving the upper stratum less disguised by the false images of the lower.

In intermediate cases, where greater penetration or focal perspective is required, with a thin glass cover, the objective lenses must be proportionately separated by an increased interval, the searcher being traversed towards the objective; and in general confused images of both upper and lower strata can be obtained by opposite arrangements‡.

A very interesting refinement upon the corrections for chromatic effects may be accomplished by gradually traversing either way both searching and objective lenses and closely watching the effect.

The most brilliant definition is generally obtained when the searcher (a little more overcorrected) is used as close to the objective as possible.

The overcorrection of the searcher is increased by separating its component lenses according to the divisions upon the sliding tubes of the searcher.

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* Nearly one-fourth of an inch thick.

† With a "Kelner" two-thirds of an inch focal length, a very clear, very large, and flat field is presented to the eye, notwithstanding the increased power with the searcher. A one-and-a-half-inch objective by Ross was used generally for a condensing illuminating apparatus more or less stopped off.

‡ Such as separating the objective lenses and traversing the searcher further from them.

It will be seen that an exceedingly small pencil engages the surface of the searcher diverging from a point in the image $p_{\prime} q_{\prime}$, which is inverted again at $p_{\prime\prime} q_{\prime\prime}$. As the searcher is traversed nearer the eye the pencils become less divergent, and the effect of the searcher is diminished. On the contrary, as it approaches the objective, $p_{\prime} q_{\prime}$ being formed nearer to the latter after refocusing, a *more* divergent pencil engages a greater aperture of the searcher, and this now automatically causes a stronger overcorrection than before. The essential action of the searcher is to apply a rapid variable correction by a traversing movement (fig. 2, Plate LII.).

The use of this instrument will be facilitated by first setting the microscope for ordinary use without the searcher, adjusting an eyepiece, the focus, and screw-collar to the most distinct vision, and then applying the draw tube containing the searcher placed at a point nearest to the eyepiece E. As the searcher is traversed towards the objective, the lenses of the objective may *require separation*.

The change in the general aberration is shown by the divided index of the milled head actuating the movement of the searcher (M, fig. 1).

The power obtained is in general from two and a half to four times greater than that given with the third eyepiece C of 1 inch focal length: with a very fine eighth of Messrs. POWELL and LEALAND's *new* construction, a clear and satisfactory definition of the beading of the *Pleurosigma formosum* was exhibited to them, by means of the aplanatic searcher, at a power *estimated* at 4000 diameters*. Several inferior objectives have acquired a fine definition by the application of the searcher.

This paper perhaps will hardly be complete if I omit to add, that the instrument will be most effectively employed by considering it as a conjugate portion or integral part of the objective itself, in which the minute traversing adjustment of the objective lenses finds its counterpart in the more extended and therefore more delicate adjusting traverse of the searcher itself. So that, in short, during minute microscopical research each adjustment should be intelligently applied, according to the nature of the research in hand. The indications of the one adjustment should be employed to verify those of the other. Correlative movements by the aid of the searcher may introduce aplanatic images, whilst a violation of their correlation will exhibit deformity.

I ought also to state that I have found in every case, either an extra thickness of glass cover or a deeper immersion of a given object in the film of Canada balsam (or other fluid used for mounting it) to require for a precise definition additional adjustment; the searcher should be made in this case to traverse towards the object to attain the new correction requisite. The same remark is applicable to immersion lenses. Further slight improvement can be effected in the precision of definition by separating more or less the component glasses of the Huyghenian eyepiece (the power of which is preferred as low as 3-inch focal length for the $\frac{1}{20}$ "immersion") or by substituting for it a single achromatic combination slightly overcorrected for spherical aberration of 2 inches focal length, or less according to the power required*.

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* The usual power of the one-eighth with a C eyepiece is 800; a power of 4000 is given by an eyepiece of one-fifth of an inch focal length.

An additional cap containing a supplementary achromatic lens is sometimes advantageously fixed upon the lenses of the searcher, when (for instance) a power of 700 diameters is desired to be developed by a half-inch objective (for test *Podura* beading).

In conclusion, the experiments detailed in this paper, selected from a great number made within the last few years, it is hoped will induce more able observers to repeat them in a more general form; but, so far as they are detailed, they appear satisfactorily to demonstrate the detection of residuary aberration of considerable amount in the very finest microscopes, and enable one to measure it and to suggest means of diminishing the errors of the glasses whilst greatly increasing the power. Whether a similar method can be applied also to telescopes has been some time under the author's consideration, with results which he hopes on a future occasion to have the honour of communicating to the Society.

APPENDIX.

The law of displacement followed by the final focal image corresponding to a minute displacement of the internal lenses of a complex objective, the front lenses or facet *remaining fixed*, possesses some interest and may thus be expressed:—

Let F be the distance of the final focal image when the objective lenses are closed together.

$F+\delta F$ its distance when the front sets of the objective are displaced by a quantity δx.

Then it will be found if f_1 be the distance of the virtual image conjugate with the *object* as formed by the front set of lenses,

$$\delta F : \delta x :: -F^2 : f_1^2;$$

and consequently every slight change of the screw-collar of an adjusting objective produces comparatively a very large displacement in the final focal image, and therefore of the traversing image-searcher; so that the searcher-traverse represents a movement conjugate with the objective index. Again, since this traverse towards the objective encounters rays of increasing divergence, an increasing breadth of pencil is encountered by the lenses of the searcher, and its own peculiar aberration receives an instantaneous increase, which introduces an important new element in definition, it having been observed that the glasses must be very gradually overcorrected as the image is formed nearer the objective, within the tube of the microscope.

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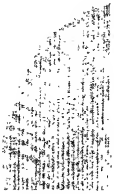
* I may be permitted to add a note here (Nov. 7, 1870), that a WRAY one-fifth, made expressly, admitted of as great amplification as an ordinary one-twelfth. In fact these researches appear to point decisively to greater advantages to be expected from raising the quality of the lower objectives rather than deepening focal length. Observers are more numerous every year who prefer the one-eighth to the one-twenty-fifth and one-fiftieth.

Fig. 7

Lepisma



×1600

×750

Fig. 1 *2500 diameters*

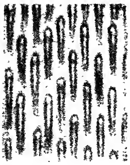


Fig. 2

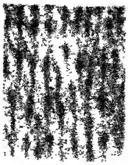
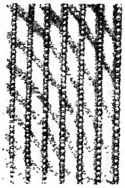


Fig. 6

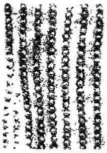
Lepisma



×1666

1/16 objective

Fig. 3



×2300

Fig. 4

×900

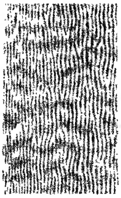


Fig. 5

×1800

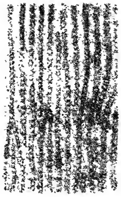


Fig. 8

Structure of the Podura Scale

Jos. S. Aldous del. et lith. ad Nat. with Camera Lucida

Royston Pigott

Phil. Trans MDCCCLXX Plate LII

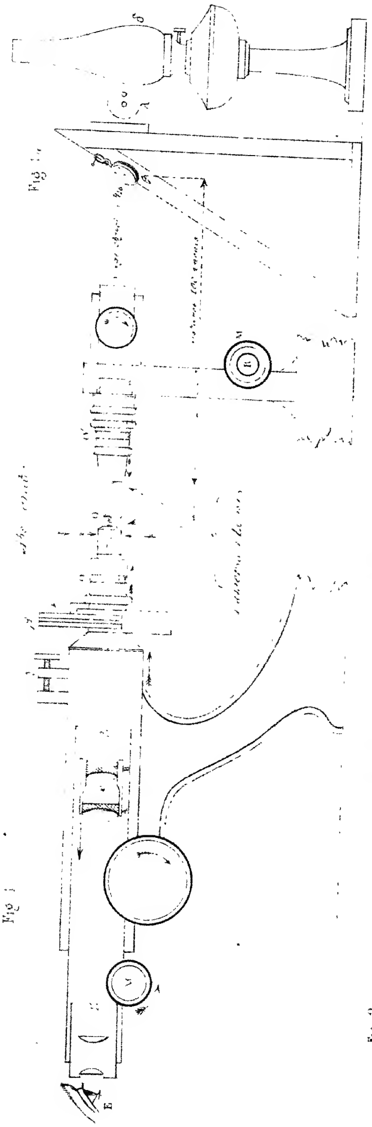


Fig 1a

Fig 1

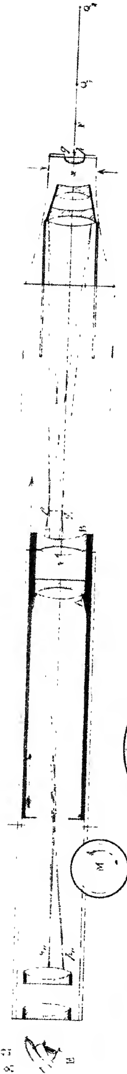
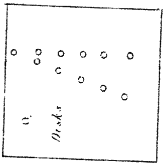
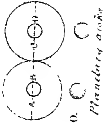


Fig 2

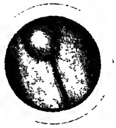


9. Disks



8\.

10. Planetary disks



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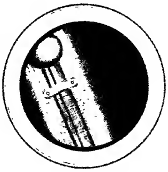
6



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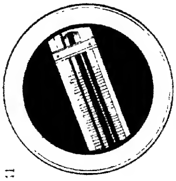

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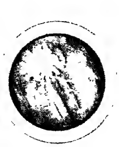

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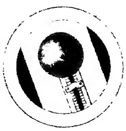

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W. West ... lith

EXPLANATION OF THE PLATES.

PLATE LI.

Plate LI. is intended to represent the working powers gained by the use of the Aplanatic Searcher by means of comparative outline drawings of a given "scale" taken by Mr. ALDOUS with the Camera under the magnifying-powers and objectives indicated.

Fig. 1. The standard appearance of the Podura under the $\frac{1}{50}$. $\frac{1}{25}$, and $\frac{1}{8}$ POWELL and LEALAND objectives.

Fig. 2. Resolution of the lower beads of Podura.

Fig. 3. The beads of the upper stratum.

Fig. 4. Comparative magnifying-power of the $\frac{1}{8}$ objective with the searcher, and also in the ordinary way with a "third" eyepiece, C, of 1 inch focal length.

Fig. 5. General appearance of the wavy markings of the Podura, consisting of beaded ribbing faintly visible here with a pocket-lens.

Fig. 6. Both sets of beading exhibited at once.

Figs. 7, 8. Ordinary and extraordinary appearance of Lepisma.

PLATE LII.

This Plate shows the image-test arrangements of the objectives and object of which a miniature is desired, and also the construction of the searcher.

M. The divided milled head of the traversing aplanatic searcher, consisting of separable lenses, A, B, having a variable interval, x', between them. The searcher traverses the draw tube, into which is fixed the eyepiece E. R, M are adjusting milled heads of the stage supporting the image objective O′ (fig. 1 *a*).

O, O′, fig. 1, fig. 1 *a*, are the objective to be tested and the miniature-forming $\frac{1}{16}$ immersion objective, giving an image α of the object θ, or double disks λ, illuminated by a lamp, δ.

γ represents the focal adjustment, and

β the aberrameter inserted into the nose of the microscope containing two revolving disks forming central and peripheral stops.

Fig. 2 represents the course of the rays from the object Q to the last focal image $q_{\prime\prime}p_{\prime\prime}$ erected.

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